Non-Geometric Fluxes in Exceptional Field Theory

Marc Syväri[1]

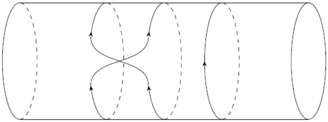
[1] Department of Physics
Ludwig-Maximilians-University

12.03.2018 / IMPRS Workshop

Based on Lüst, Malek, MS arXiv: 1710.05919

Outline

T-Duality



- Compactification over $S^1 \Rightarrow x \sim x + 2\pi R$
- Invariant mapping: $R \to \frac{\alpha'^2}{R}$
- Generalisation to T^d (with B-field)
- T-duality is a O(d,d) symmetry

$$\mathcal{H}_{MN}(g,B) \to {h_M}^P \mathcal{H}_{PQ} {h_N}^Q = \mathcal{H}'_{MN}(g',B') \text{ with } h \in O(d,d)$$

Advantages of Double Field Theory

- SUGRA with manifest invariant T-duality
  [Hull, Zwiebach, (2009)]
- Doubling of coordinates $\Rightarrow$ theory with explicit O(d,d)-symmetry
  - called DFT (Double Field Theory)
- Consequences (M=1,...,2d, i=1,...,d)
  - Doubling of coordinates (and derivatives) $\Rightarrow X^M = \begin{pmatrix} x_i \\ \tilde{x}^j \end{pmatrix}$
  - Introduction of Section Conditions:
    $Y^M{}_P{}^N{}_Q \partial_M \otimes \partial_N = Y^M{}_P{}^N{}_Q \partial_M \partial_N = 0$ with $Y^M{}_P{}^N{}_Q = \eta^{MN} \eta_{PQ}$
- Generalized diffeomorphisms (infinitesimal symmetries):
  $\delta_U \left(V^M\right) = L_U \left(V^M\right) + Y^M{}_P{}^N{}_Q V^P \partial_N U^Q$

Exceptional Field Theory

- Consideration of S-duality: $g_s \to \frac{1}{g_s}$
- Extension of Duality Group: T-duality + S-duality = U-duality
  $\Rightarrow$ Symmetry group: $E_{d(d)}$ [Berman, Perry, (2010)]
  - Extended spacetime
  - manifest invariant U-duality
  - Generalized diffeomorphisms (with different $Y^M{}_P{}^N{}_Q$)
  - Section condition $Y^M{}_P{}^N{}_Q \partial_M \otimes \partial_N = Y^M{}_P{}^N{}_Q \partial_M \partial_N = 0$

What are fluxes? (DFT/ EFT)

- Spacetime tensors: $\delta_\xi (T) = L_\xi (T)$ with $U^M = (\xi^i, 0)$
- Used for string phenomenology
  - Generating potentials in string compactification (e.g. "moduli stabilisation") [Flournoy, Wecht, Williams (2004)]
- Non-geometric fluxes can lead to non-associativity

Example for non-geometric fluxes in DFT

- Start with a 3-torus with H-flux, $H_{123} = N$: [Kachru, Schulz, Tripathy, Trivedi (2002)]

$$ds^2 = \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 \,, \; B_{12} = Nx^3$$

- After a T-duality in x^1-direction, ${f^1}_{23} = N$:

$$ds^2 = \left(dx^1 - Nx^3\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 \,, \; B_{12} = 0$$

- A second T-duality transformation in x^2-direction:

$$ds^2 = \frac{\left(dx^1\right)^2 + \left(dx^2\right)^2}{1 + N^2\left(x^3\right)^2} + \left(dx^3\right)^2 \,, \; B_{12} = \frac{Nx^3}{1 + N^2\left(x^3\right)^2}$$

- Perform the redefinition: $\left(\hat{g}^{-1} + \beta\right)^{-1} = g + b$

Example for non-geometric fluxes in DFT

- 3-Torus with globally non-geometric Q-flux ${\mathcal{Q}_3}^{12} = N$:

$$d\hat{s}^2 = \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 , \; \beta^{12} = Nx^3$$

- In DFT T-duality transformation in direction x^3 is still possible as generalized transformation:

$$d\hat{s}^2 = \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 , \; \beta^{12} = N\tilde{x}_3$$

$\Rightarrow$ Locally non-geometric flux: $R^{123} = N$

- Duality Chain:

$$H_{123} \overset{T_1}{\longleftrightarrow} {f^1}_{23} \overset{T_2}{\longleftrightarrow} {Q_3}^{12} \overset{T_3}{\longleftrightarrow} R^{123}$$

Different dimensions of EFT

- Every dimension in EFT is different

Group	dim	Wrapping modes	$Y^{M}{}_{N}{}^{P}{}_{Q}$
O(d,d)	2d	$x^M \to (x^i, x_i)$	$\eta^{MP}\eta_{NQ}$
$E_{4(4)} = SL(5)$	10	$x^M \to (x^i, x_{ij})$	$\epsilon^{\alpha MP}\epsilon_{\alpha NQ}$
$E_{5(5)} = SO(5,5)$	16	$x^M \to (x^i, x_{ij}, x^z)$	$\frac{1}{2}\left(\gamma^\alpha\right)^{MN}\left(\gamma_\alpha\right)_{PQ}$
$E_{6(6)}$	27	$x^M \to \left(x^i, x_{ij}, x^{\bar{i}}\right)$	$10 d^{MPR} d_{NQR}$
$E_{7(7)}$	56	$x^M \to (x^i, x_{ij}, x^{ij}, x_i)$	$-12\left(t_\alpha\right)^{M}{}_{N}\left(t_\alpha\right)^{P}{}_{Q} + \delta^{M}{}_{Q}\delta^{P}{}_{P}$

[Berman, Cederwall, Kleinschmidt, Thompson (2002)]

Example of $SO(5,5)$

- Fluxes: $\delta_\xi(T) = L_\xi(T)$ with $U^M = (\xi^i, 0)$
- Coordinates: $X^M \to (x^i, x_{ij}, x^z)$ with $M = 1, ..., 16$ and $i, j = 1, ..., 5$
- Transformation behavior of non-geometric fields (g and Ω_3)
  - $\delta_\xi(e^i{}_{\bar{i}}) = L_\xi(e^i{}_{\bar{i}})$
  - $\delta_\xi(\Omega^{ijk}) = L_\xi(\Omega^{ijk}) - 3\partial^{[ij}\xi^{k]}$
- Modification of derivatives: $\delta_\xi(\partial^{ij}\phi) \neq L_\xi(\partial^{ij}\phi)$
  - $\hat{\partial}^{ij} = \partial^{ij} + \Omega^{ijk}\partial_k$
  - $\hat{\partial}^z = |e|\partial^z - \frac{1}{6}\epsilon_{i_1 i_2 i_3 i_4 i_5}\Omega^{i_1 i_2 i_3}\partial^{i_4 i_5} - \frac{1}{12}\epsilon_{i_1 i_2 i_3 i_4 i_5}\Omega^{i_1 i_2 i_3}\Omega^{i_4 i_5 k}\partial_k$
- Locally non-geometric flux in SO(5,5):

$$R^{i,jklm} = 4\hat{\partial}^{i[j}\Omega^{klm]} - e^{[j}{}_{\bar{i}}\epsilon^{klm]ip}\hat{\partial}^z e^{\bar{i}}{}_p$$

Results in $E_{7(7)}$

- 4D embedding tensor $\in \mathbf{912} \oplus \mathbf{56}$ of $E_{7(7)}$, under geometric GL(7)

$$\begin{aligned} \mathbf{912} \oplus \mathbf{56} \rightarrow & \mathbf{1}_{-14} \oplus \mathbf{35}_{-10} \oplus \overline{\mathbf{140}}_{-6} \oplus 2 \cdot \overline{\mathbf{7}}_{-6} \oplus \mathbf{224}_{-2} \oplus 2 \cdot \mathbf{21}_{-2} \\ & \oplus \mathbf{28}_{-2} \oplus \overline{\mathbf{28}}_{2} \oplus \overline{\mathbf{21}}_{2} \oplus \overline{\mathbf{21}}_{2} \oplus \overline{\mathbf{224}}_{2} \oplus 2 \cdot \mathbf{7}_{6} \oplus \mathbf{140}_{6} \\ & \oplus \overline{\mathbf{35}}_{10} \oplus \overline{\mathbf{1}}_{14} \end{aligned}$$

- More non-geometric fluxes in higher dimensions:

$$\begin{aligned} R^{i,jklm} &= 4\hat{\partial}^{i[j}\Omega^{klm]} - e^{[j}{}_{\bar{i}}\epsilon^{klm]inpq}\hat{\partial}_{pq}e^{\bar{i}}{}_{n} \\ R^{ij}{}_{k} &= \hat{\partial}_{kl}\Omega^{ijl} - \frac{1}{72}\epsilon_{klmnpqr}\hat{\partial}^{ij}\Omega^{lmnpqr} + \ldots \\ R^{i} &= \hat{\partial}_{jk}\Omega^{ijk} - 4e_{\bar{i}}{}^{j}\hat{\partial}^{i}e^{\bar{i}}_{j} - 8e_{\bar{i}}{}^{i}\hat{\partial}^{j}e^{\bar{i}}_{j} \\ R^{ijklmnp} &= \hat{\partial}^{[i}\Omega^{jklmnp]} - 2\Omega^{[ijk}\hat{\partial}^{l}\Omega^{mnp]} \\ R^{ijkl} &= \hat{\partial}^{[i}\Omega^{jkl]} - \frac{5}{8}\hat{\partial}_{pq}\Omega^{ijklpq} - \frac{5}{4}\Omega^{[ijk}\hat{\partial}_{pq}\Omega^{lpq]} \end{aligned}$$

Summary and Outlook

- EFT is a SUGRA which has a manifest T- and S- duality.
- Fluxes promises solutions to many problems of current research
- Existence of fluxes has interesting consequences (not discussed)
  - Non-associativity
  - Missing momenta modes in flux frames

- Outlook
  - Connection to L_∞?
  - Consequences of the missing momenta modes

Thank you for your attention!