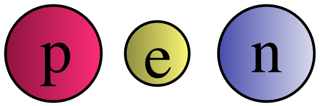
QCD and Jet Physics at e^+e^- accelerators

- History of Strong Interactions
- QCD; confinement; asymptotic freedom
- hadronisation and hadron-jets
- Quark-Spin
- Gluon-Spin
- gluon self-coupling
- asymptotic freedom from jet rates
- determinations of α_s

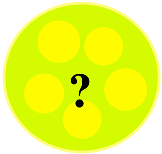
QCD at hadron accelerators: –> WS

History of Strong Interactions (1)

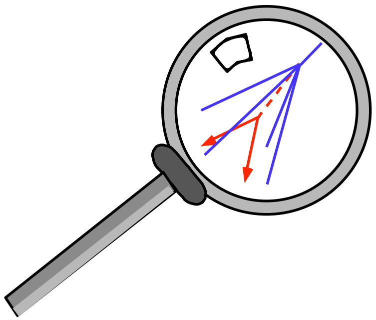
1932: discovery of neutrons



1933: $\vec{\mu} \cong 2.5 \, \frac{e}{2 \, m_p} \, \vec{\sigma} \Rightarrow$ substructure of the protons

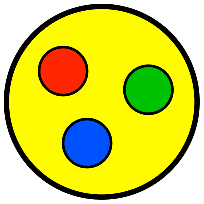


1947: discovery of π-mesons and long-living V-particles (K^0, Λ) in cosmic rays

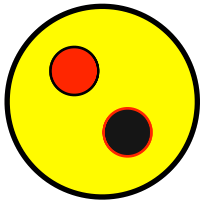


1953: V-particles produced at accelerators
new inner quantum number ("strangeness").

1964: static quark-model;
new inner quantum number: colour



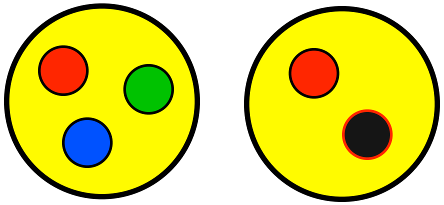
Baryon
(p,n, Λ,...)



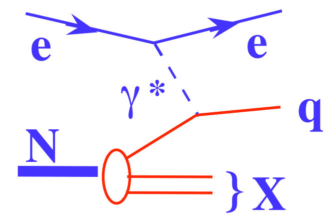
Meson
(π,K,...)

History of Strong Interactions (2)

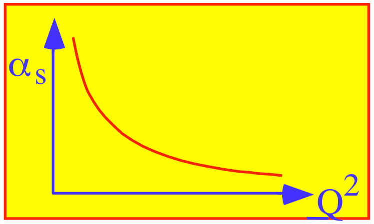
1964: static quark model ;
new inner quantum number: colour.



1969: dynamic parton model :

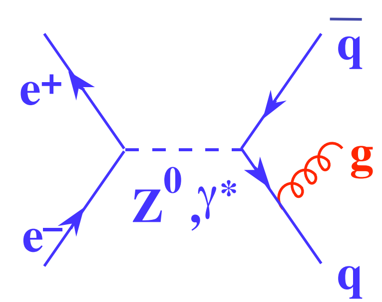


1973: concept of asymptotic freedom ;
Quantum Chromo Dynamics.



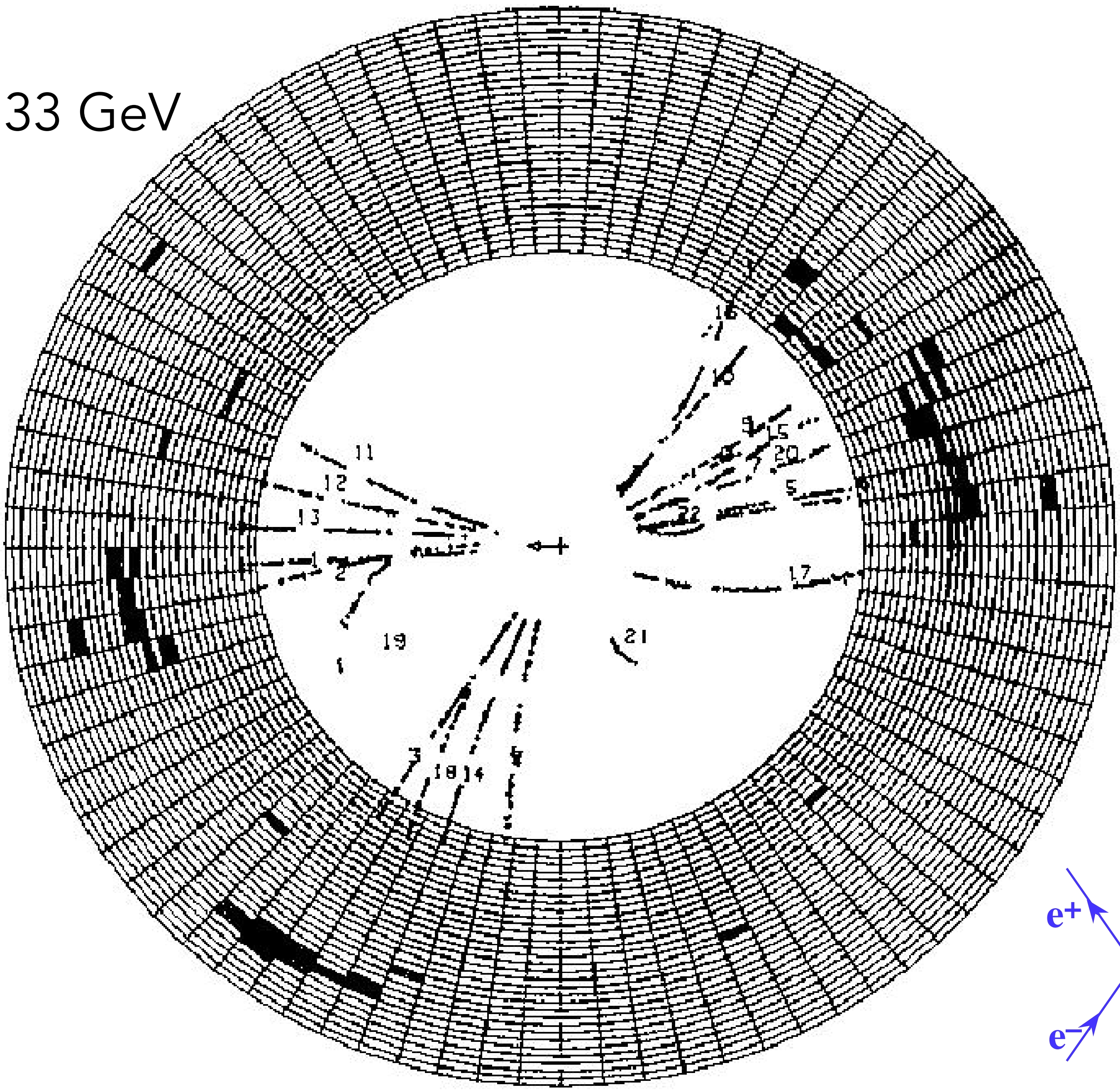
1975: 2-Jet structure in $e^+ e^-$ - annihilation:
confirmation of quark-parton-model.

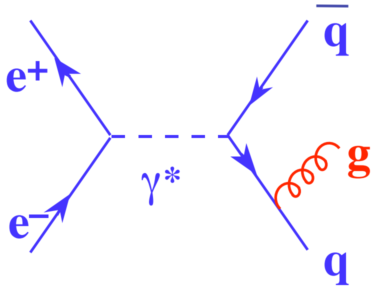
1979: discovery of gluons in 3-Jet-events
of $e^+ e^-$ -annihilations.



3-Jet event recorded with the JADE Detector (1979-1986)

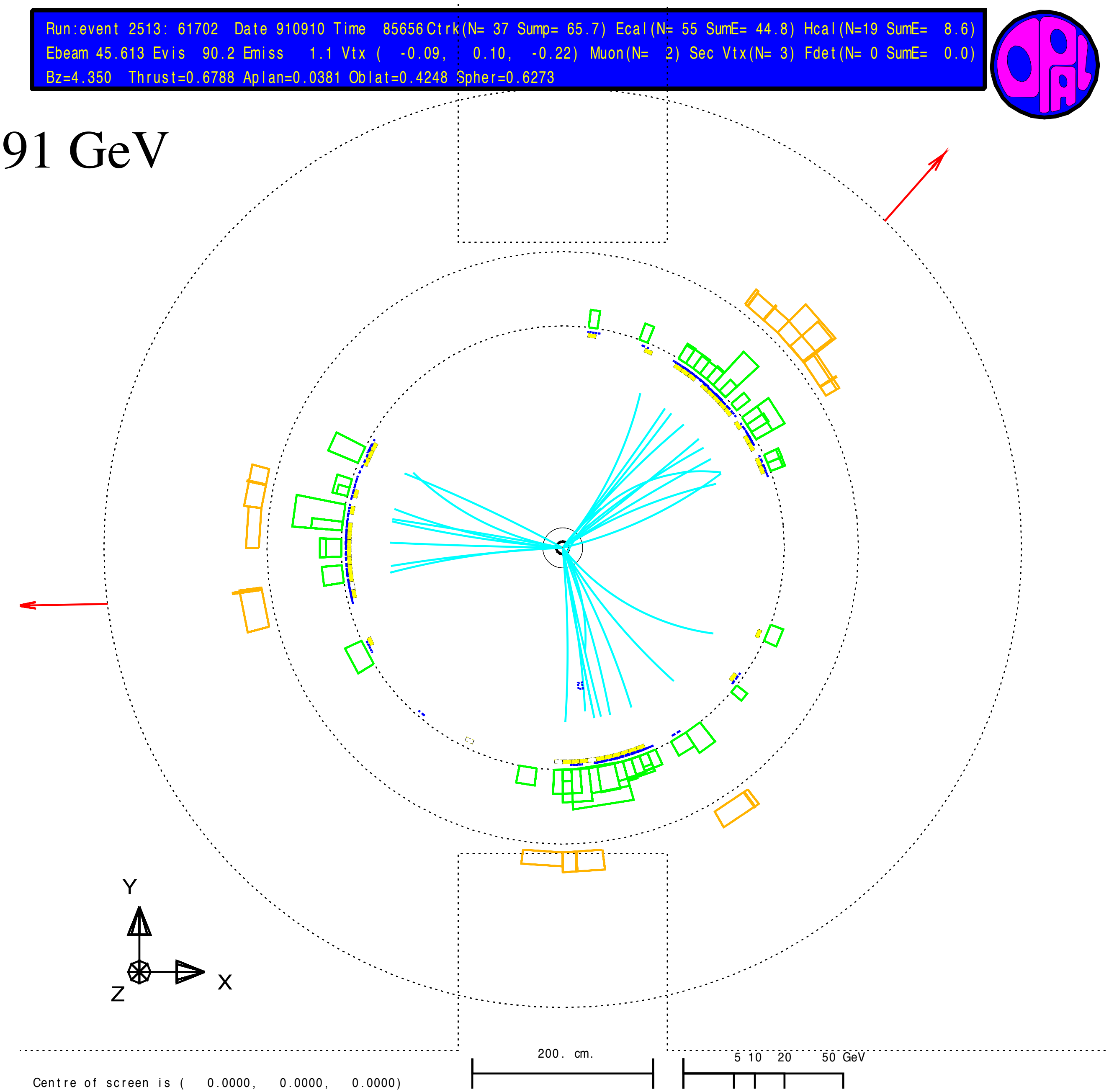
E_{cm} ~33 GeV



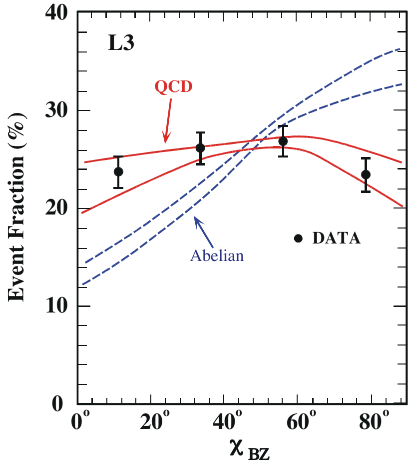


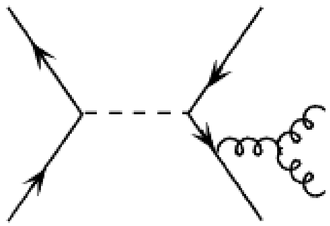
3-Jet event recorded with the OPAL Detector (1989-2000)

$E_{cm} = 91$ GeV

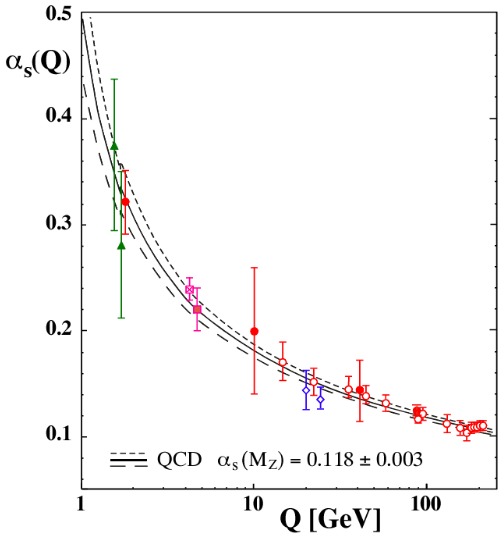


History of Strong Interactions (3)



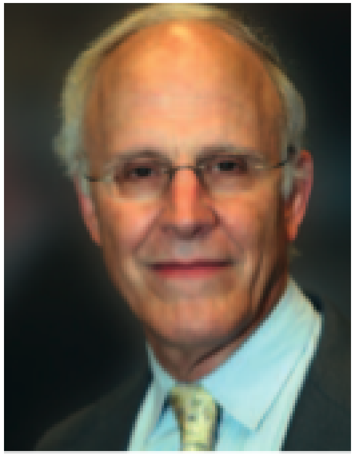
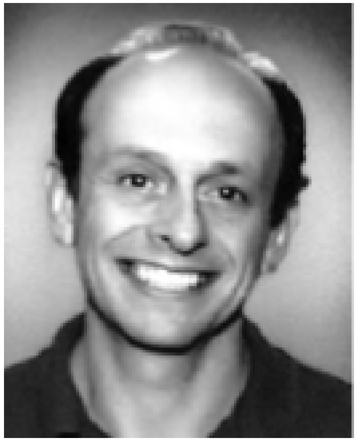


1991: exp. signature of the gluon self coupling



1990-2000: confirmation of asymptotic freedom

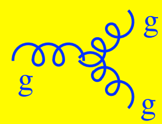
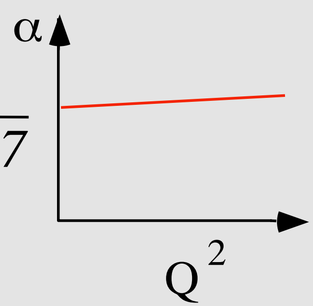
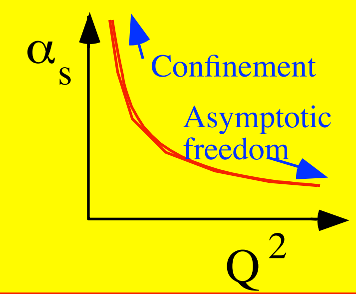
2004: Nobel Prize (concept of A.F.) to D. Gross, H.D. Politzer und F. Wilczek


QCD:

- gauge-field theory of Strong Interactions
- underlying gauge group: SU(3) ; non-abelian
- force mediating particles/quanta: gluons
- self-coupling of gluons
- renormalised coupling constant α_s is energy dependent:
- α_s large at small energies (large distances):
  confinement of quarks
- α_s small at large energies (small distancies):
  asymptotic freedom of quarks

properties of QED and QCD:

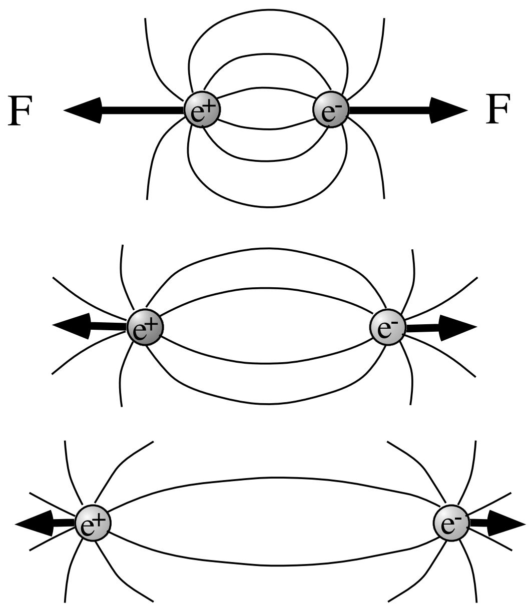
	QED	QCD
fermions	*leptons* (e, μ, τ)	*quarks* (u, d, s, c, b, t)
force couples to	*electric charge*	*3 color-charges*
exchange quantum	*photon* (γ) (carries no charge)	*gluons* (g) (carry 2 color charges) $\Rightarrow$  is possible
coupling "constant"	$\alpha(Q^2=0) = \frac{1}{137}$ 	$\alpha_s(Q^2 = M_Z^2) \approx 0.12$ 
free particles	*leptons* (e, μ, τ)	(color neutral bound states of q and $\bar{q}$) *Hadronen*
theory	*perturbation theory up to* $O(\alpha^5)$	*perturbation theory up to* $O(\alpha_s^4)$
precision achieved	10^{-6} .... 10^{-7}	0.1% ... 20%

Warum gibt es keine freien Quarks?

QED

Elektrische Ladungen:

Kraft $F \propto 1/r^2$; Energiedichte $\propto 1/r$



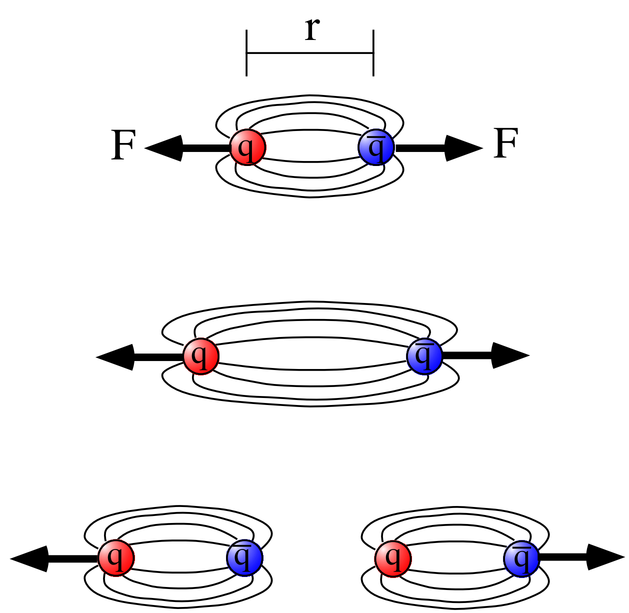
Kraft- und Energiedichte zwischen Ladungsträgern nimmt ab.

⇒ Träger elektrischer Ladung sind <u>freie</u> Teilchen

QCD

Farbladungen:

Kraft $F \propto$ const ; Energiedichte $\propto r$



Kraft- und Energiedichte steigen an, bis ein neues Quark- Antiquark-Paar aus dem Vakuum erzeugt wird.

⇒ Träger von Farbladung kommen nur in gebundenen, '<u>farbneutralen'</u> Zuständen vor.

"Confinement"

energy dependence of coupling "constant":

renormalisation group equation ("β-function")

- in leading order perturbation theory:

$$\mu \frac{d}{d\mu} \alpha_i(\mu) = -\beta_0 \, \alpha_i^2$$

with $\beta_0 = \frac{1}{2\pi}\left[\frac{11}{3}\begin{pmatrix} N_c \equiv 0 \\ N_c \equiv 2 \\ N_c \equiv 3 \end{pmatrix} - \frac{4}{3}\begin{pmatrix} N_{fam} \\ N_{fam} \\ N_f/2 \end{pmatrix} - N_{Higgs}\begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix}\right]$ ← QED ← weak ← QCD

- integration ⇒

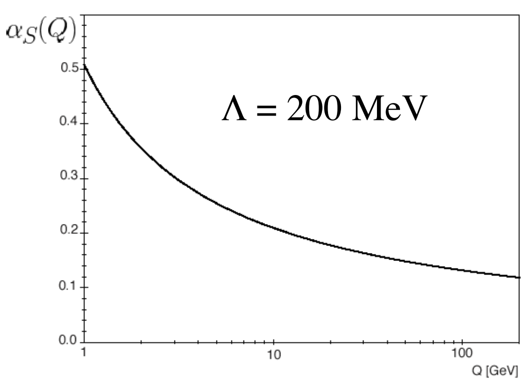
$$\alpha_i\left(q^2\right) = \frac{\alpha_i\left(\mu^2\right)}{1 + \frac{\beta_0}{2}\alpha_i\left(\mu^2\right)\ln\frac{q^2}{\mu^2}}$$

or

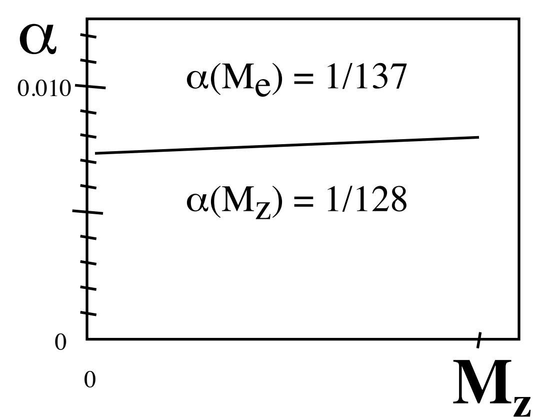
$$\alpha_i\left(q^2\right) = \frac{2}{\beta_0 \ln\frac{q^2}{\Lambda^2}}$$

with $\Lambda^2 = \frac{\mu^2}{e^{2/\beta_0 \alpha_s\left(\mu^2\right)}}$

QCD: $N_c = 3$; $\beta_0 = \frac{23}{6\pi}$



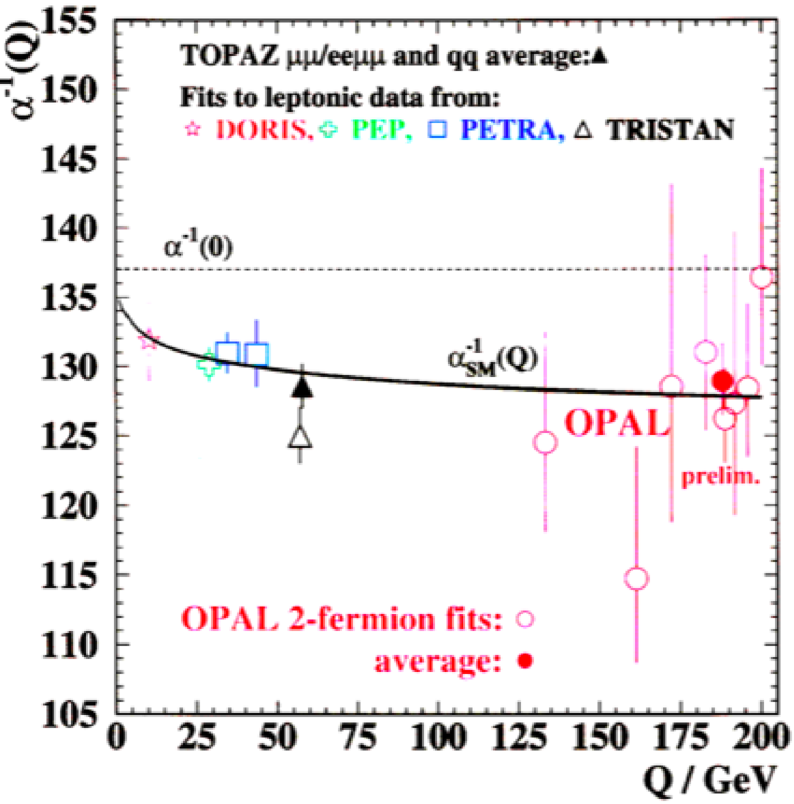
QED: $N_c = 0$; $N_{fam} = 3$ $\beta_0 = -\frac{12}{6\pi}$



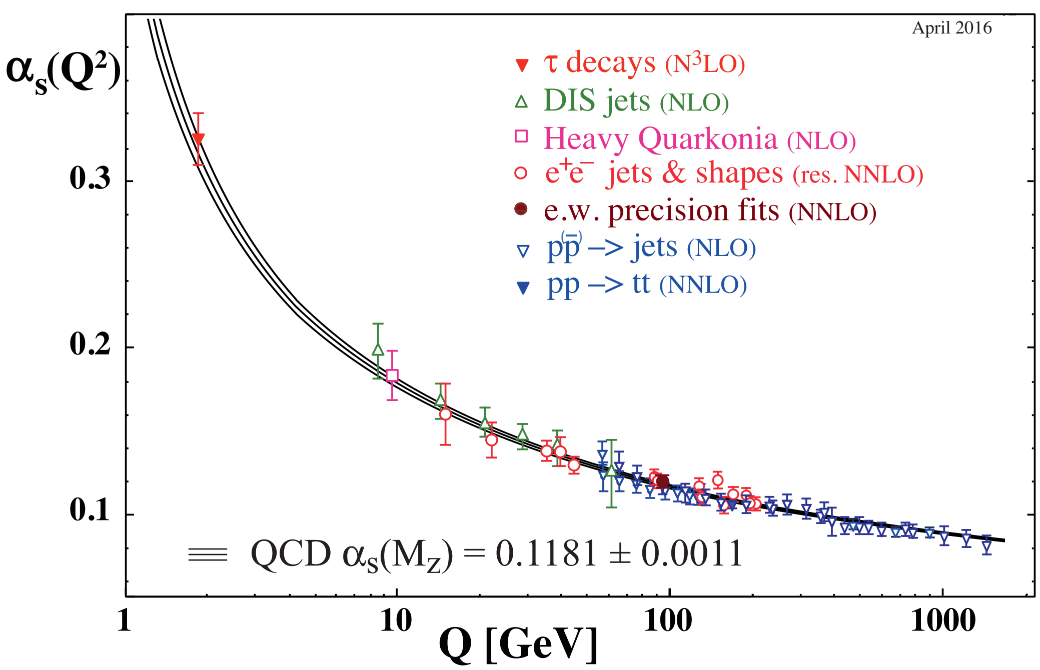
energy dependence of coupling "constants":

- experimentally verified with high precision

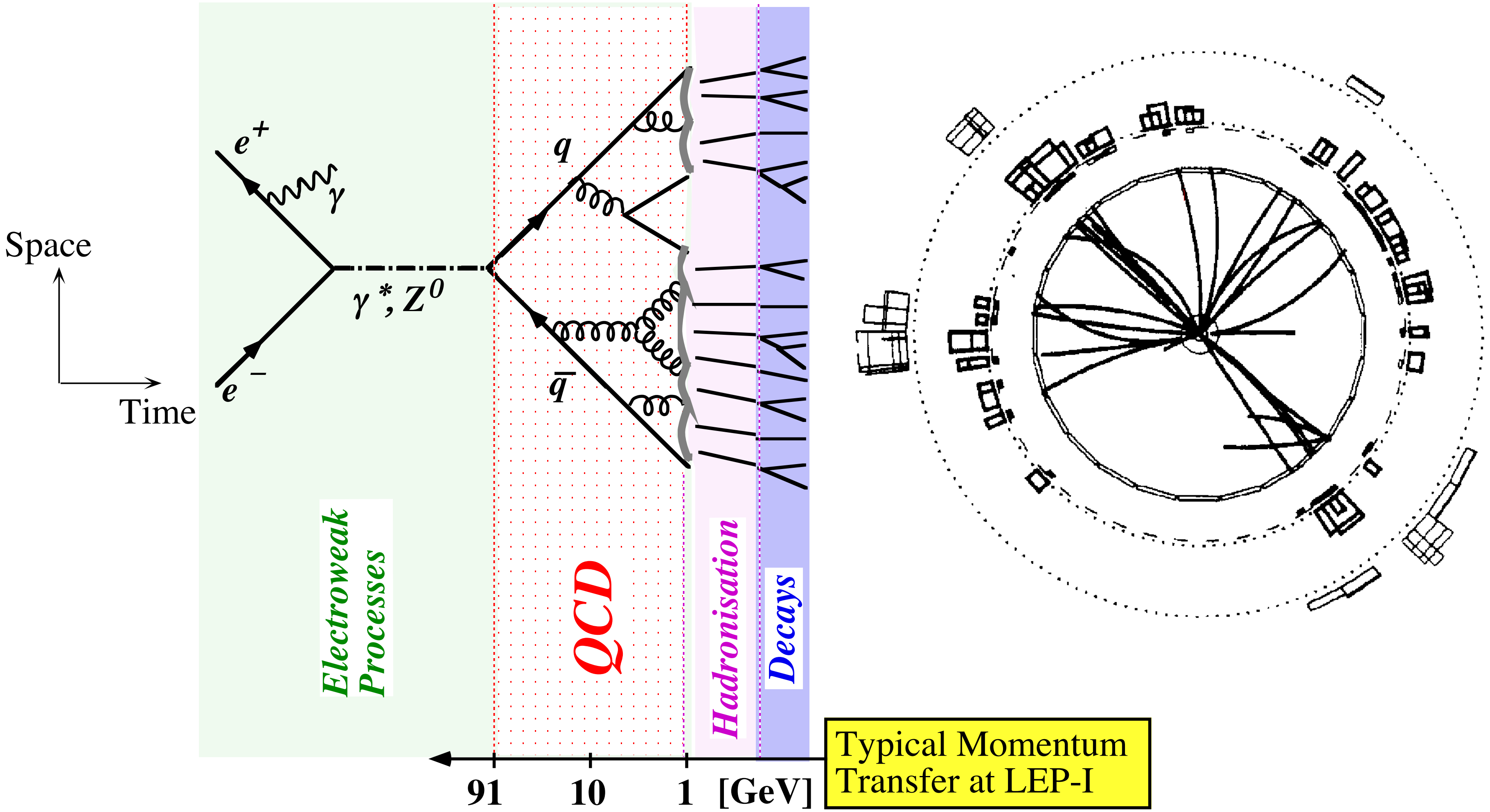
QED



QCD



Anatomy of hadronic events in e^+e^- annihilation



- QCD: shower development calculated in perturbation theory (fixed order; (N)LLA)
- Hadronisation: phenomenological models of string-, cluster- or dipole fragmentation
- Decays: randomized according to experimental decay tables

physics of hadron-jets

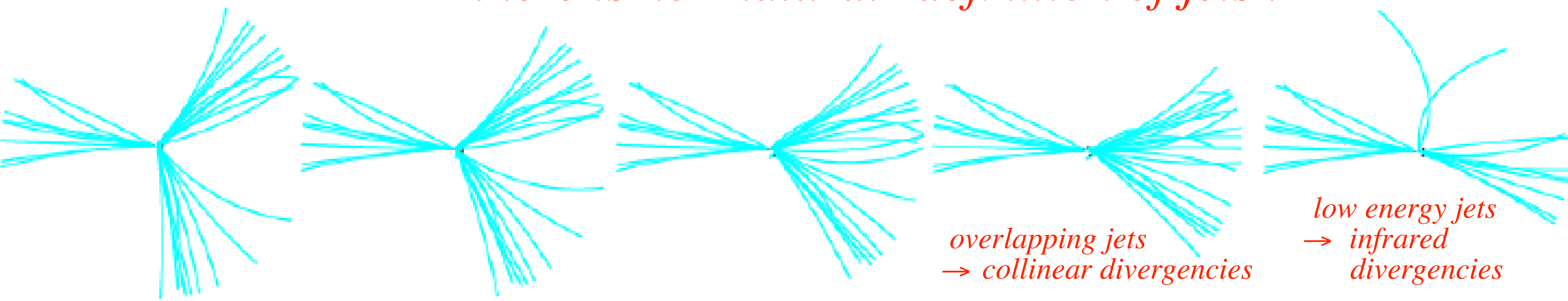
In order to compare hadron-jets with analytic QCD - calculations (quark- and gluon dynamics) one must define resolvable particle jets in theory and in experiment

→

for this one needs:

- definition of resolution criteria (e.g. minimal invariant pair mass, minimal angle, minimal energies ..)
- prescription of how to combine non-resolvable jets

however: *there is no "natural" definition of jets !*



Durham - jet definition: (mostly used in $e^+ e^-$ - annihilation)

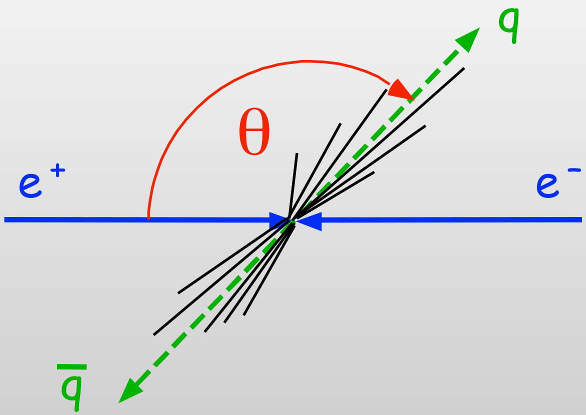
2 groups of particles, i and j, can be resolved if the minimal transversal energy of the 4-vectors, $y_{ij} = 1/2 \min(E_i^2, E_j^2) \cdot (1 - \cos(\theta_{ij}))$, satisfies: $y_{ij} \geq y_{cut}$

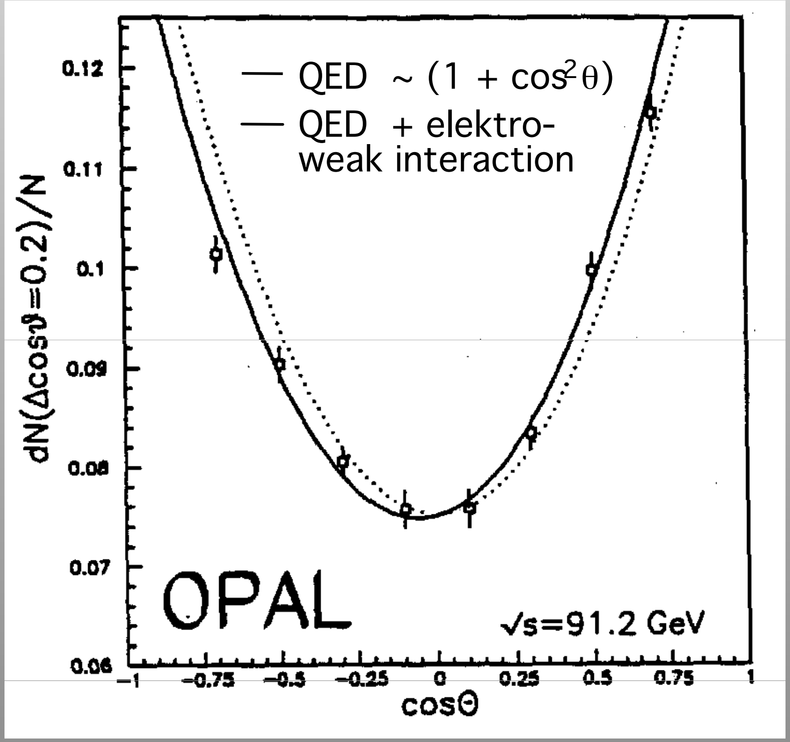
If $y_{ij} < y_{cut}$, the 'proto-jets' i and j are replaced by a new proton-jet k (recombination)

$p_k = p_i + p_j$ (recursive procedure, starting with smallest y_{ij}, until all $y_{ij} \geq y_{cut}$).

Test of basic quantum numbers (q-, g-spin):

Quark-Spin = 1/2 $\leftrightarrow$ $\frac{d\sigma}{d\theta} \sim (1 + \cos^2\theta)$





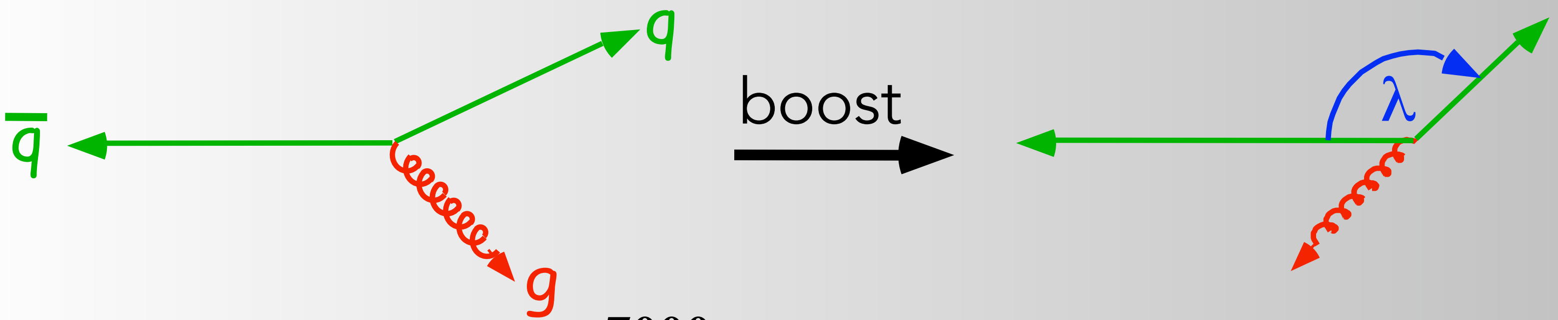
coarse structure: quarks have spin 1/2

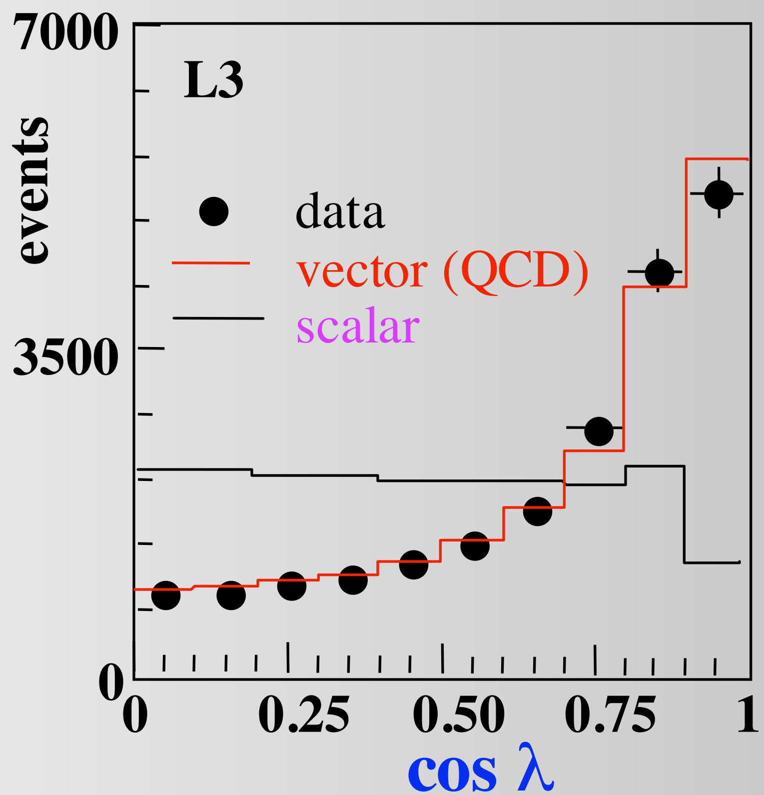
fine structure: deviation from $1 + \cos^2\theta$ is due to electro-weak interference contributions of 4.5%;

$\sin^2\theta_w = 0.2255 \pm 0.00212$

Orientation of Gluon-Jets in 3-Jet-Events:

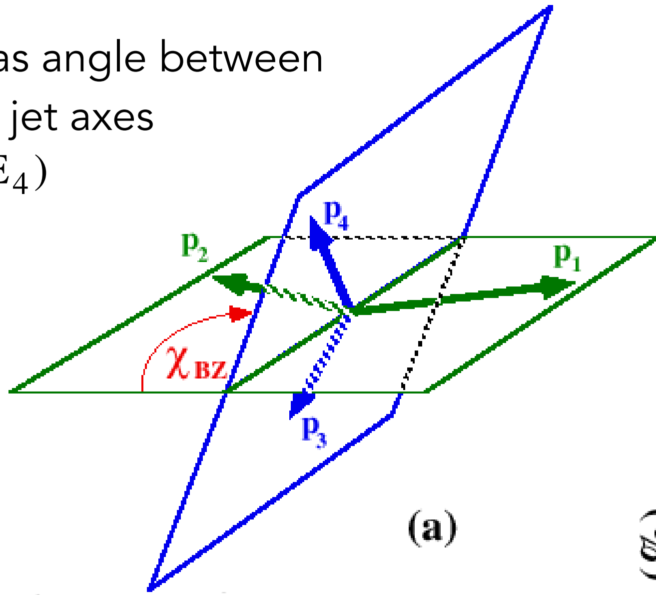
Test of the Gluon-Spin (QCD: g-spin = 1)

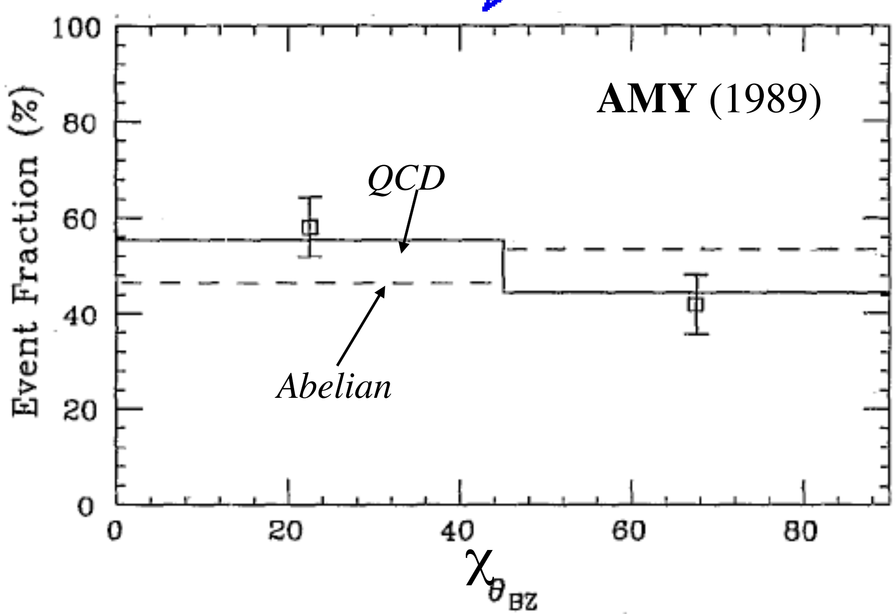


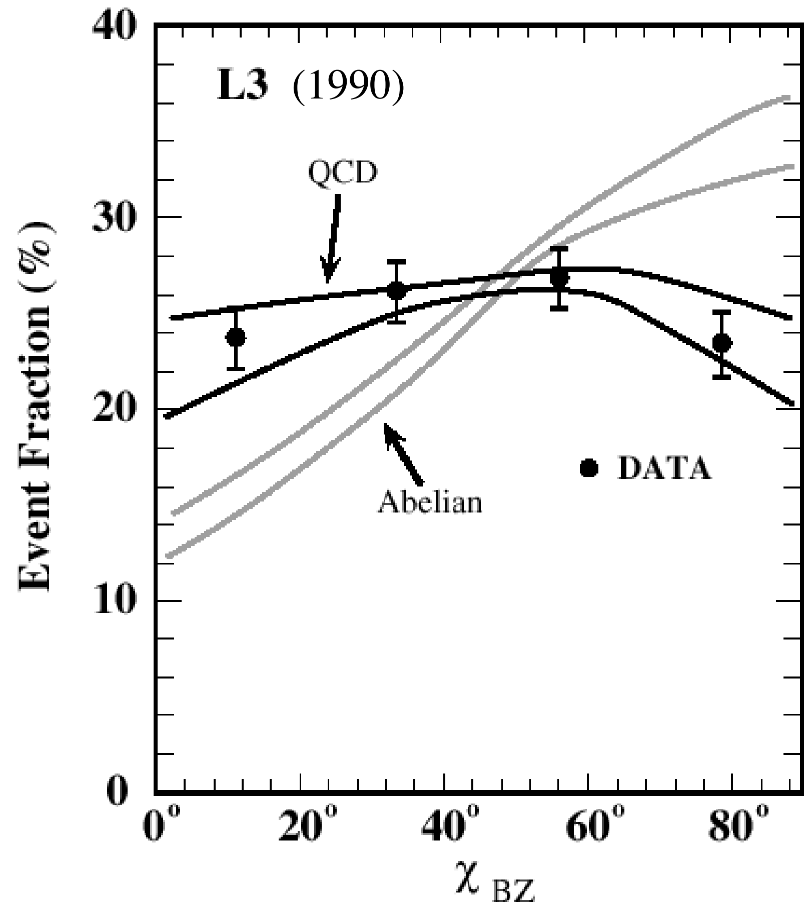


Non-Abelian gauge structure from 4-jet events

Bengtson-Zerwas angle between
energy-ordered jet axes
($E_1 \geq E_2 \geq E_3 \geq E_4$)







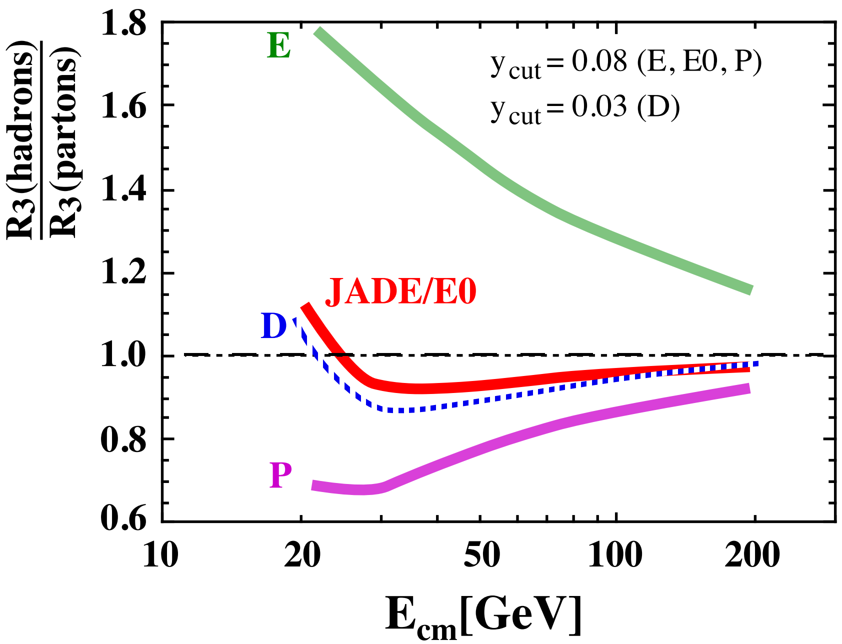
Asymptotic Freedom (running α_s)

Historically (1987):

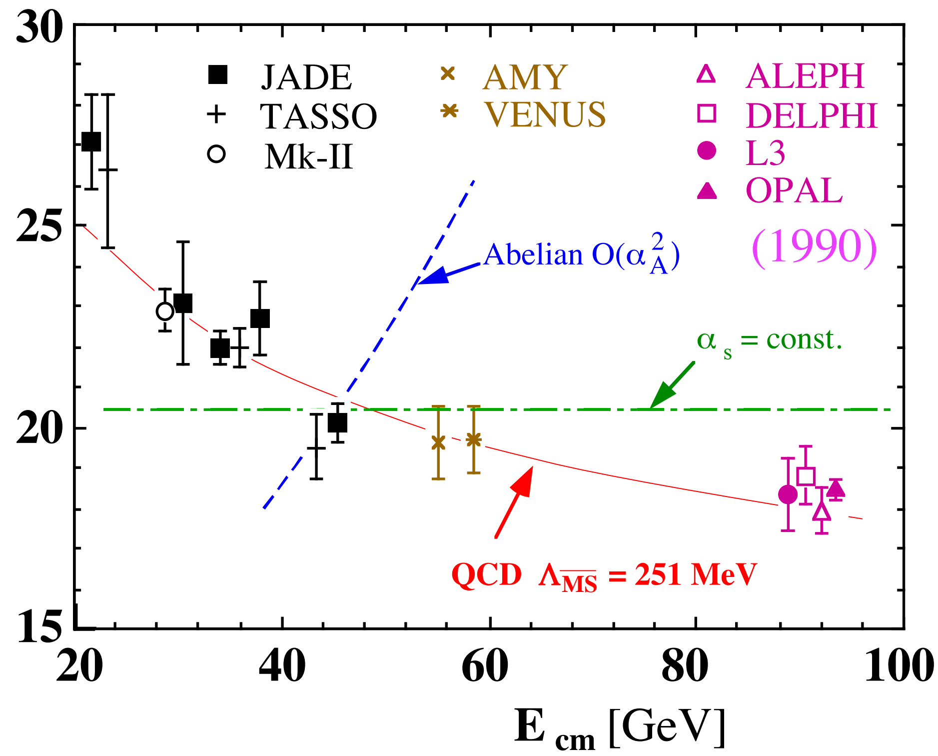
energy dependence of 3-jet production rates (R_3):

$$R_3 = C_1(y_{cut}) \cdot \alpha_s(\mu) + C_2(y_{cut}) \cdot \alpha_s^2(\mu)$$

JADE Jet finder:
small and (almost) energy independent hadronisation corrections:

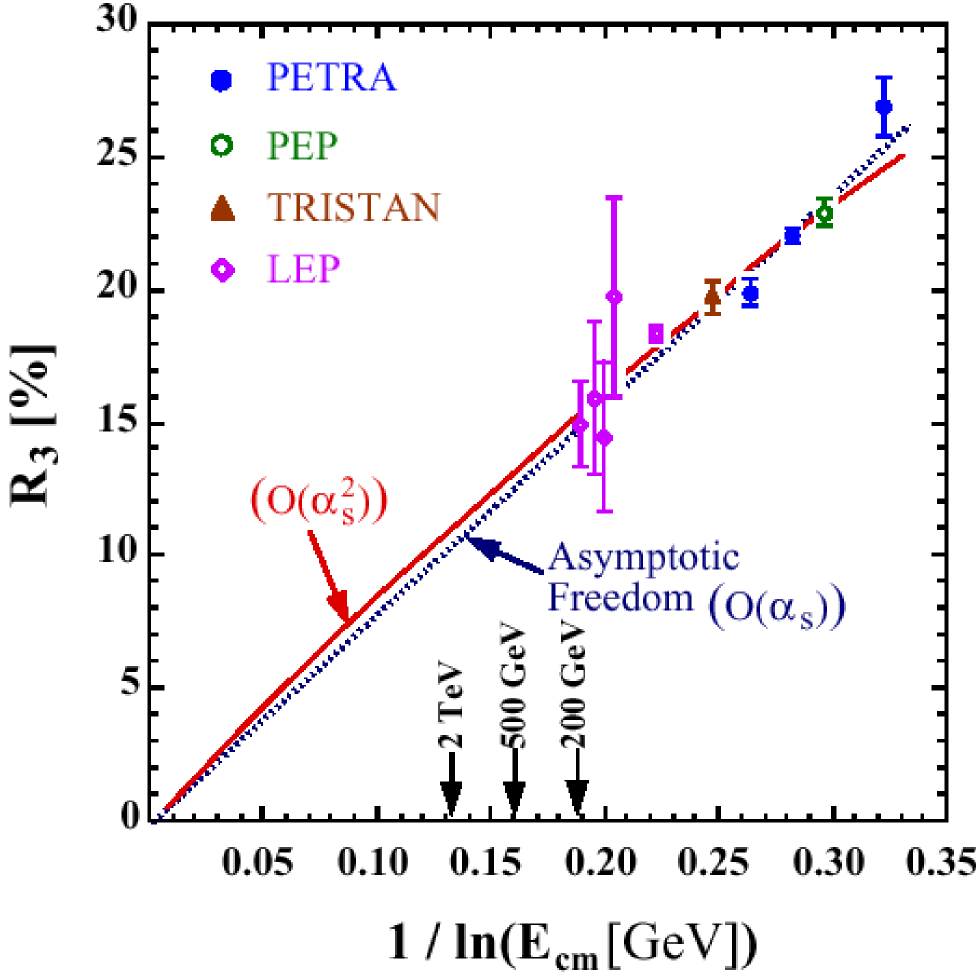


$R_3(y_{cut} = 0.08)$ [%]



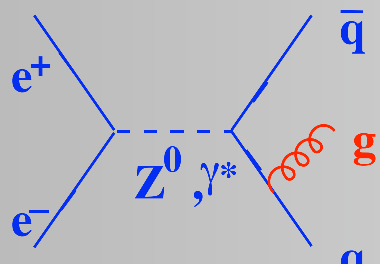
Asymptotic Freedom from jet rates

$$R_3 \equiv \frac{\sigma_{3-\text{jet}}}{\sigma_{\text{tot}}} \propto \alpha_s(E_{cm}) \propto \frac{1}{\ln E_{cm}}$$

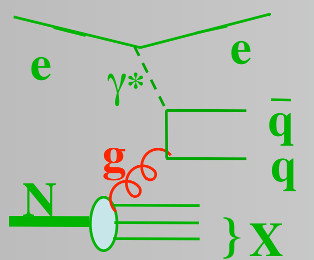


Experimental Determination of α_s

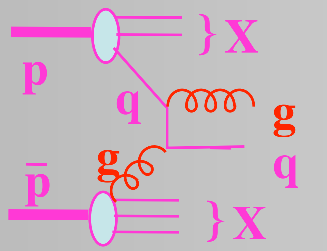
in all processes in which gluons occur:



- e^+e^- annihilations
  - total hadronic production cross section
  - hadronic decay widths of the Z^0 and of the τ
  - jet rates and shape variables



- deep inelastic lepton-nucleon-scattering
  - scaling violations of structure functions
  - sum rules of structure functions
  - jet rates and shape variables



- proton-(anti-)proton collisions
  - jet rates
  - photoproduction
  - t-quark production cross section

running α_s up to 4th order:

$$Q^2 \frac{\partial \alpha_s(Q^2)}{\partial Q^2} = \beta\left(\alpha_s(Q^2)\right)$$

$$\beta(\alpha_s(Q^2)) = -\beta_0 \alpha_s^2(Q^2) - \beta_1 \alpha_s^3(Q^2) - \beta_2 \alpha_s^4(Q^2) - \beta_3 \alpha_s^5(Q^2) + \mathcal{O}(\alpha_s^6)$$

$$\beta_0 = \frac{33 - 2N_f}{12\pi} \,,$$

$$\beta_1 = \frac{153 - 19N_f}{24\pi^2} \,,$$

$$\beta_2 = \frac{77139 - 15099N_f + 325N_f^2}{3456\pi^3} \,,$$

$$\beta_3 \approx \frac{29243 - 6946.3N_f + 405.089N_f^2 + 1.49931N_f^3}{256\pi^4}$$

$$\alpha_s(Q^2) = \frac{1}{\beta_0 L} - \frac{1}{\beta_0^3 L^2}\beta_1 \ln L + \frac{1}{\beta_0^3 L^3}\left(\frac{\beta_1^2}{\beta_0^2}\left(\ln^2 L - \ln L - 1\right) + \frac{\beta_2}{\beta_0}\right) + \frac{1}{\beta_0^4 L^4}\left(\frac{\beta_1^3}{\beta_0^3}\left(-\ln^3 L + \frac{5}{2}\ln^2 L + 2\ln L - \frac{1}{2}\right) - 3\frac{\beta_1\beta_2}{\beta_0^2}\ln L + \frac{\beta_3}{2\beta_0}\right)$$

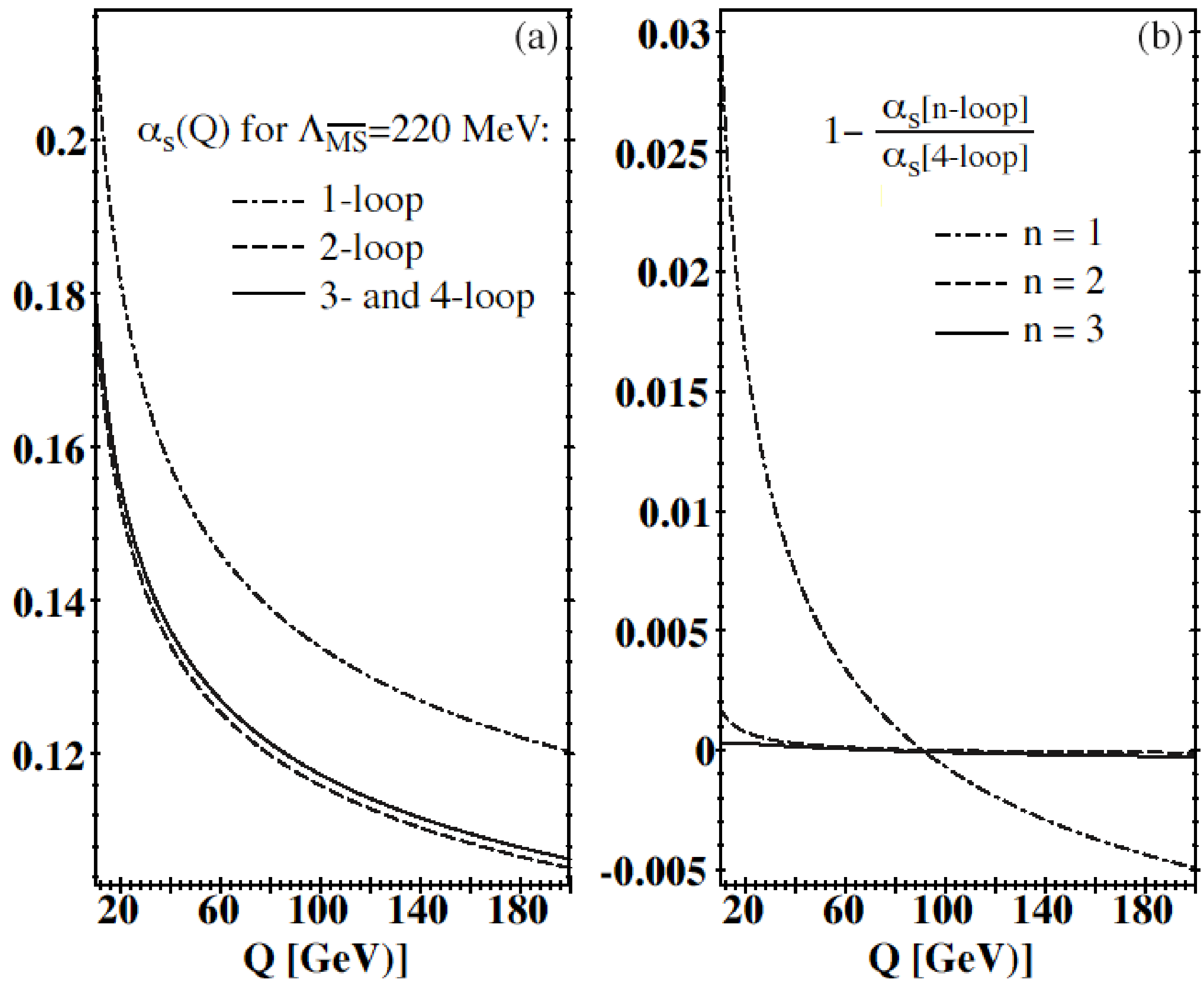
$$L = \ln \frac{Q^2}{\Lambda^2_{\overline{MS}}}$$

Ritbergen, Vermaseren, Larin

β_0 and β_1 do not depend on renormalisation scheme; β_2 and β_3 … do !

choose $\overline{\text{MS}}$ scheme for all of the following discussion.

relative size of higher order corrections



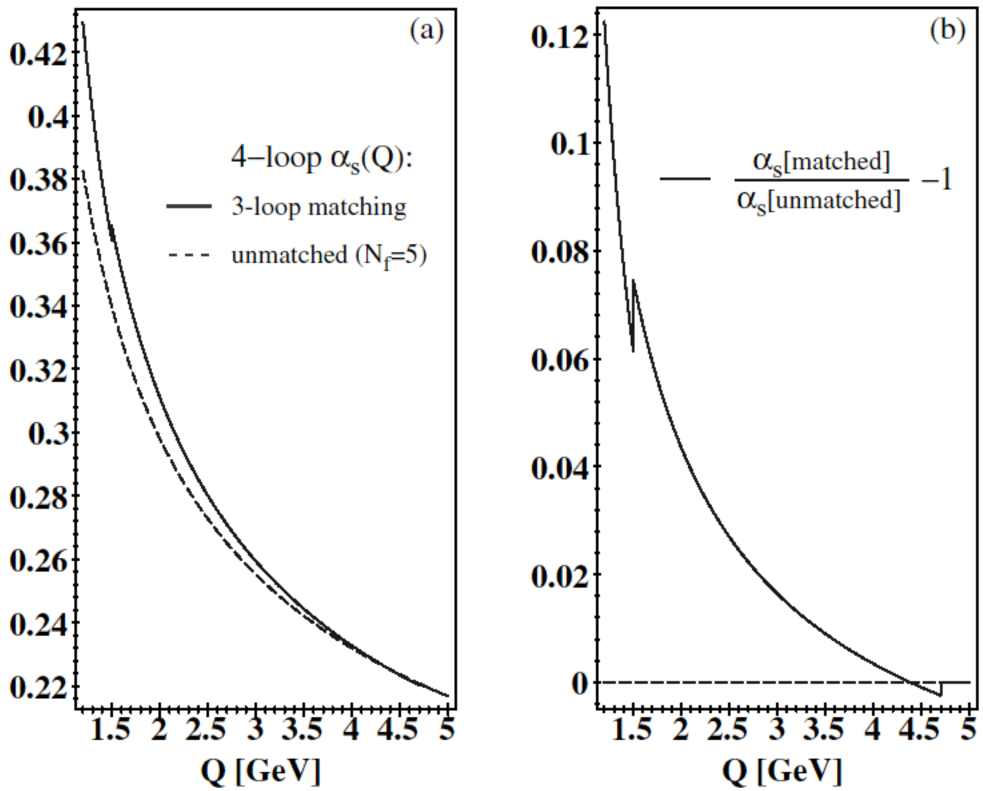
heavy quark threshold matching

Matching conditions for the choice $\mu^{(Nf)} = M_q$ (pole mass definition):

$$\frac{a'}{a} = 1 + C_2\ a^2 + C_3\ a^3$$ (with $a' = \alpha_s^{(Nf-1)}/\pi$; $a = \alpha_s^{(Nf)}/\pi$)

$$C_2 = -0.291667 \text{ and } C_3 = -5.32389 + (N_f - 1) \cdot 0.26247$$

(3-loop condition; Chetyrkin, Kniehl, Steinhauser)



perturbative predictions for physical quantities

$$\mathcal{R}(Q^2) = P_l \sum_n R_n \alpha_s^n$$
$$= P_l \left(R_0 + R_1 \alpha_s(\mu^2) + R_2(Q^2/\mu^2)\alpha_s^2(\mu^2) + \ldots\right)$$

in n^{th} order perturbation theory

R_1 : "leading order coefficient" (lo)

R_2 : "next to leading coefficient" (nlo)

R_3 : "next-next-to leading" (nnlo)

Resummation of logs arising from soft and collinear singularities:

$$\Sigma(\mathcal{R}) \equiv \int_0^{\mathcal{R}} \frac{1}{\sigma}\frac{d\sigma}{d\mathcal{R}} d\mathcal{R} = C(\alpha_s)\exp\left[G(\alpha_s, L)\right] + D(\alpha_s, \mathcal{R}) \qquad L = \ln(1/\mathcal{R}) \qquad C(\alpha_s) = 1 + \sum_{n=1}^{\infty} C_n \hat{\alpha}_s^n$$

$$G(\alpha_s, L) = \sum_{n=1}^{\infty}\sum_{m=1}^{n+1} G_{nm}\hat{\alpha}_s^n L^m$$
$$\equiv L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L)\cdots$$

	Leading logs	Next-to-Leading logs	Subleading logs	Non-log. terms	
$\ln\Sigma(\mathcal{R}) =$	$G_{12}\hat{\alpha}_s L^2$	$+G_{11}\hat{\alpha}_s L$		$+\alpha_s \mathcal{O}(1)$	$\mathcal{O}(\alpha_s)$
	$+G_{23}\hat{\alpha}_s^2 L^3$	$+G_{22}\hat{\alpha}_s^2 L^2$	$+G_{21}\hat{\alpha}_s^2 L$	$+\alpha_s^2 \mathcal{O}(1)$	$\mathcal{O}(\alpha_s^2)$
	$+G_{34}\hat{\alpha}_s^3 L^4$	$+G_{33}\hat{\alpha}_s^3 L^3$	$+G_{32}\hat{\alpha}_s^3 L^2 + \cdots$	$+\cdots$	$\mathcal{O}(\alpha_s^3)$
	$+\cdots$	$+\cdots$	$+\cdots$	$+\cdots$	$\vdots$
$=$	$L g_1(\alpha_s L)$	$+g_2(\alpha_s L)$	$+\cdots$	$+\cdots$	

renormalisation scale dependence

$$\mathcal{R} \equiv \mathcal{R}(Q^2/\mu^2, \alpha_s); \;\; \alpha_s \equiv \alpha_s(\mu^2)$$

since choice of μ is arbitrary, physical observables $\mathcal{R}$ should not depend on μ

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \mathcal{R}(Q^2/\mu^2, \alpha_s) = \left(\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s} \right) \mathcal{R} \stackrel{!}{=} 0$$

$$\begin{aligned} 0 = \mu^2 \frac{\partial R_0}{\partial \mu^2} + \alpha_s(\mu^2) \mu^2 \frac{\partial R_1}{\partial \mu^2} &+ \alpha_s^2(\mu^2) \left[\mu^2 \frac{\partial R_2}{\partial \mu^2} - R_1 \beta_0 \right] \\ &+ \alpha_s^3(\mu^2) \left[\mu^2 \frac{\partial R_3}{\partial \mu^2} - [R_1 \beta_1 + 2 R_2 \beta_0] \right] \\ &+ \mathcal{O}(\alpha_s^4) \; . \end{aligned}$$

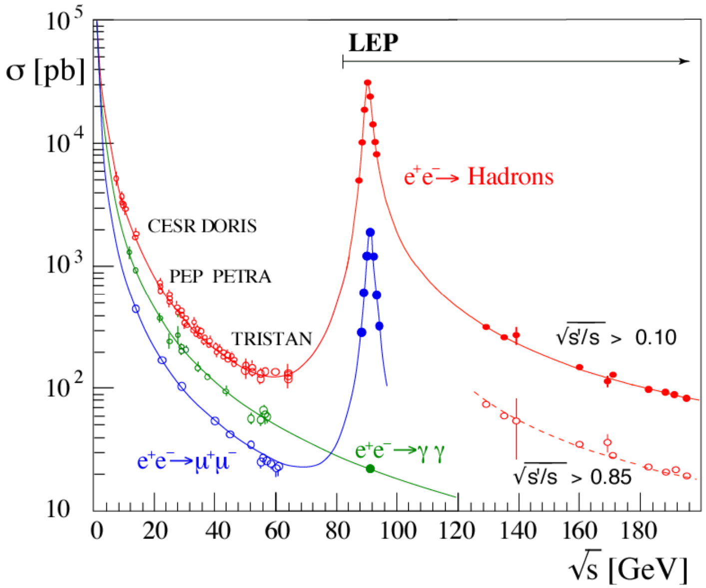
–>

$$\begin{aligned} R_0 &= \text{const.} \; , \\ R_1 &= \text{const.} \; , \\ R_2 \left(\frac{Q^2}{\mu^2} \right) &= R_2(1) - \beta_0 R_1 \ln \frac{Q^2}{\mu^2} \; , \\ R_3 \left(\frac{Q^2}{\mu^2} \right) &= R_3(1) - [2 R_2(1) \beta_0 + R_1 \beta_1] \ln \frac{Q^2}{\mu^2} + R_1 \beta_0^2 \ln^2 \frac{Q^2}{\mu^2} \end{aligned}$$

Perturbative QCD coefficients beyond leading order become renormalisation scale dependend !

This dependence is used to quantify theoretical uncertainties due to unknown higher orders.

hadronic width of Z^0 boson



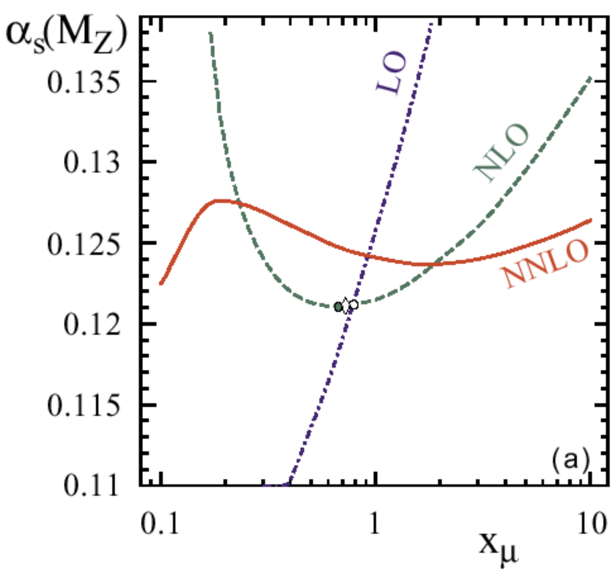
$$R_Z = \frac{\Gamma(Z^0 \rightarrow \text{hadrons})}{\Gamma(Z^0 \rightarrow \text{leptons})} = 20.768 \pm 0.0024$$

$$R_Z = 19.934\left[1 + 1.045\frac{\alpha_s(\mu)}{\pi} + 0.94\left[\frac{\alpha_s(\mu)}{\pi}\right]^2 - 15\left[\frac{\alpha_s(\mu)}{\pi}\right]^3\right]$$

$$\Rightarrow \quad \alpha_s(M_Z) = 0.124 \pm 0.004 \quad (\text{exp.})$$

~~$\pm\ 0.002\ (M_H, M_{top})$~~

$$^{+\,0.003}_{-\,0.001} \quad (\text{QCD})$$



$x_\mu = \mu / E_{cm}$

error source	$\Delta\alpha_s(M_{Z^0})$
$\Delta M_{Z^0} = \pm 0.0021$ GeV	± 0.00003
$\Delta M_t = \pm 5$ GeV	± 0.0002
~~$M_H = 100 \ldots 1000$ GeV~~	~~± 0.0017~~
$\mu = (\frac{1}{4} \ldots 4)\ M_{Z^0}$	$^{+\,0.0028}_{-\,0.0004}$
renormalization schemes	± 0.0002
total	$^{+\,0.003}_{-\,0.002}$

α_s from τ-decays

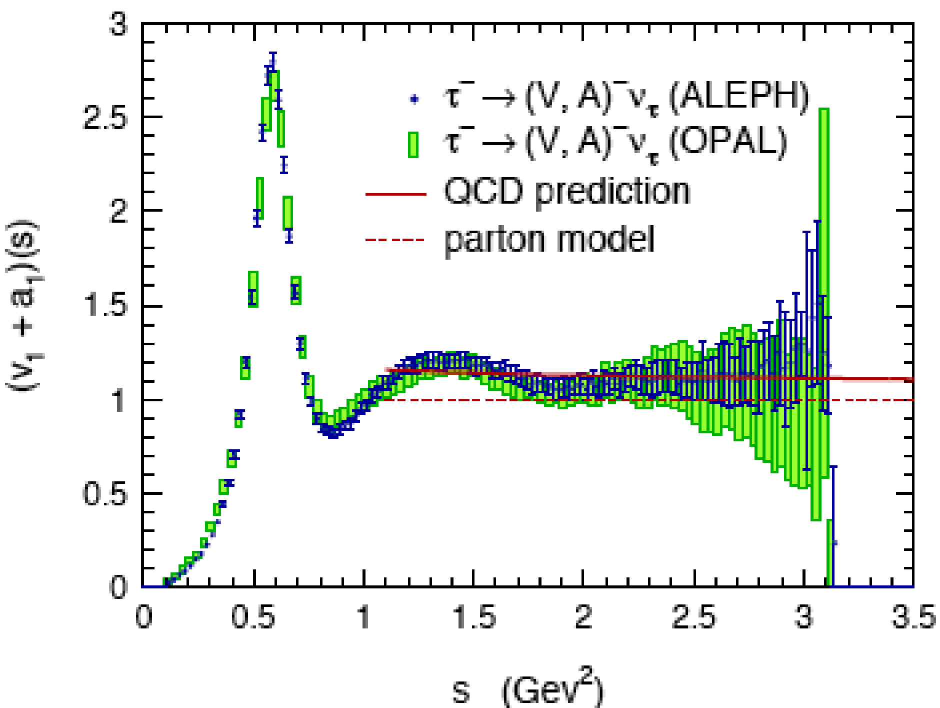
$$R_\tau = \frac{\Gamma(\tau \to \text{hadrons } \nu_\tau)}{\Gamma(\tau \to e\nu_e \nu_t)}$$

$$QCD: \quad R_\tau = 3.058(1.001 + \delta_{pert} + \delta_{nonpert})$$

$$\delta_{pert} = \frac{\alpha_s(m_\tau)}{\pi} + 5.20\left(\frac{\alpha_s(m_\tau)}{\pi}\right)^2 + 26.37\left(\frac{\alpha_s(m_\tau)}{\pi}\right)^3$$

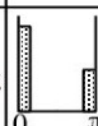
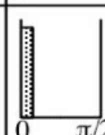
measurements of R as well as the mass spectra of hadronic τ-decays and comparison

with $O(\alpha_S^3)$ perturbative QCD results in $\alpha_S(M_\tau)$ also provides an independent determination of the leading nonperturbative contributions $\delta_{nonpert}$

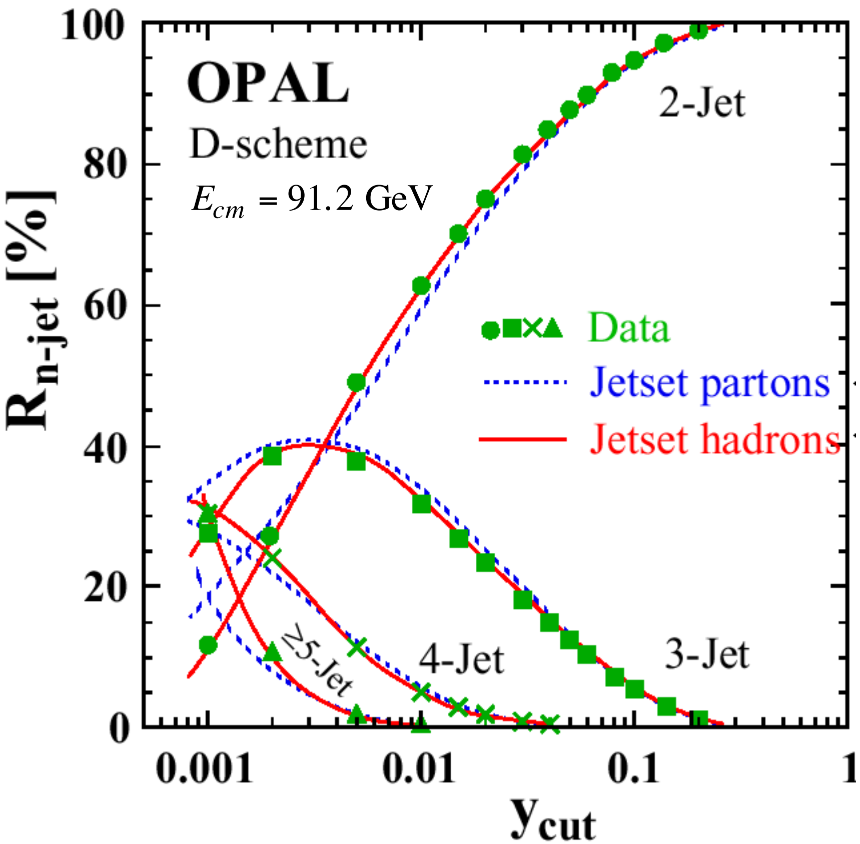


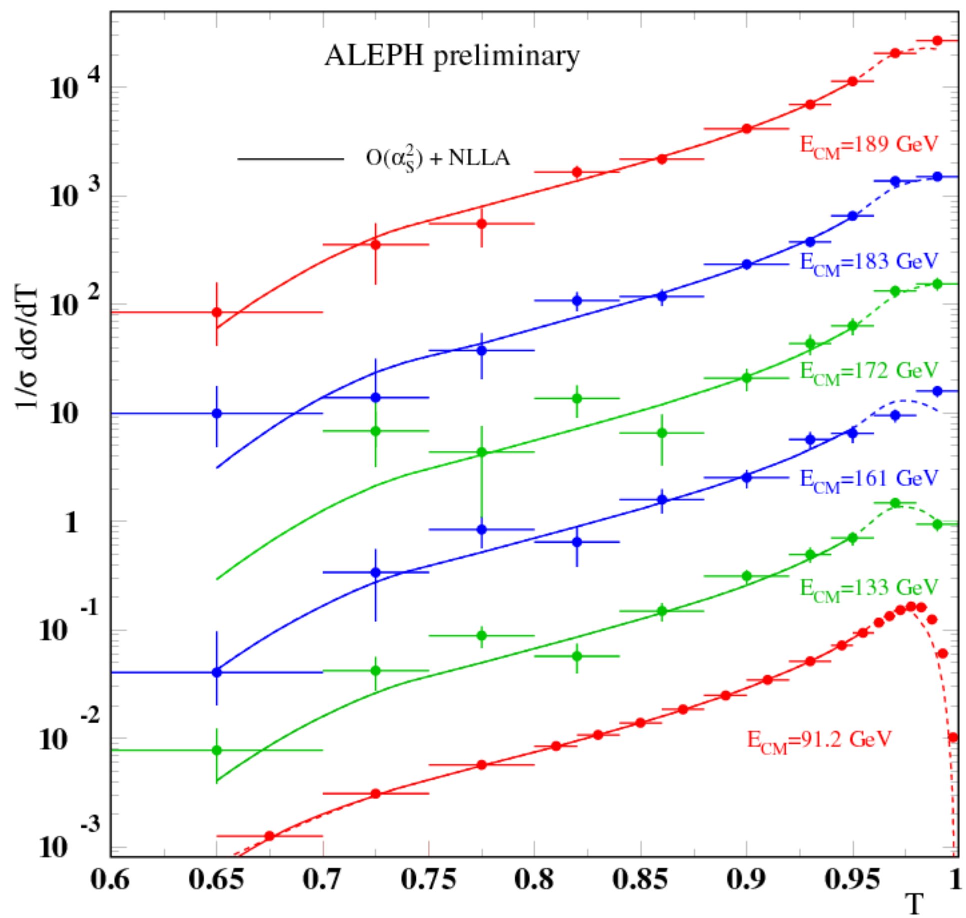
$\alpha_s(M_z) = 0.1213 \pm 0.0006_{\text{exp}} \pm 0.0010_{\text{theo}}$

Event Shape Observables

Name of Observable	Definition	Typical Value for: 			QCD calculation
Thrust	$T = \max\limits_{\vec{n}} \left(\frac{\Sigma_i \lvert \vec{p}_i \vec{n} \rvert}{\Sigma_i \lvert \vec{p}_i \rvert} \right)$	1	≥2/3	≥1/2	(resummed) $O(\alpha_s^2)$
Thrust major	Like T, however T_{maj} and $\vec{n}_{maj}$ in plane ⊥ $\vec{n}_T$	0	≤1/3	≤1/√2	$O(\alpha_s^2)$
Thrust minor	Like T, however T_{min} and $\vec{n}_{min}$ in direction ⊥ to $\vec{n}_T$ and $\vec{n}_{maj}$	0	0	≤1/2	$O(\alpha_s^2)$
Oblateness	$O = T_{maj} - T_{min}$	0	≤1/3	0	$O(\alpha_s^2)$
Sphericity	$S = 1.5\,(Q_1 + Q_2)$; $Q_1 \le \ldots \le Q_3$ are Eigenvalues of $S^{\alpha\beta} = \frac{\Sigma_i\, p_i^{\alpha}\, p_i^{\beta}}{\Sigma_i\, p_i^2}$	0	≤3/4	≤1	none (not infrared safe)
Aplanarity	$A = 1.5\, Q_1$	0	0	≤1/2	none (not infrared safe)
Jet (Hemis-phere) masses	$M_\pm^2 = \left(\Sigma_i E_i^2 - \Sigma_i \vec{p}_i^{\,2}\right)_{i \in S_\pm}$ ($S_\pm$: Hemispheres ⊥ to $\vec{n}_T$) $M_H^2 = \max(M_+^2, M_-^2)$ $M_D^2 = \lvert M_+^2 - M_-^2 \rvert$	0 0	≤1/3 ≤1/3	≤1/2 0	(resummed) $O(\alpha_s^2)$
Jet broadening	$B_\pm = \frac{\Sigma_{i \in S_\pm} \lvert \vec{p}_i \times \vec{n}_T \rvert}{2\, \Sigma_i \lvert \vec{p}_i \rvert}$; $B_T = B_+ + B_-$ $B_w = \max(B_+, B_-)$	0 0	≤1/(2√3) ≤1/(2√3)	≤1/(2√2) ≤1/(2√3)	(resummed) $O(\alpha_s^2)$
Energy-Energy Correlations	$EEC(\chi) = \sum\limits_{events} \int_{\chi+\frac{\Delta\chi}{2}}^{\chi-\frac{\Delta\chi}{2}} \sum\limits_{i,j} \frac{E_i E_j}{E_{vis}^2}\, \delta(\chi - \chi_{ij})\, d\chi$				(resummed) $O(\alpha_s^2)$
Asymmetry of EEC	$AEEC(\chi) = EEC(\pi - \chi) - EEC(\chi)$				$O(\alpha_s^2)$
Differential 2-jet rate	$D_2(y) = \frac{R_2(y - \Delta y) - R_2(y)}{\Delta y}$				(resummed) $O(\alpha_s^2)$

Jet production and hadronic event shapes





Pert. QCD

- in NLO: $\frac{1}{\sigma_0}\frac{d\sigma}{dy} = R_1(y)\,\alpha_s(\mu^2) + R_2\left(y,\frac{\mu^2}{Q^2}\right)\alpha_s^2(\mu^2)$

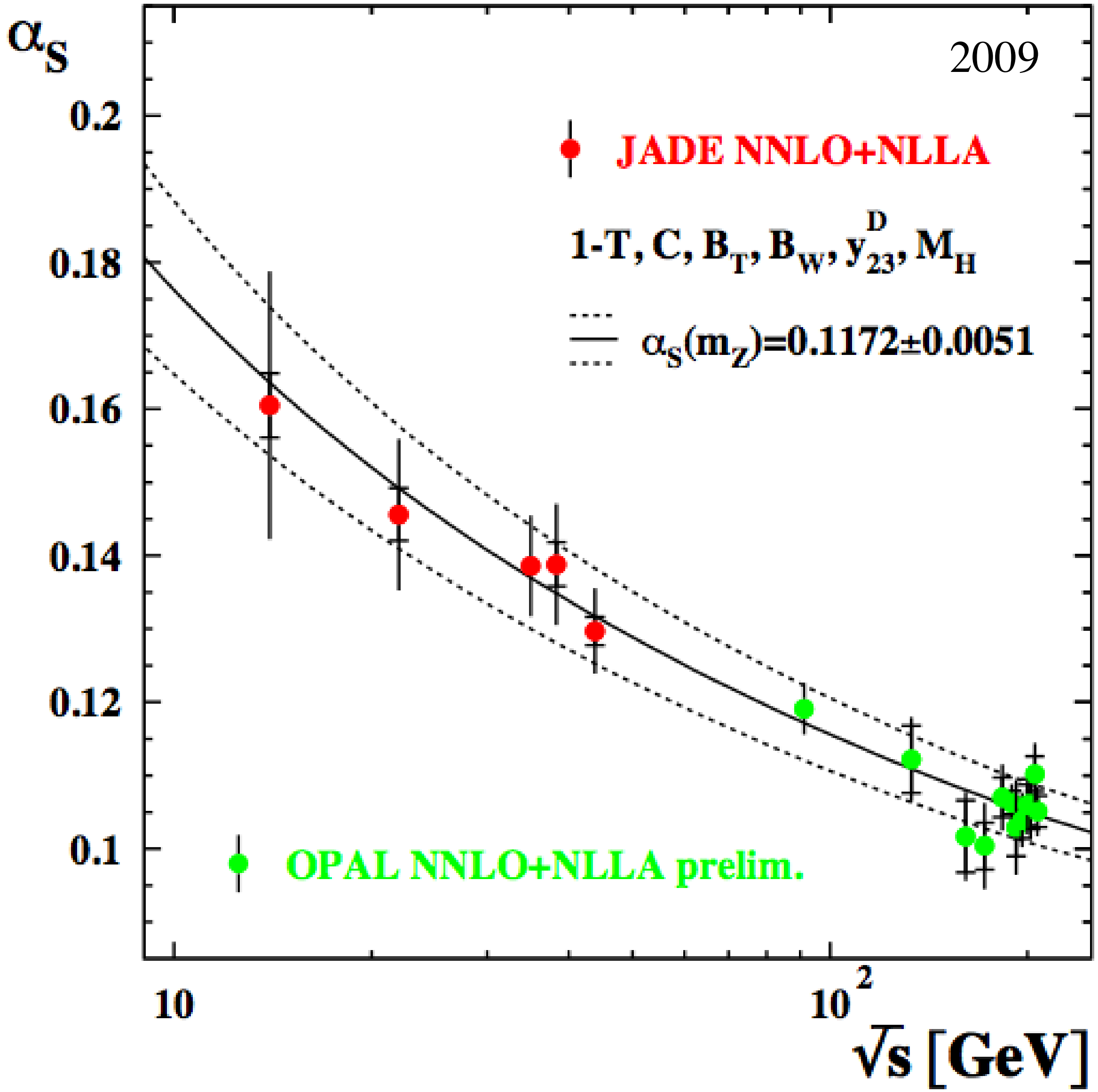
Ellis, Ross & Terrano (ERT);
Kunszt & Nason, Catani & Seymour

in NNLO: A. Gehrmann-de Ridder et al., 2007

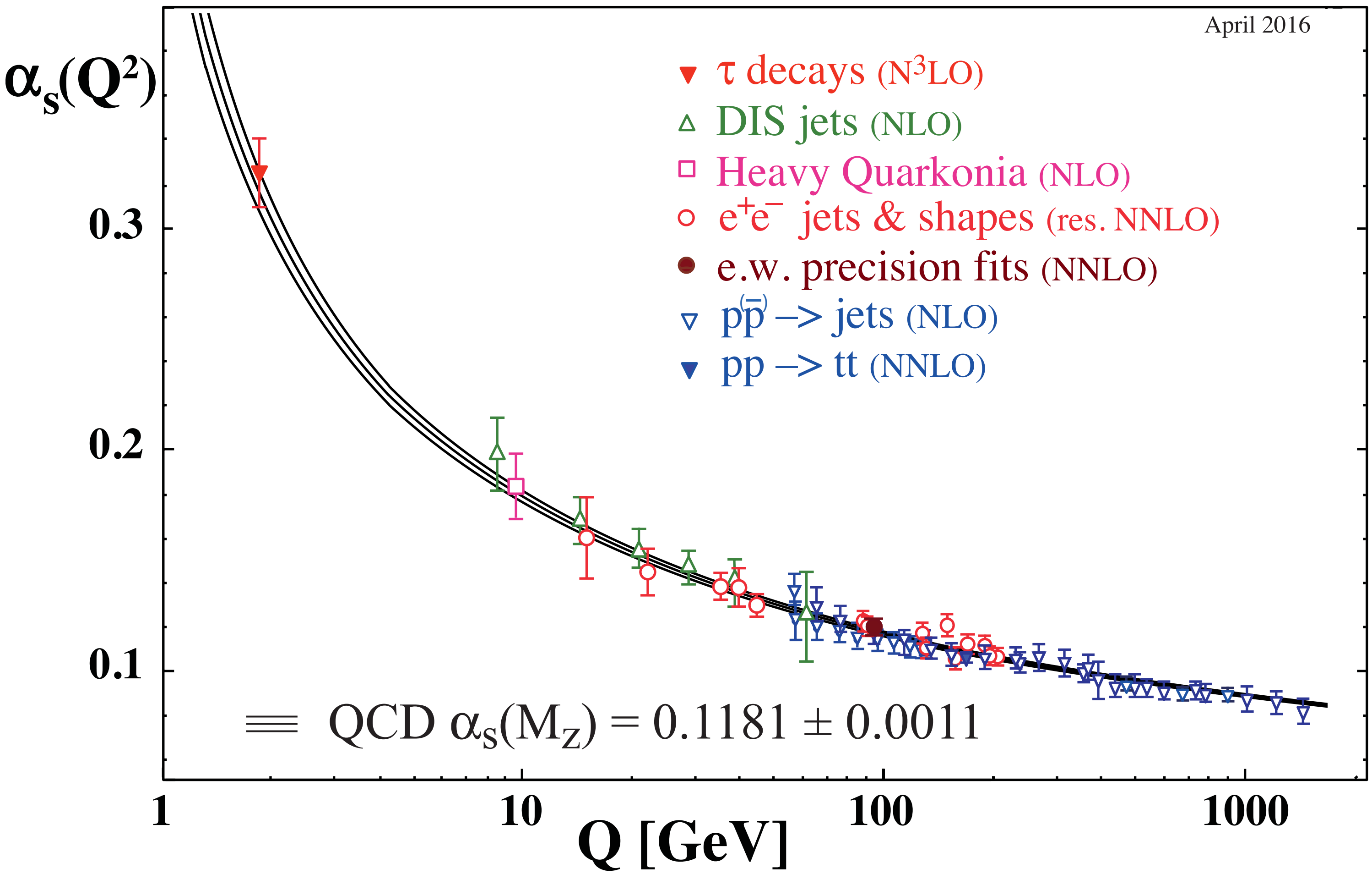
- plus resummation of leading and next-to-leading logarithms (NLLA) –> "matching schemes"

Catani, Trentadue, Turnock, Webber

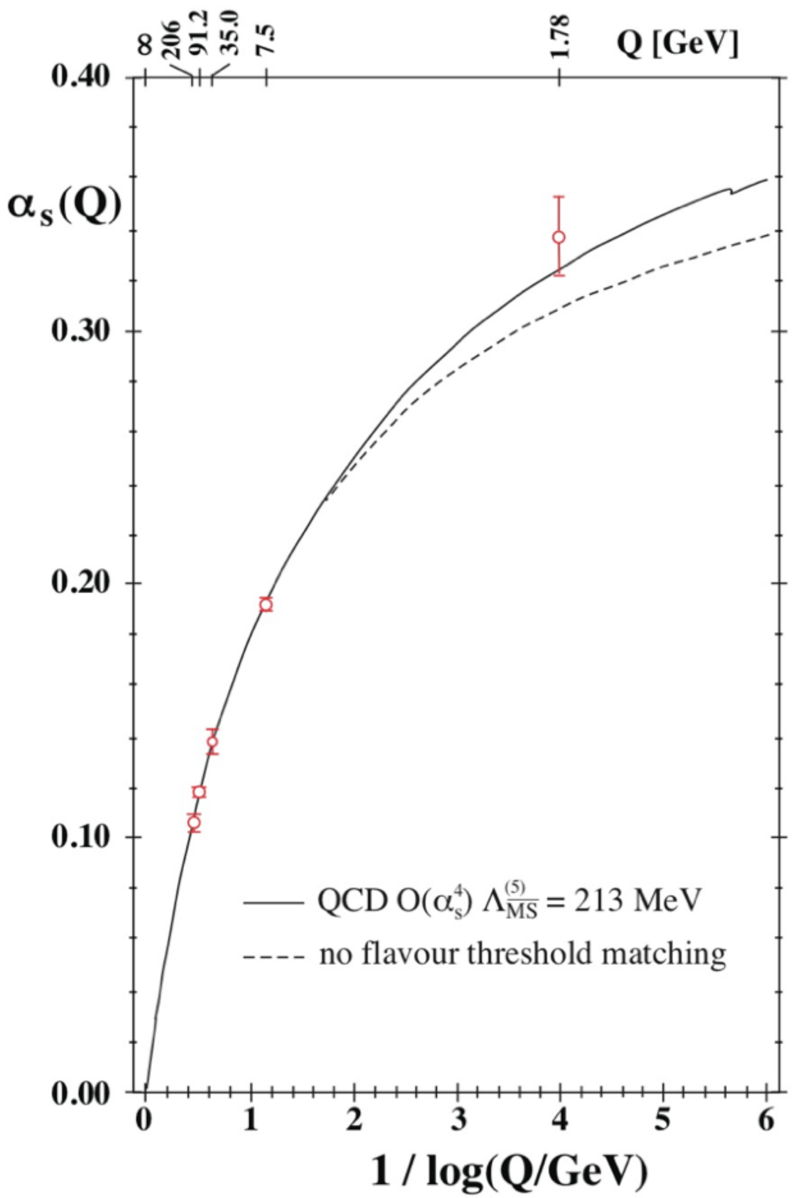
α_s from jet rates und event shapes in NNLO QCD:



global summary of α_s determinations:



Evidence for Asymptotic Freedom:



Summary:

- QCD established as gauge field theory of Strong Interactions:
  - asymptotic freedom from energy dependence of jet rates and of α_s
  - colour charge of gluons established
  - spins der Quarks (1/2) and gluons (1) verified

- quarks and gluons don't exist as free particles, but only in bound, „colourless" states (hadrons)

- at high energies, hadrons resemble the directions of produced primary quarks and gluonen („jets")

- precise measurements of properties of jets provide quantitative Tests der QCD

- determination of α_s from many reactions:
  $\alpha_s(M_Z) \sim 0.12 \ (0.1181 \pm 0.0011)$

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