A brief overview on the issue of Bose-Einstein condensation of dark matter axions

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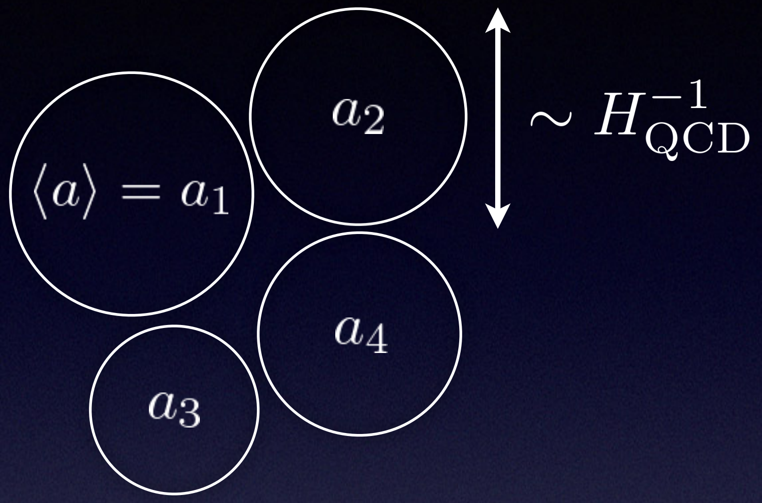
Peculiarities of axion dark matter

- Non-thermal production



The axion field starts to oscillate at

$$H \lesssim m_a$$

Typical momentum dispersion $\delta p \sim H_{\rm QCD}$

- Large occupation number

$$\mathcal{N} \sim n_a \frac{(2\pi)^3}{\frac{4\pi}{3}(\delta p)^3} \sim 10^{60}$$

n_a : number density of axions

- Axions have a capacity to form a Bose-Einstein condensate (BEC).

Bose-Einstein condensation of axion dark matter ?

- Usual picture of BEC:
  In equilibrium, interacting bosons migrate to the lowest energy state.

- Sikivie argued that axions can thermalize due to gravitational interactions to form a BEC.

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Bose-Einstein Condensation of Dark Matter Axions

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We show that cold dark matter axions thermalize and form a Bose-Einstein condensate (BEC). We obtain the axion state in a homogeneous and isotropic universe, and derive the equations governing small axion perturbations. Because they form a BEC, axions differ from ordinary cold dark matter in the nonlinear regime of structure formation and upon entering the horizon. Axion BEC provides a mechanism for the production of net overall rotation in dark matter halos, and for the alignment of cosmic microwave anisotropy multipoles.

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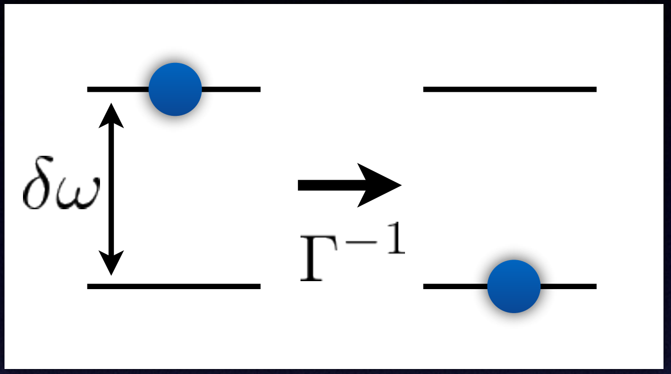
Estimate of the transition rate

Erken, Sikivie, Tam, and Yang (2012)

- Time evolution of quantum oscillators due to interactions

$$H = \sum_i \omega_i a_i^\dagger a_i + \sum_{ijkl} \frac{1}{4} \Lambda_{kl}^{ij} a_k^\dagger a_l^\dagger a_i a_j$$

$$\dot{a}_i = i[H, a_i]$$



- Change of occupation number due to the transition…

$$\Gamma \sim \dot{\mathcal{N}}/\mathcal{N} \qquad \mathcal{N}_k = a_k^\dagger a_k / V$$

λa^4 $\quad H_I = -\frac{\lambda}{4!}\int d^3x a^4(x) \quad \Rightarrow \quad \Gamma_\lambda \sim \frac{1}{4}\lambda m_a^{-2} n_a$

gravity $\quad H_I = -\frac{G}{2}\int d^3x d^3x' \frac{\rho_a(x)\rho_a(x')}{|\mathbf{x}-\mathbf{x}'|} \quad \Rightarrow \quad \boxed{\Gamma_g \sim 4\pi G m_a^2 n_a / (\delta p)^2}$

Note that $\Gamma_g \propto R(t)^{-1}$ while $\Gamma_\lambda \propto R(t)^{-3}$

$\delta p \sim m_a \delta v \propto 1/R(t)$

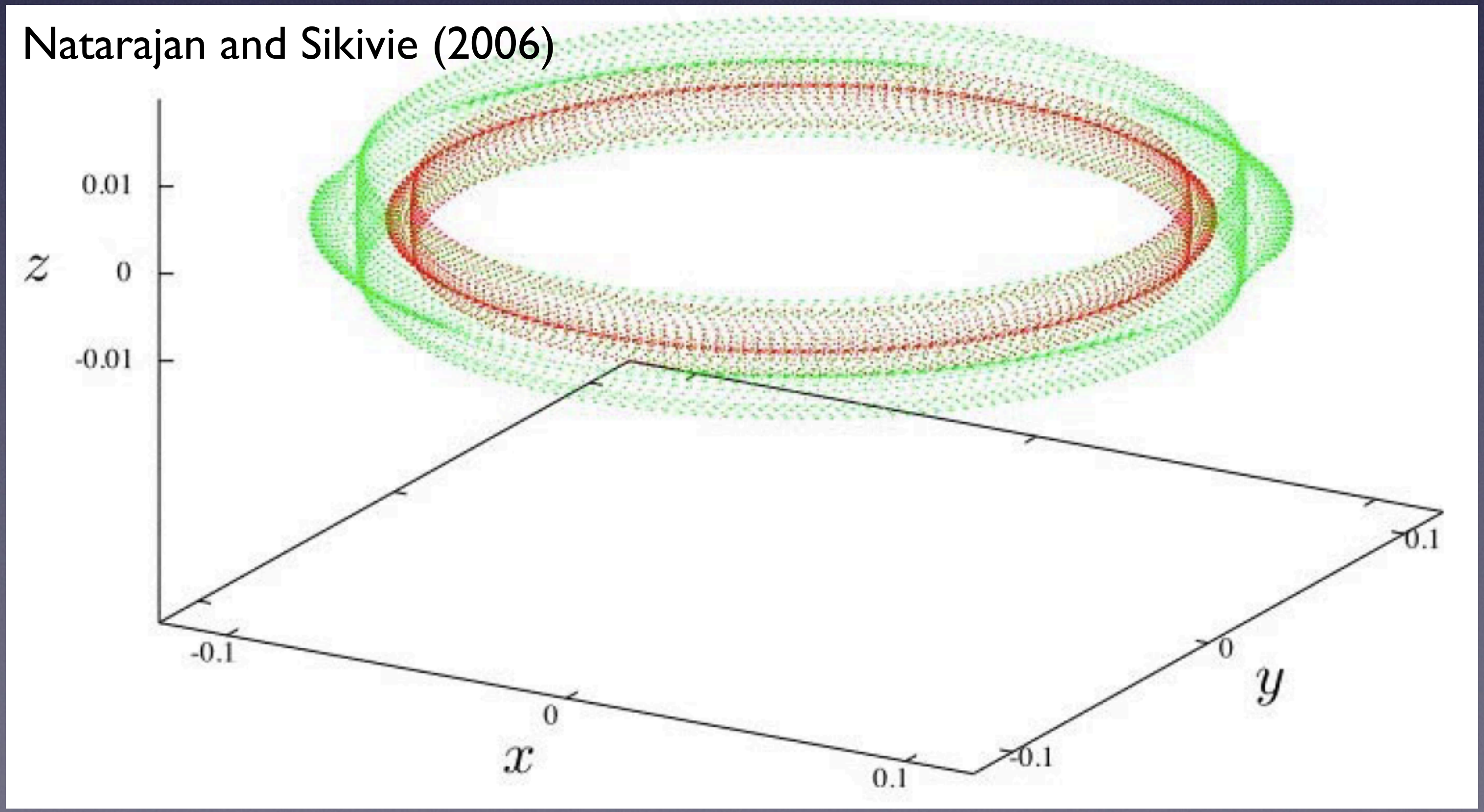
$R(t)$: scale factor of the universe

- Γ_g exceeds the expansion rate H at $T \sim \mathrm{keV}$.
  After that axions form a BEC, evolving somewhat differently ?

Effects on phase space structure of galactic halos

Sikivie (2011), Banik and Sikivie (2013), Chakrabarty and Sikivie (2018)

- The thermalization would lead to a large scale correlation of axion dark matter (larger than a galactic scale !?)
- Axions may fall into a galactic potential with net overall rotation. → Formation of “caustic rings”.
- Signatures in matter distribution in galaxy !?
  Possibility to distinguish axions from other dark matter candidates.



Thermal contact with other particle species

Erken, Sikivie, Tam, Yang (2012)

- Gravity is universal:
  Speculation that axion BEC may also enter into thermal contact with other species in a similar rate

- If photons reach thermal contact with axions

$$\rho_{\gamma i} = \frac{\pi^2}{15} T_{\gamma i}^4 = \rho_{\gamma f} + \rho_{a f} = \frac{\pi^2}{30} T_{\gamma f}^4 (2+1)$$

$$\Rightarrow \quad T_{\gamma f} = (2/3)^{1/4} T_{\gamma i}$$

baryon-to-photon ratio at BBN $\eta_{\mathrm{BBN}} = (2/3)^{3/4} \eta_{\mathrm{std.}}$

effective # of neutrino d.o.f. $N_{\mathrm{eff}} = 6.77$ (obs. $N_{\mathrm{eff}} \simeq 3-4$)

- Does axion dark matter conflict with standard cosmology !?

More formal calculations

KS and Yamaguchi (2013)
Noumi, KS, Sato, and Yamaguchi (2014)

- Identify the rate as $\Gamma \sim \langle \dot{\mathcal{N}}_p \rangle / \langle \mathcal{N}_p \rangle$
  and calculate the time evolution of the quantum expectation value of the number operator $\mathcal{N}_p = a_p^\dagger a_p / V$

$$\langle \mathrm{in} | \mathcal{N}_p(t) | \mathrm{in} \rangle = \langle \mathcal{N}_p \rangle + i \int_{t_0}^{t} dt' \langle [H_I(t'), \mathcal{N}_p] \rangle + \mathcal{O}(H_I^2) + \dots$$

- If initial axions are described by coherent states,
  it reproduces Sikivie's result

$$|\{\alpha\}\rangle = \prod_i e^{-\frac{1}{2}|\alpha_i|^2} \sum_{n=0}^{\infty} \frac{\alpha_i^n}{n! \sqrt{V^n}} (a_i^\dagger)^n |0\rangle$$

$$\Gamma_g \simeq \frac{4\pi G m_a^2 n_a}{(\delta p)^2}$$

$$a_i |\alpha_i\rangle = V^{1/2} \alpha_i |\alpha_i\rangle$$

- Terms of first order in $H_I(t)$ vanishes if number states are used for other particle species.
  → A large value of N_{eff} was fictitious (?)

$$|\{\mathcal{N}\}\rangle = \prod_i \frac{1}{\sqrt{\mathcal{N}_i! V^{\mathcal{N}_i}}} (a_i^\dagger)^{\mathcal{N}_i} |0\rangle$$

- The use of coherent states appears to be crucial.

Do axions "gravitationally thermalize" ?

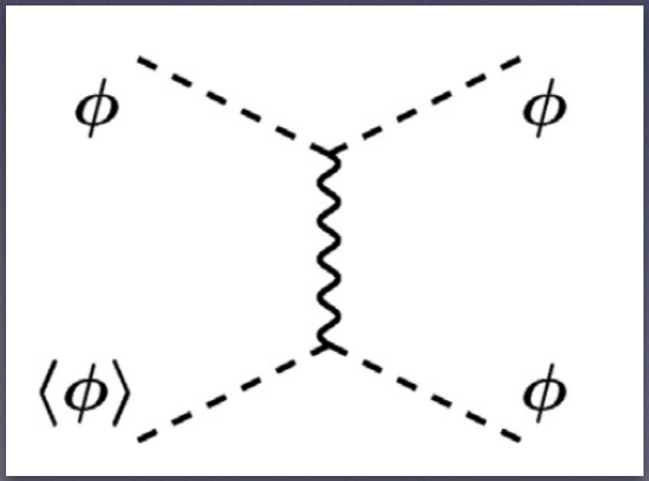
Davidson and Elmer (2013)
Davidson (2015)

- Doubts on the interpretation of the result.

- The "interaction rate" Γ_g can be reproduced just by using classical linearized Einstein equations with classical axion field. (coherent state = classical field !)

- Seems to be irrelevant to "thermalization":
Solutions of classical Einstein equations imply density fluctuations grow, which means that the axion field becomes inhomogeneous, rather than homogeneous (BEC).

- The rate of processes relevant to dissipation appears to be negligible compared to the expansion rate of the universe. Davidson (2015)



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Do dark matter axions form a condensate with long-range correlation?

Alan H. Guth,[1,*] Mark P. Hertzberg,[1,2,†] and C. Prescod-Weinstein[3,‡]

- If classical description is valid, one can consider 2 distinctive cases for the evolution of the system, according to the sign of interaction:
  - Repulsive
    → chance to have a homogeneous configuration
  - Attractive
    → instability in low momentum modes
    no chance to develop a large-scale correlation
- Self-interactions of axions are attractive, corresponding to the latter case.
- The final state would be some complicated configuration of many clumps with a shorter correlation length. (i.e. axion stars and/or miniclusters…)

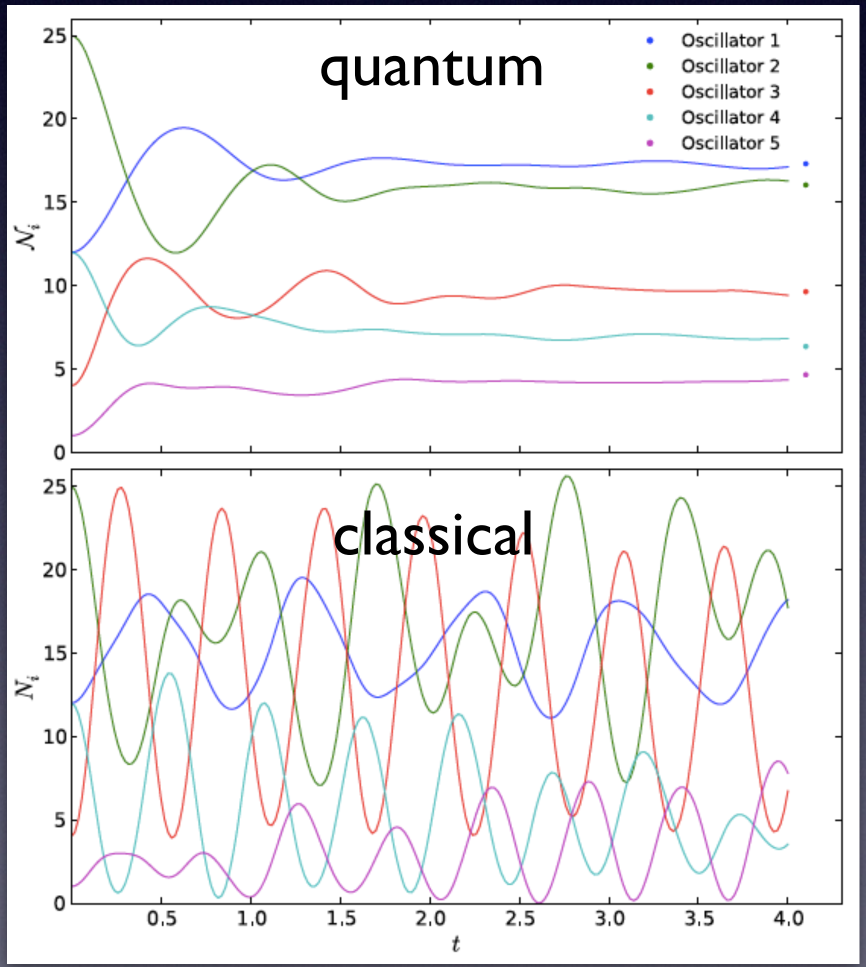
Controversy on classical/quantum description

- Which is the correct description of the axion field after $\Gamma_g > H$?
  - Is classical description enough ?
  - Or could it no longer obey classical field equations ?
- Sikivie's numerical simulations of a simplified system of interacting bosons

$$H = \sum_i \omega_i a_i^\dagger a_i + \frac{1}{4} \sum_{ijkl} \Lambda_{ij}^{kl} a_i^\dagger a_j^\dagger a_k a_l$$

supported the latter.

Sikivie and Todarello, 1607.00949

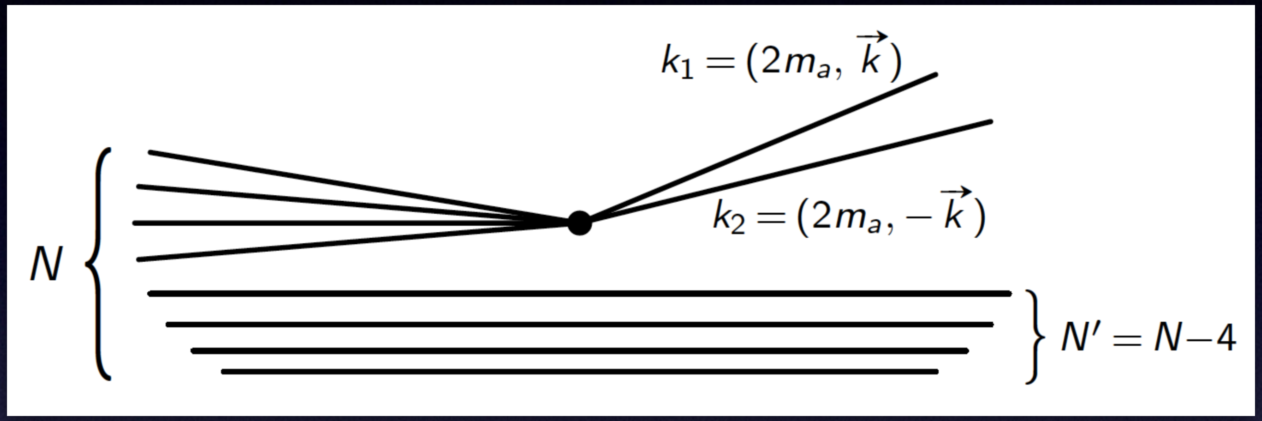


- Inappropriate comparison ?
  - One may have to compare with an ensemble average over classical states, rather than a specific classical state. Hertzberg, 1609.01342

Investigation of the validity of classical description

Dvali and Zell (2018)

- Axions initially described as a coherent state (classical field) can evolve into some non-coherent state due to the rescatterings.



$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 + \frac{1}{4!}\alpha a^4 + \dots \qquad \alpha \equiv m_a^2/f_a^2$$

- Corresponding rate

$$\Gamma \sim m_a \alpha^4 \mathcal{N}^4$$

implies the "quantum break-time" for axions

$$t_q \sim \mathcal{N}\Gamma^{-1} = m_a^{-1}\frac{f_a^8 m_a^4}{\rho_a^3} \sim 10^{173}\,\mathrm{sec}$$

- This time scale is 10^{156} times larger than the age of the universe! Classical description could be quite accurate.

Summary

- The scenario of gravitationally thermalizing axion BEC dark matter possesses interesting observational implications.
  - Effects on phase space structure of galactic halo
  - Possible modifications of cosmological parameters
- Several criticisms on the claim on deviations from classical description.
- Counter arguments appear to converge, justifying experimental approaches based on the classical description.