Gauge-top unification

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based on Pierre Hosteins, RK, M. Ratz, K. Schmidt-Hoberg
arXiv:0905.3323, JHEP 0907:029,2009

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Outline

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- Gauge-top unification in string derived models is interesting
- Consequences can give a chance to exclude models

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- Orbifolds are the 'simplest' extra dimensional setup with chiral spectrum

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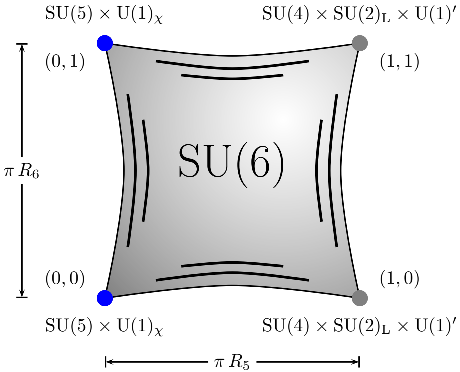

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We get the tree level relation $y_t = g$ (large top Yukawa coupling)

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$$Y_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathcal{O}(g) \end{pmatrix} + \begin{pmatrix} s^{n_{11}} & s^{n_{12}} & s^{n_{13}} \\ s^{n_{21}} & s^{n_{22}} & s^{n_{23}} \\ s^{n_{31}} & s^{n_{32}} & s^{n_{33}} \end{pmatrix}$$

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Topic of this talk!

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- Bulk fields charged under this U(1) get non-trivial profile through the local FI term [Lee, Nilles, Zucker]
- Effect even occurs when the effective FI term in 4D vanishes $\Rightarrow$ Local effect! (compare to F-theory)

Zero mode profile

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- Zero mode profile:

$$\varphi \simeq f \prod_I \left| \vartheta_1 \left(\frac{z - z_I}{2\pi} \middle| \tau \right) \right|^{\frac{1}{2\pi} g_6 q_\varphi \xi_I}$$

$$\exp\left(-\frac{1}{8\pi^2 \tau_2} g_6 q_\varphi \xi_I (\mathrm{Im}(z - z_I))^2 \right)$$

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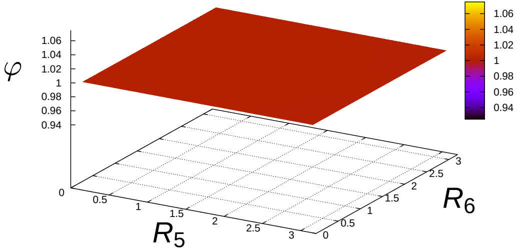
- f is a normalization constant, z the torus coordinate, z_I labels the fixed points, τ is the Teichmüller parameter of the torus and $\vartheta_1(z|\tau)$ the Jacobi ϑ-function
- ξ_I is the FI term:

$$\xi_I = \frac{1}{16\pi^2} g_6 \Lambda^2 \operatorname{tr}(q_I), \quad \Lambda = \text{UV cutoff}$$

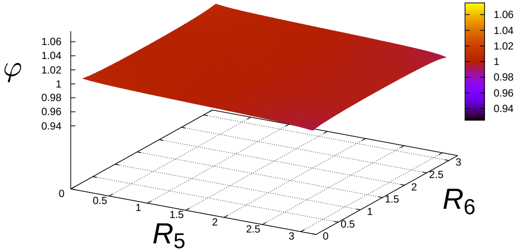
Localization of a zero mode

$$q = 0$$



Localization of a zero mode

$$q = -1 \cdot 10^{-2}$$



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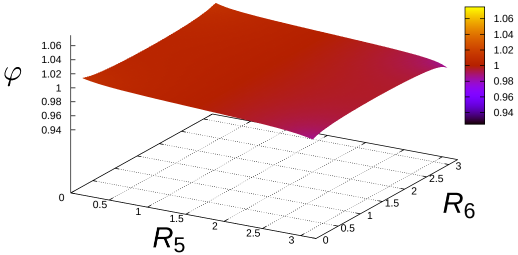

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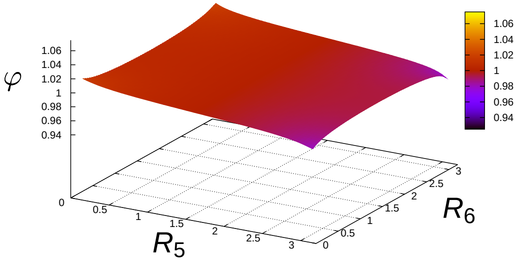
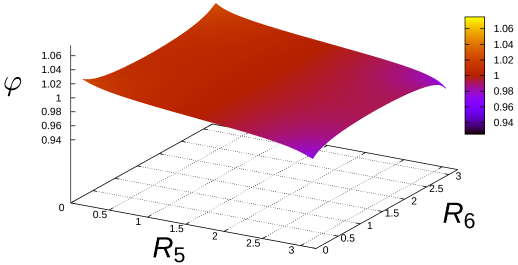


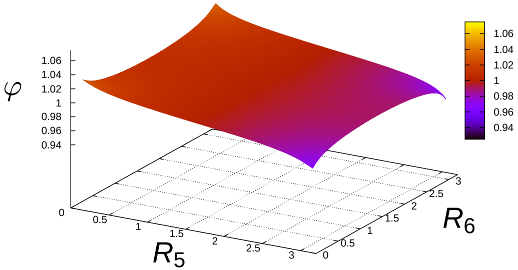
Localization of a zero mode

$$q = -4 \cdot 10^{-2}$$



Localization of a zero mode

$$q = -5 \cdot 10^{-2}$$



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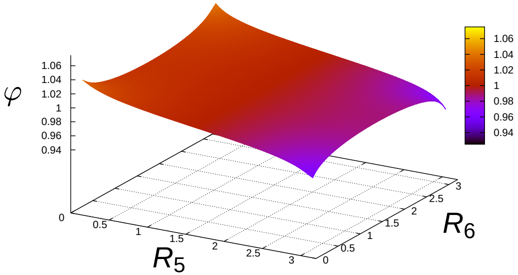
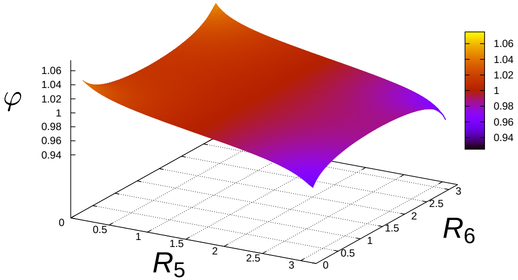


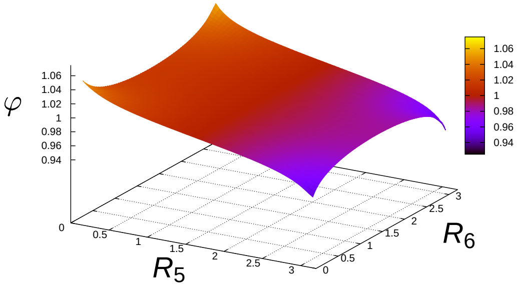
Localization of a zero mode

$$q = -7 \cdot 10^{-2}$$



Localization of a zero mode

$$q = -8 \cdot 10^{-2}$$



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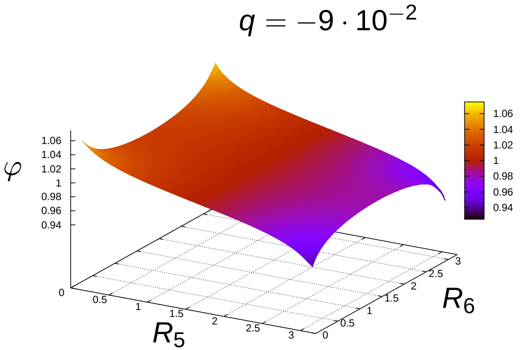

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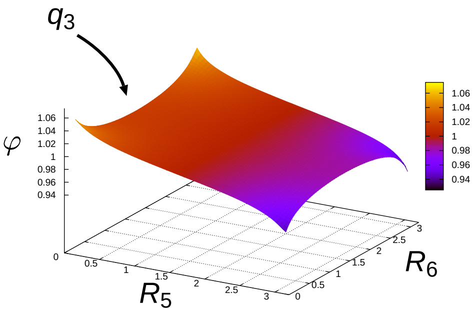

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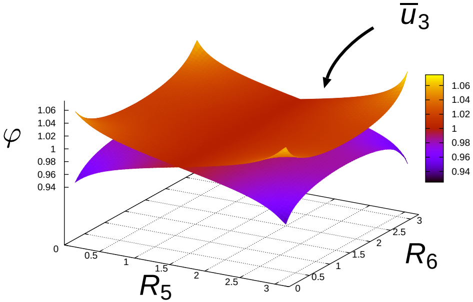

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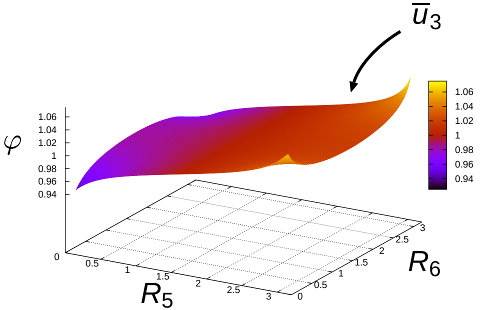

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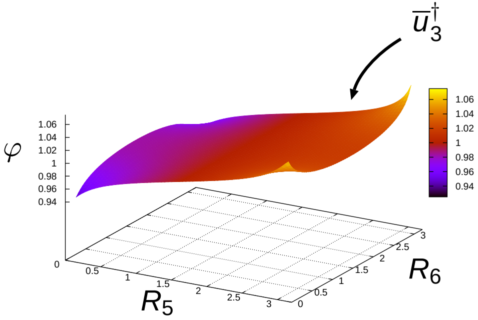

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Overlap integrals differ $\Rightarrow y_t < g$

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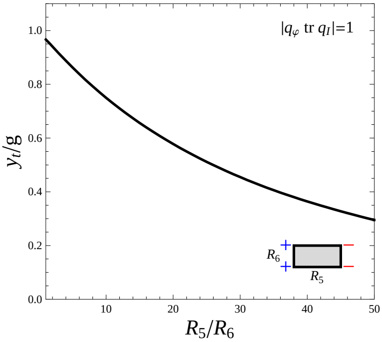

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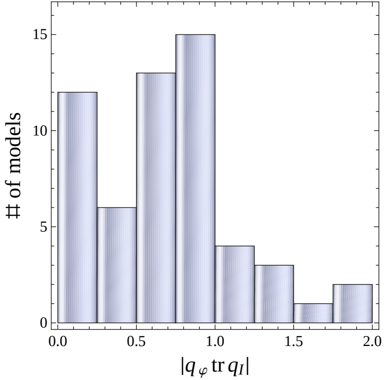
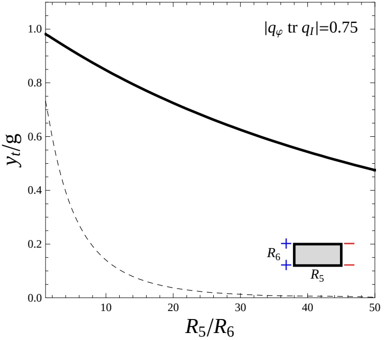
- Similar effects should also occur in non heterotic GUTs

Different models in heterotic string theory

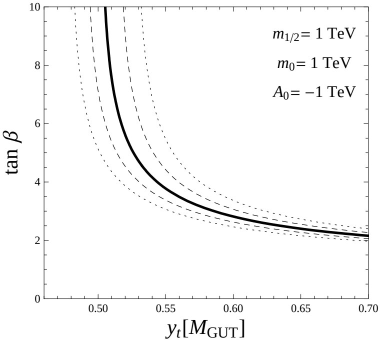


(a) $|q_\varphi \ \mathrm{tr}\, q_I|$ at $I = (0, 0)$.

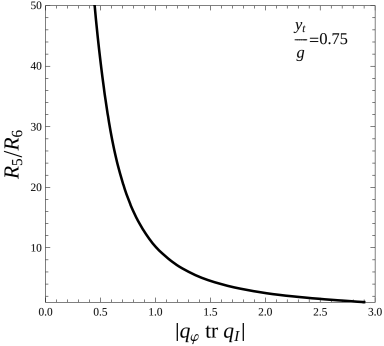


(b) y_t/g.

$\tan\beta$ can be related to the extradimensions



(c) y_t vs. $\tan\beta$.



(d) R_5/R_6 vs. $|q_\varphi \ \mathrm{tr}\, q_I|$.

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This has other important implications $\Rightarrow$ Yukawa pattern, Gauge thresholds, etc.

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Thank you for your attention!