The parameter space of locked inflation and corrections to modulated reheating

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Introduction to Inflation

Model Universe as a cosmological perfect fluid with equation of state $p \approx -\rho$. To preserve the equation of state impose the potential slow-roll conditions:

$$\epsilon \simeq M_p^2 \left(\frac{V'}{V}\right)^2 \ll 1, \qquad \eta \simeq M_p^2 \frac{V''}{V} \ll 1 \tag{1}$$

it follows that for successfull slow-rolling

$$V'' = m^2 \ll H^2 \tag{2}$$

Is it possible to have an inflationary model with $m \geq H$?

Locked Inflation[1]

$$V(\Phi,\phi) = M_\Phi^2\Phi^2 + \lambda\phi^2\Phi^2 + \alpha\left(\phi^2 - M_\star^2\right)^2$$

$$\alpha \sim \frac{M^4}{M_p^4}, \quad M_\Phi^2 \sim \frac{M^4}{M_p^2}, \quad M_\star \sim M_p, \quad \lambda \sim 1, \quad M \sim O(\text{TeV})$$

Vacuum at $\phi = M_\star$, $\Phi = 0$ and saddle point at $\phi = 0$, $\Phi = 0$.

$$\phi_0 = 0, \qquad \Phi_0 \sim M_p \gg \alpha M_\star$$

In this way

$$H_0^2 \simeq \alpha M_\star^4/M_p^2 \sim M_\Phi^2, \quad \Phi(t) \simeq M_p e^{-3/2H_0 t}\cos M_\Phi t$$

But why ϕ does not roll down to the true minimum?

[1]Dvali and S. Kachru, "New old inflation", (2003)

Locked Inflation

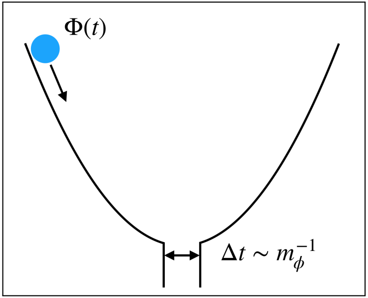
$$V(\phi) = \lambda\phi^2\Phi^2 + \alpha\left(\phi^2 - M_\star^2\right)^2$$

$$m_\phi^2(t) = V'' \simeq \lambda\langle\Phi^2\rangle(t) - \alpha M_\star^2$$

where the substitution $\Phi^2 \to \langle\Phi^2\rangle$ is justified as long as Φ crosses zero fast enough i.e.

$$\Delta t \sim \frac{\Delta\Phi e^{3/2H_0 t}}{\Phi_0 M_\Phi} \leq \frac{1}{\sqrt{\alpha}M_\star}$$

$$\overset{equality}{\Longrightarrow} N \sim -\frac{1}{3}\ln\alpha \sim 50$$



Imposing $m_\phi^2(t_f) \sim 0$ also gives $N \sim 50$

Consistency Problem: Saddle Inflation

At the end of locked inflation there might be another period of slow-roll inflation. Using small angle approximation

$$V(\phi) \simeq \alpha M_\star^4 - \alpha \phi^2 M_\star^2$$

Imposing initial condition $\phi_0 = H_0$ and $\phi_0' = 0$:

$$\phi(N) \approx \frac{H_0}{2\delta}(\delta+1)\exp\left(\frac{3}{2}(\delta-1)N\right), \quad \delta \doteq \sqrt{1+\frac{4}{3}\frac{M_p^2}{M_\star^2}}$$

Inflation will terminate when $\epsilon \doteq \frac{1}{2}\frac{\dot{\phi}^2}{V} \sim 1$ giving $\phi(N_f) \simeq \frac{M_p}{\frac{3}{2}(\delta-1)}$:

$$N_f = \frac{2}{3(\delta-1)} \ln \frac{2M_p\delta}{3(\delta+1)^2 H_0} \leq 1 \implies M_\star \leq 10^{-2} M_p$$

Lower bound on $M_\star$: $T_r \sim \sqrt{H\, M_p} \geq 10^{-2}\text{GeV} \implies M_\star \geq 10^{-5} M_p$

Taking the upper bound changes the number of e-folds by $\Delta N/N \ll 1$

Consistency Problem: one loop correction

Colemann-Weinberg one loop correction:

$$\Delta V \approx \frac{\alpha M_\star^2}{64\pi^2}\Phi^2 \ln\left(\frac{\Phi^2}{\mu^2}\right)$$

Moves Φ's vacuum to $\tilde{\Phi} \neq 0$

$$\tilde{\Phi} > \sqrt{\alpha} M_\star \implies \text{Locked Inflation will last forever}$$

Fix μ via the condition: $V''(\sqrt{\alpha} M_\star) = M_\Phi^2$

Require: $V'(\sqrt{\alpha} M_\star) > 0 \implies M_\Phi^2 \geq 10^{-2}\alpha M_\star^2$

Taking the lower bound gives a negligible correction to the number of e-folds

Consistency Problem: parametric resonances

$$\chi'' + [2q(\tau)(1-\cos(2\tau)) - b]\chi = 0$$

where: $b \doteq \frac{\alpha M_\star^2}{M_\Phi^2} + \frac{9H^2}{4M_\Phi^2} \overset{1Loop}{\leq} 100, \quad \tau \doteq M_\Phi t, \quad q(\tau) \doteq \frac{\Phi_0^2}{4M_\Phi^2} e^{-3\frac{H}{M_\Phi}\tau}$

$$\chi \doteq e^{\frac{3}{2}Ht}\phi$$

Mathieu's equation has a general solution of the form

$$\chi(\tau) = f(\tau)e^{s\tau} \implies \phi(t) \sim e^{\left(s - \frac{3H}{2M_\Phi}\right)\tau}$$

where $s \sim 0.1$ implying $H \geq 0.07 M_\Phi$

- Numerical study, but only to first order in: Copeland, Rajantie "The end of locked inflation", (2005)

To avoid constraint Φ could be a complex scalar field

Production of topological defects

Due to spontaneous symmetry breaking at the end of inflation topological defects might be formed. We can not use temperature to find the correlation length ξ. Let us be guided by non adiabaticity

$$\frac{|\dot{\omega}_k|}{\omega^2}\Big|_{t=t_c} \geq 1, \quad \omega_k(t)^2 = k^2 + \langle \Phi^2 \rangle(t) - \alpha M_\star^2$$

Implying $\quad k_{crit}^3 \sim \alpha H M_\star^2 \sim \xi^{-3} \sim 10^{-30}\text{GeV}^3$

The number density of produced topological defects turns out to be negligible:

$$n \sim k_{crit}^3 \sim 10^{-30}\text{GeV}^3$$

Due to the low inflationary scale $O(10^{-3}\,\text{eV})$ topological defects are practically zero.

Modulated Reheating

Dvali, Gruzinov, Zaldarriaga, "A new mechanism for generating density perturbations from inflation", (2003)

Consider an effective Yukawa-like coupling with a Λ cutoff and a modulating field S

$$\lambda\bar{\psi}\phi\psi, \qquad \lambda = \lambda(S) = \lambda_0\left(1 + \frac{S}{\Lambda} + \ldots\right)$$

	Universe 1	Universe 2
Coupling	λ_1	λ_2
Decay rate	$\Gamma_1 \sim \lambda_1^2 M_\star \sim H_{in}$	$\Gamma_2 \sim \lambda_2^2 M_\star \sim H_{fin}$
Evolution	$\rho_1(t_f) = \frac{\rho_0}{a^4}$	$\rho_2(t_f) = \frac{\rho_0}{a^3}$

$$\rho_1(t_f) = \left(\frac{\lambda_2}{\lambda_1}\right)^{4/3}\rho_2(t_f) \implies \frac{\delta\rho}{\rho} \sim \frac{\delta\lambda}{\lambda} \sim \frac{\delta\Gamma}{\Gamma}$$

Necessary Assumption: $M_S \ll H$ during inflation
General Prediction: $f_{NL} \sim O(1)$

Non Gaussianities in modulated reheating

$$ds^2 = (1+2\phi)dt^2 + 2(B_{,i} + S_i)dx^i dt - a^2 e^{-2\psi}(e^h)_{ij} dx^i dx^j$$

$$(e^h)_{ij} \doteq \delta_{ij} + 2E_{,ij} + F_{i,j} + F_{j,i} + h_{ij}$$

Fix $F_i = E = 0$. Consider the two following gauges

$$\psi = 0 \doteq \text{Flat Gauge}, \qquad \delta\rho = 0 \doteq \text{Uniform Energy Gauge}$$

Under the shift: $\tau \to T = \tau + \delta\tau$, $x^i \to X^i = x^i + \xi^i$

$$a^2 e^{-2\psi}(e^h)_{ij}(\tau, \vec{x}) \simeq a^2 e^{-2\bar{\psi}}(e^{\bar{h}})_{ij}(T, \vec{X})$$

$$\psi|_{\delta\rho=E=F_i=0}(T, \vec{X}) = -\ln\frac{a(T)}{a(\tau)}$$

$$\boxed{\zeta \doteq -\psi|_{\delta\rho=E=F_i=0} \doteq N - \hat{N} \doteq \delta N}$$

Non Gaussianities in modulated reheating

$$\zeta = \delta N \overset{SR}{\simeq} \frac{\partial N}{\partial S}\delta S + \frac{1}{2}\frac{\partial^2 S}{\partial S^2}\delta S^2 + ... - \hat{N}$$

Expand δS in modes and compute non gaussianities:

$$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = \left(\frac{\partial N}{\partial S}\right)^2 \frac{\partial^2 N}{\partial S^2}\left(\langle \delta S_{\vec{k}_1}\delta S_{\vec{k}_2}\delta S_{\vec{k}_3}\delta S_{\vec{k}_3}\rangle + \text{Perms}\right)$$

$$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle \doteq (2\pi)^3\delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)B_\zeta(k_1, k_2, k_3)$$

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5}f_{NL}\left(P_1P_2 + 2\text{Perms}\right)$$

Matching $\quad f_{NL} = \frac{5}{6}\frac{\partial^2 N}{\partial S^2}\left(\frac{\partial N}{\partial S}\right)^{-2}$

Non Gaussianities in modulated reheating

$$f_{NL} = \frac{5}{6}\frac{\partial^2 N}{\partial S^2}\left(\frac{\partial N}{\partial S}\right)^{-2}$$

$$N \supset \int_{a_f}^{a_r} d\ln a + \int_{a_r}^{a_m} d\ln a = -\frac{1}{3}\ln\rho_r + \frac{1}{4}\ln\rho_r \overset{\rho\propto\Gamma^2}{=} -\frac{1}{6}\ln\Gamma(S(t_r))$$

$$f_{NL} = 5\left(1 - \frac{\Gamma''\Gamma}{\Gamma'^2} - \frac{\Gamma}{\Gamma'}\frac{\partial^2 S_r}{\partial S_m^2}\left(\frac{\partial S_r}{\partial S_m}\right)^{-2}\right)$$

$$\xrightarrow{\text{Static S}} f_{NL} = 5\left(1 - \frac{\Gamma''\Gamma}{\Gamma'^2}\right) = \frac{5}{2} \sim O(1)$$

where $\lambda = \frac{S}{\Lambda}$ and $\Gamma \propto \lambda^2$ have been used

Conclusion and outlook

- Parameter space: $M_\star \leq 10^{-2} M_p$ and $H_0 \leq M_\Phi \leq 10\, H_0$
- $f_{NL} \sim O(1)$ with enhancement/suppression due to the evolution of the modulating field
- Sensibility to current experiments
- Numerical study of parametric resonances needed