Relics from the early universe

We know there is strong evidence from cosmological observations that there is not enough known matter/energy in the universe to explain all observations

- Dynamics of individual galaxy types (rotation curves, velocity dispersions),
- Haloes of galaxies: formation of galaxies by gravitational collapse can only be explained for overall mass content bigger than Jeans mass (not consistent with Ω_b only)!
- Binary galaxies: dynamics and tidal effects,
- Galaxy groups: dynamics of galaxies,
- Gravitational lensing of background galaxies by a cluster (strong lensing),
- Deformation of typical galaxy shapes by foreground cluster galaxies (weak lensing),
- Measurement of temperature of intergalactic gas assuming that gas is bound,
- Structure formation from CMB anisotropies only explainable if there is additional non baryonic matter.

Explain all observations
→ Assume existence of other type(s) of weakly interacting particles not contained in the standard model of particle physics.
If these *decouple or are produced non-thermally in the early universe* and are *still present today* as (**thermal) relic** of the early universe
→ Could fulfill the requirements to explain the missing matter problem.

1. Freeze out of particles:

How does an expanding universe behave evolve as a function of its particle content
→ need to understand the microscopic behavior of individual components with respect to statistical mechanics for a significant number of particles/quanta

→ Quantum mechanics
→ Thermodynamics
→ Cosmology

The description of a system is simplest if it is in **thermal equilibrium**. This is in general the case if all reactions that the particles in equilibrium undergo go both directions with the same rate and if the reaction rates are high enough.

The history of the universe can be described as evolution through different phases in which different contents were in thermal equilibrium. The different phases are connected by phase transitions, where individual particle species, i.e. part of the content left equilibrium.

→ Universe can be described by thermo-dynamical parameters:

T – Temperature s – Entropy density ρ – energy density g – ndf , etc.

In **thermal equilibrium** (Each process occurs in both directions with same rate): "Dilute, weakly interacting gas approximation

[note: natural units in the following, incl. k_B]

→ Distribution of particle species *i* follows:

$$f_i(\mathbf{p}) = \left[\exp\left(\frac{E_i-\mu_i}{T} \pm 1\right)\right]^{-1},$$

Where $+1$ is valid for Fermi-Dirac and -1 for Bose-Einstein statistics and μ_i chemical potential, $E_i = \sqrt{\mathbf{p}^2 + m_i^2}$

For number density of particle *i*:

$$n_i = \frac{g_i}{(2\pi)^3} \int f_i(\mathbf{p}) d^3 p$$

$$\text{With } d^3p \rightarrow 4\pi \sqrt{E^2 - m_i^2} E dE$$

→ For approximations of non-relativistic ($T/m \ll 1$) particles relation between Temperature and number/Energy density can be obtained – both for Bose Einstein and Dirac particles:

$$n_{NonRel} = g_i \left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{\boldsymbol{m}}{\boldsymbol{T}}}$$

→ For approximations of relativistic ($T/m \gg 1$):

$$n_{Rel} \propto g_i \boldsymbol{T}^{\mathbf{3}}$$

Total energy density in universe as function of T:

Non-relativistic particles: exponentially suppressed
→ Sum over relativistic particles in equilibrium at given T (Stefan Boltzmann law):

$$\rho_{Rel} \propto g_{eff}(T) T^4$$

To get g_{eff} (total number of effectively massless degrees of freedom, species with $m_i \ll T$): Need to introduce specific temperature for each particle T_i (difference if not in thermal equilibrium with other particle species!)

$$g_{eff} = \sum_{i=bosons} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{j=ferminons} g_j \left(\frac{T_j}{T}\right)^4$$

Understand which particles are in thermal equilibrium at given T:
→ use entropy density: "Number of possible microstates in given volume (use co-moving!) equivalently describing a given thermodynamical/ quantum-mechanical macro-state.
→ Very important: (effective) Number of degrees of freedom of involved partices!

From definition of entropy:

$$dS(V,T) = \frac{1}{T}[d(\rho(T)V) + p(T)dV]$$

Together with FLRW metric (remember: expanding universe) it can be shown: entropy per co-moving volume is conserved:

$$a^3 \frac{dp}{dt} = \frac{d}{dt}[a^3(\rho + p)]$$

The entropy density

$$s(T) \equiv \frac{S(V,T)}{V} = \frac{\rho(T) + p(T)}{T}$$

can be shown to be given by (using equation of state for radiation dominated universe $p = \frac{\rho}{3}$ and expression for relativistic energy density and pressure):

$$s = \frac{2\pi^2}{45} g_{eff}^s \boldsymbol{T^3}$$

where

$$g_{eff}^s = \sum_{i=bosons} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{j=ferminons} g_j \left(\frac{T_j}{T}\right)^3 \qquad (2)$$

Remember number density $\propto \boldsymbol{T^3}$
→ Relation: $n_\gamma \propto s$

For phase transitions the Boltzmann Transport Equations (BTE)

$$\hat{\boldsymbol{L}}[f] = \boldsymbol{C}[f]$$

have to be solved, with $\hat{\boldsymbol{L}}$ the Liousville operator for phase space density and $\hat{\boldsymbol{C}}$ the collision operator containing all possible reactions any particle can perform. This gives the evolution of the particle's phase space functions $f_i = \left[\exp(\frac{E-\mu}{T} \pm 1)\right]^{-1}$,

With the number density $n_i(T)$and using the representation of the Liousville operator in the FLRW metric the Boltzmann equation can be written as

(5)

$$\dot{n}_i + 3\frac{\dot{R}}{R}n_i = \frac{g_i}{(2\pi)^3}\int \boldsymbol{C}[f]\frac{d^2p}{E}$$

where g_i describe the number of internal degrees of freedom of specific particle species i.

General case: this is a coupled set of integral-differential equations for the phase space distributions of all particle species i under consideration!

We are interested in the particle densities n_p as a function of time while leaving thermal equilibrium. All other content/particles are in the following assumed to be in thermal equilibrium.

In order to scale out the effect of expansion of the universe i.e. to look at the evolution of the number of particles in a co-moving volume, the particle density is normalized to entropy:

$$Y \equiv \frac{n_p}{s}. \tag{7}$$

If one takes into account that entropy is conserved in a co-moving volume, one can show that

$$\dot{n}_p + 3Hn_p = s\dot{Y}. \tag{8}$$

As interaction terms between particles will explicitly depend on temperature T rather than on time, it is also useful to introduce the variable $x = \frac{m}{T}$ with m an arbitrary energy scale but usually chosen as the mass of the particle under consideration. Note that during the evolution of the universe x has the same direction as time.

The Boltzmann equation can then be simplified by writing it in terms of

$$\frac{dY}{dx} = -\frac{m\, m_{pl}c_{eff}}{x^2}\cdot C(g_i, f_i, M^2), \tag{9}$$

with m_{pl} the Planck mass and $C(g, f_i, M^2)$, the collision term as a function the g_i for all particle species involved and M^2 the matrix element for the (set of) reaction(s) under consideration. Note that the collision term is a function of x.

Let's now consider a reaction of a stable particle p. Only Annihilation and creation can occur $p\bar{p} \leftrightarrow X\bar{X}$. Here X denotes all possible states the particle can decay to. These are assumed to stay in thermal equilibrium during the whole process (almost always a good assumption) and to have vanishing chemical potential μ.

Making use of the fact that all particles represented with X stay in good approximation in thermal equilibrium, the collision operator can be simplified and one can write:

$$\frac{dY}{dx} = -\frac{m\, m_{pl}c_{eff}}{x^2}\langle\sigma_{p\bar{p}\to X\bar{X}}|v|\rangle s\left(Y^2 - Y_{eq}^2\right)$$

or

$$\dot{n}_p + 3Hn_p = -\langle\sigma_{p\bar{p}\to X\bar{X}}|v|\rangle\left[n_p^2 - \left(n_p^{eq}\right)^2\right],$$

where $\langle\sigma_{p\bar{p}\to X\bar{X}}|v|\rangle$ describes the thermally averaged annihilation cross-section for all available channels times the velocity and $Y_{eq} = n_p^{eq}/s$ is the equilibrium number of particles n_p per co-moving volume.

This is intuitive:
The term $\dot{n}_p$ is amended by the term $3Hn_p$, taking care of the dilution of the particle species due to expansion of the universe. (12)
The right hand side can be interpreted as:
In thermal equilibrium $n_p = n_p^{eq}$, meaning that all reactions go both directions with the same rate.
If the particle density n_p is increasing, the rate $p\bar{p} \to X\bar{X}$ of disappearing particles will be given by $n_p^2\langle\sigma_{p\bar{p}\to X\bar{X}}|v|\rangle$, corrected for the number of particles created in equilibrium $X\bar{X} \to p\bar{p}$. The square in the density appears since $n_p = n_{\bar{p}}$.
After some rearrangement:

$$\frac{x}{Y_{eq}}\frac{dY}{dx} = -\frac{\Gamma_A}{H}\left[\left(\frac{Y}{Y_{eq}}\right)^2 - 1\right],$$

to describe the change of n_p per co-moving volume with decreasing T (increasing time and x). Here $\Gamma_A = n_p^{eq}\langle\sigma_{p\bar{p}\to X\bar{X}}|v|\rangle$ is the annihilation rate for the particle species under consideration. The expression is controlled by the "effectiveness of annihilations", given by the ratio $\frac{\Gamma_A}{H}$ times a measure for deviation from thermal equilibrium.

We now have to distinguish between particles that are relativistic and non-relativistic during deviation from thermal equilibrium. remember:

$$n_{non-rel} = g_i\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m}{T}} \tag{13}$$

for the non-relativistic species (i.e. $x \ll 3$) and

$$n_{rel} \propto T^3 \tag{14}$$

For relativistic particle species (i.e. $x \gg 3$).
For both cases Γ_A decreases as T decreases, exponentially for non-relativistic particles, as some power of T in the relativistic case. This means that annihilations at some value x_f becomes ineffective, roughly once $\Gamma_A \cong H$. Such that for $x \lesssim x_f$ we get $Y \cong Y_{eq}$.

<u>Freeze out of relativistic particles: Limit on the neutrino mass</u>

For relativistic particle ν: Freeze out occurs at $x_f \lesssim 3$. (f for frezze out)

We see that

$$Y_{eq,rel} = \frac{n_{eq,rel}}{s} \propto \frac{T^3}{s} \propto \frac{T^3}{T^3} = const.$$

is temperature independent, hence not changing in time as long as the particle is relativistic. Hence, the asymptotic value $Y(x \to \infty) \equiv Y_\infty$ is the equilibrium value at freeze out.
For Y_∞ we find:

$$Y_\infty = Y_{eq}(x_f) = \frac{n_{eq}(x_f)}{s(x_f)} = \frac{g_{\nu,eff}}{g^s_{eff}(x_f)} \cdot const.$$

Using this expression, assuming constant entropy per co-moving volume, the abundance of a particle today can then be calculated

$$n_{p,0} = s_0 Y_\infty = 2970\ Y_\infty cm^{-3} = 825 \frac{g_{\nu,eff}}{g^s_{eff}(x_f)} cm^{-3}$$

(s_0: present entropy density)
For the present relic mass density $\rho_{p,0}$this translates into

$$\rho_{p,0} = s_0 Y_\infty m = 2970\ Y_\infty \left(\frac{m}{eV}\right) eV\ cm^{-3}$$

or in terms of critical density

$$\Omega_p h^2 = 0.078 \frac{g_{\nu,eff}}{g^s_{eff}(x_f)} \left(\frac{m}{eV}\right). \quad (20)$$

This can be used to derive an upper limit on the mass density due to known neutrino species. We know $\Omega_0 h^2 \lesssim 1$. Using the decoupling temperature of neutrinos $T\sim$ few MeV and effective number of degrees of freedom for neutrinos this implies:

$$\Omega_{\nu\bar{\nu}} h^2 = \frac{m_\nu}{91.5\ eV} \quad (21)$$

or

$$\sum m_{\nu\bar{\nu}} \lesssim 91.5\ eV. \quad (22)$$

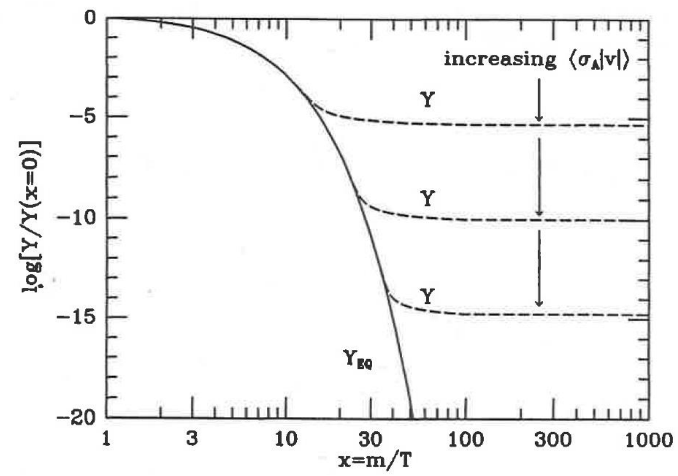
Note that this solution is only very mildly dependent on the exact process of freeze out, due to the flatness of Y_{eq} as a function of T for $x_f \lesssim 3$.

<u>Freeze out of non-relativistic particles, cold relics:</u>

For non-relativistic freeze out the situation is more complicated and solutions to the equations above have to be found numerically as Y_{eq} is decreasing exponentially with decreasing temperature (see Figure below).

Note that the higher the thermally averaged cross section times velocity of particles $\langle\sigma_{p\bar{p}\to X\bar{X}}|v|\rangle$, the lower the relic abundance will be. This is because deviation from equilibrium will happen for a higher x, where Boltzmann suppression of Y_{eq} becomes more and more relevant.

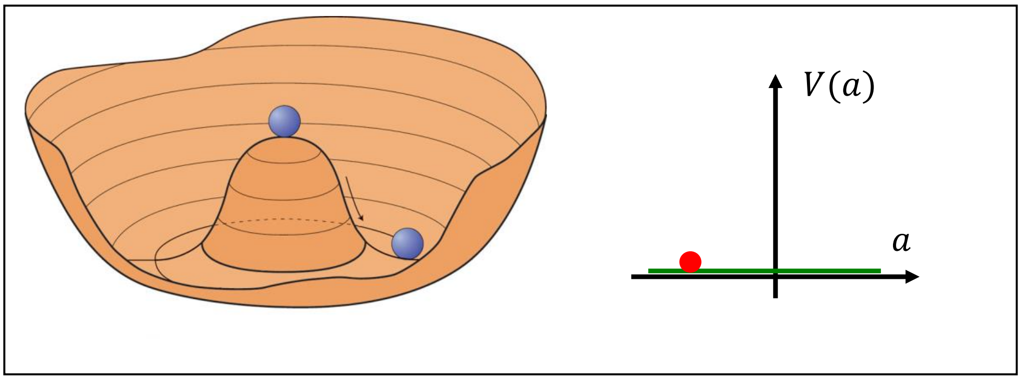
It turns out that for a $\langle\sigma_{p\bar{p}\to X\bar{X}}|v|\rangle$ characteristic for the weak interaction the relic abundance for WIMPs with mass $10 - 1000\ \mathrm{GeV}$ is approximately $\Omega_{p\bar{p}}h^2 \sim 1$. This is sometimes called the **WIMP miracle**.



Freeze out of massive particles. Taken from E.Kolb and S. Turner, The early universe

1.Non-thermal production of cold dark matter particles (axions – ALPs):

Considers spontaneous symmetry breaking:
broken complex scalar U(1) symmetry (remember Higgs?)



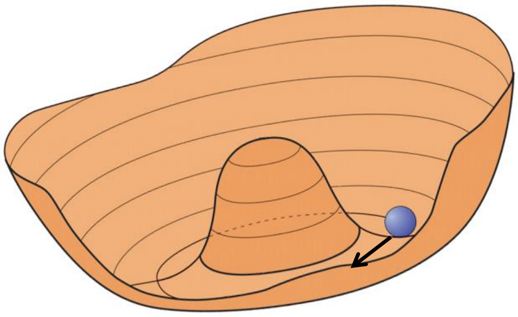
If energy density low enough: Field relaxes into minimum of potential. Due to complex field: Phase θ is arbitrary!

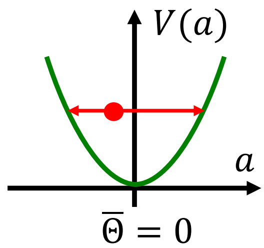
In QFT corresponds to introduction of two particles:

1) "Movement" around potential trough (massless)
2) Along same θ (massive)

Excitation of field → particle with mass (second derivative at minimum of potential)

If Mexican hat potential tilted, for example by non-perturbative QCD effects:





→ Field is non symmetric about Phase θ → generation of mass

Mass suppressed by distance between origin of field and minimum of trough: Energy scale of symmetry breaking!

→ For very high E → very small mass →WISPs (axions, ALPs)

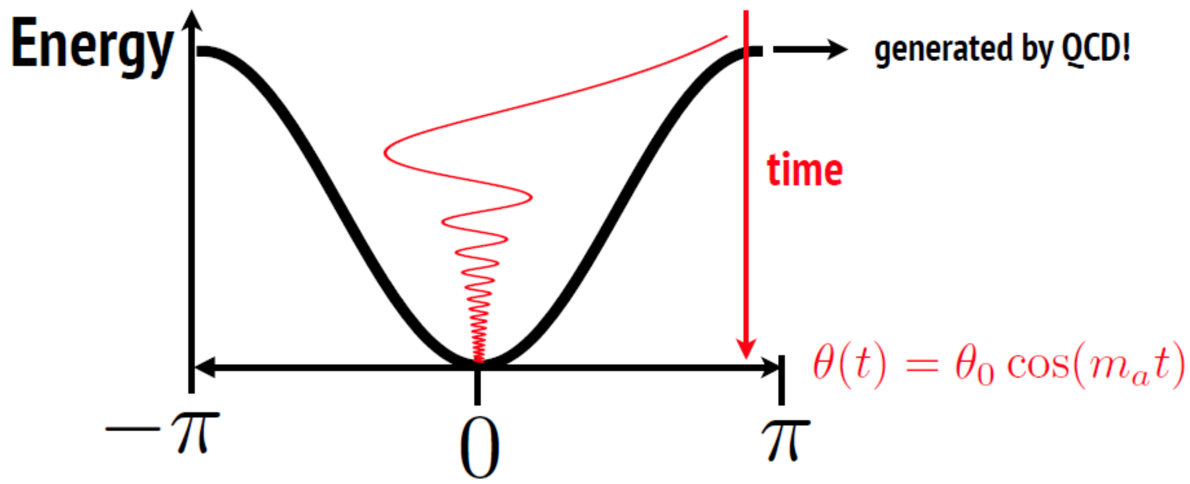
Movement around minimum is "frictionless"

→relic oscillations expected!

Remember expansion of universe → Oscillations are damped by now (very small)

Note: NON-THERMAL Production of local field oscillations,
i.e. particle population without initial momentum: NON RELATIVISTIC!

Number density depends on initial alignment of θ after symmetry breaking.


Energy
generated by QCD!
time
$\theta(t) = \theta_0 \cos(m_a t)$
$-\pi$
0
π