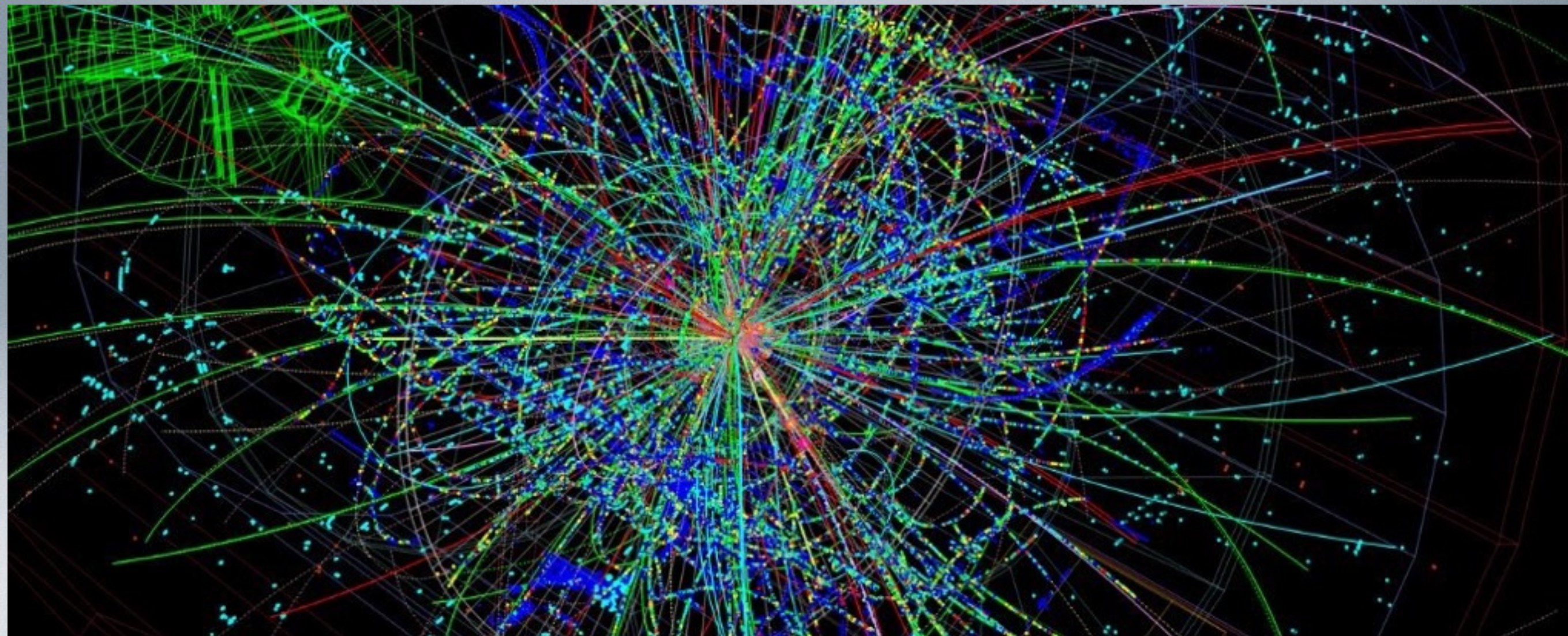


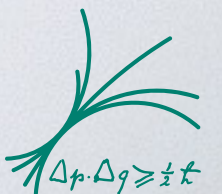
Pushing the precision frontier in Collider Physics

Gudrun Heinrich

Max Planck Institute for Physics, Munich



IMPRS Young Scientists workshop
Castle Ringberg, December 2018



Outline

- Motivation: Why to push the precision further?
- Basic building blocks of precision calculations
- Overview of recent developments:
methods and phenomenology



Motivation

if the Standard Model is an effective theory
valid up to a scale Λ

$$\text{data} = \text{SM} + \mathcal{O}(v^2/\Lambda^2) \simeq \text{SM} + \mathcal{O}(6\% \times 1\text{TeV}^2/\Lambda^2)$$

so New Physics at the TeV scale is not excluded yet
need to reach this level of precision!



Status of the Standard Model today

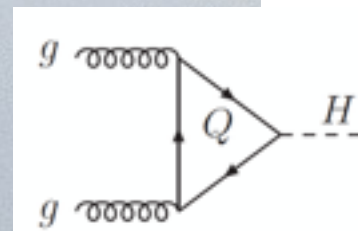
ATLAS-CONF-2018-031

Higgs production and decay

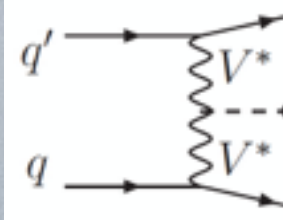
most uncertainties
not below 10% level

statistics will improve

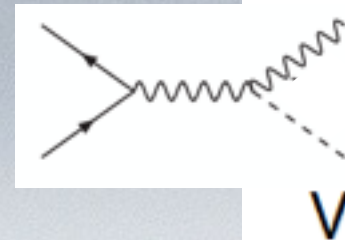
systematic uncertainty
contains
theory uncertainty!



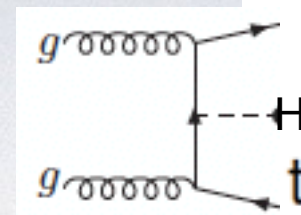
ggF



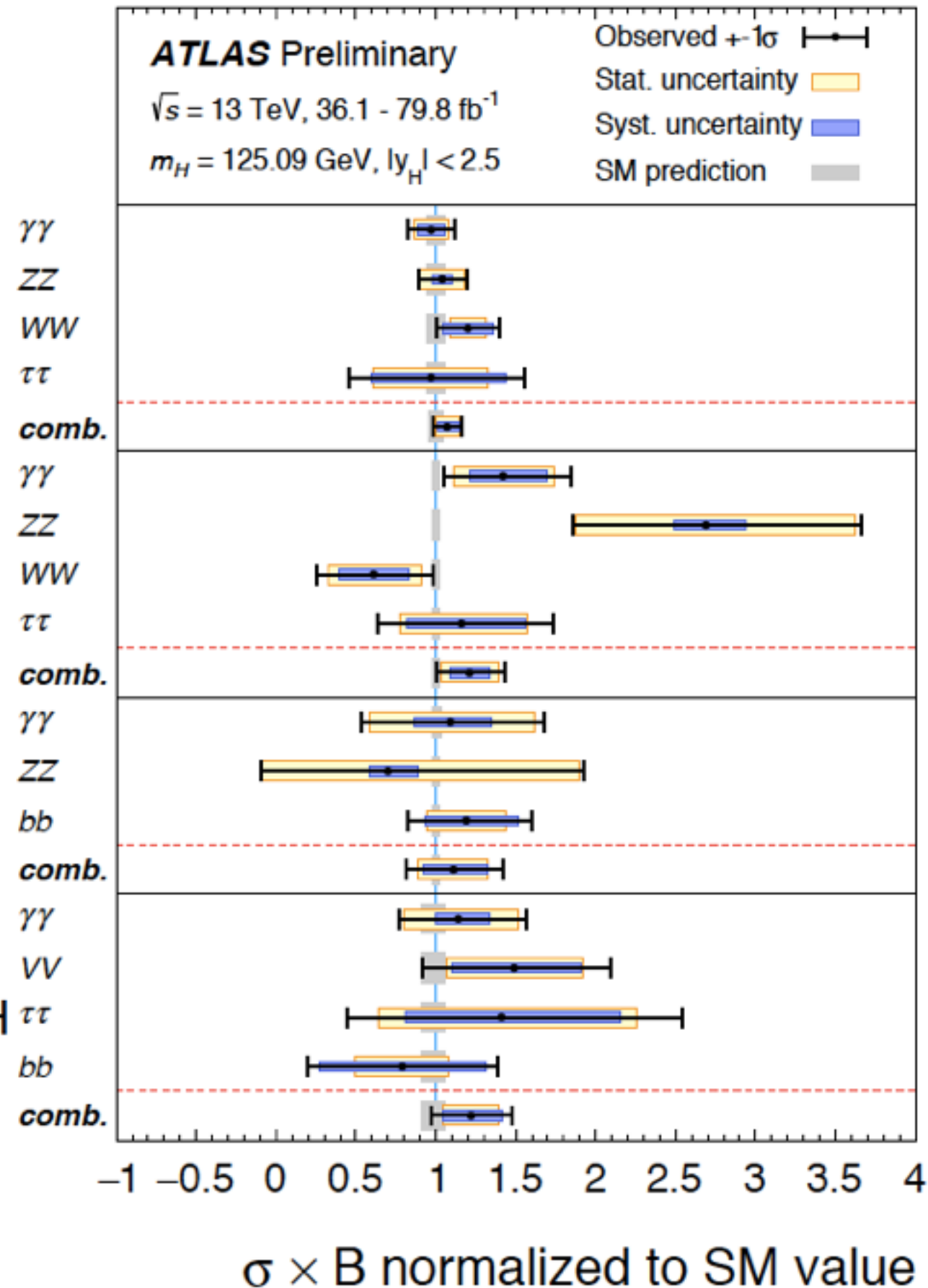
VBF



VH



ttH+tH



Status of the Standard Model today

July 2018

CMS Preliminary

CMS measurements
vs. NNLO (NLO) theory

7 TeV CMS measurement (stat,stat+sys)

8 TeV CMS measurement (stat,stat+sys)

13 TeV CMS measurement (stat,stat+sys)

$\gamma\gamma$

$W\gamma$, (NLO th.)

$Z\gamma$, (NLO th.)

$Z\gamma$, (NLO th.)

$WW+WZ$

WW

WW

WW

WZ

WZ

WZ

ZZ

ZZ

ZZ

$1.06 \pm 0.01 \pm 0.12$

$1.16 \pm 0.03 \pm 0.13$

$0.98 \pm 0.01 \pm 0.05$

$0.98 \pm 0.01 \pm 0.05$

$1.01 \pm 0.13 \pm 0.14$

$1.07 \pm 0.04 \pm 0.09$

$1.00 \pm 0.02 \pm 0.08$

$0.96 \pm 0.05 \pm 0.08$

$1.05 \pm 0.07 \pm 0.06$

$1.02 \pm 0.04 \pm 0.07$

$0.96 \pm 0.02 \pm 0.05$

$0.97 \pm 0.13 \pm 0.07$

$0.97 \pm 0.06 \pm 0.08$

$1.14 \pm 0.04 \pm 0.05$

5.0 fb^{-1}

5.0 fb^{-1}

5.0 fb^{-1}

19.5 fb^{-1}

4.9 fb^{-1}

4.9 fb^{-1}

19.4 fb^{-1}

2.3 fb^{-1}

4.9 fb^{-1}

19.6 fb^{-1}

35.9 fb^{-1}

4.9 fb^{-1}

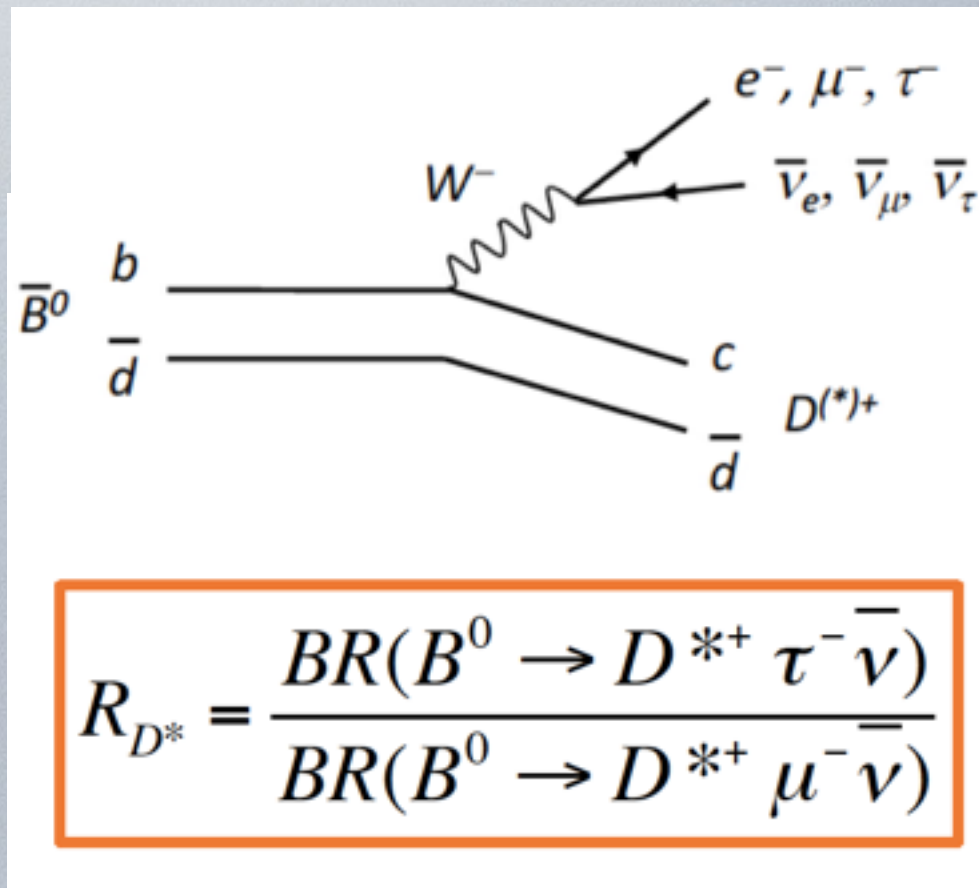
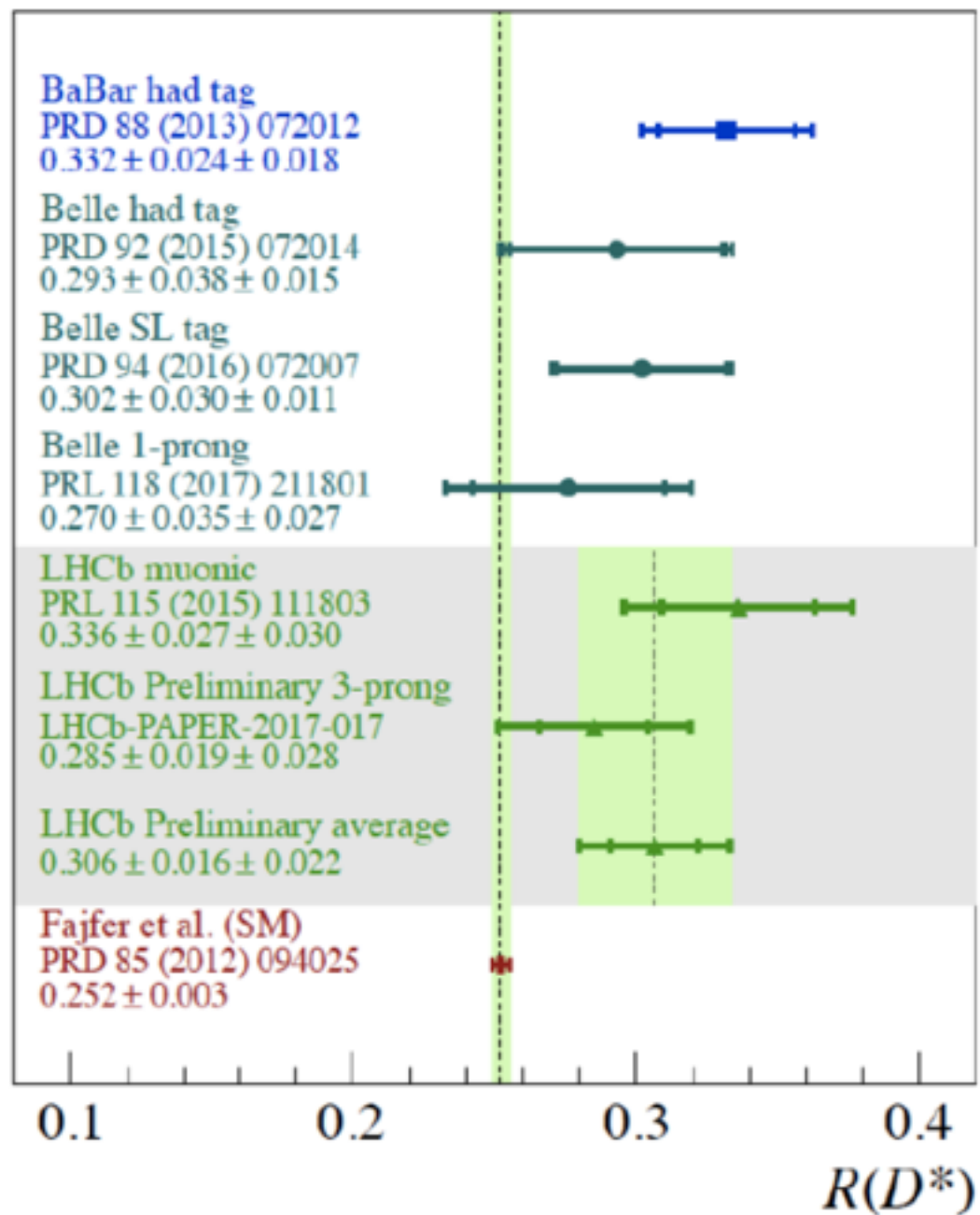
19.6 fb^{-1}

35.9 fb^{-1}

Production Cross Section Ratio: $\sigma_{\text{exp}} / \sigma_{\text{theo}}$

All results at:
<http://cern.ch/go/pNj7>

some “anomalies”

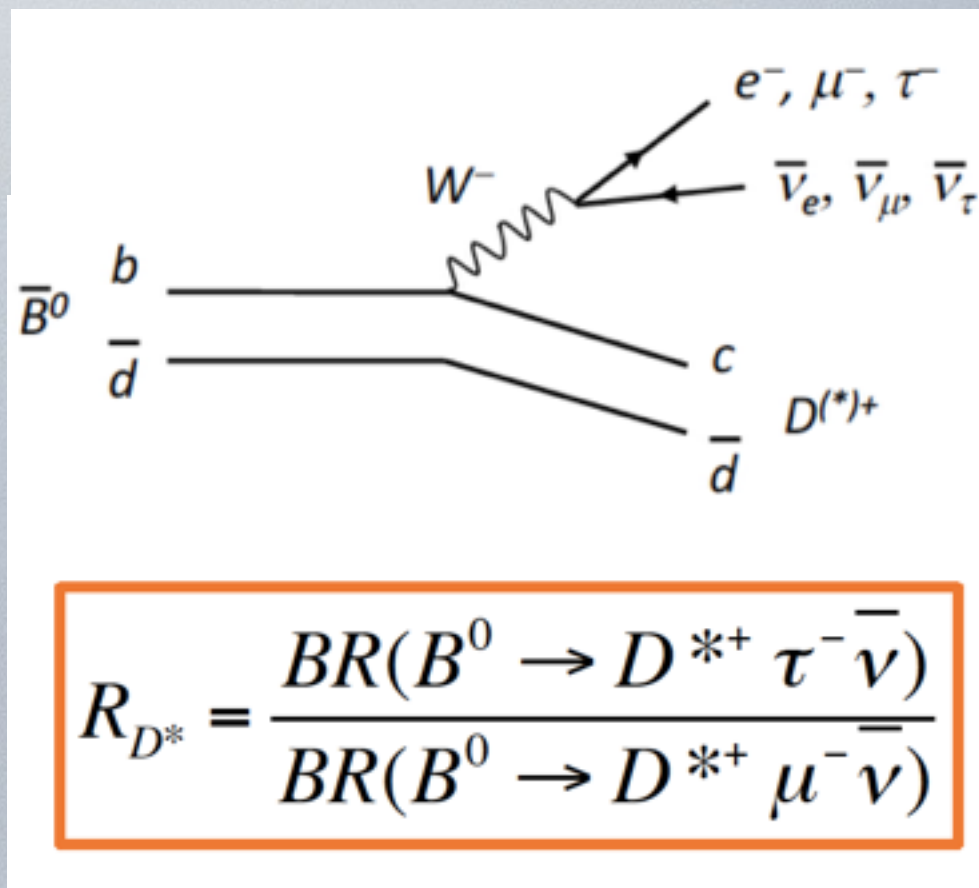
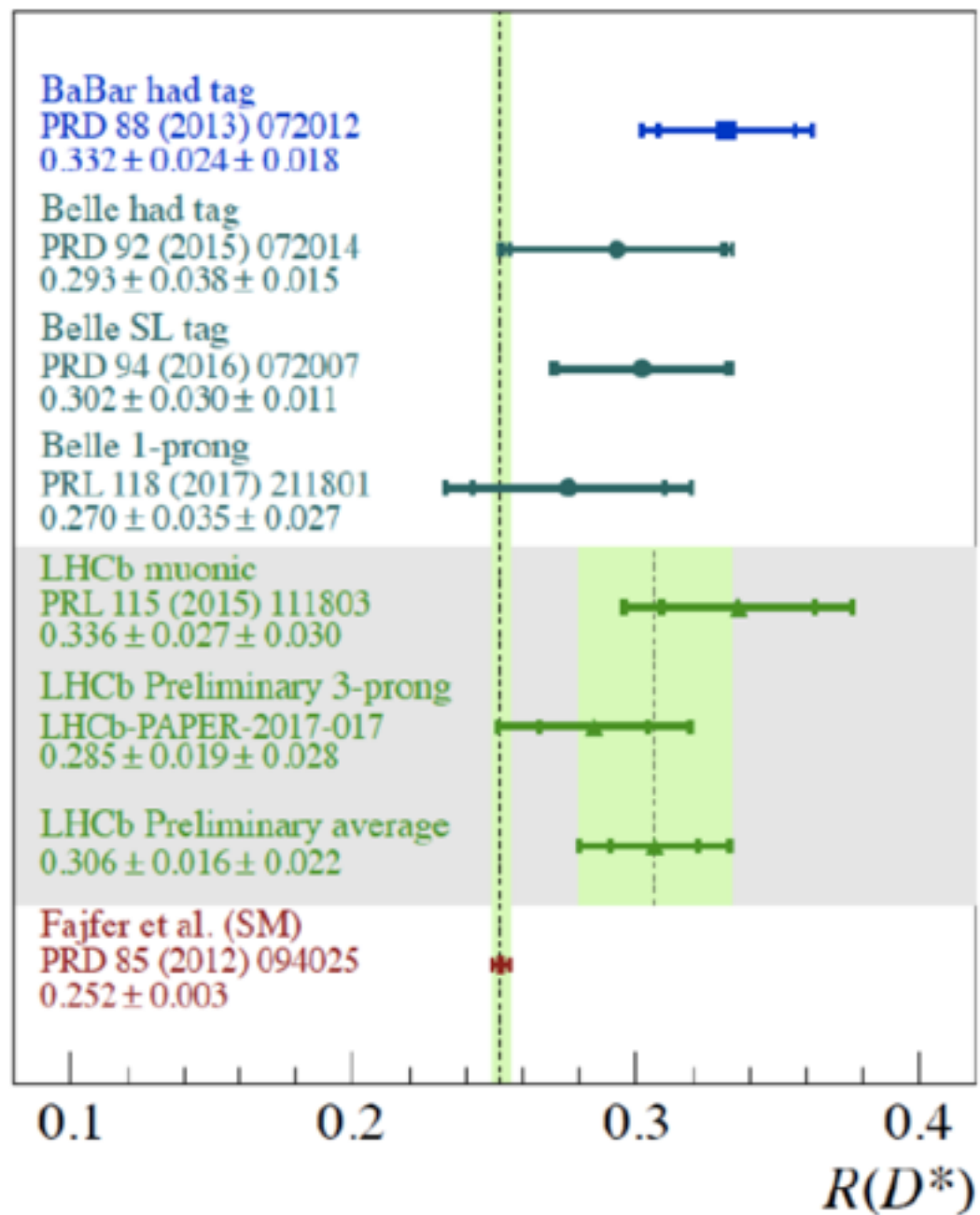


combination of R(D) and R(D*) about 4 sigma deviation from SM prediction

lepton flavour universality
violated?

J.Albrecht, Moriond 2018

some “anomalies”



combination of $R(D)$ and $R(D^*)$ about 4 sigma deviation from SM prediction

lepton flavour universality
violated?

theory predictions
very well under control

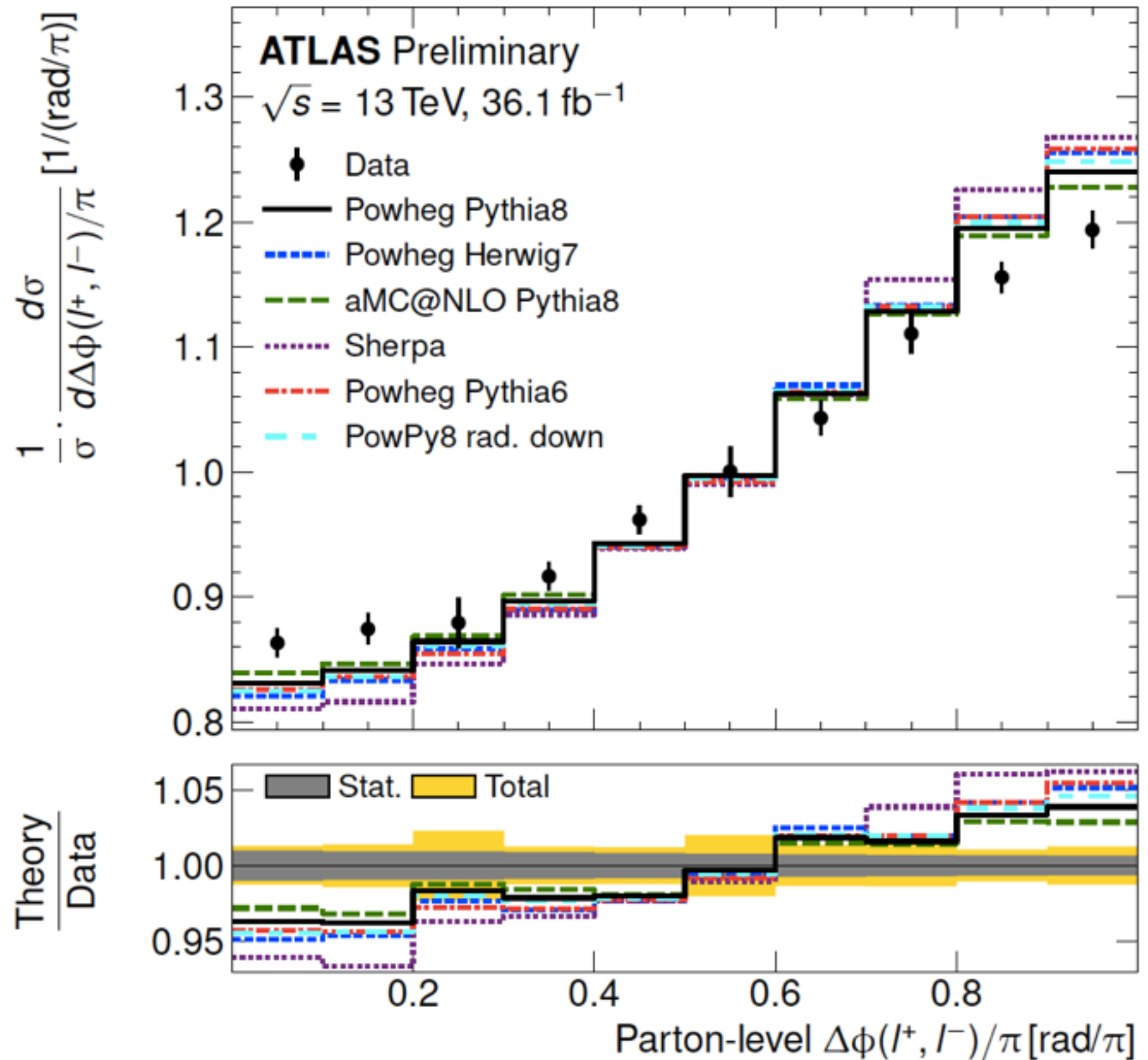
J.Albrecht, Moriond 2018

some “anomalies”

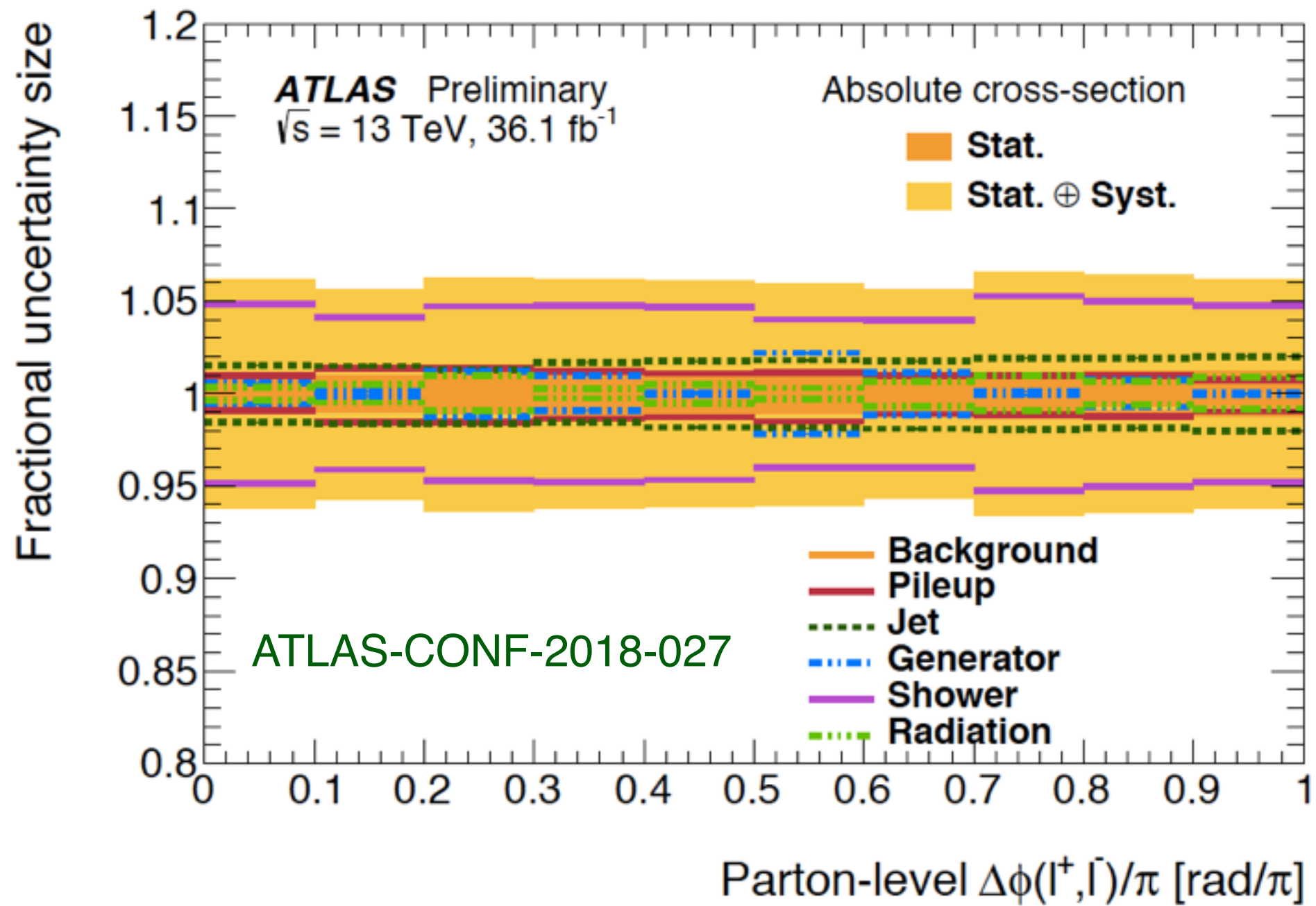
ATLAS-CONF-2018-027

spin correlations in
top pair production
with leptonic decays

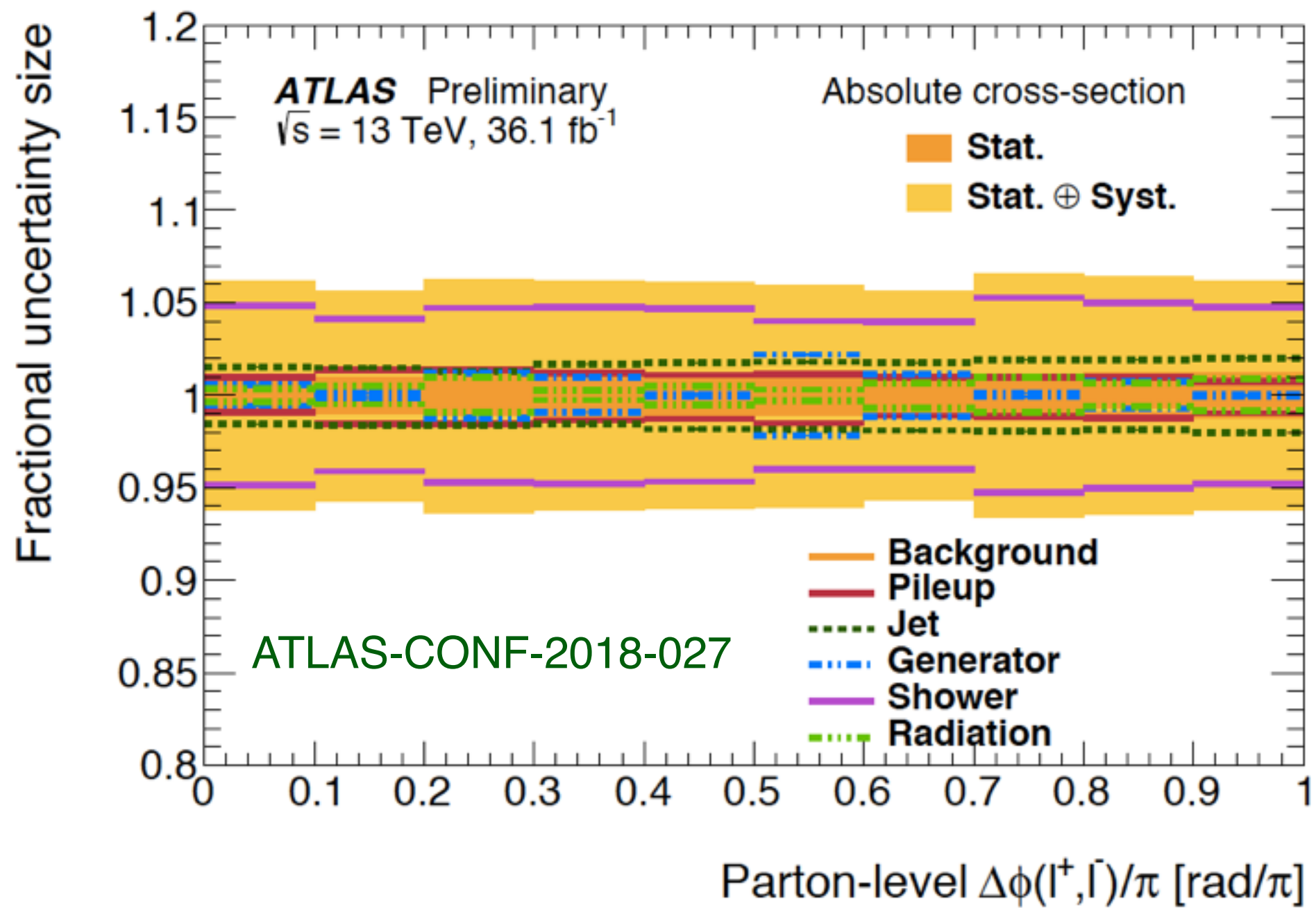
3.2 sigma deviation
from SM prediction



some “anomalies”



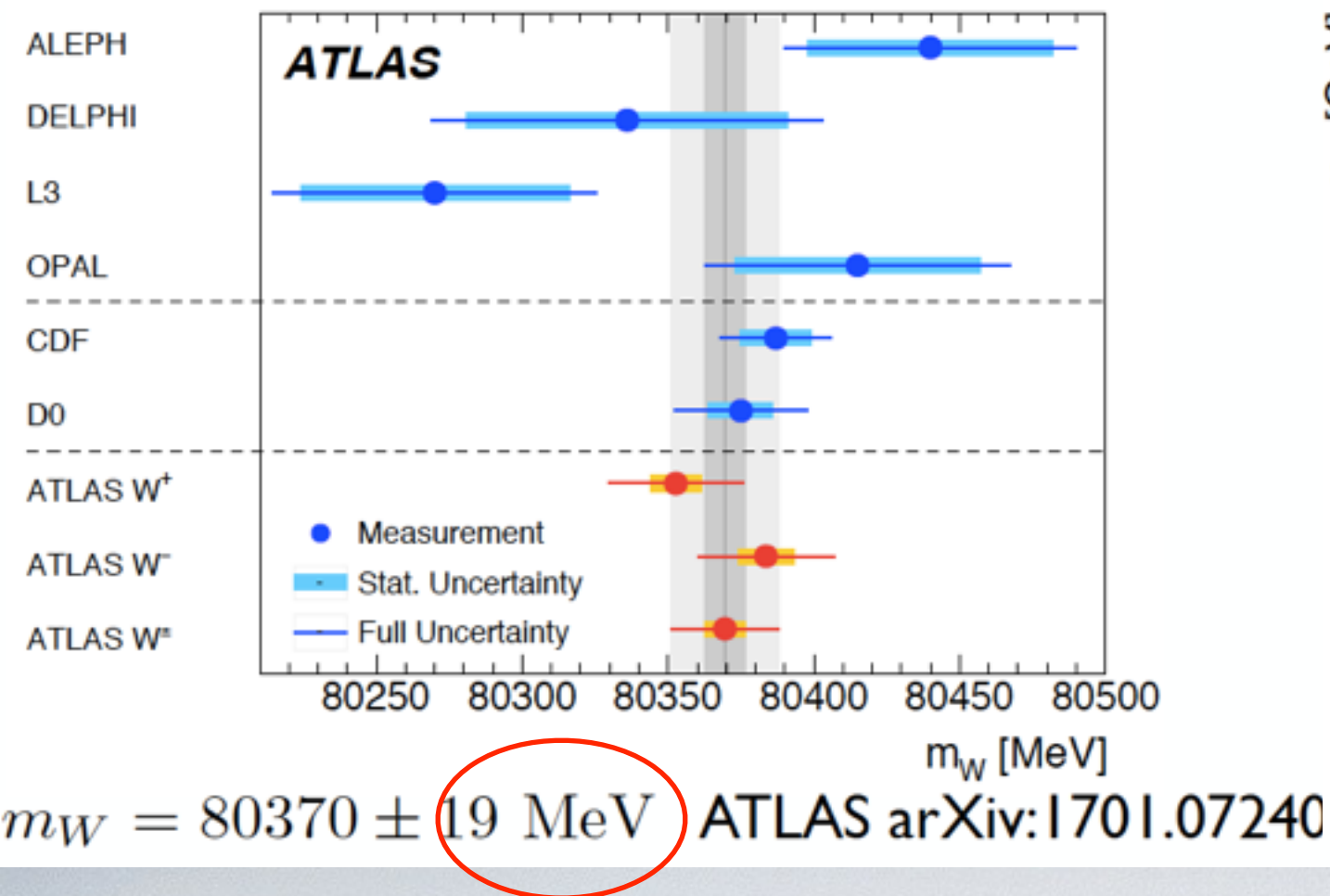
some “anomalies”



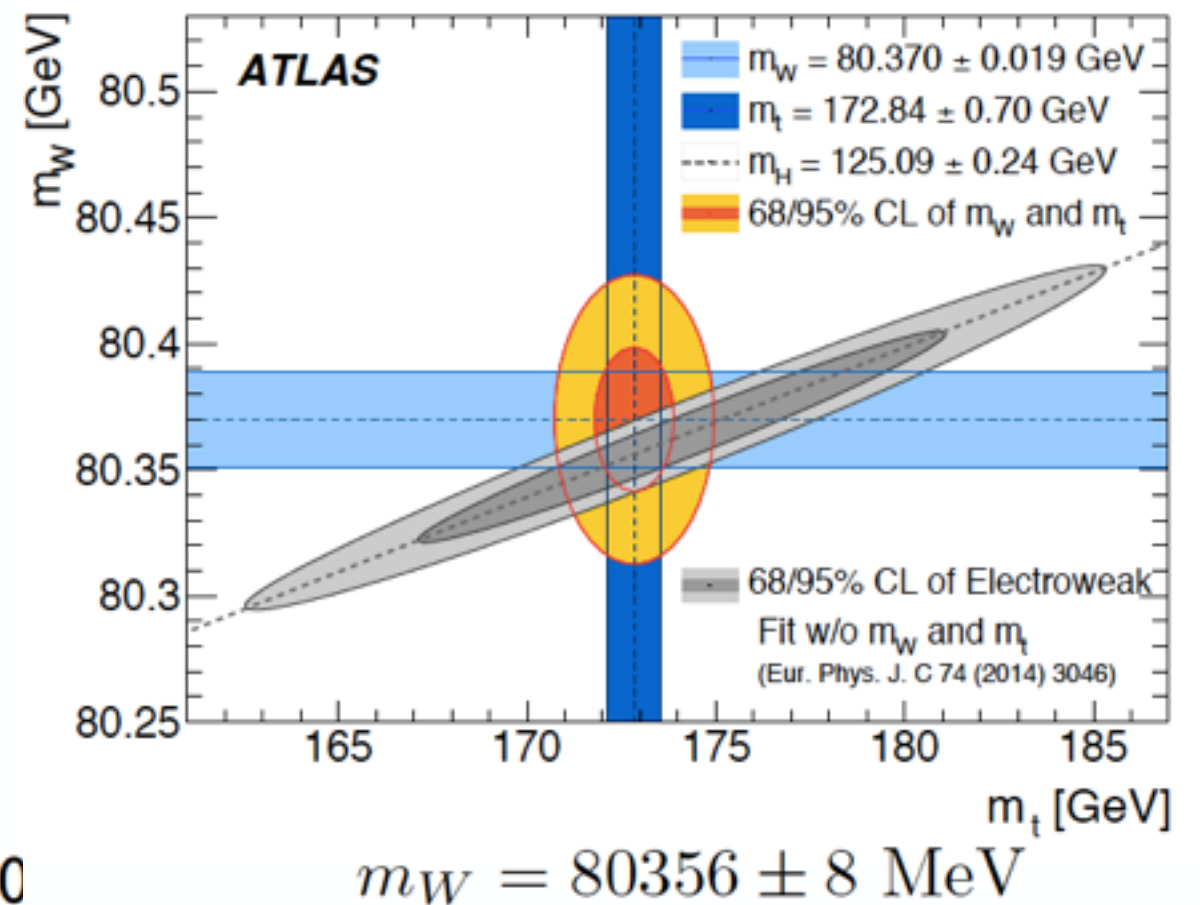
need better control of
theory predictions

Precision measurements: W boson mass

MW experimental values



Global EW fit compared to ATLAS results



A. Vicini, LHCP 2017

extraction of M_W from **shape** of p_T^l distribution

distortion of shape at permil level leads to $\mathcal{O}(10 \text{ MeV})$ shift in mass

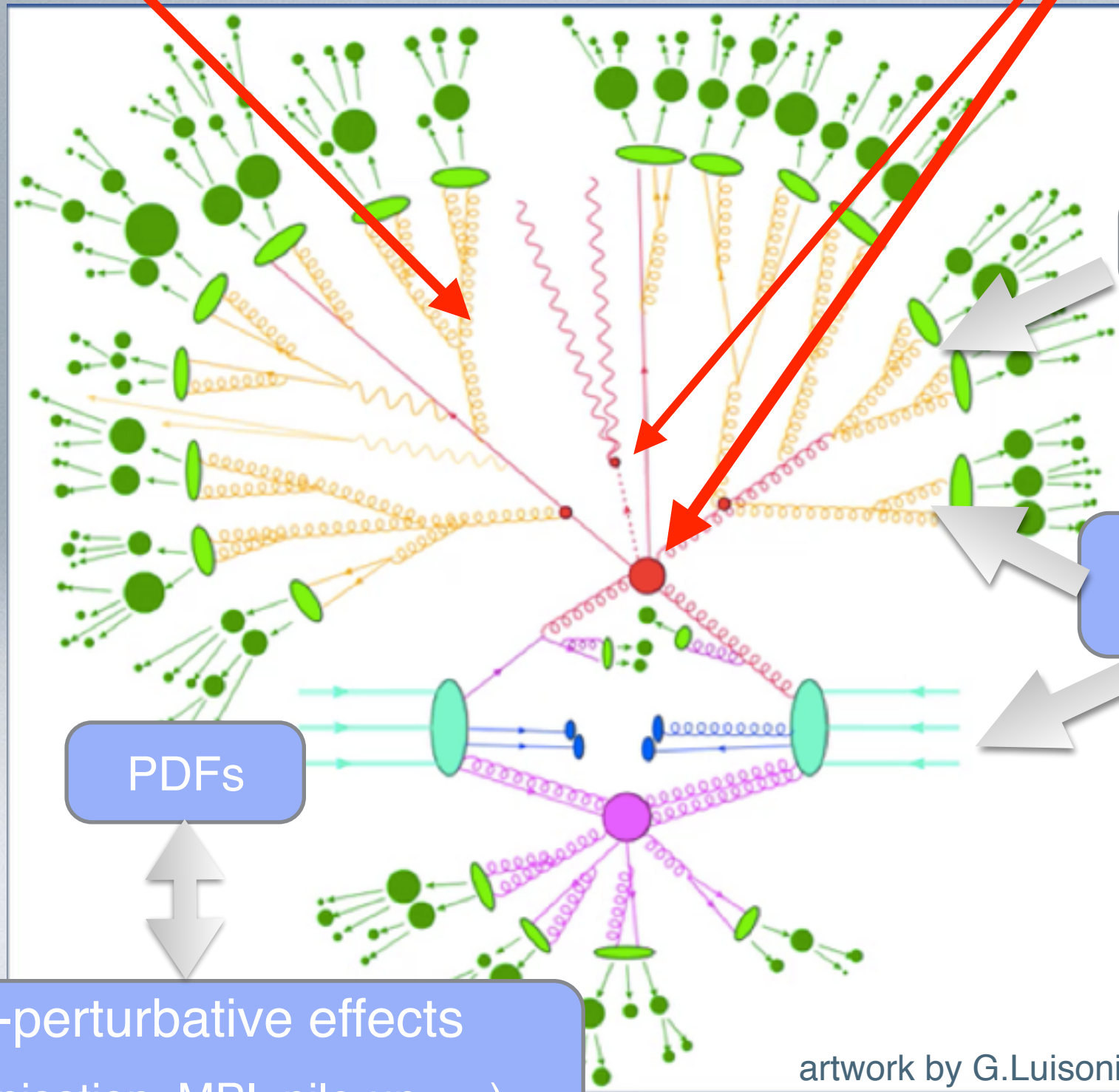
⇒ control of radiative corrections distorting the shape extremely important!

The precision frontier

parton shower

resummation

fixed order calculations
NLO (QCD+EW), NNLO, ...



non-perturbative effects
(hadronisation, MPI, pile-up, ...)

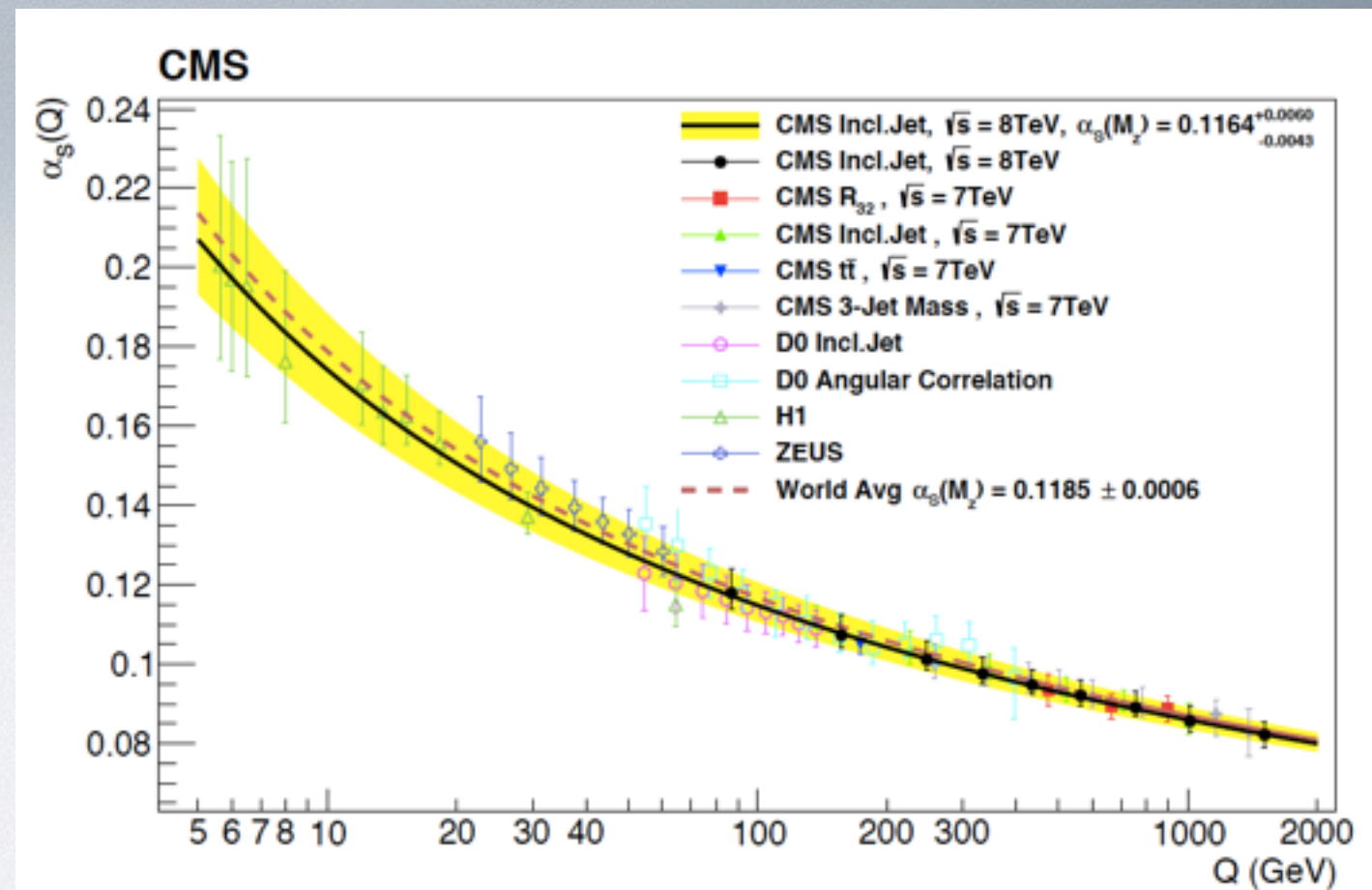
Theorist's basic toolbox

- local gauge invariance $SU(2) \times U(1) \times SU(3)_c$
- renormalizability
- perturbative expansions, e.g.

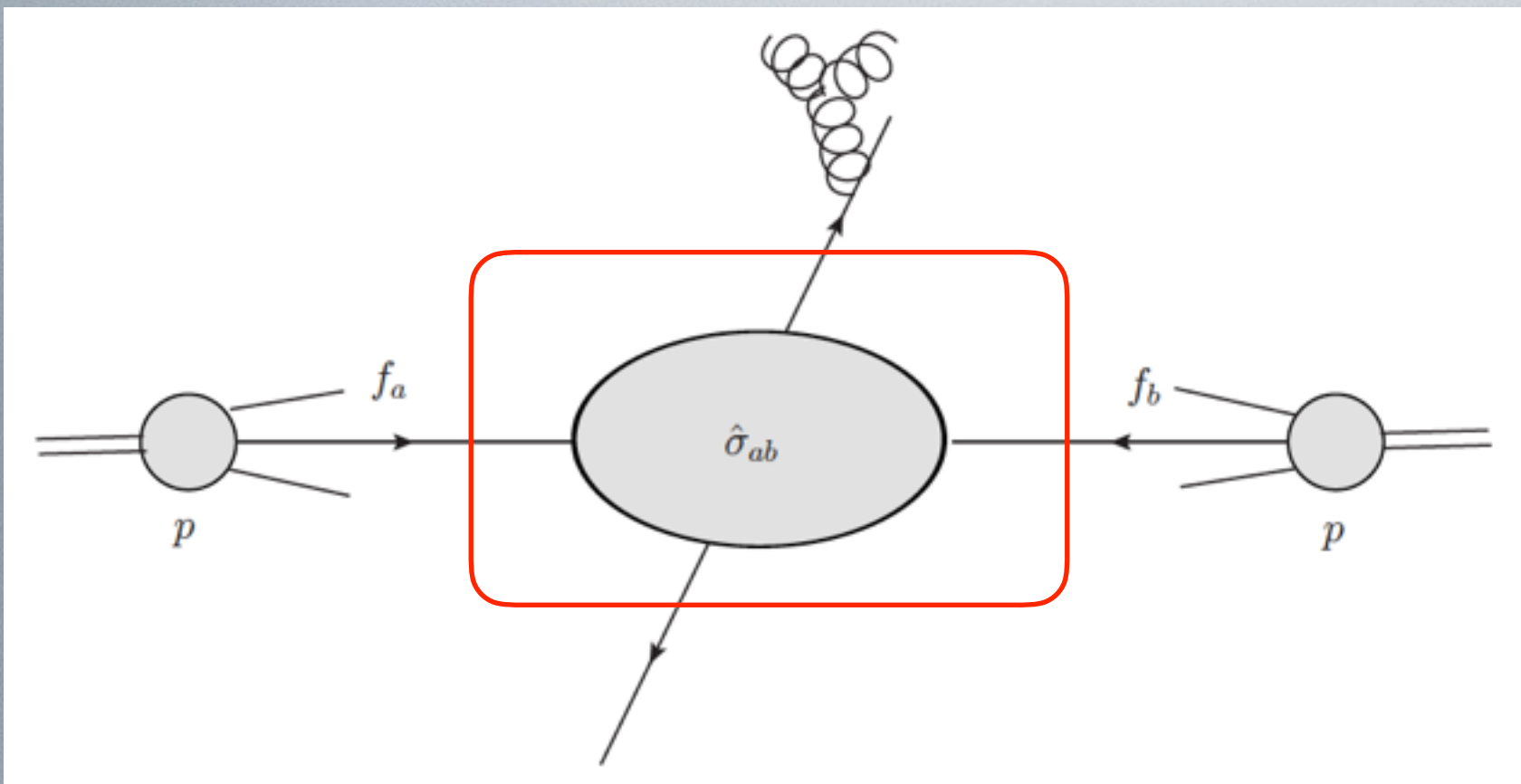
$$\hat{\sigma} = \alpha_s^k(\mu) \left[\hat{\sigma}^{\text{LO}} + \alpha_s(\mu) \hat{\sigma}^{\text{NLO}}(\mu) + \alpha_s^2(\mu) \hat{\sigma}^{\text{NNLO}}(\mu) + \dots \right]$$

important principles of QCD:

- asymptotic freedom
quarks and gluons almost free particles at large energy scales
- factorisation
short- and long distance effects can be separated



Factorisation



separate long-distance
from short-distance
dynamics

parton distribution functions (PDFs)

factorisation scale

$$d\sigma_{pp} = \sum_{a,b} \int_0^1 dx_1 f_{a/p_1}(x_1, \alpha_s, \mu_f) \int_0^1 dx_2 f_{b/p_2}(x_2, \alpha_s, \mu_f) \\ \times d\hat{\sigma}_{ab}(x_1, x_2, \alpha_s(\mu_r), \mu_r, \mu_f) + \mathcal{O}\left(\frac{\Lambda}{Q}\right)^b$$

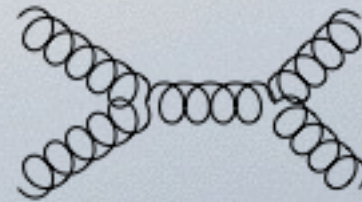
partonic cross section
(calculable in perturbation theory)

power corrections

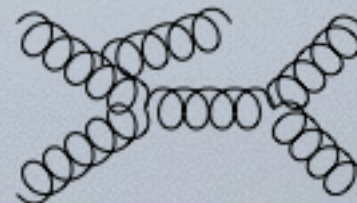
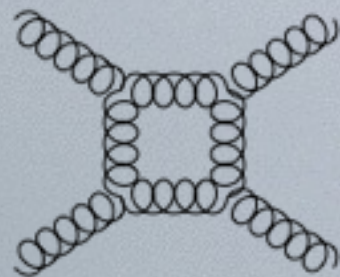
Perturbative expansion (in QCD)

example 2 to 2 scattering

LO: usually tree level diagrams

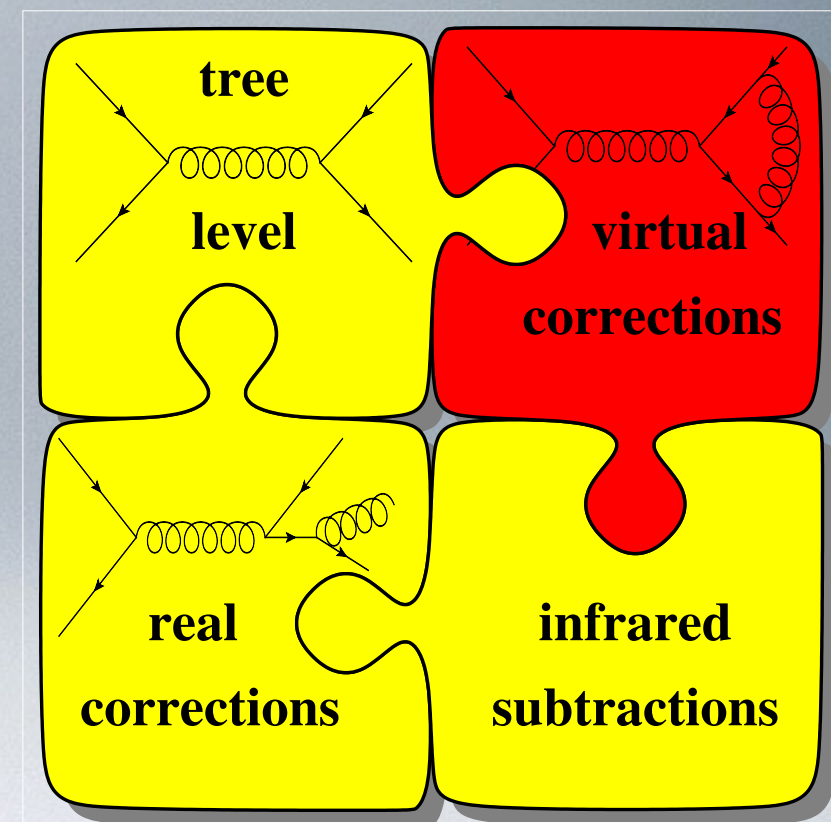


NLO: one loop (virtual) + extra real radiation + subtraction terms



individual contributions are **divergent**

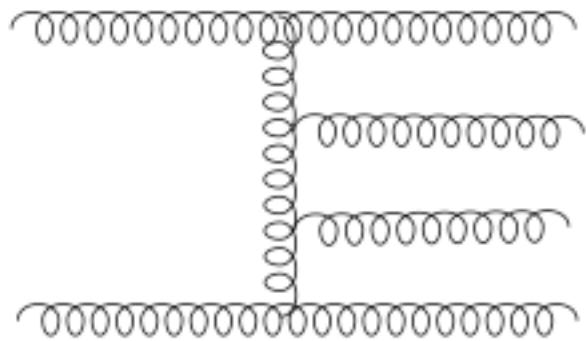
- requires the **isolation of the singularities**
dimensional regularisation: $D = 4 - 2\epsilon$
- need a good **subtraction method** for singularities of individual contributions



$$\sigma^{NLO} = \underbrace{\int_{m+1} \left[d\sigma^R - d\sigma^S \right]_{\epsilon=0}}_{\text{numerically}} + \underbrace{\int_m \left[\underbrace{d\sigma^V}_{\text{cancel poles}} + \underbrace{\int_S d\sigma^S}_{\text{analytically}} \right]_{\epsilon=0}}_{\text{numerically}}$$

QCD corrections: building blocks

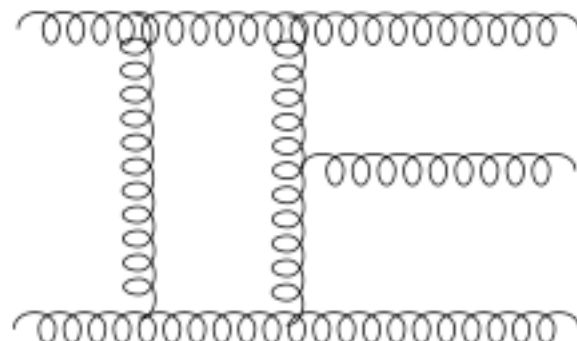
NNLO:



double real



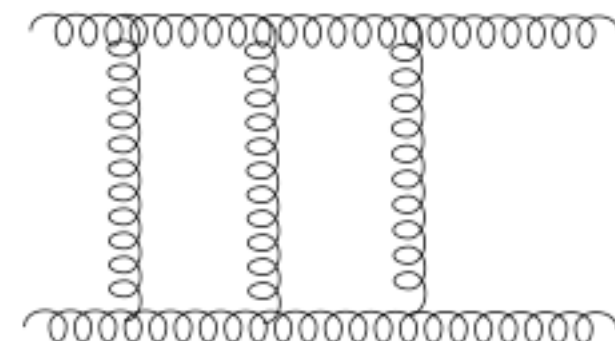
implicit IR poles (PS integration)



1-loop virtual
 $\otimes$ single real



explicit and implicit poles



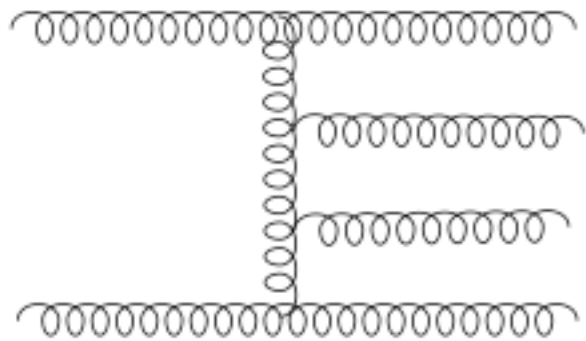
2-loop virtual



explicit poles $1/\epsilon^{2L}$

QCD corrections: building blocks

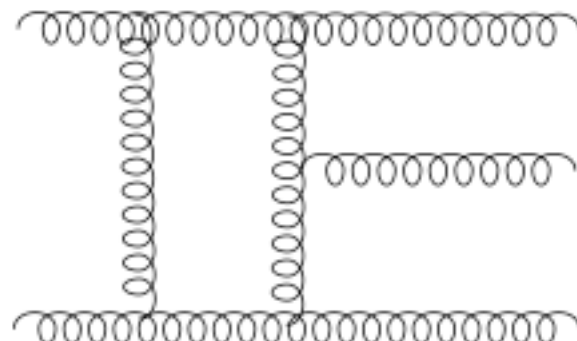
NNLO:



double real



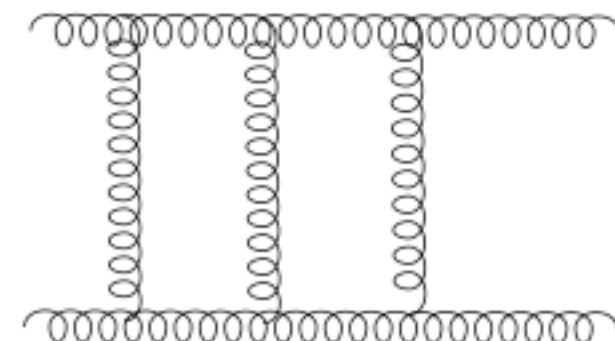
implicit IR poles (PS integration)



1-loop virtual
⊗ single real



explicit and implicit poles



2-loop virtual



explicit poles $1/\epsilon^{2L}$

bottlenecks: IR subtraction



harder with more massless particles
(intricate IR singularity structure)

two-loop integrals



harder with more massive/off-shell particles
(more scales → more complicated
analytic structure)

Status

- **NLO automation:**

phase after

“industrial revolution”

- various automated tools
- NLO QCD matched to parton shower is state of the art



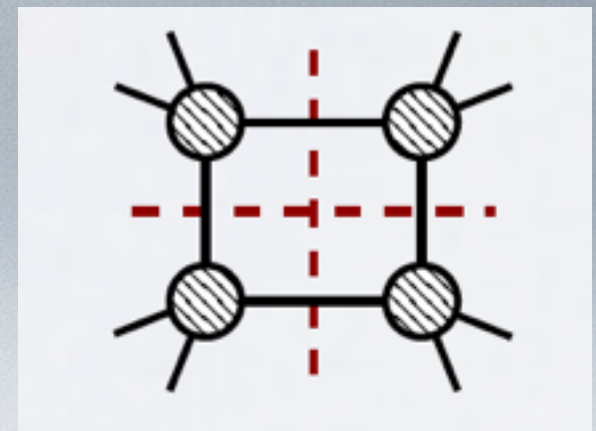
- **NNLO:** (partial) automation starts to become reality



- **NNNLO:** some results available!

What caused the NLO revolution?

- automation of subtraction methods for IR divergent real radiation
Frixione, Kunszt, Signer '95; Catani, Seymour '96 → MadDipole, AutoDipole, FxFx, ...
- unitarity-inspired methods for virtual corrections
gauge dependent off-shell states introduce “spurious” terms
→ try to use on-shell quantities as building blocks
- construct N-point one-loop amplitudes from tree amplitudes
Bern, Dixon, Dunbar, Kosower '94
- use complex momenta in generalised cuts
Britto, Cachazo, Feng '04
- numerical reduction at integrand level
Ossola, Papadopoulos, Pittau '06
- D-dimensional unitarity
Anastasiou, Britto, Feng, Kunszt, Mastrolia '06;
Forde '07; Giele, Kunszt, Melnikov '08

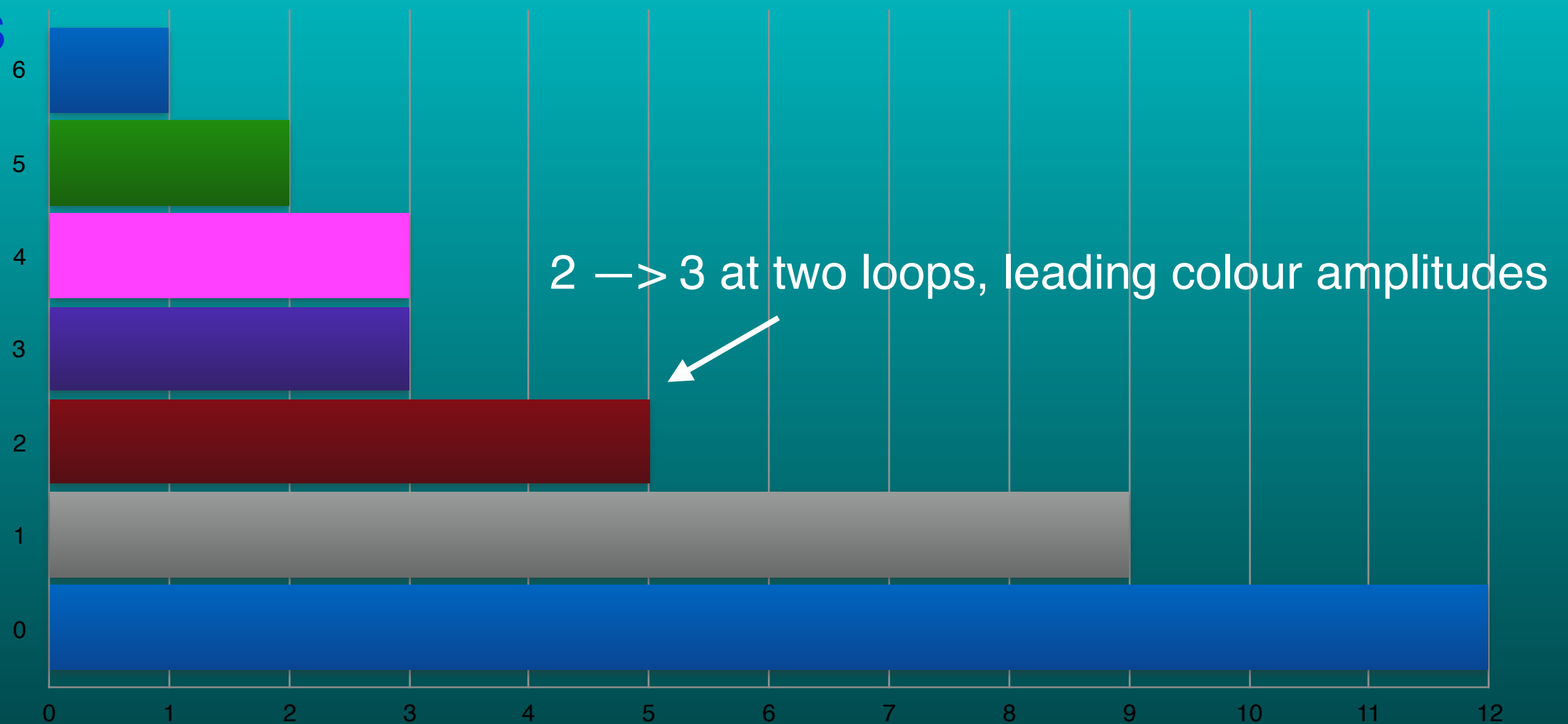


measure of complexity

#loops + #legs + #scales (masses, off-shellness)

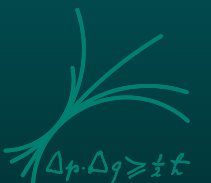
complexity does not scale linearly!

loops



legs

(refers to physical results, not individual integrals)



Calculations beyond one loop



typical procedure and corresponding tools

1. automated amplitude generation

public **tools** e.g. QGRAF [P.Nogueira], FeynArts [T.Hahn et al.]

saturation of Lorentz/spin indices: helicity amplitudes or projectors to form factors

2. reduction of the loop amplitudes to coefficients $\otimes$ master integrals

reduction highly non-trivial; master integrals not known before reduction

multi-purpose **tools** e.g. Reduze [C.Studerus, A.v.Manteuffel], FIRE [A.V.Smirnov], LiteRed [R.N.Lee], KIRA [Maierhöfer, Usovitsch, Uwer '17]

mostly based on integration by parts (IBP) relations

$\Rightarrow$ solve large linear systems

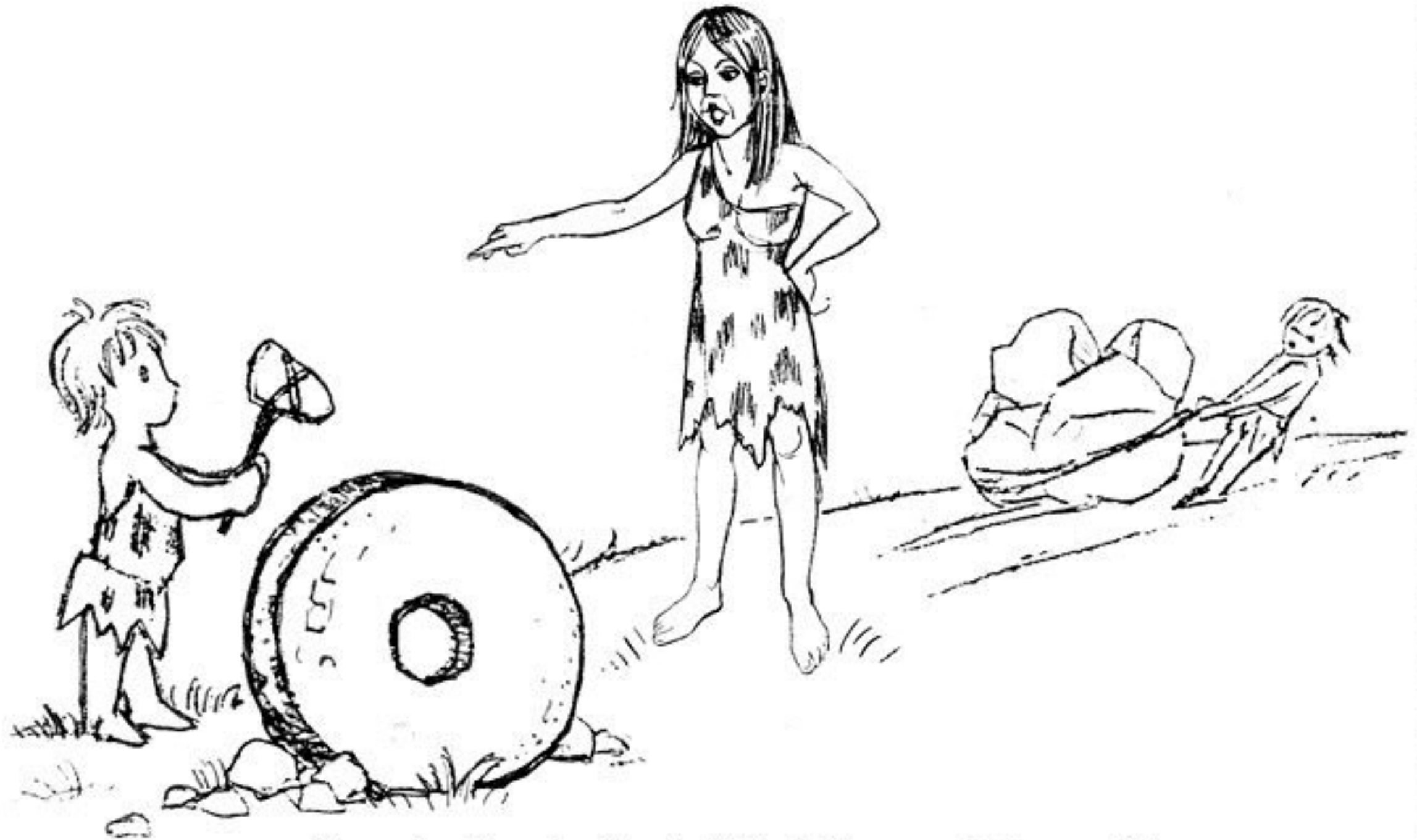
$$\int d^D k \frac{\partial}{\partial k^\mu} v^\mu f(k, p_i) = 0$$

two-loop **integrand** reduction:

very interesting new developments, but not ready for automation yet (?)

Mastrolia, Ossola '11; Badger, Frellesvig, Zhang '12; Kosower, Larsen '12,
Mastrolia, Mirabella, Ossola, Peraro '12; Feng, Huang '12;
Papadopoulos et al.'12; Ita '15; Larsen, Zhang '16; Mastrolia, Peraro, Primo '16,
Peraro '16; Larsen, Rietkerk '17, Badger et al '17,'18, Abreu et al '17, Boels et al '18,
Chawdry, Lim, Mitov '18, ...

Development of new methods is very important!



All you've done is chisel all day! Do something useful,
like helping your brother drag those rocks up the hill.

© Marjorie Sarnat

3. calculation of the master integrals

- analytically? may not always be possible

most efficient method: **differential equations**

Kotikov '91; Remiddi '97, Gehrmann, Remiddi '00 , ...

J.Henn '13: canonical form, caused “2-loop revolution” $d\vec{f}(x, \epsilon) = \epsilon dA(x) \vec{f}(x, \epsilon)$

- numerically? may not always be accurate/fast enough
also lots of recent progress

4. subtraction of IR divergent real radiation

interesting recent NNLO developments (see later)

5. stable and fast Monte Carlo program

(or if not fast: how to make results available in a flexible format)



3. calculation of the master integrals

- analytically? may not always be possible

most efficient method: **differential equations** see talk of Leila Maestri

Kotikov '91; Remiddi '97, Gehrmann, Remiddi '00, ...

J.Henn '13: canonical form, caused “2-loop revolution” $d\vec{f}(x, \epsilon) = \epsilon dA(x) \vec{f}(x, \epsilon)$

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(or if not fast: how to make results available in a flexible format)



multi-loop integrals

processes not (yet) known precisely enough typically involve

- several mass scales (EW corrections, quark masses, BSM particles)
- more than two loops

analytic results for multi-scale two-loop integrals are sparse

numerical evaluation:

- often considered as “poor man’s solution”
as long as analytic results are not available
- but easier extendible to many mass scales



pro's and con's

analytic

numerical

pole cancellation

exact

with numerical uncertainty

fast evaluation

☒ (mostly)

depends

control of integrable singularities

☒ control of analytic regions

difficult

extension to more scales

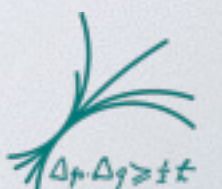
difficult

less difficult

automation

difficult

promising



pro's and con's

analytic

numerical

pole cancellation

exact

with numerical uncertainty

fast evaluation

☒ (mostly)

depends

control of integrable singularities

☒ control of analytic regions

difficult

extension to more scales

difficult

less difficult

automation

difficult

promising

see talk of Stephan Jahn



NNLO real radiation subtraction methods

- **antenna subtraction** analytically integrated subtraction terms
[Gehrmann-DeRidder, Gehrmann, Glover '05]
- **qt “subtraction”** slicing, (colourless final states)
[Catani, Grazzini '07]
- **N-jettiness** slicing [Gaunt, Stahlhofen, Tackmann, Walsh '15]
[Boughezal, Focke, Liu, Petriello '15]
- **sector-improved residue subtraction** numerically integrated subtraction terms
[Czakon, Heymes, Mitov '10; Czakon, Heymes '14] [Boughezal et al. '11]
- **nested subtraction** [Caola, Melnikov, Röntschi '17]
- **projection to Born / structure function approach** only special kinematics
[Gao, Li, Zhu '12] [Brucherseifer, Caola, Melnikov '14] [Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15]
- **colorful** only final state colour so far [Del Duca, Somogyi, Trocsanyi et al '05, '16]
- **local analytic sector subtraction**
[Magnea, Maina, Pelliccioli, Signorile-Signorile, Torricelli, Uccirati '18]
- **geometric subtraction** (slicing so far)
[Herzog '18]

NNLO results

● antenna

● qt

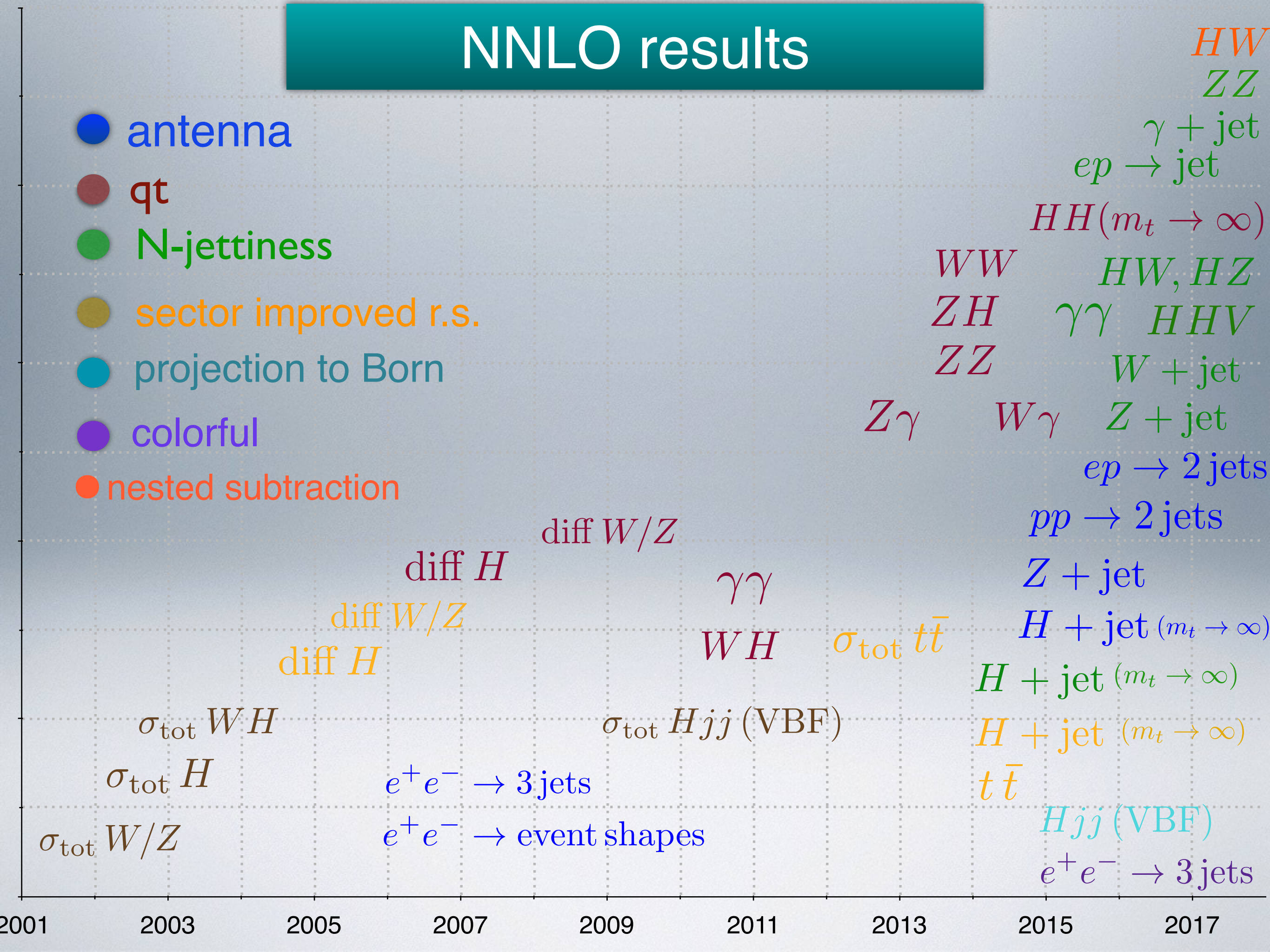
● N-jettiness

● sector improved r.s.

● projection to Born

● colorful

● nested subtraction



NNLO results

● antenna

● qt

● N-jettiness

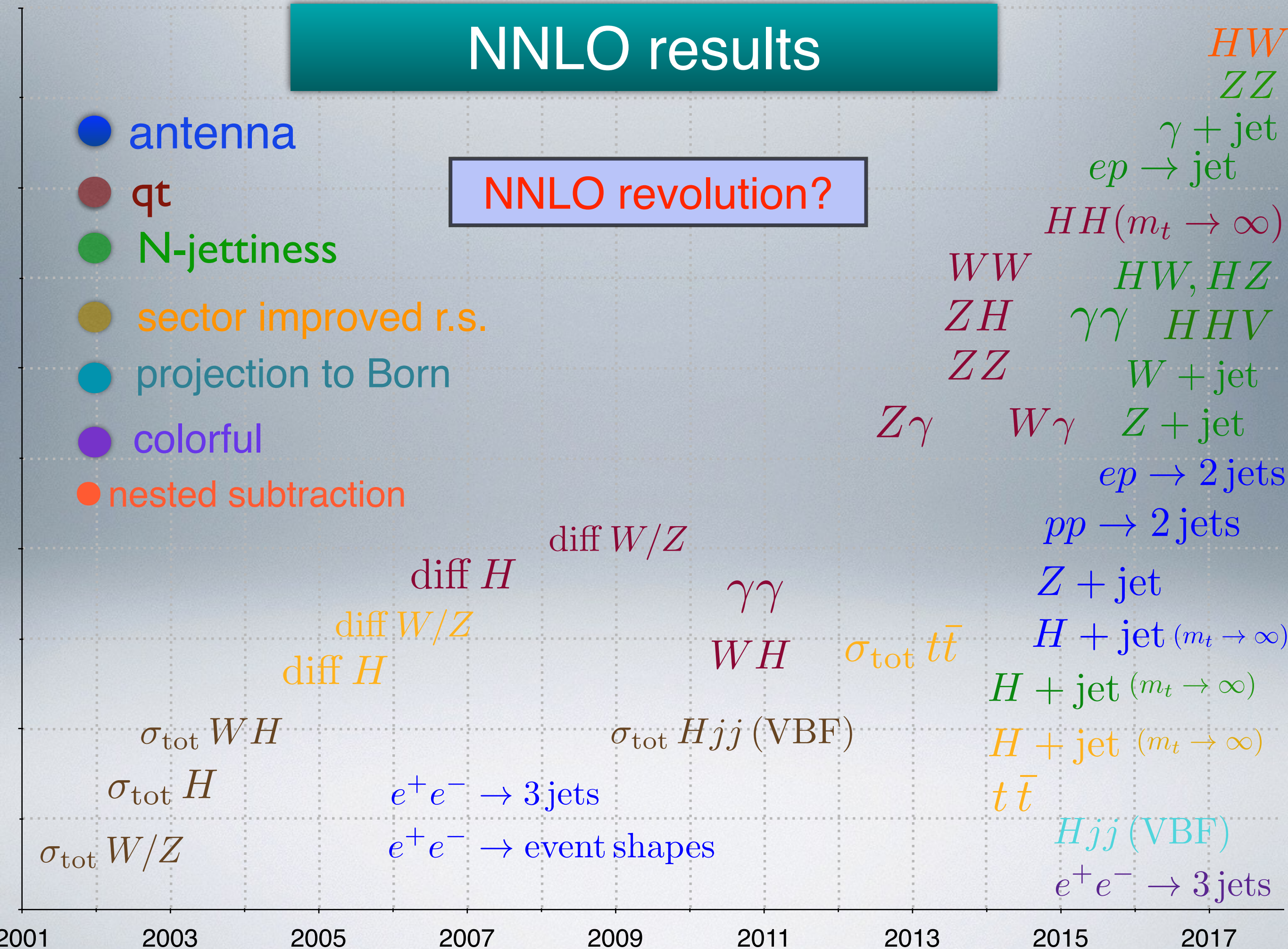
● sector improved r.s.

● projection to Born

● colorful

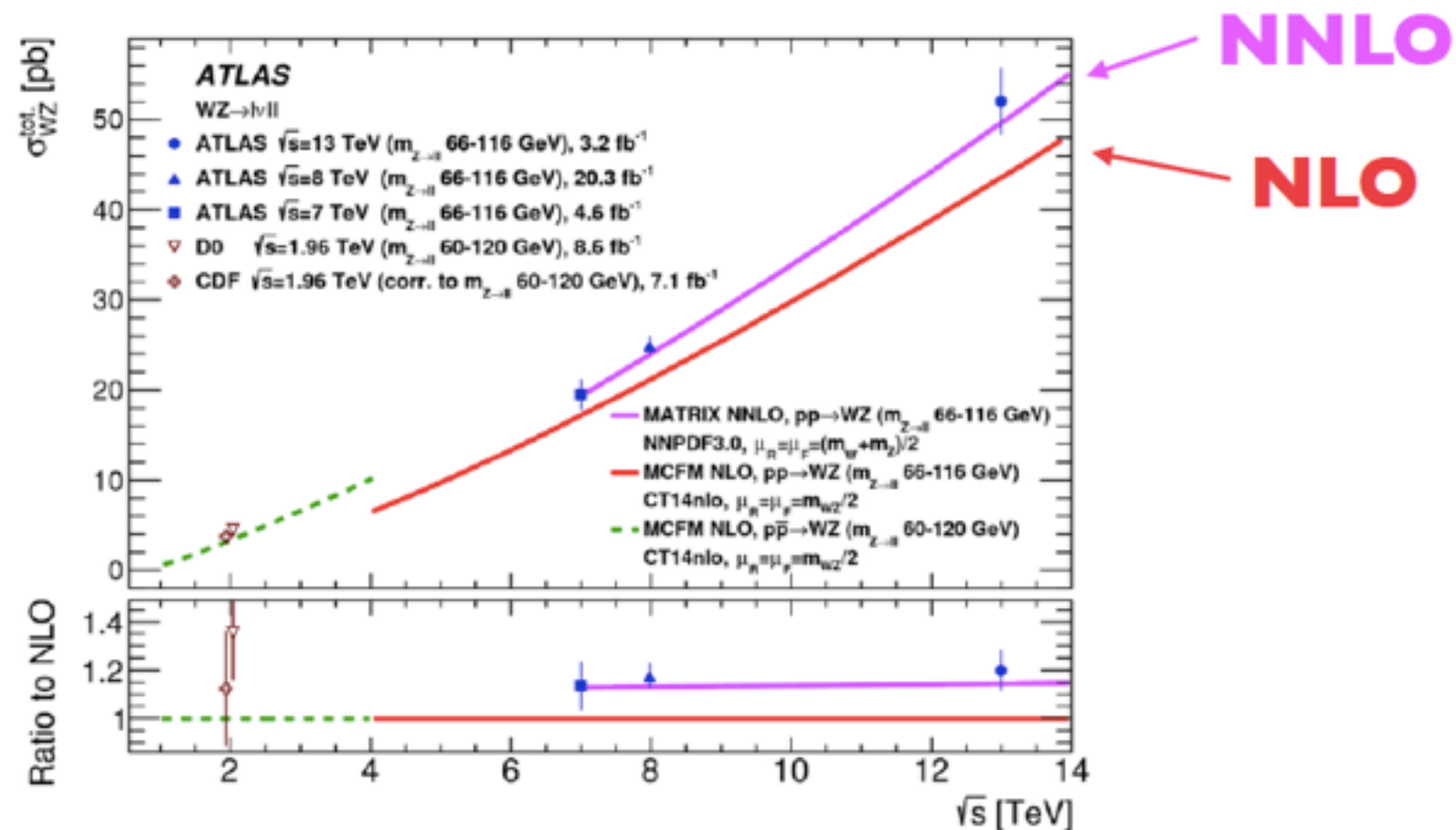
● nested subtraction

NNLO revolution?



Precision phenomenology: some recent results

- N³LO
- mixed QCD-EW corrections
- NNLO
- NLO for loop-induced processes



Higgs production in gluon fusion

status before 2018: N3LO in $m_t \rightarrow \infty$ limit

expansion around soft limit [Anastasiou et al. 1602.00695](#)



E_{CM}	σ	$\delta(\text{theory})$	$\delta(\text{PDF})$	$\delta(\alpha_s)$
13 TeV	48.58 pb	$+2.22\text{pb} (+4.56\%)$ $-3.27\text{pb} (-6.72\%)$	$\pm 0.90 \text{ pb } (\pm 1.86\%)$	$+1.27\text{pb} (+2.61\%)$ $-1.25\text{pb} (-2.58\%)$

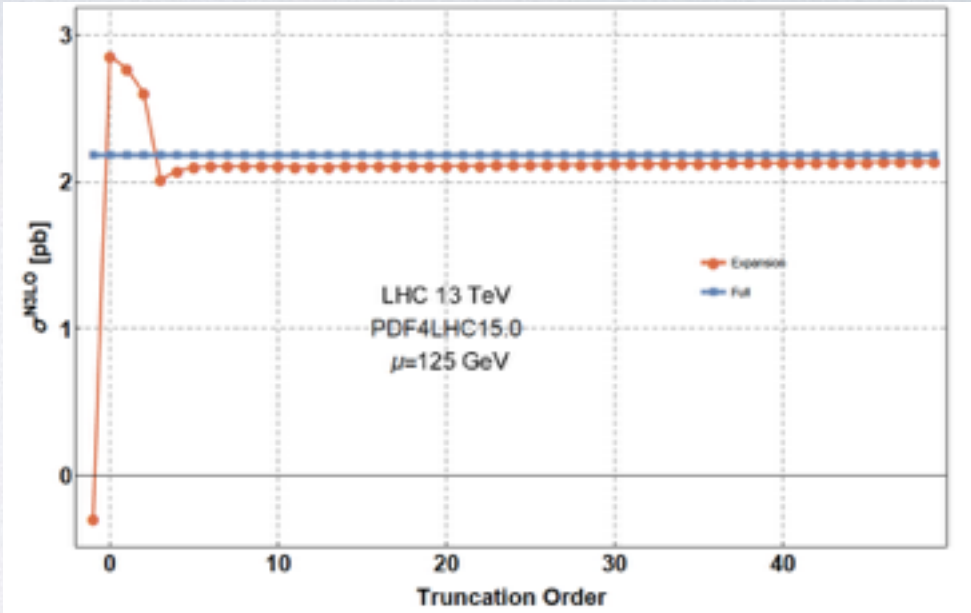
$\delta(\text{theory})$

$\delta(\text{scale})$	$\delta(\text{trunc})$	$\delta(\text{PDF-TH})$	$\delta(\text{EW})$	$\delta(t, b, c)$	$\delta(1/m_t)$
$+0.10 \text{ pb}$ -1.15 pb	$\pm 0.18 \text{ pb}$	$\pm 0.56 \text{ pb}$	$\pm 0.49 \text{ pb}$	$\pm 0.40 \text{ pb}$	$\pm 0.49 \text{ pb}$
$+0.21\%$ -2.37%	$\pm 0.37\%$	$\pm 1.16\%$	$\pm 1\%$	$\pm 0.83\%$	$\pm 1\%$

gone

no expansion around soft limit anymore

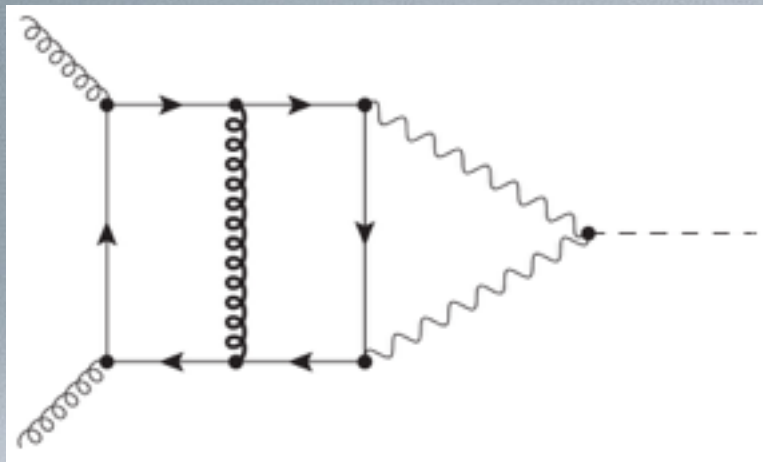
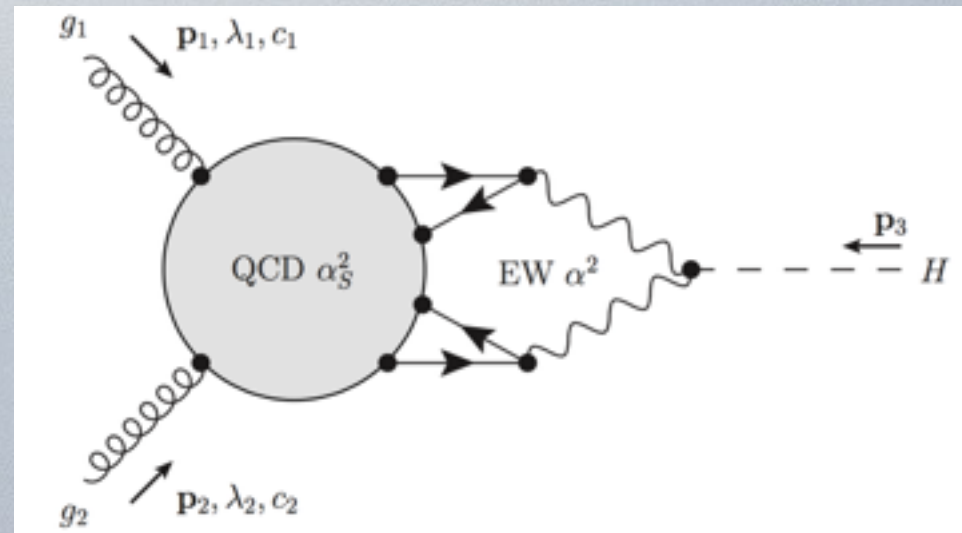
[B.Mistlberger 1802.00833](#)



Higgs production in gluon fusion

calculation of NLO QCD corrections to mixed QCD-EW corrections

Bonetti, Melnikov, Tancredi
1711.11113, 1801.10403



multi-scale 3-loop diagrams

corresponding real radiation contribution
very challenging $\rightarrow$ soft approximation

$$\delta\sigma_{\text{QCD-EW}}^{\text{NLO}} / \sigma_{\text{QCD,full}}^{\text{NLO}} \sim (4.7 - 5.5) \times 10^{-2}$$

confirming previous estimates

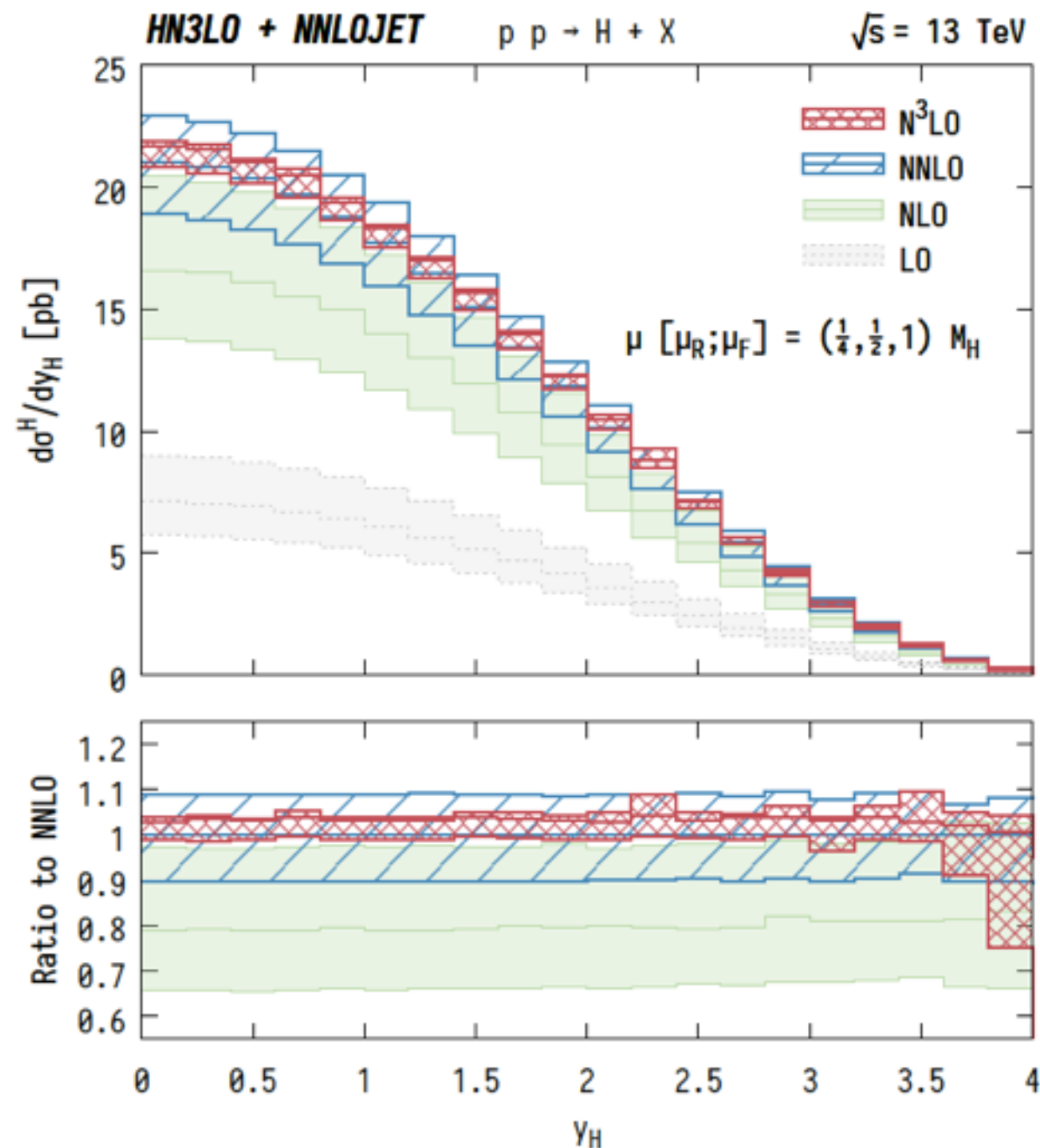
see also Anastasiou et al. 1811.11211

Higgs production in gluon fusion

differential N3LO Higgs boson production

Cieri, Chen, Gehrmann, Glover, Huss 1807.11501

(see also Dulat, Mistlberger, Pelloni '17 for partial results)



qT subtraction extended
to N3LO

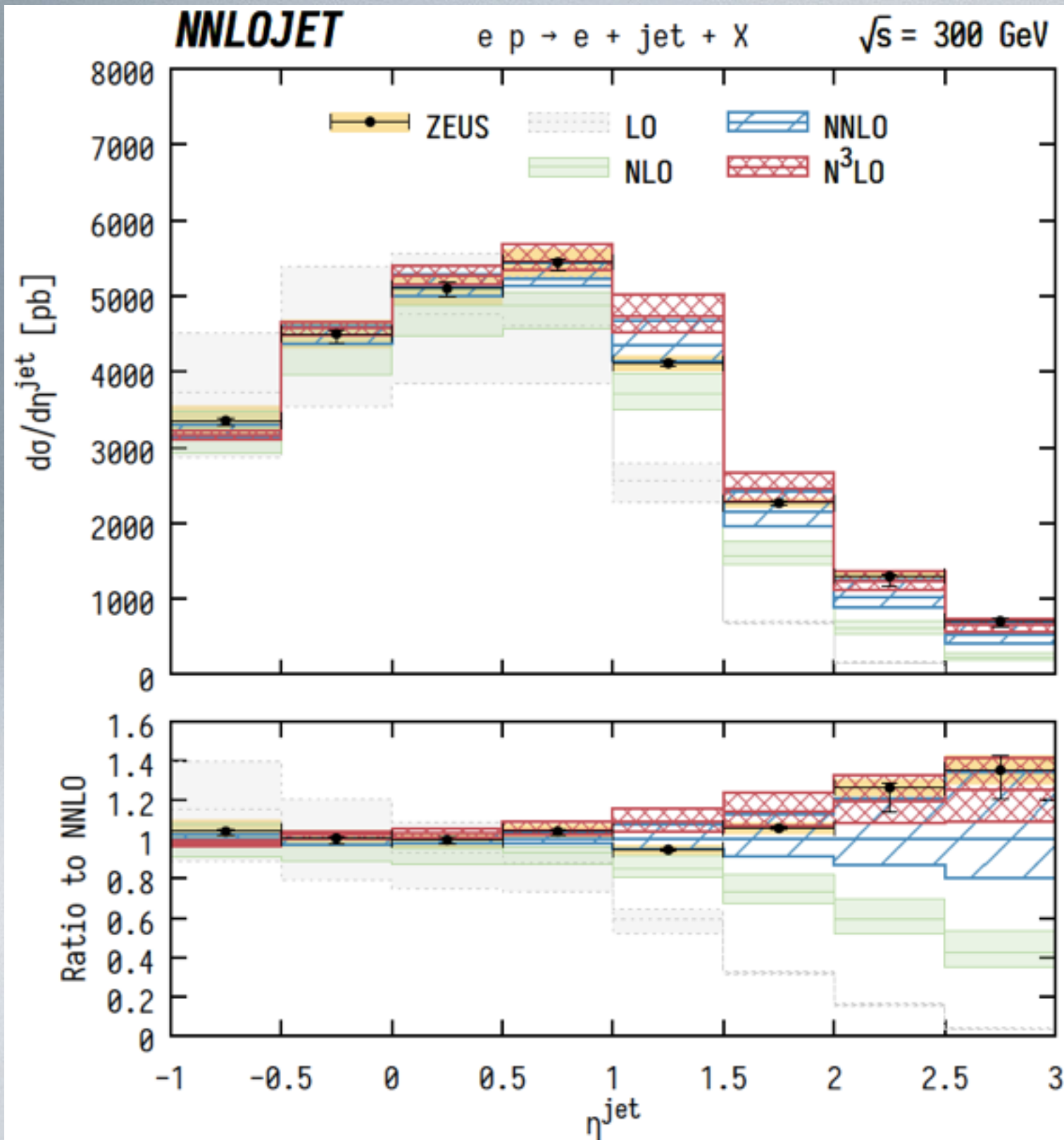
collinear subtraction term
extracted numerically

used some results from ihixs2
Dulat, Lazopoulos, Mistlberger 1802.00827

substantial reduction of
scale uncertainties

Differential N3LO jet production in DIS

Currie, Gehrmann, Glover, Huss, Niehues, Vogt 1803.09973



subtraction scheme:
projection to Born

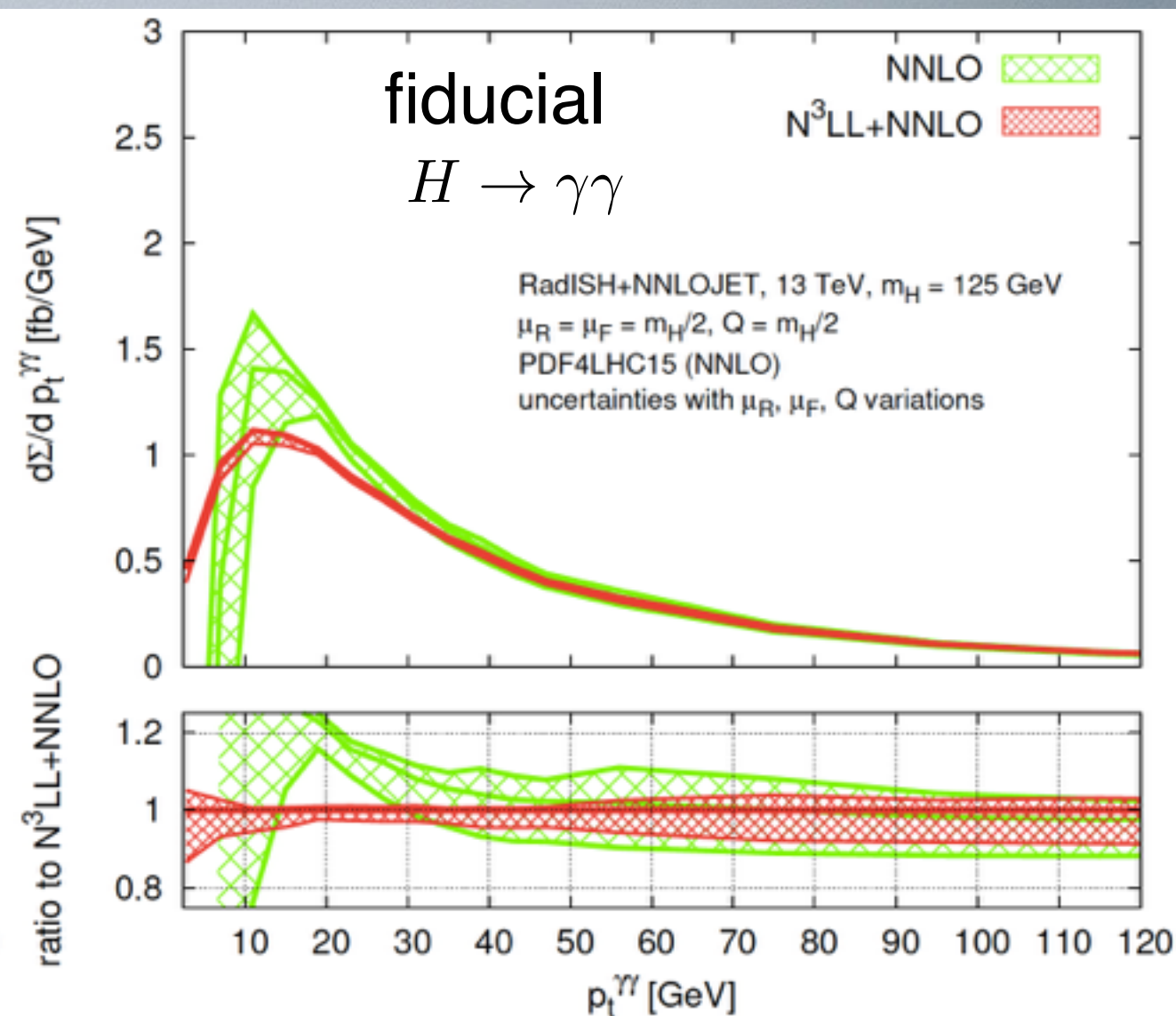
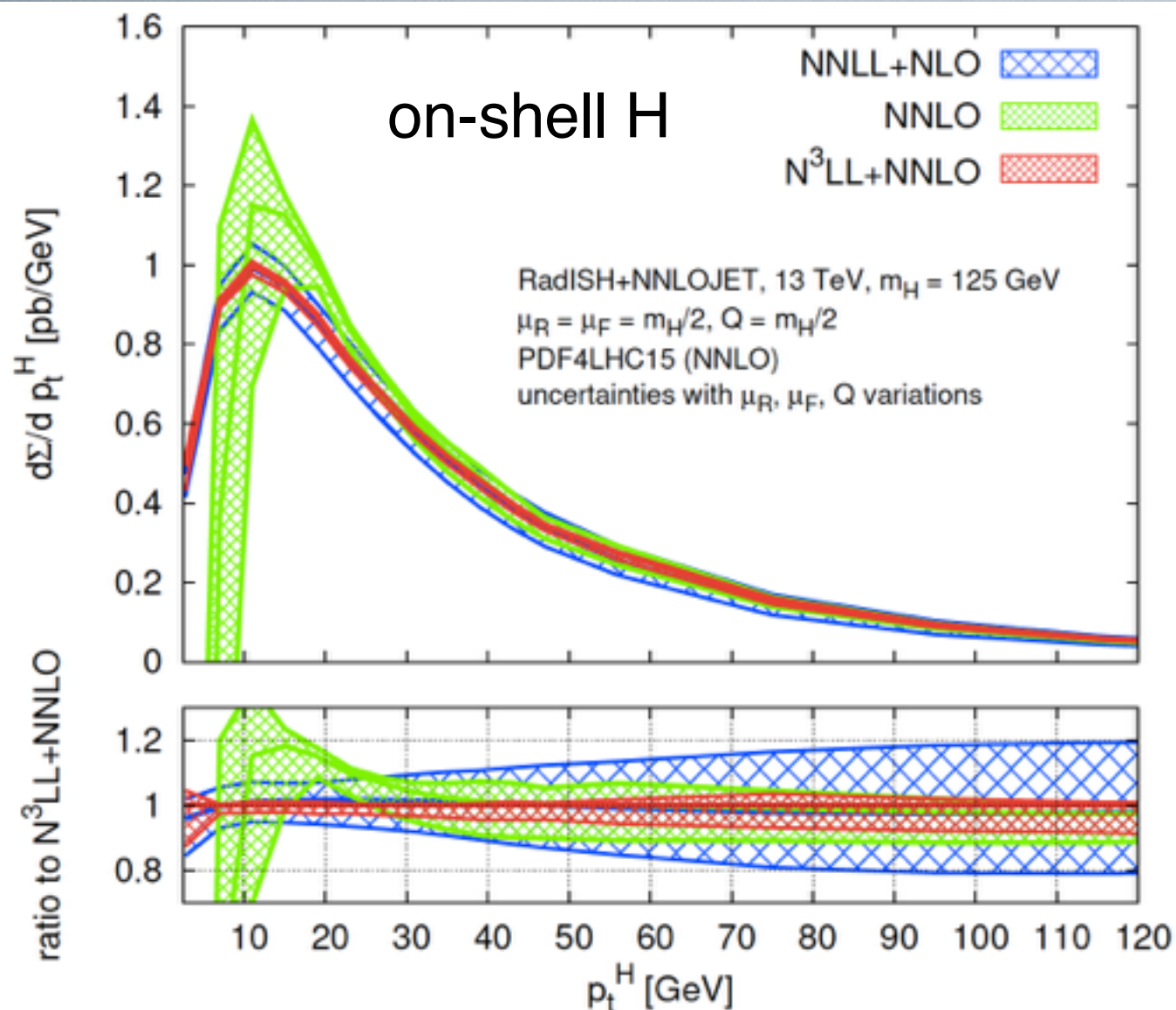
Brucherseifer, Caola, Melnikov '14,
Cacciari, Dreyer, Karlberg,
Salam, Zanderighi '15

showpiece of perturbation theory
agreement with data improved

Higgs pT spectrum

Higgs pT and fiducial distributions at N3LL+NNLO in $m_t \rightarrow \infty$ limit

Bizon, Chen, Gehrmann-De Ridder, Gehrmann, Glover,
Huss, Monni, Re, Rottoli, Torrielli 1805.05916



perturbative uncertainties now at a level where other effects (quark masses)
become equally important in whole pT range

Higgs production in Vector Boson Fusion

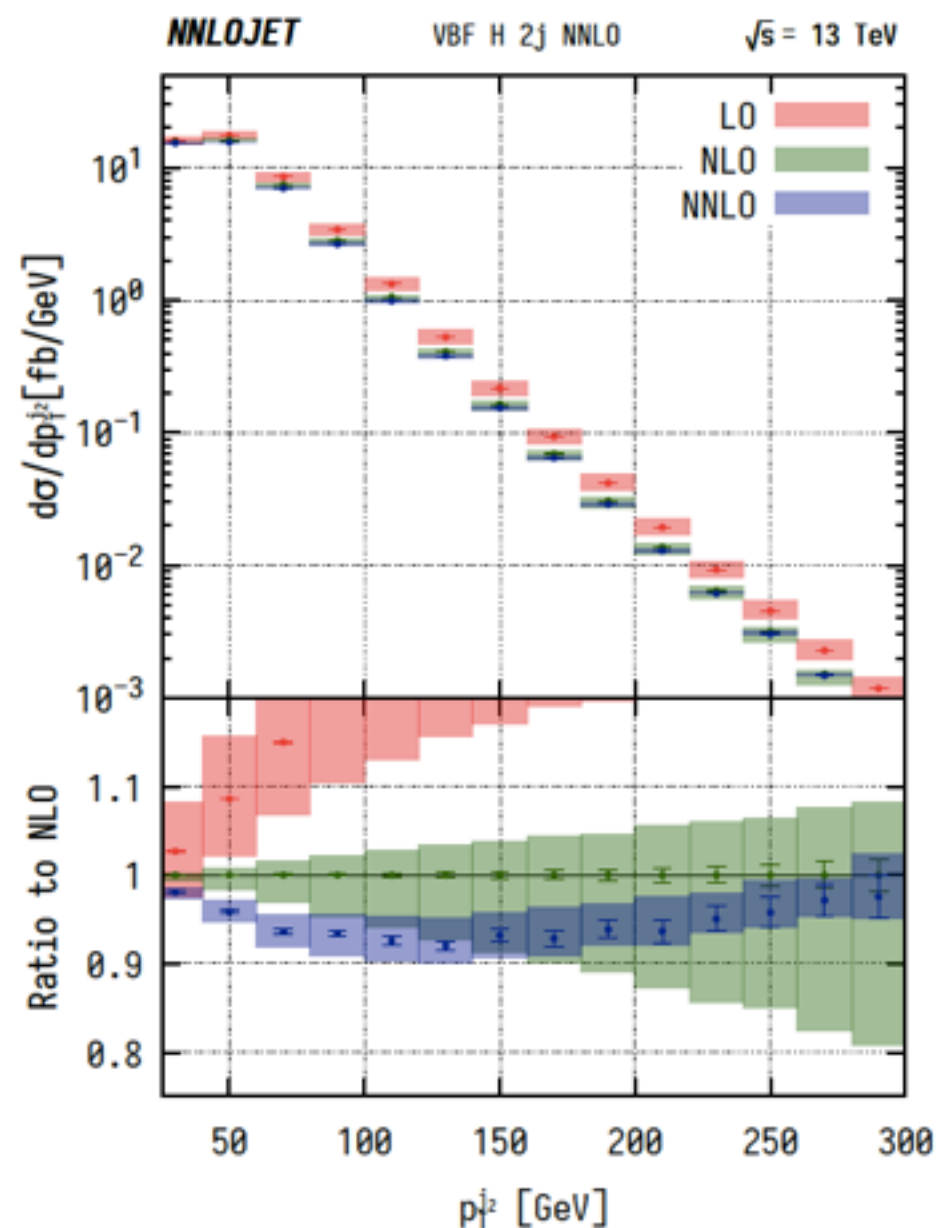
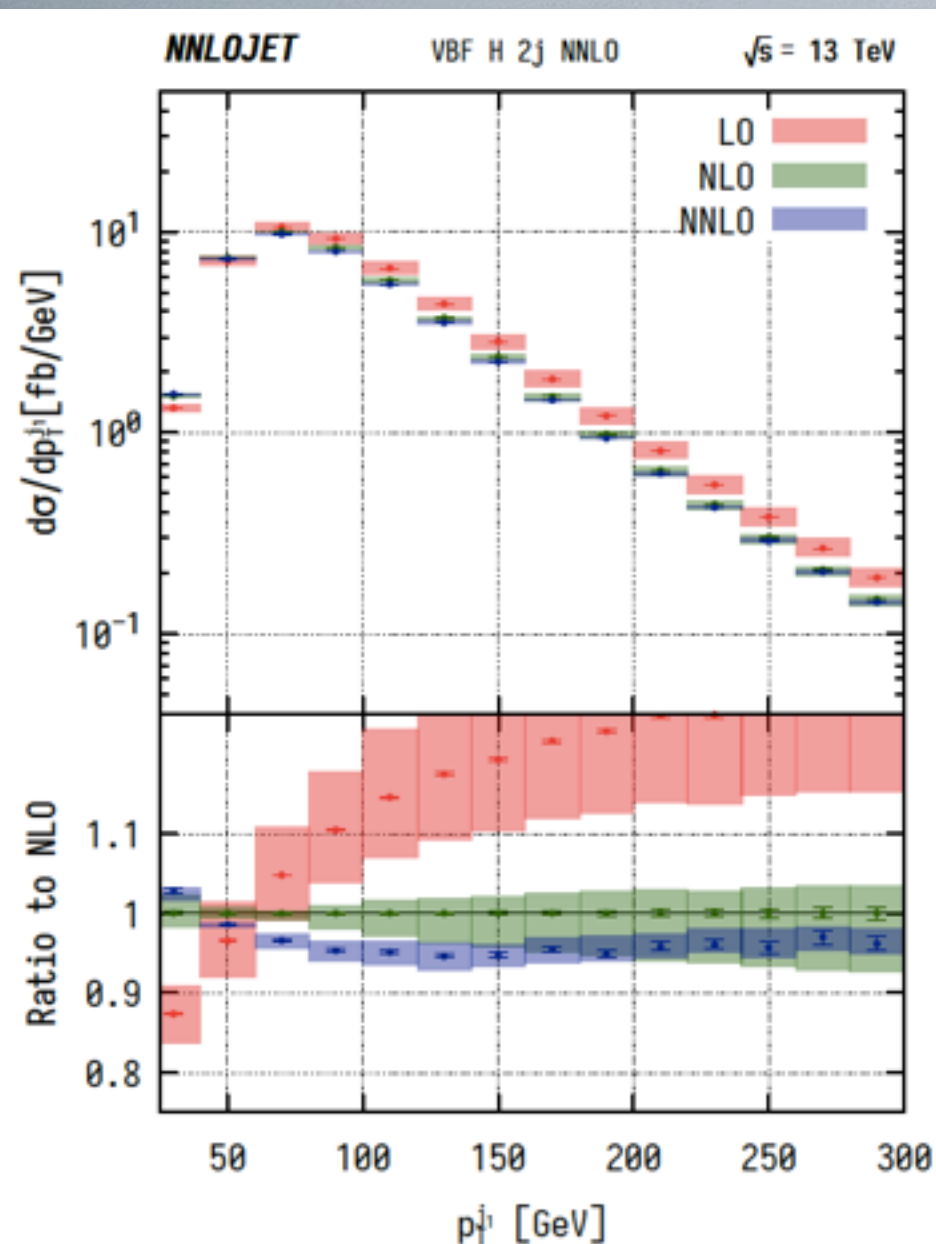
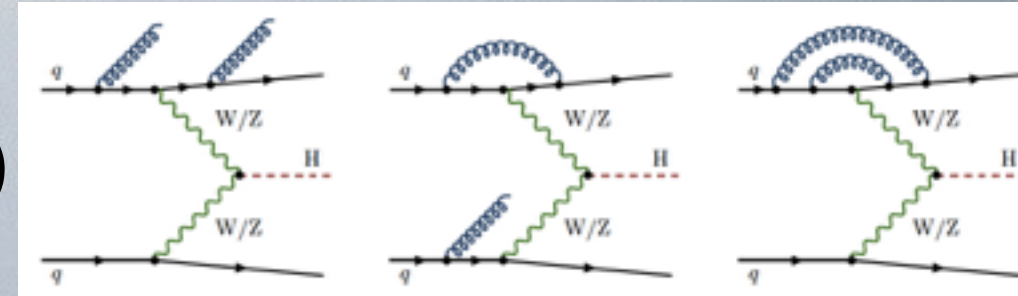
- fully differential NLO : VBFNLO **Arnold-Zeppenfeld '08-'18**

- N3LO total cross section **Dreyer, Karlberg '16**

- differential NNLO (in “DIS-approximation”)

Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15

Cruz-Martinez, Gehrmann, Glover, Huss '18 → revision of previous calculations



impact on
efficiency of
VBF cuts

NNLO

- enormous progress in the last few years
- caused mainly by development of IR subtraction schemes and improved techniques for loop integrals
- automation possible to some extent
 - collections of processes in same framework available
 - NNLOJet Durham/Uni Zurich++
 - MCFM v8 Campbell, Ellis, Giele, Neumann, Williams et al.
 - MATRIX Grazzini, Kallweit, Wiesemann et al.

NNLO

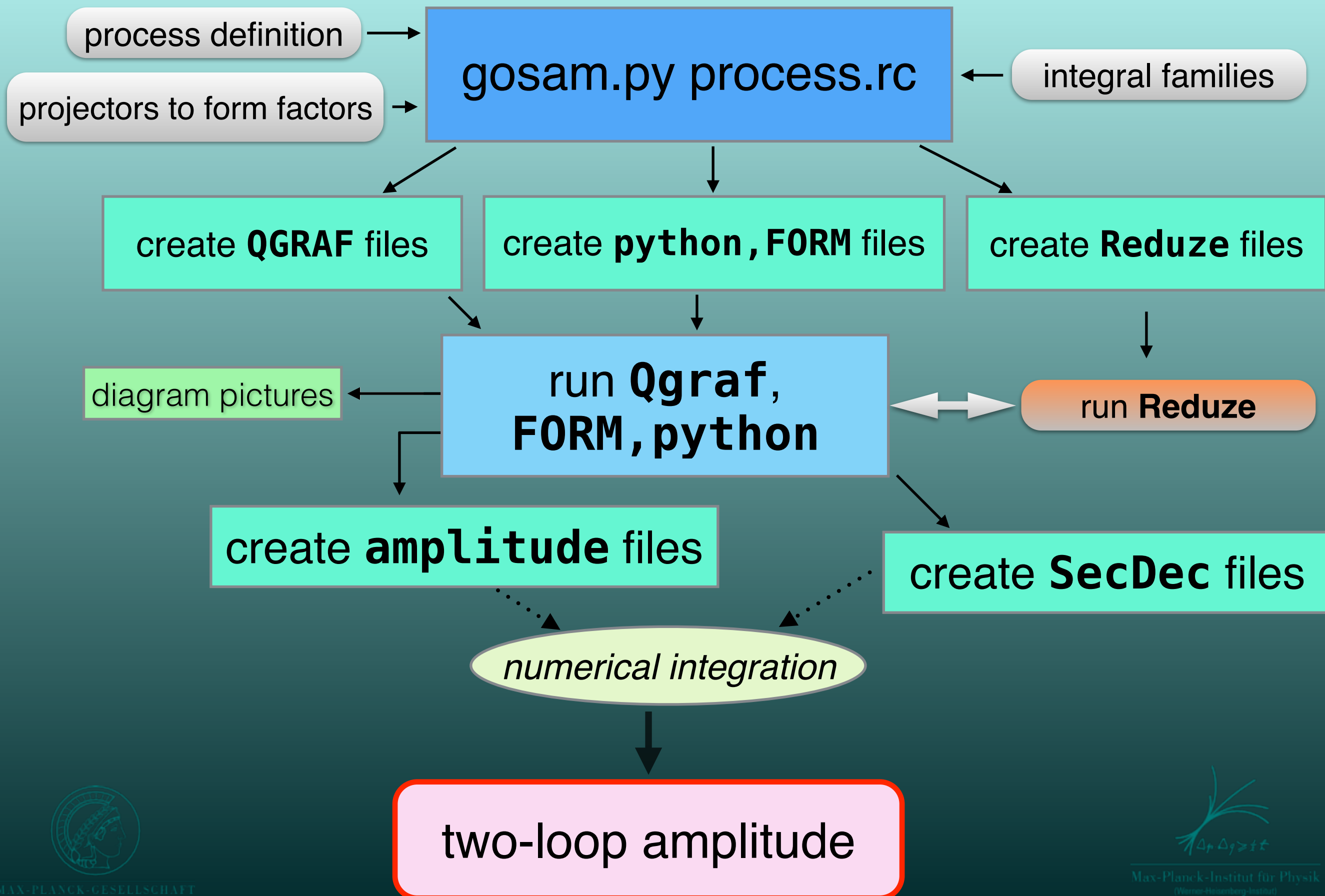
- enormous progress in the last few years
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→ collections of processes in same framework available

- NNLOJet Durham/Uni Zurich++
- MCFM v8 Campbell, Ellis, Giele, Neumann, Williams et al.
- MATRIX Grazzini, Kallweit, Wiesemann et al.

our plan: automated generation of 2-loop amplitudes “on the fly”

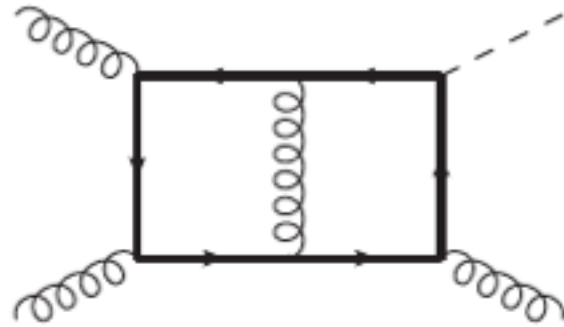
automated 2-loop amplitudes: GoSam @ 2 loops



Loop induced processes at NLO

Stephen Jones,
QCD@LHC '18

HJ Production

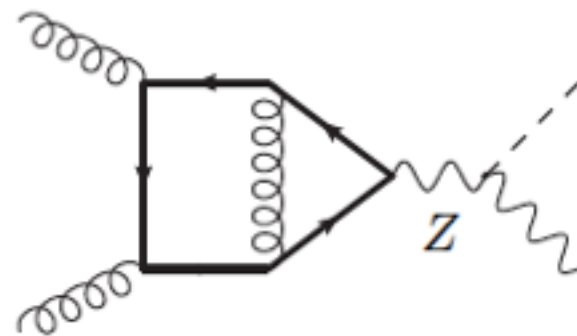
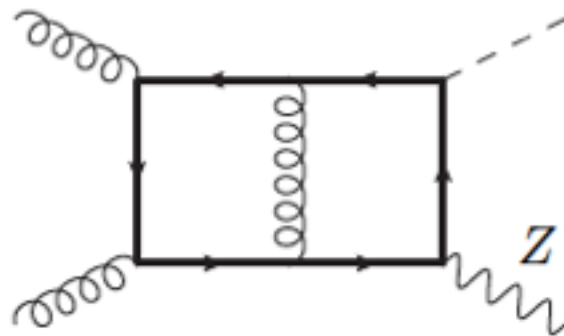


Contributes to Higgs p_T distribution

Important for large $p_T \gtrsim m_T$

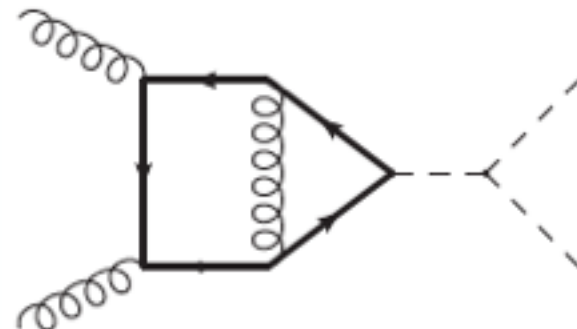
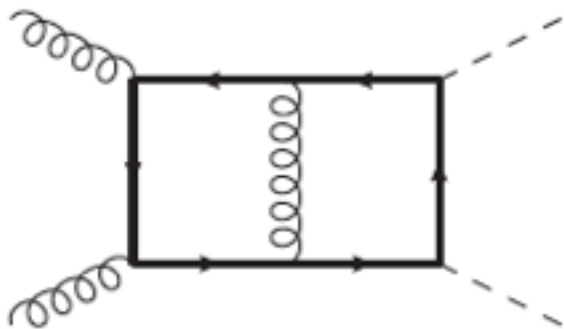
At large p_T can "probe inside the loop", distributions can be modified by heavy BSM particles

HZ Production



Measurement of ZH coupling
 $gg \rightarrow ZH$ channel formally NNLO
Contributes $\sim 10\%$ @ 14 TeV

HH Production



Sensitive to Higgs self coupling,
probe of EW symmetry breaking

$$\frac{m_H^2}{2} H^2 + \boxed{\frac{m_H^2}{2v} H^3} + \frac{m_H^2}{8v^2} H^4$$

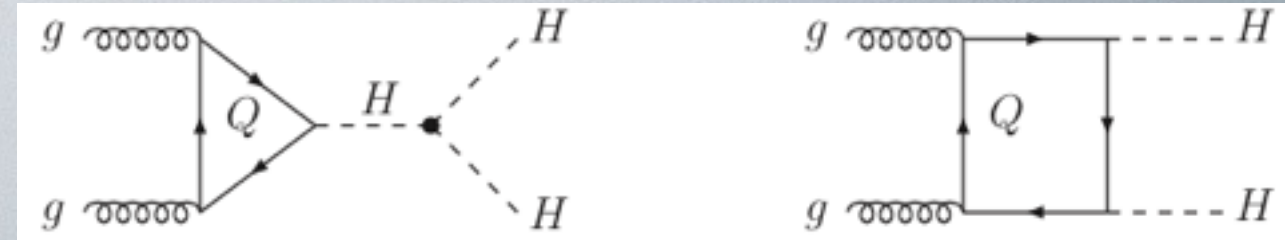
Top-quark mass corrections to e.g. $gg \rightarrow \gamma\gamma$, $gg \rightarrow ZZ$

hard due to several mass scales

HH in gluon fusion

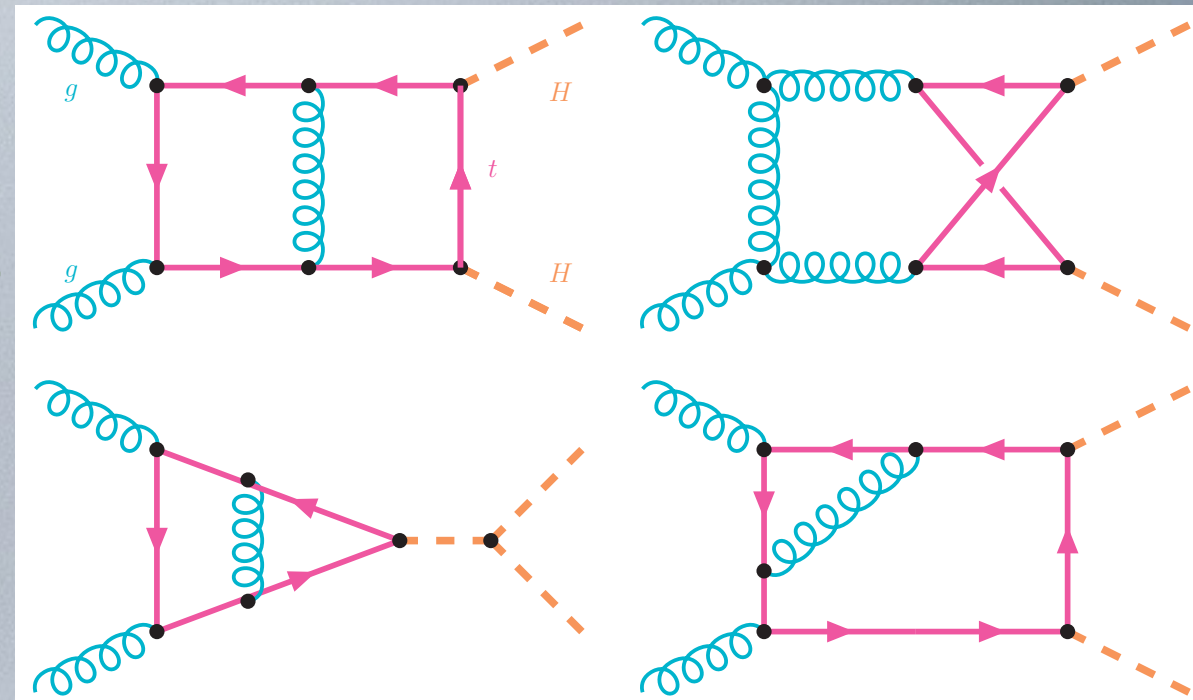
LO with full heavy quark mass dependence

Glover, van der Bij '88, Plehn, Spira, Zerwas '96



NLO with full top quark mass dependence

Borowka, Greiner, GH, Jones, Kerner, Schlenk, Schubert, Zirke '16



approximations:

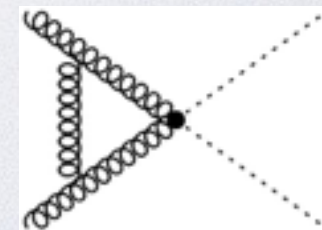
$m_t \rightarrow \infty$ limit:

“Higgs Effective Field Theory” (HEFT)



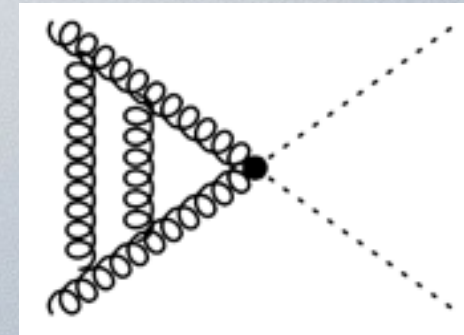
“Born-improved NLO HEFT”: rescale by $\mathcal{M}^{LO}(m_t)/\mathcal{M}_{\text{HEFT}}^{LO}$

NLO in Born-improved HEFT Dawson, Dittmaier, Spira '98 (**HPAIR**)



rescaled

HH in gluon fusion



NNLO in $m_t \rightarrow \infty$ limit:

- **total xs NNLO** De Florian, Mazzitelli '13
- **NNLO including all matching coefficients** Grigo, Melnikov, Steinhauser '14
- **differential NNLO** De Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev '16

Note:

HEFT strictly valid only for $\sqrt{\hat{s}} \ll 2m_t$ } $\Rightarrow$ validity of HEFT limited to
HH production threshold: $2m_H < \sqrt{\hat{s}}$ } $250 \text{ GeV} < \sqrt{\hat{s}} < 340 \text{ GeV}$

“best” approximation:

“approximate Full Theory” (FTapprox)

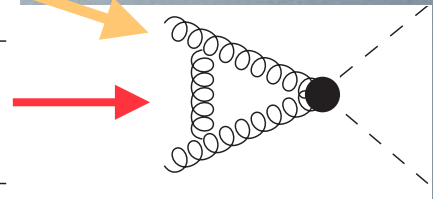
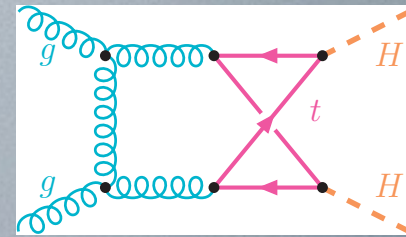
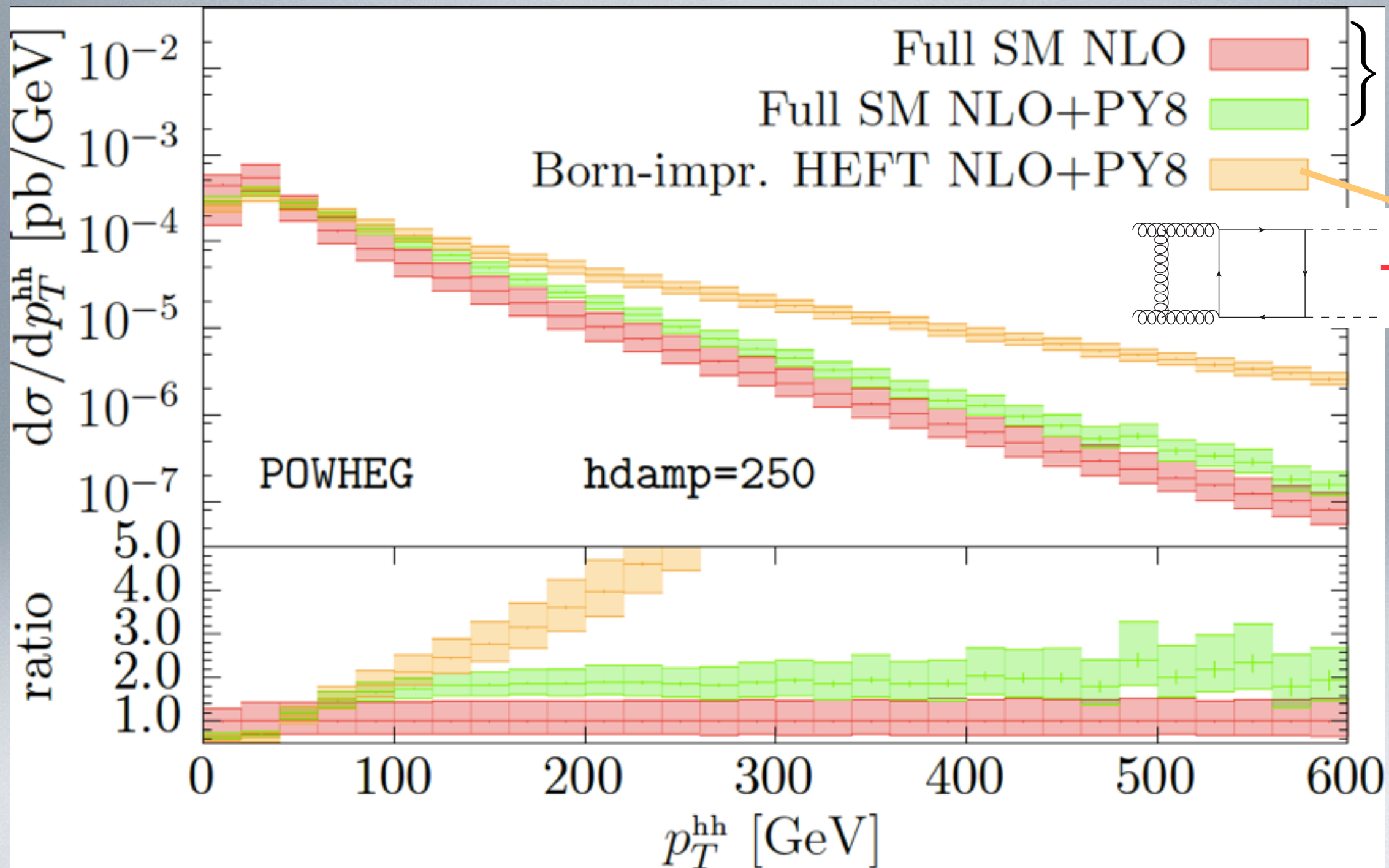
- **full mass dependence in NLO real radiation**

Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14;
Maltoni, Vryonidou, Zaro '14

- **NNLO_FTapprox** Grazzini, Kallweit, GH, Jones, Kerner, Lindert, Mazzitelli '18

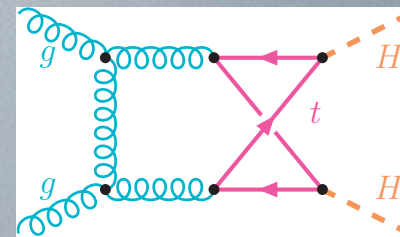
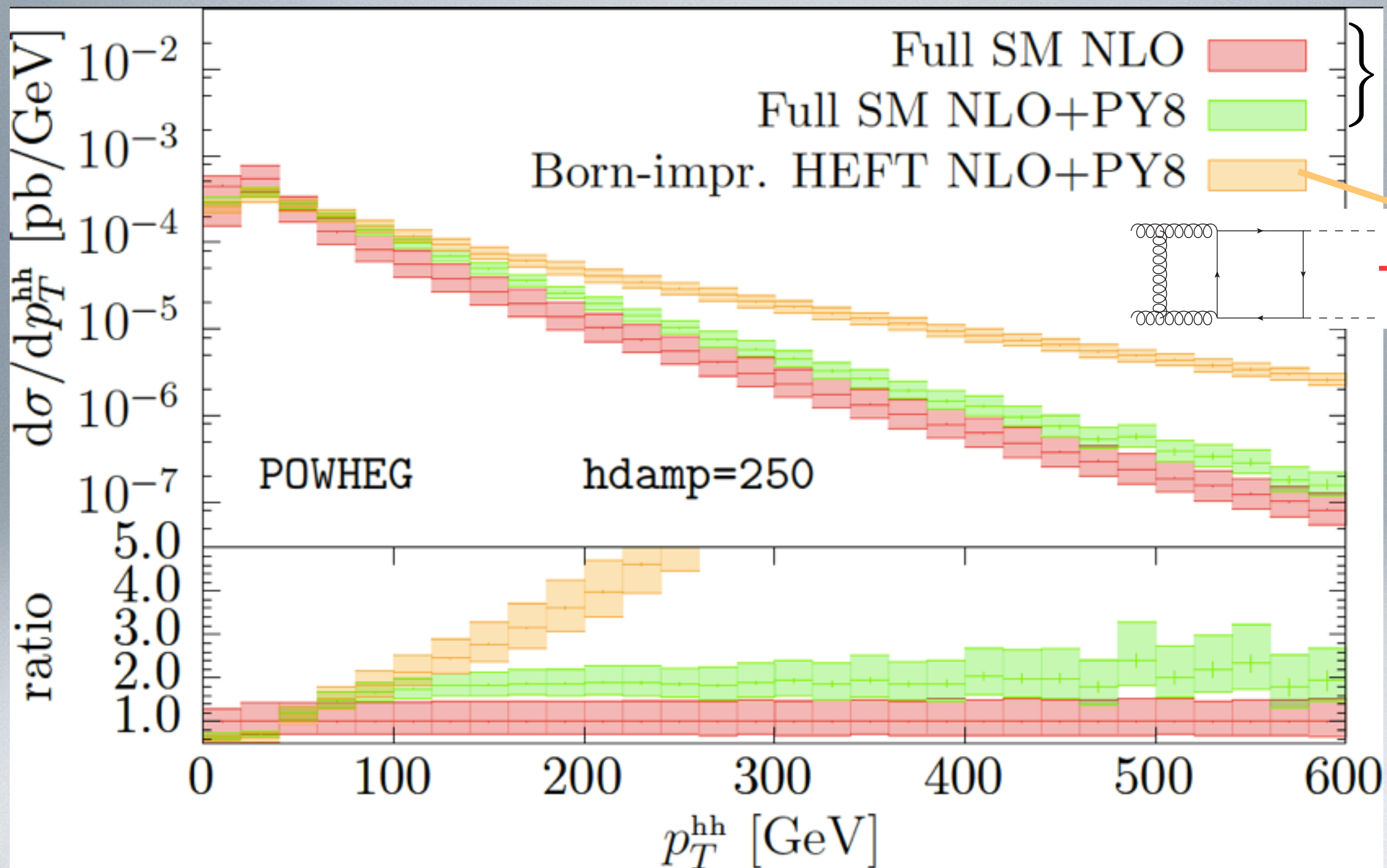
mass effects versus parton shower effects

GH, S.Jones, M.Kerner, G.Luisoni, E.Vryonidou 1703.09252



mass effects versus parton shower effects

GH, S.Jones, M.Kerner, G.Luisoni, E.Vryonidou 1703.09252

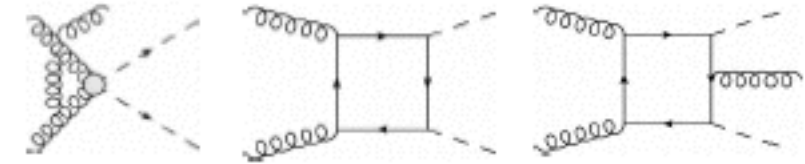


shower effects large but order(s) of magnitude smaller than difference to Born-improved HEFT

HH production: combination of full NLO with NNLO

Grazzini, Kallweit, GH, Jones, Kerner, Lindert, Mazzitelli; 1803.02463

Technical ingredients

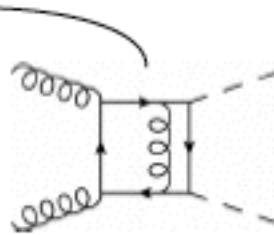


Tree-level and one-loop amplitudes (HEFT and full- M_t) → OpenLoops

[Cascioli, Lindert, Maierhofer, Pozzorini]

Full NLO (two-loop) virtual corrections → two dimensional grid + interpolation

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zirke, '16]



Analytical results for NNLO two-loop corrections in the HEFT

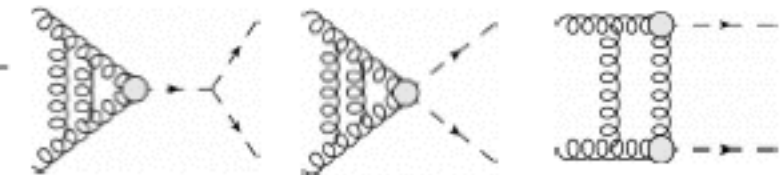
[de Florian, JM, '13]

NNLO subtraction formalism: q_T -subtraction

[Catani, Grazzini, '07]

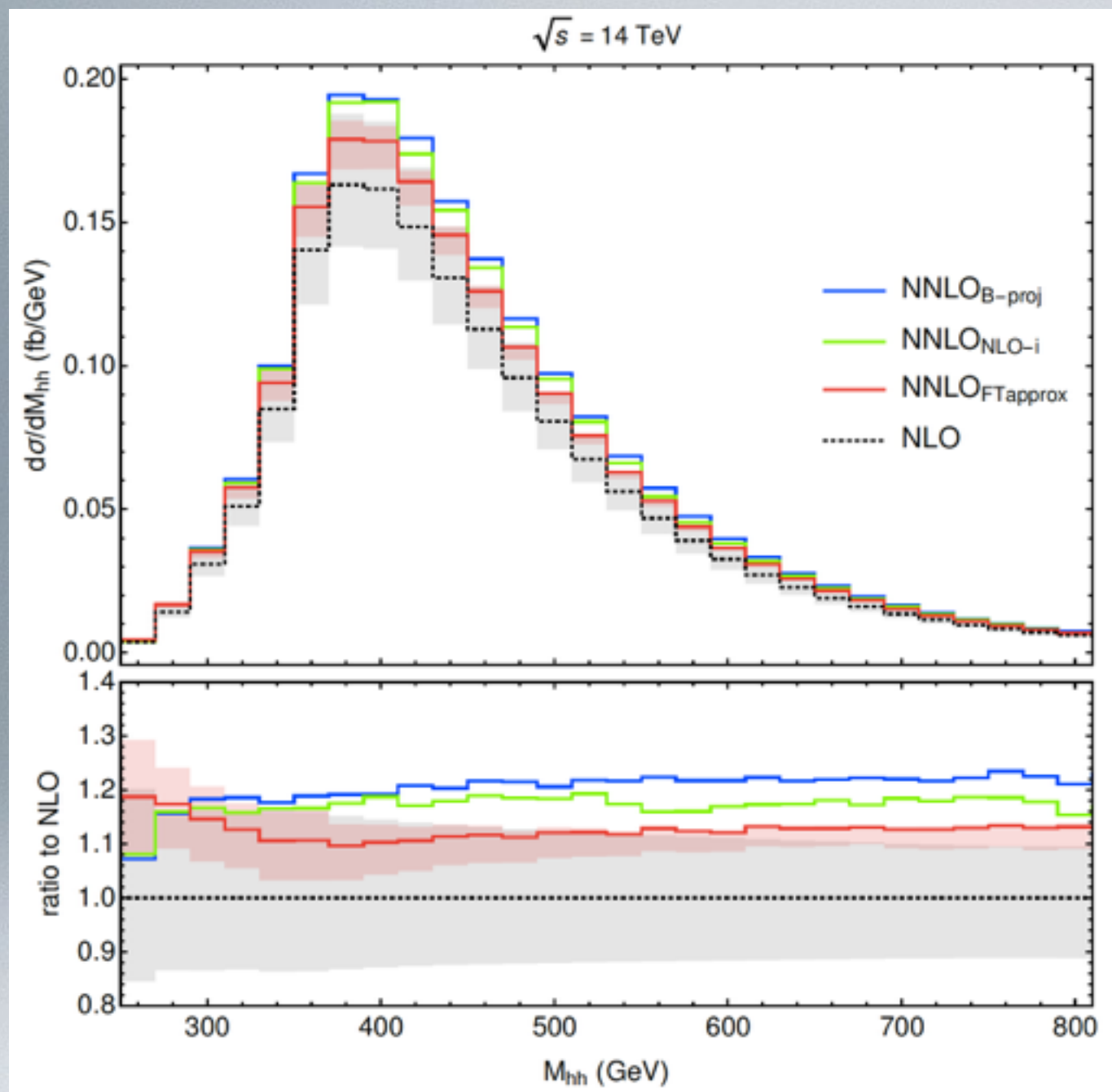
Implementation based on public code MATRIX

[Kallweit, Grazzini, Wiesemann, '17]



Javier Mazzitelli

HH production



NNLO_FTapprox:

$\mathcal{O}(\alpha_s^4)$ part: at n-loops in HEFT,

$X=2-n$ extra partons:

reweight

$$\mathcal{A}_{\text{HEFT}}^{(n)}(ij \rightarrow HH + X)$$

with

$$\mathcal{R}(ij \rightarrow HH + X) = \frac{\mathcal{A}_{\text{Full}}^{\text{Born}}(ij \rightarrow HH + X)}{\mathcal{A}_{\text{HEFT}}^{(0)}(ij \rightarrow HH + X)}$$

considerable reduction of
scale uncertainties

remaining mt uncertainty
estimated to be $\sim 5\%$

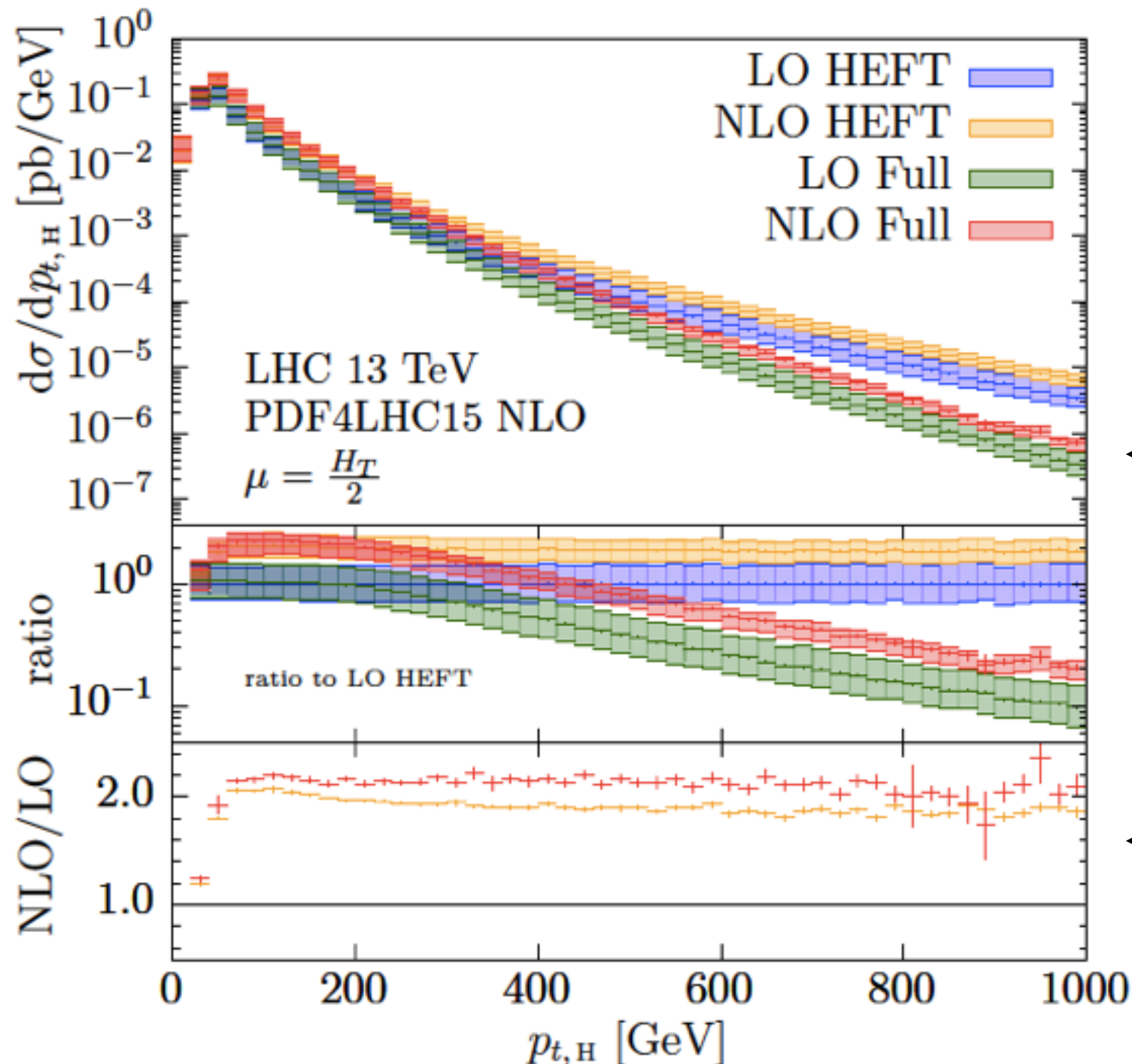
soft gluon resummation on top of NNLO_FTapprox also available

De Florian, Mazzitelli 1807.03704 resummation effects below 1% for $\mu = m_{hh}/2$

gluon fusion: Higgs pT spectrum

H+jet at NLO including full top quark mass dependence

Jones, Kerner, Luisoni, 1802.00349



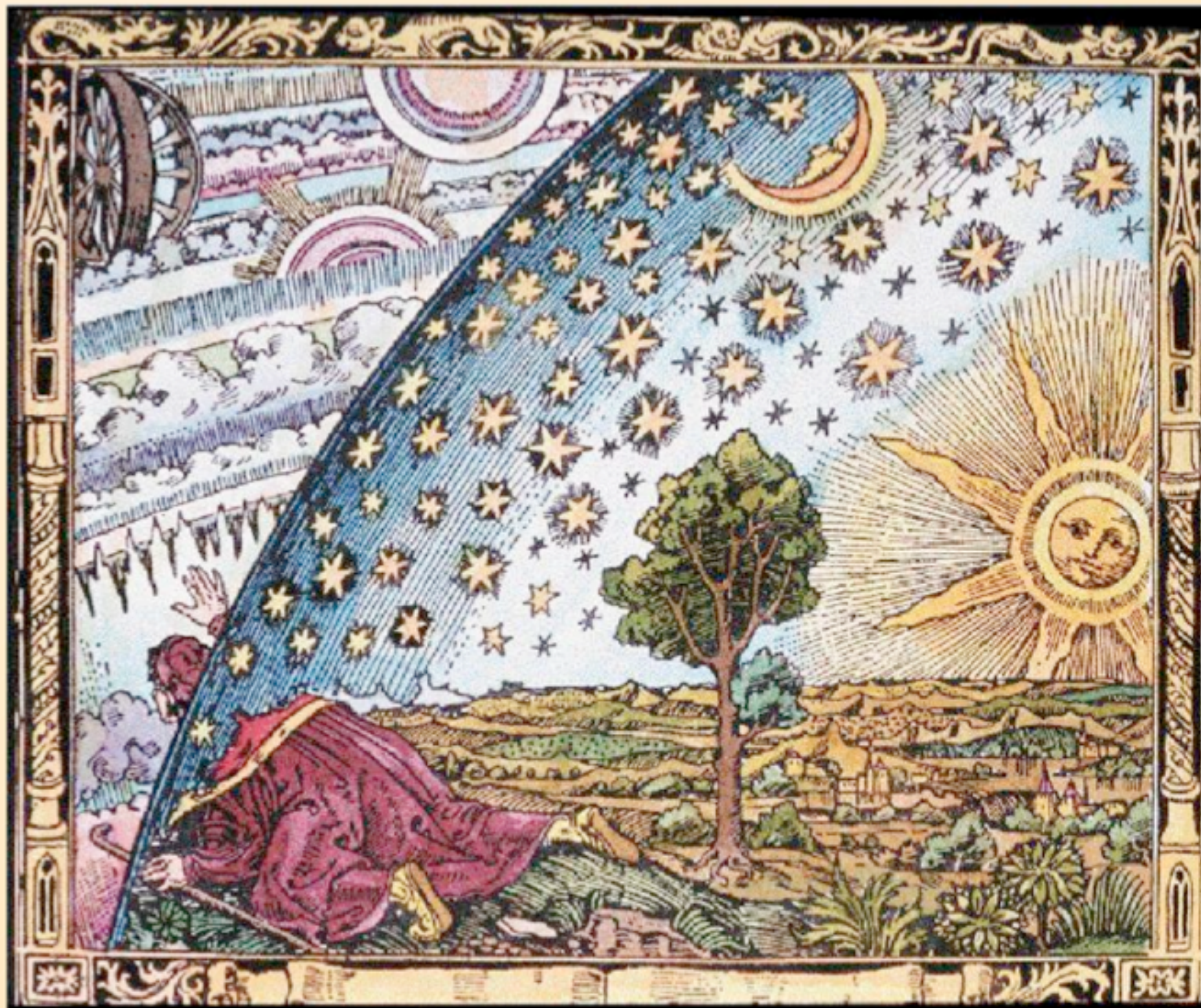
settles a longstanding question about the uncertainties due to unknown top mass effects at NLO

← full NLO: different scaling behaviour at large pT

← K-factors full vs HEFT: similar shape for $\mu = H_T/2$...

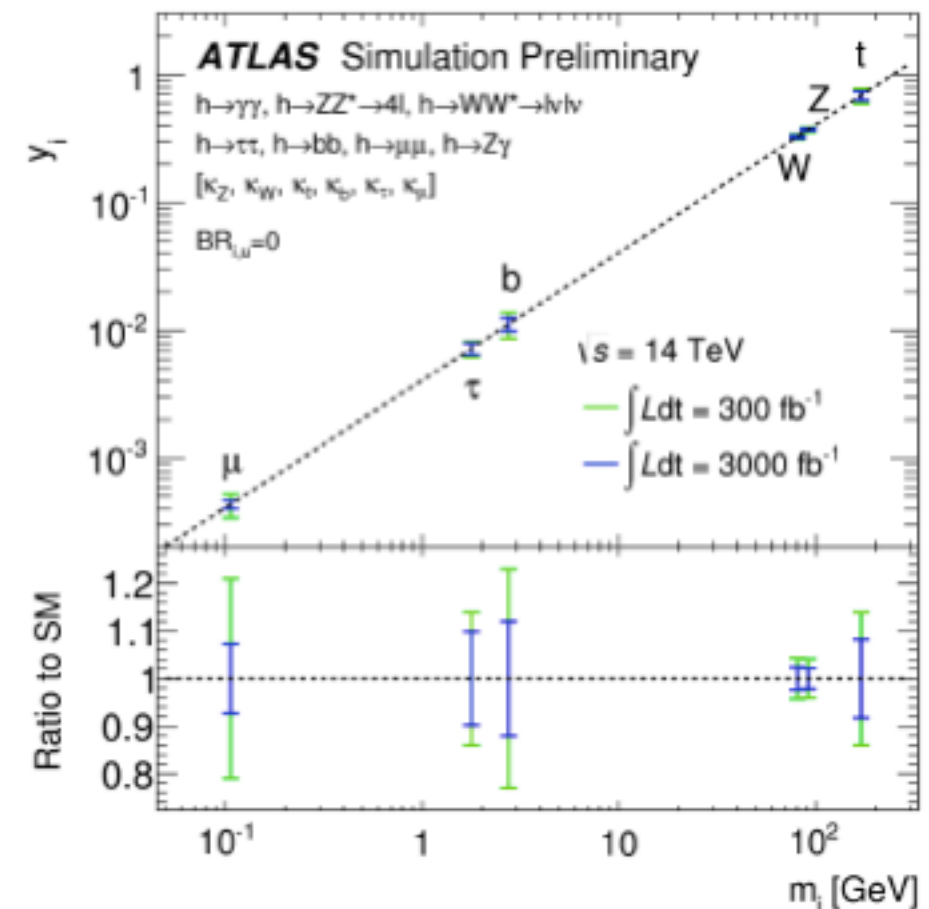
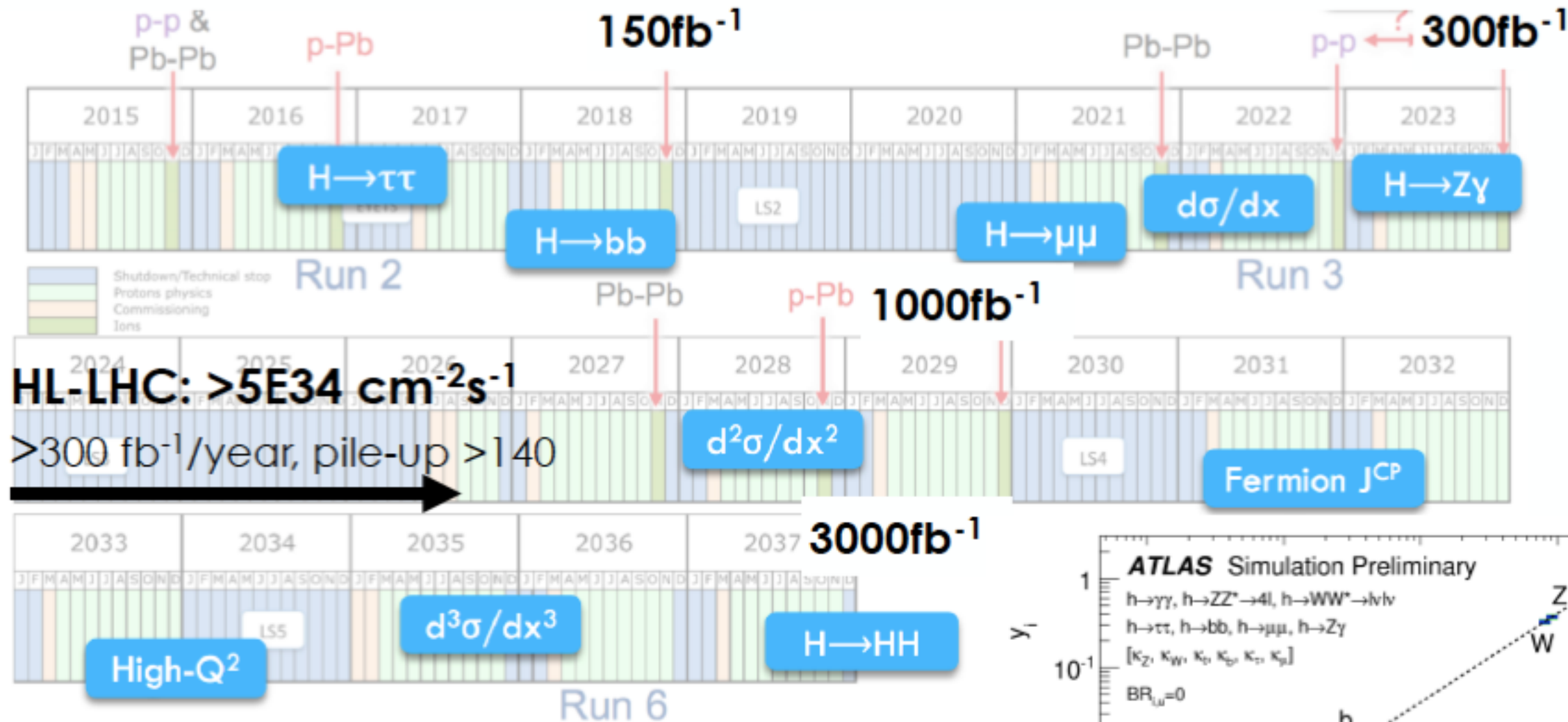
Summary

- the Higgs sector is just starting to get explored
- it is likely that New Physics is hiding in small deviations
- precision calculations and -measurements become vital
- the calculation of higher order predictions saw an amazing boost due to
 - new ideas how to calculate
 - loop integrals
 - IR divergent real radiationboth **analytically** and **numerically**
 - deeper insights into the structure of scattering amplitudes



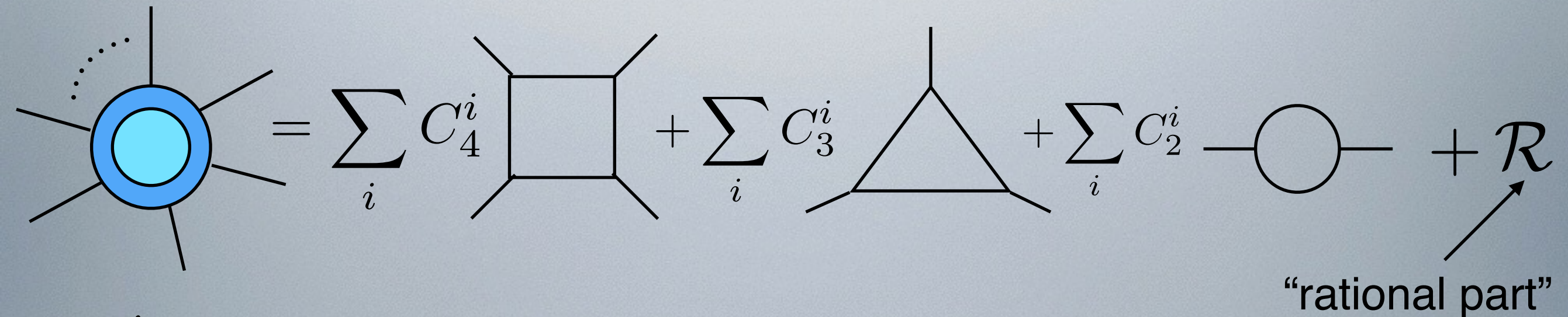
TIMELINE BEYOND RUN2

Credits: A. David @ GRC 2017



P.Meridiani, EPS 2017

One-loop n-point amplitudes



The diagram shows a general one-loop n-point amplitude on the left, represented by a blue circle with a smaller blue circle inside, and several external lines. This is equated to a sum of three terms: a box integral (a square with four external lines) multiplied by $\sum_i C_4^i$, a triangle integral (a triangle with three external lines) multiplied by $\sum_i C_3^i$, and a bubble integral (a circle with two external lines) multiplied by $\sum_i C_2^i$. The sum is followed by a plus sign and a symbol $\mathcal{R}$, which is labeled "rational part" with an arrow pointing to it.

$$\text{One-loop } n\text{-point amplitude} = \sum_i C_4^i \text{ (box)} + \sum_i C_3^i \text{ (triangle)} + \sum_i C_2^i \text{ (bubble)} + \mathcal{R}$$

“rational part”

C_n^i can be obtained by numerical reduction at integrand level

“master integrals”: boxes, triangles, bubbles, tadpoles

most complicated functions are dilogarithms

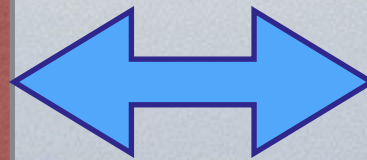
very different at two loops (and beyond)!

master integrals not a priori known

NLO automation

Monte Carlo program

- tree amplitudes
- infrared subtractions
- phase space integration/
event generation
- parton shower (optional)



BLHA or
custom made

One-loop provider

- virtual amplitude

- Powheg
- Sherpa
- Herwig7/Matchbox
- Geneva
- Vincia

- Blackhat
- FeynArts
- GoSam
- Madloop
- NJet
- OpenLoops
- Recola

all in one:

- MG5_aMC@NLO
- Helac-NLO
- Grace

collection of pre-computed processes:

- MCFM
- VBFNLO

NNLO real radiation subtraction methods

two main categories: “subtraction” and “slicing”

subtraction:

subtract piece which leads to IR divergences and add it back in integrated form (analytic integration over factorized phase space)

$$\begin{aligned} d\hat{\sigma}_{ab,NNLO} = & \int_{n+2} \left[d\hat{\sigma}_{ab,NNLO}^{RR} - d\hat{\sigma}_{ab,NNLO}^S \right] \\ & + \int_{n+1} \left[d\hat{\sigma}_{ab,NNLO}^{RV} - d\hat{\sigma}_{ab,NNLO}^T \right] \\ & + \int_n \left[d\hat{\sigma}_{ab,NNLO}^{VV} - d\hat{\sigma}_{ab,NNLO}^U \right] \end{aligned}$$

$$\begin{aligned} d\hat{\sigma}_{ab,NNLO}^T &= d\hat{\sigma}_{ab,NNLO}^{V,S} - d\hat{\sigma}_{ab,NNLO}^{MF,1} - \int_1 d\hat{\sigma}_{ab,NNLO}^S, \\ d\hat{\sigma}_{ab,NNLO}^U &= -d\hat{\sigma}_{ab,NNLO}^{MF,2} - \int_1 d\hat{\sigma}_{ab,NNLO}^{V,S} - \int_2 d\hat{\sigma}_{ab,NNLO}^S \end{aligned}$$

$$\begin{aligned} d\hat{\sigma}_{ab,NNLO} = & \int_{n+2} d\hat{\sigma}_{ab,NNLO}^{RR} + \int_{n+1} \left[d\hat{\sigma}_{ab,NNLO}^{RV} + d\hat{\sigma}_{ab,NNLO}^{MF,1} \right] \\ & + \int_n \left[d\hat{\sigma}_{ab,NNLO}^{VV} + d\hat{\sigma}_{ab,NNLO}^{MF,2} \right], \end{aligned}$$

NNLO real radiation subtraction methods

slicing: define a cut parameter to separate IR sensitive part and split cross section into contributions above and below the cut

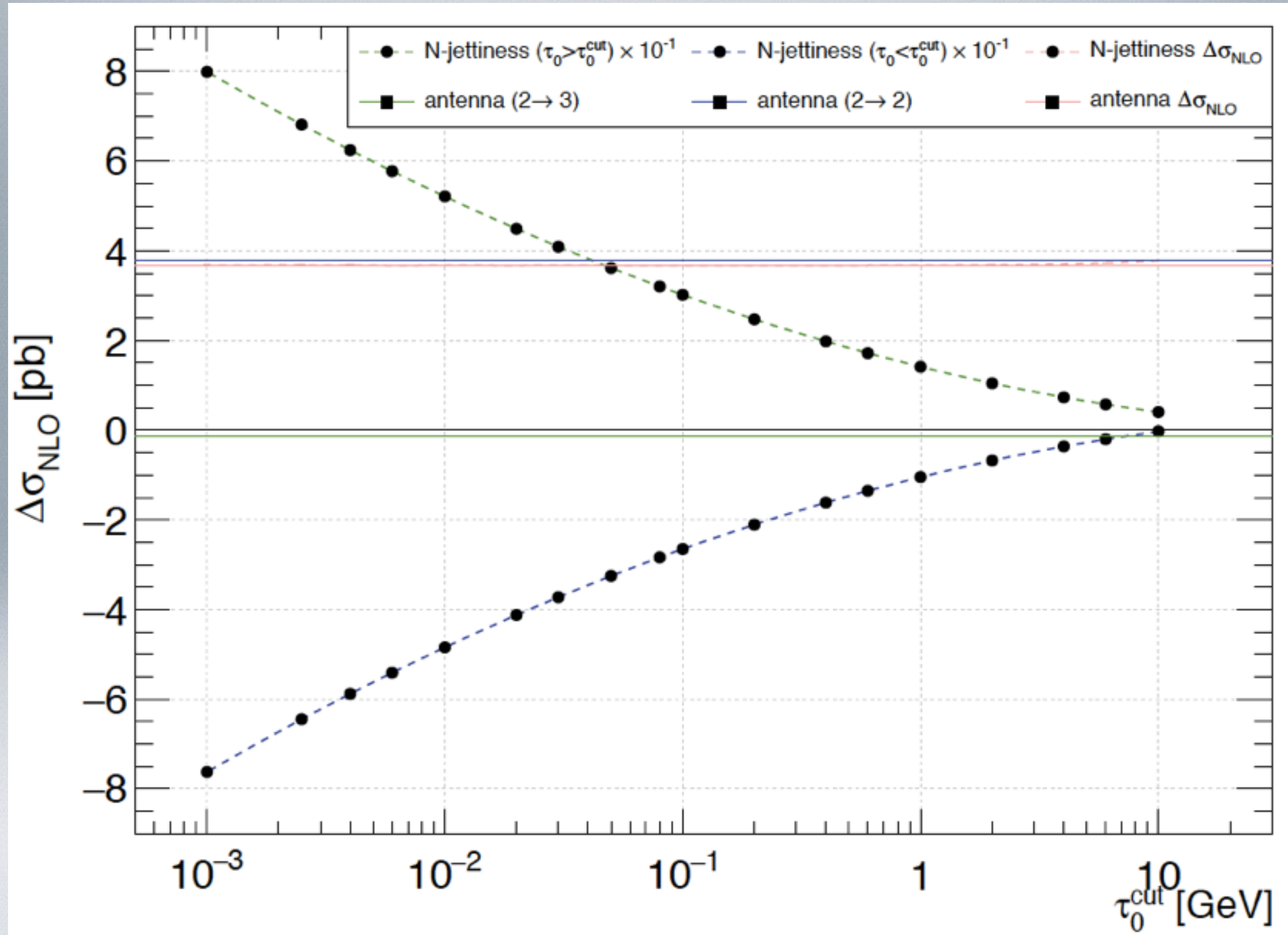
$$\begin{aligned}\sigma_{NNLO} &= \int d\Phi_N |\mathcal{M}_{VV}|^2 + \int d\Phi_{N+1} |\mathcal{M}_{RV}|^2 \theta_0^< \\ &+ \int d\Phi_{N+2} |\mathcal{M}_{RR}|^2 \theta_0^< + \int d\Phi_{N+1} |\mathcal{M}_{RV}|^2 \theta_0^> \\ &+ \int d\Phi_{N+2} |\mathcal{M}_{RR}|^2 \theta_0^> \\ &\equiv \sigma_{NNLO}(\mathcal{T}_0 < \mathcal{T}_0^{cut}) + \sigma_{NNLO}(\mathcal{T}_0 > \mathcal{T}_0^{cut}) .\end{aligned}$$

$$\theta_0^< = \theta(\mathcal{T}_0^{cut} - \mathcal{T}_0)$$

below $\mathcal{T}_0^{cut}$: cross section can be obtained from universal IR behaviour (SCET, resummation)

note: both parts depend logarithmically on $\mathcal{T}_0^{cut} \Rightarrow$ large cancellations
residual cutoff dependence

comparison local vs non-local subtraction method



example ZZ production GH, Jahn, Jones, Kerner, Pires '17

MATRIX

process	status	comment
$pp \rightarrow \mathbf{Z}/\gamma^*(\rightarrow \ell\ell/\nu\nu)$	✓	validated analytically + FEWZ
$pp \rightarrow \mathbf{W}(\rightarrow \ell\nu)$	✓	validated with FEWZ , NNLOjet
$pp \rightarrow \mathbf{H}$	✓	validated analytically (by SusHi)
$pp \rightarrow \gamma\gamma$	✓	validated with 2γNNLO
$pp \rightarrow \mathbf{Z}\gamma \rightarrow \ell\ell\gamma$	✓	[Grazzini, Kallweit, Rathlev '15]
$pp \rightarrow \mathbf{Z}\gamma \rightarrow \nu\nu\gamma$	✓	[Grazzini, Kallweit, Rathlev '15]
$pp \rightarrow \mathbf{W}\gamma \rightarrow \ell\nu\gamma$	✓	[Grazzini, Kallweit, Rathlev '15]
$pp \rightarrow \mathbf{ZZ}$	✓	[Cascioli et al. '14]
$pp \rightarrow \mathbf{ZZ} \rightarrow \ell\ell\ell\ell$	✓	[Grazzini, Kallweit, Rathlev '15]
$pp \rightarrow \mathbf{ZZ} \rightarrow \ell\ell\ell'\ell'$	✓	[Grazzini, Kallweit, Rathlev '15]
$pp \rightarrow \mathbf{ZZ} \rightarrow \ell\ell\nu'\nu'$	✓	[Kallweit, MW '18]
$pp \rightarrow \mathbf{ZZ}/\mathbf{WW} \rightarrow \ell\ell\nu\nu$	✓	[Kallweit, MW '18]
$pp \rightarrow \mathbf{WW}$	✓	[Gehrmann et al. '14]
$pp \rightarrow \mathbf{WW} \rightarrow \ell\nu\ell'\nu'$	✓	[Grazzini, Kallweit, Pozzorini, Rathlev, MW '16]
$pp \rightarrow \mathbf{WZ}$	✓	[Grazzini, Kallweit, Rathlev, MW '16]
$pp \rightarrow \mathbf{WZ} \rightarrow \ell\nu\ell\ell$	✓	[Grazzini, Kallweit, Rathlev, MW '17]
$pp \rightarrow \mathbf{WZ} \rightarrow \ell'\nu'\ell\ell$	✓	[Grazzini, Kallweit, Rathlev, MW '17]
$pp \rightarrow \mathbf{HH}$	(✓)	not in public release yet

NNLOJET

The method of antenna subtraction is implemented in the NNLOJET program, a semi-automated Monte Carlo for NNLO phenomenology.

Processes

Many processes are already included at NNLO:

- ▶ $pp \rightarrow H + 0,1 \text{ jets}$ [[arXiv:1408.5325](#)],
- ▶ $pp \rightarrow Z(l^+l^-) + 0,1 \text{ jets}$ [[arXiv:1607.01749](#)],
- ▶ $pp \rightarrow W(l^+l^-) + 0,1 \text{ jets}$ [[arXiv:1712.07543](#)],
- ▶ NC & CC DIS single/dijets [[arXiv:1606.03991](#)],
- ▶ NC DIS single jet ($N^3\text{LO}$) [[arXiv:1803.09973](#)],
- ▶ $pp \rightarrow \text{dijets}$ [[arXiv:1611.01460](#)] (Joao tomorrow).
- ▶ $e^+e^- \rightarrow 3 \text{ jets}$ [[arXiv:1709.01097](#)],
- ▶ VBF at NNLO [[arXiv:1802.02445](#)]

Development and optimization of MCFM

Downloadable from mcfm.fnal.gov

- A Multi-Threaded Version of MCFM, J.M. Campbell, R.K. Ellis, W. Giele, 2015
- Color singlet production at NNLO in MCFM,
R. Boughezal, J.M. Campbell, R.K. Ellis, C. Focke, W. Giele, X. Liu, F. Petriello, C. Williams, 2016

The publicly available code MCFM 8.0 provides predictions for color singlet production at Next-to-Next-to Leading Order. The code runs in parallel using a hybrid openMP/MPI version of Vegas enabling MCFM to get accurate distributions in a few hours of runtime on moderate clusters of a few hundred computing cores. Included processes in this version are *W*, *Z*, *H*, *WH*, *ZH* and *di-photon* production. In the next releases more processes will be included: *Z-photon*, *WW*, *ZZ*, *W-jet*, *Z-jet*, *H-jet* and *photon-jet*.

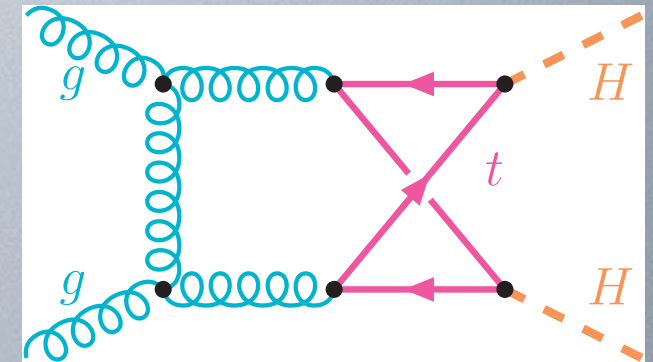
some details on virtual 2-loop diagrams

- amplitude reduction with **Reduze** [C. Studerus, A. v.Manteuffel]

- non-planar integrals computed mostly without reduction

- all integrals calculated numerically with **SecDec**

[S.Borowka et al.]



- total number of integrals after decomposition 11244,
3086 non-planar

- integration with quasi Monte Carlo method
($\mathcal{O}(1/n)$ scaling of integration errors)

[Dick, Kuo, Sloan '13] [Li, Wang, Yan, Zhao '15]

- implemented in OpenCL, evaluated on GPUs

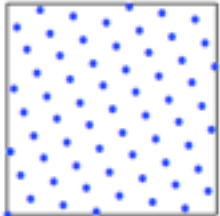
[S. Jones, M. Kerner]

- number of sampling points dynamically
set for each integral [M. Kerner]

QMC rank-1 lattice rule

$$I = \int d\vec{x} f(\vec{x}) \approx I_k = \frac{1}{n} \sum_{i=1}^n f(\vec{x}_{i,k})$$

$$\vec{x}_{i,k} = \left\{ \frac{i \cdot \vec{g}}{n} + \vec{\Delta}_k \right\}$$

$\{\dots\}$ = fractional part 

$\vec{g}$ = generating vector

$\vec{\Delta}_k$ = randomized shift

m different estimates $I_1 \dots I_m$
→ error estimate

combination with NNLO

three approximations:

- NLO-improved NNLO HEFT **NNLO_{NLO-i}**

$$\frac{d\sigma^{\text{NLO-i.NNLO HEFT}}}{dm_{hh}} = \frac{d\sigma_{\text{NLO}}}{dm_{hh}} \times \frac{d\sigma_{\text{NNLO}}^{\text{HEFT}}/dm_{hh}}{d\sigma_{\text{NLO}}^{\text{HEFT}}/dm_{hh}}$$

bin-by-bin rescaling at observable level by NNLO HEFT K-factor

- Born-projected **NNLO_{B-proj}** only piece where
mt dependence enters

reweight each NNLO event by the ratio $\text{Born}^{\text{full}}/\text{Born}^{\text{HEFT}}$

different final state multiplicities in single/double real part → need projection (not unique)
use qT recoil method Catani, De Florian, Ferrera, Grazzini 1507. 06937

- “approximate Full Theory” **NNLO_{FTapprox}**

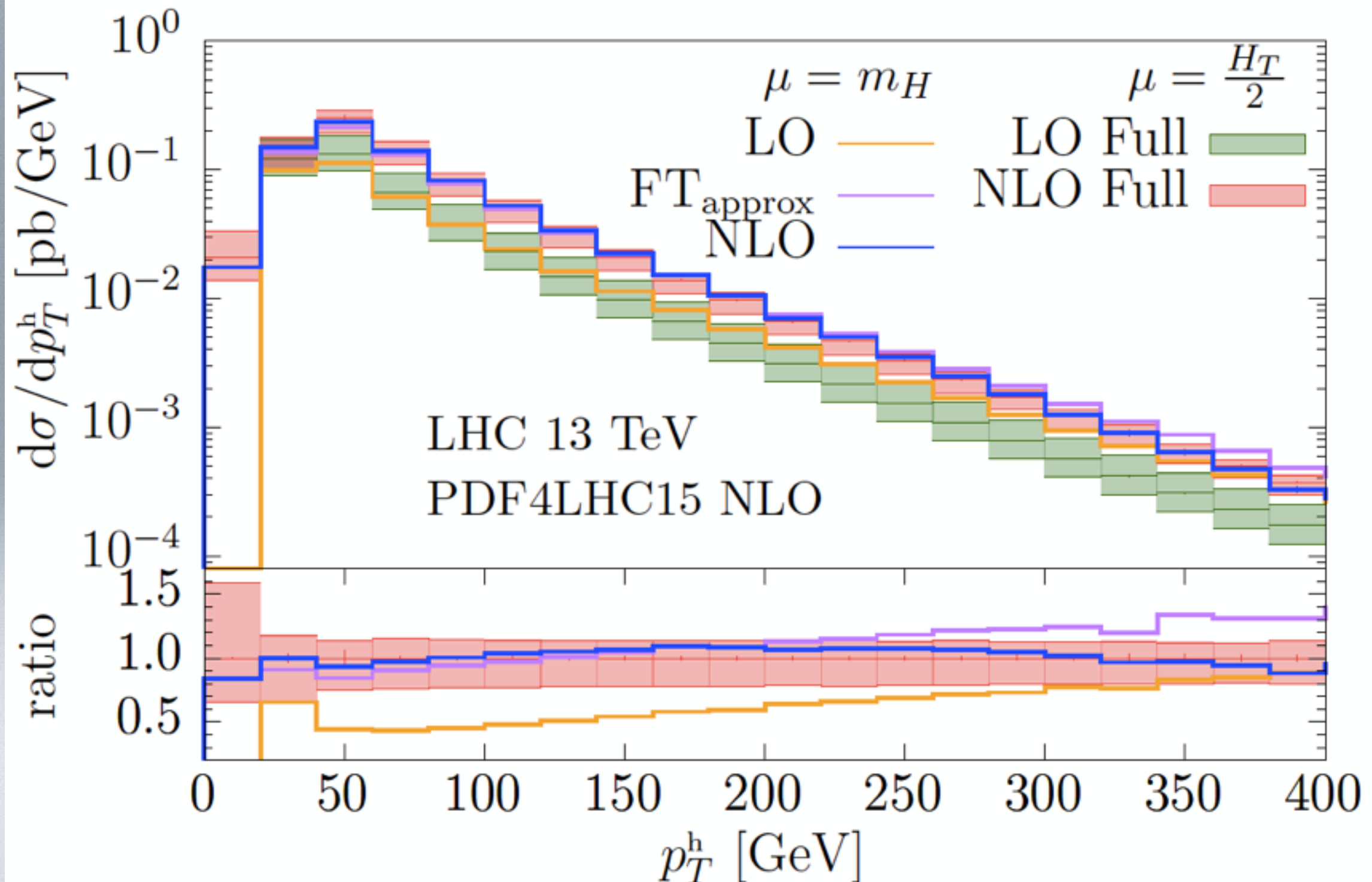
$\mathcal{O}(\alpha_s^4)$ part: at n-loops in HEFT, X=2-n extra partons: reweight $\mathcal{A}_{\text{HEFT}}^{(n)}(ij \rightarrow HH + X)$

with

$$\mathcal{R}(ij \rightarrow HH + X) = \frac{\mathcal{A}_{\text{Full}}^{\text{Born}}(ij \rightarrow HH + X)}{\mathcal{A}_{\text{HEFT}}^{(0)}(ij \rightarrow HH + X)}$$

H+jet at NLO including full top quark mass dependence

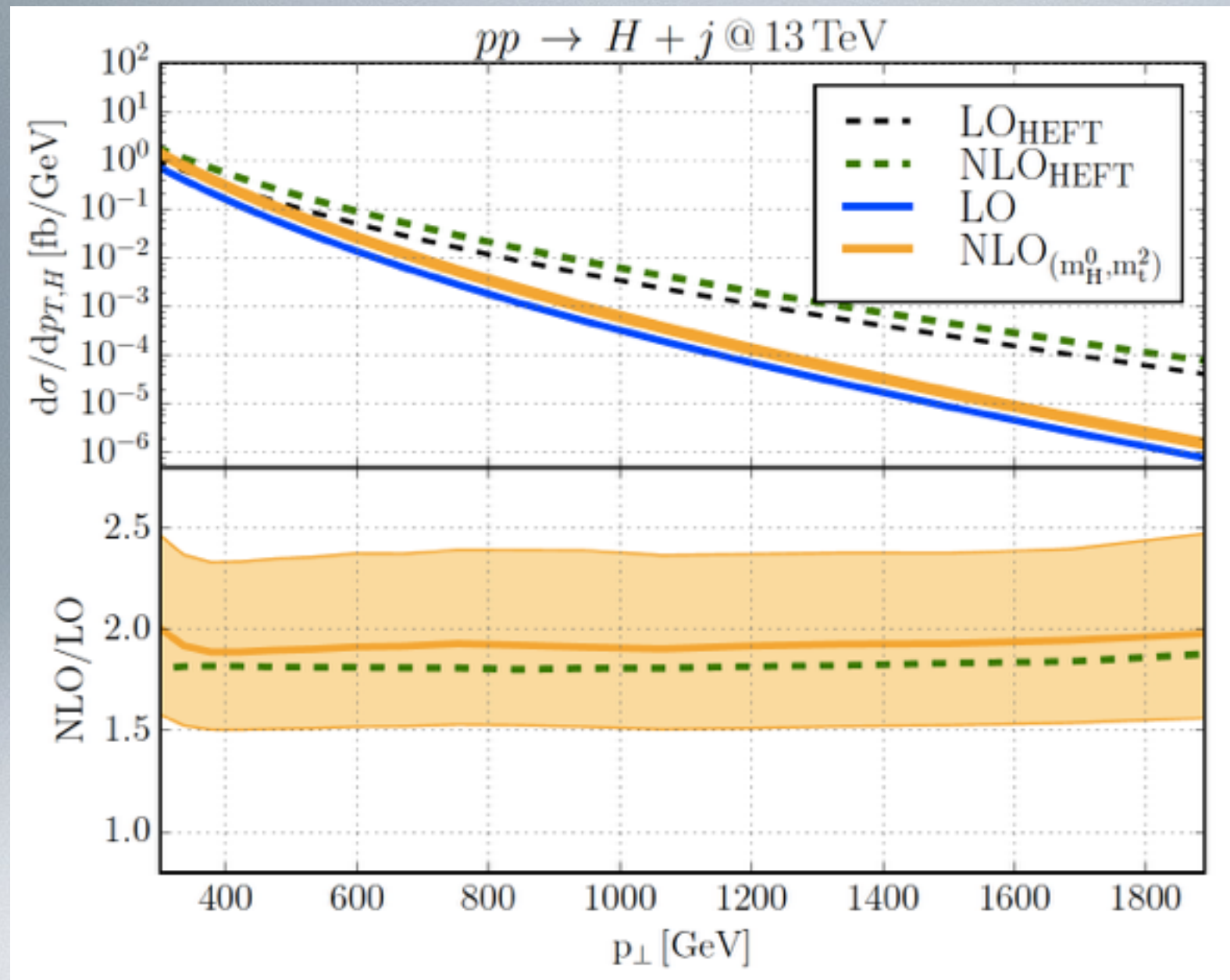
Jones, Kerner, Luisoni, 1802.00349



... but K-factors not flat for fixed scale choice $\mu = m_H$

Higgs pT spectrum

large transverse momentum expansion:



Lindert, Kudashkin, Melnikov,
Wever 1801.08226

full mass dependence in
real radiation

$$\mu = H_T/2$$

[see also Neumann1802.02981]
(MCFM)

similar behaviour of K-factors full vs HEFT (HEFT: $m_t \rightarrow \infty$ limit)
(but not of distribution!)

Higgs pT spectrum

- Top-bottom interference effects in Higgs boson production

Lindert, Melnikov, Tancredi, Wever '17

- b-quark effects in Higgs production at intermediate pT(H):

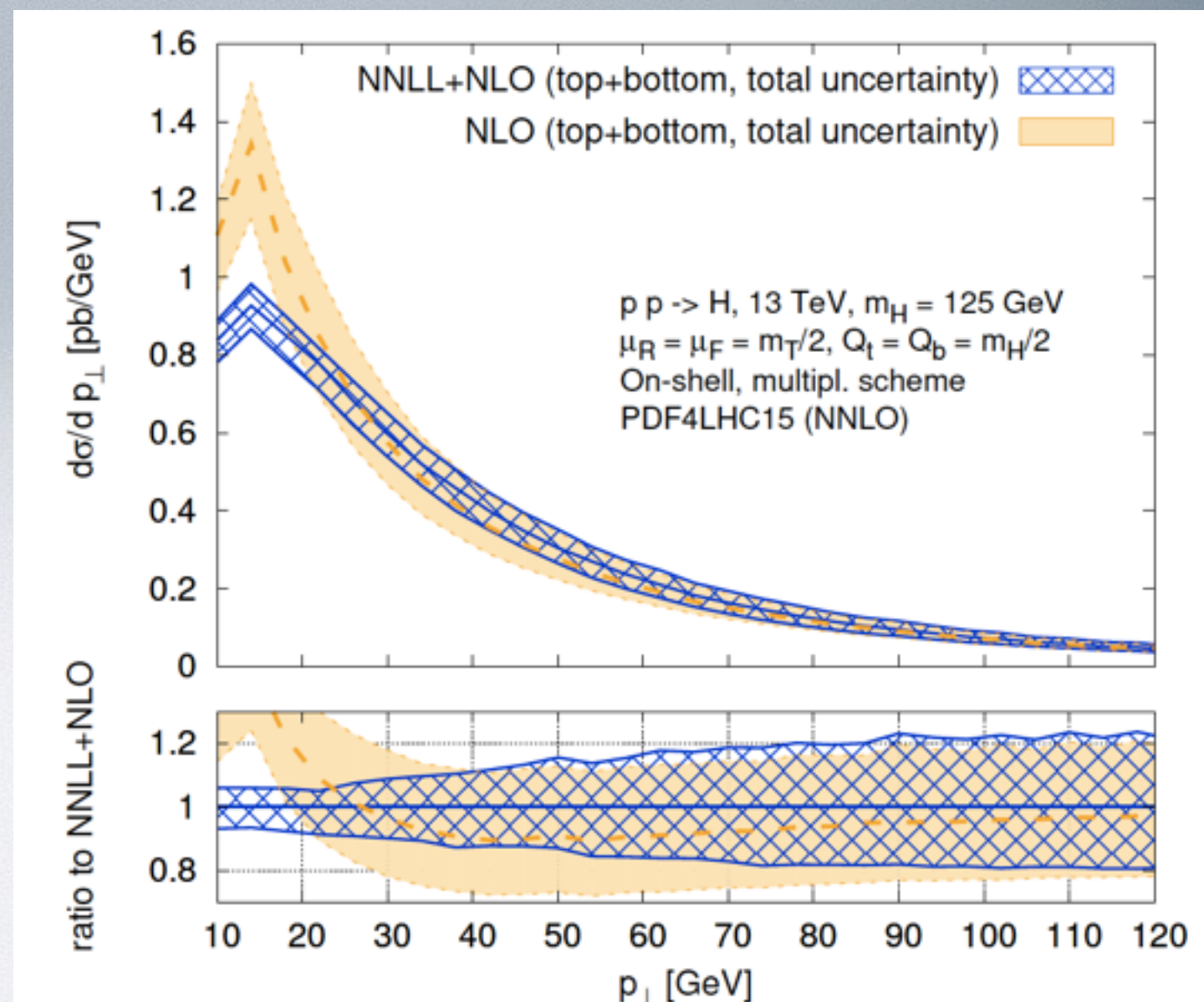
resummation in region $m_b \lesssim p_{T,H} \lesssim m_H$

Caola, Lindert, Kudashkin, Melnikov, Monni, Tancredi, Wever 1804.07632

see also Grazzini, Sargsyan '13

current uncertainty in top-bottom interference contribution to pT(H) spectrum estimated to be O(20%)

(scales, matching scheme, b-mass scheme)

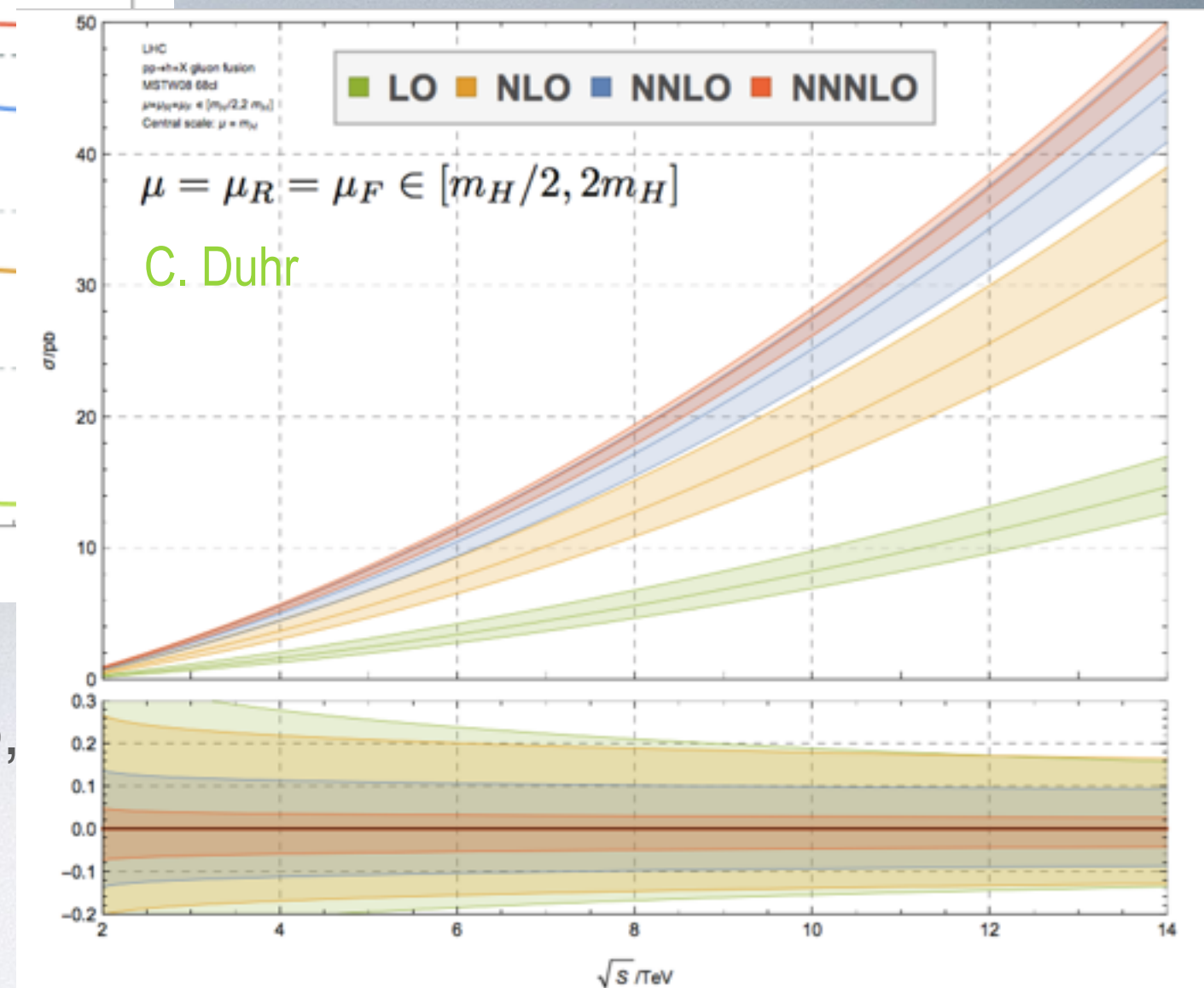
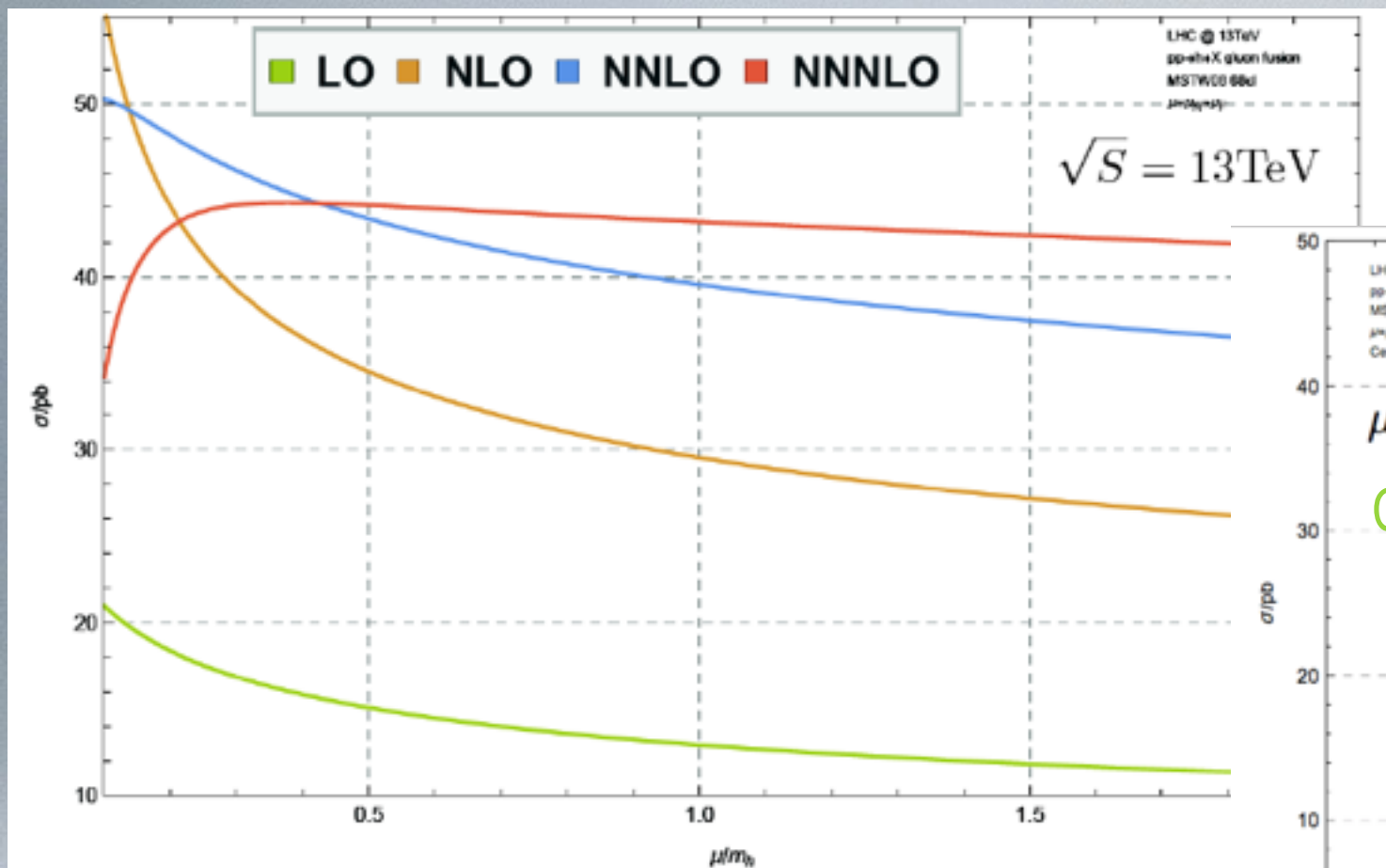


Higgs production at N3LO

$gg \rightarrow H$ inclusive

Anastasiou, Dulat, Duhr, Mistlberger '15;

Anastasiou, Dulat, Duhr, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger '16



note:
NLO and NNLO bands do not overlap,
NNLO and N³LO do

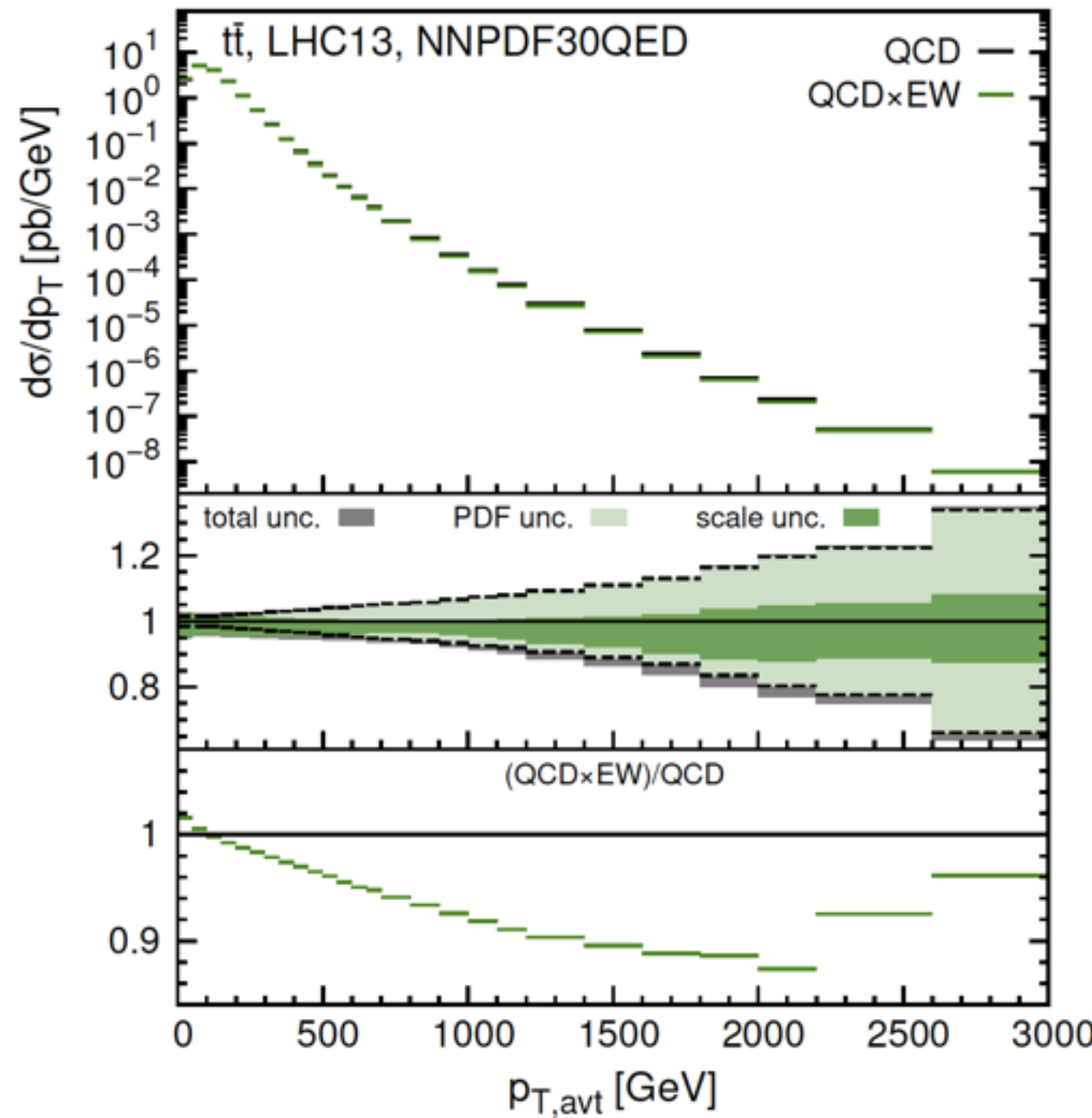
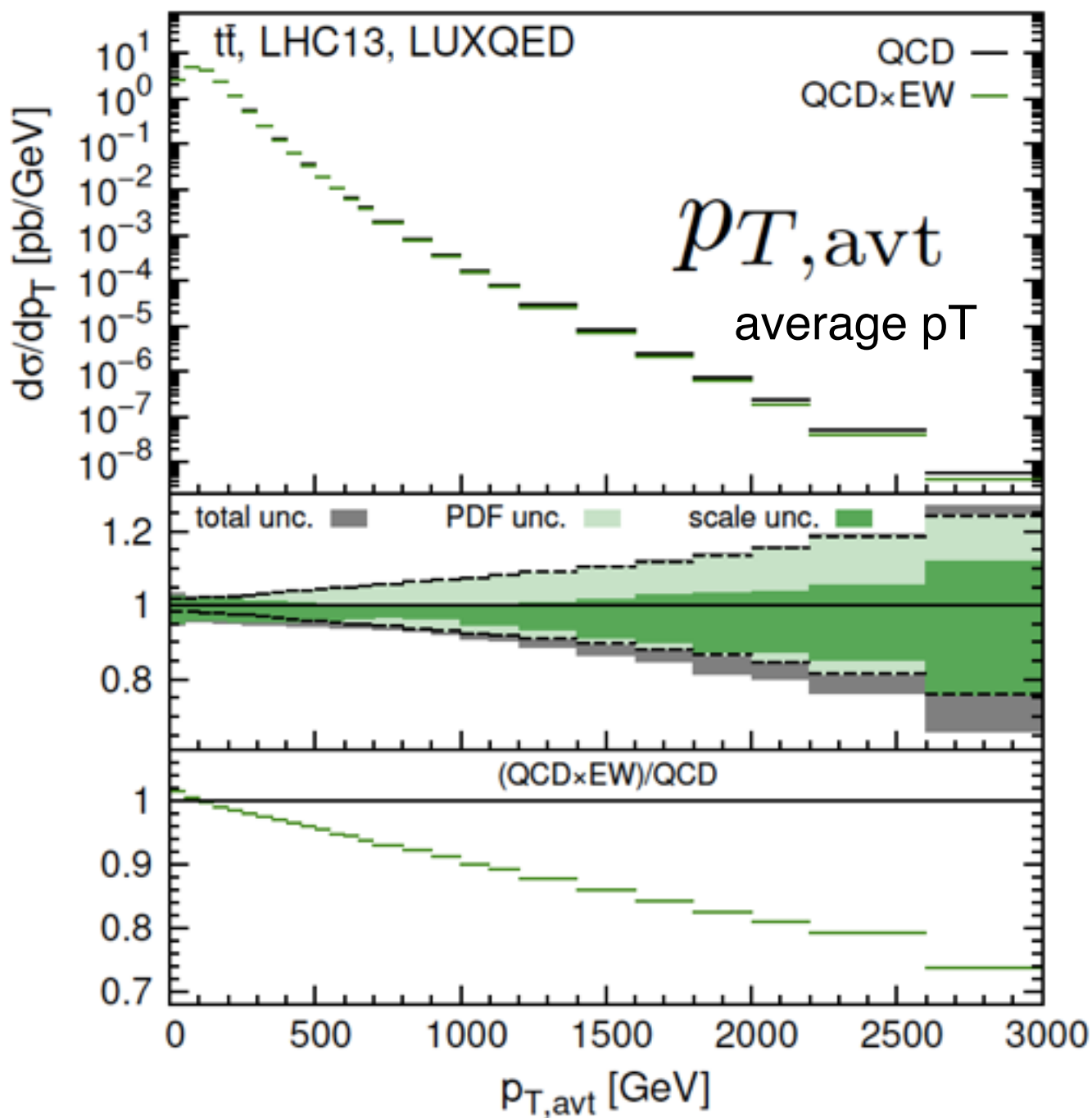
also: $qq' \rightarrow H + X$

Anzai, Hasselhuhn, Höschele, Hoff, Kilgore, Steinhauser, Ueda '15



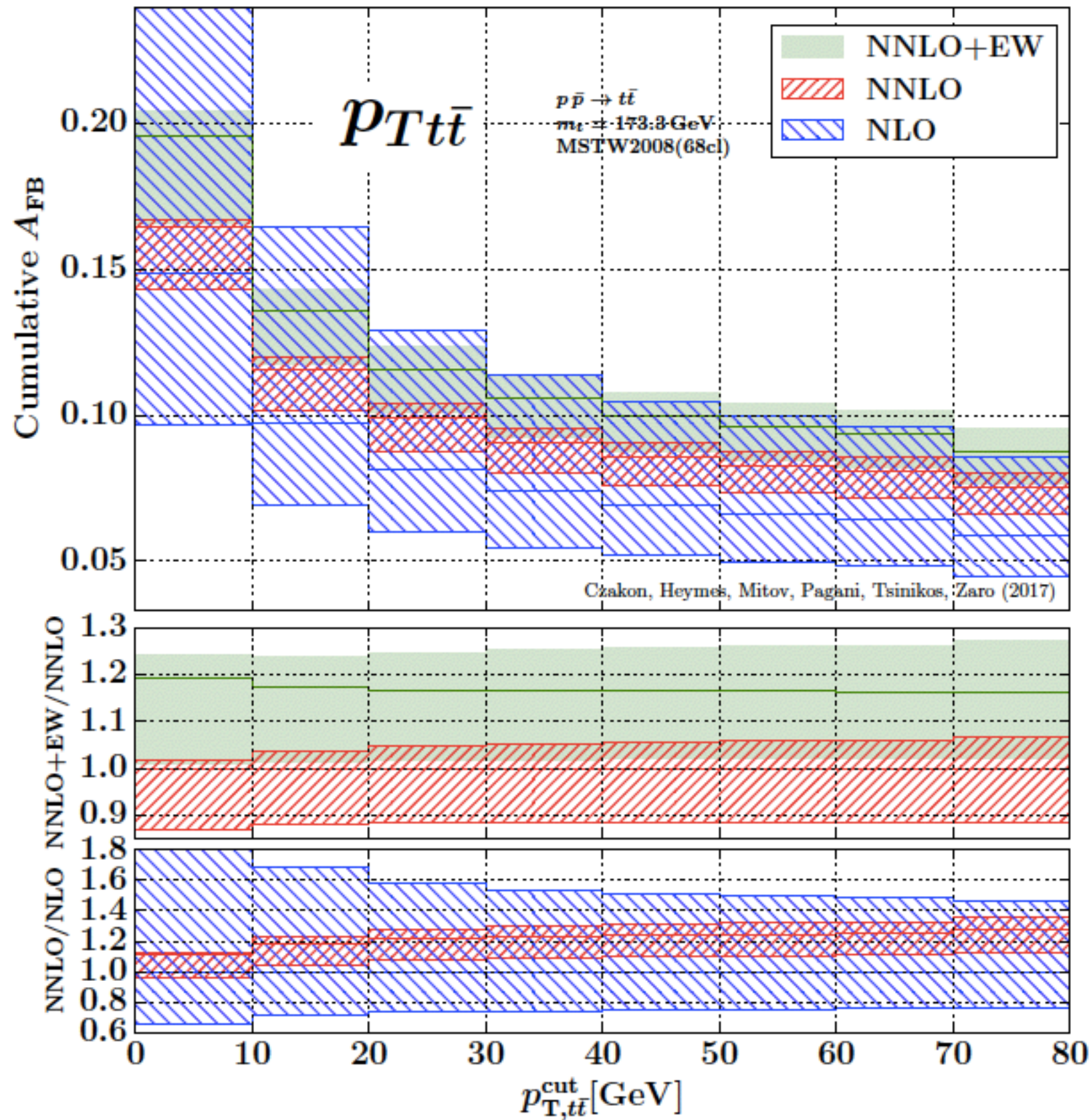
top quark pair production

NNLO QCD and NLO EW Czakon, Heymes, Mitov, Pagani, Tsinikos, Zaro '17



significant role of QED PDFs

top quark A_FB



back to
Tevatron

NNLO accuracy is present
in all bins.

EW corrections are large.
They are **outside** the
NNLO scale uncertainty
band.

$$A_{\text{FB}}(p_{T,t\bar{t}} < p_{T,t\bar{t}}^{\text{cut}})$$

Davide Pagani