Z-Boson pair production at the LHC

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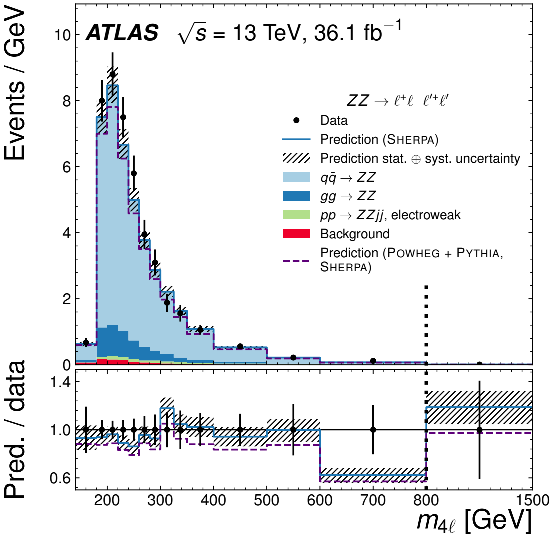
Outline

- current status
  - measurements by ATLAS and CMS
  - Standard Model predictions
- top-quark contributions at two loop level
  - interference Z- and Higgs-boson production
  - computational strategy



The ATLAS Collaboration [1709.07703]

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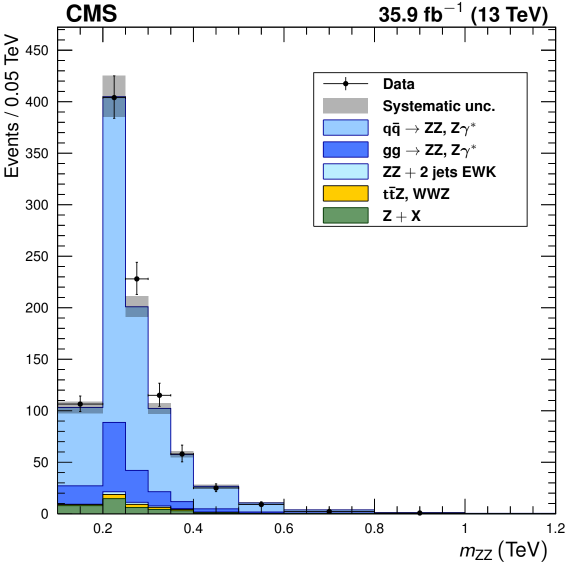
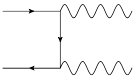


The CMS Collaboration [1709.08601]

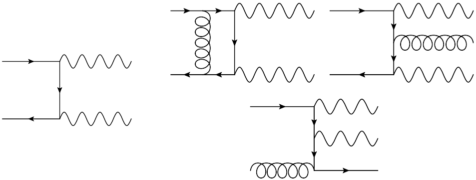
Example Diagrams for pp $\rightarrow$ ZZ in the Standard Model



LO

- order counting in at cross-section (amplitude^2) level
- corrections in quantum chromodynamics (QCD)
- *inclusive* production ($pp \rightarrow ZZ + X$)

Example Diagrams for pp → ZZ in the Standard Model



LO NLO

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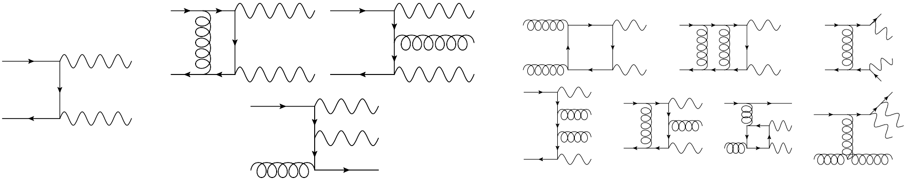


LO NLO NNLO

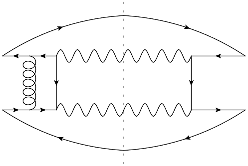
- order counting in at cross-section (amplitude^2) level
- corrections in quantum chromodynamics (QCD)
- *inclusive* production ($pp \to ZZ + X$)

Z-Boson pair production at the LHC

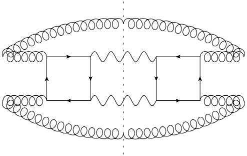
Order Counting

$$
\begin{aligned}
\sigma_{pp\longrightarrow ZZ} \propto & \left| A^{(0)}_{q\bar{q}\to ZZ} + g_s^2 A^{(2)}_{q\bar{q}\to ZZ} + \ldots \right|^2 \\
& + \left| g_s A^{(1)}_{q\bar{q}\to ZZg} + g_s^3 A^{(3)}_{q\bar{q}\to ZZg} + \ldots \right|^2 \\
& + \left| g_s A^{(1)}_{qg\to ZZq} + g_s^3 A^{(3)}_{qg\to ZZq} + \ldots \right|^2 \\
& + \left| g_s^2 A^{(2)}_{gg\to ZZ} + g_s^4 A^{(4)}_{gg\to ZZ} + \ldots \right|^2 \\
& + \ldots
\end{aligned}
$$

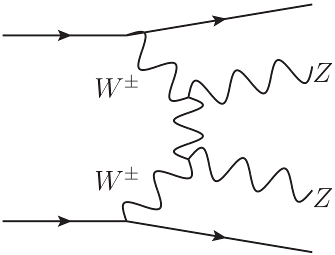
$g_s^2 \; A^{(2)}_{q\bar{q}\to ZZ} \; {A^{(0)}_{q\bar{q}\to ZZ}}^{*}$



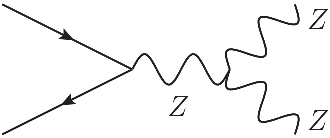
$g_s^4 \; A^{(2)}_{gg\to ZZ} \; {A^{(2)}_{gg\to ZZ}}^{*}$



Probing the Electroweak Sector of the Standard Model



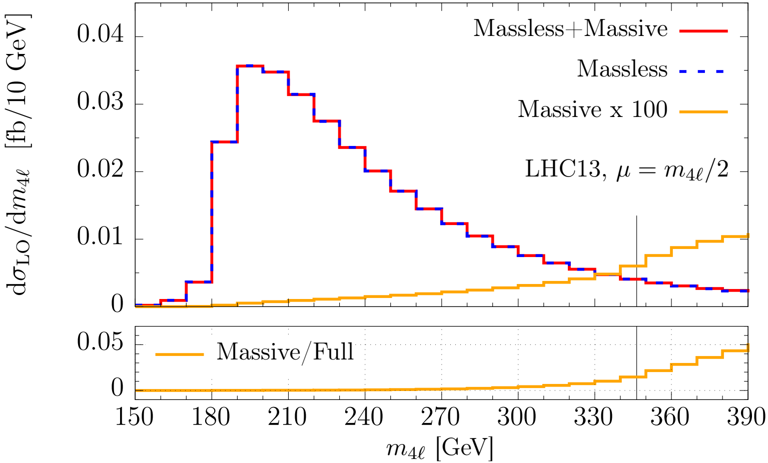
vector boson scattering
(Standard Model)



anomalous gauge couplings
(beyond Standard Model)

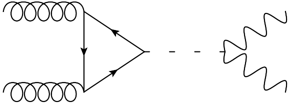
Effect of a Massive (top quark) Loop in gg $\to$ ZZ

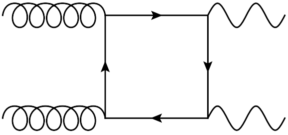
F. Caola, M. Dowling, K. Melnikov, R. Röntsch, L. Tancredi [1605.04610]



Interference with decaying Higgs boson

F. Caola, M. Dowling, K. Melnikov, R. Röntsch, L. Tancredi [1605.04610]

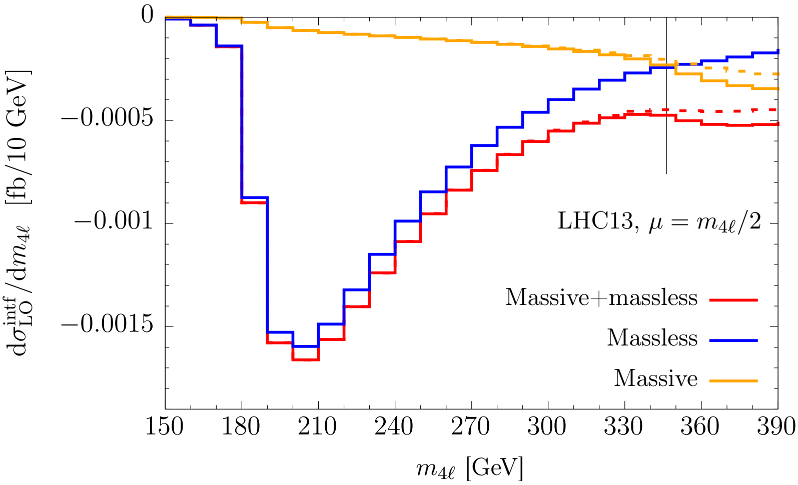




$$\sigma_{gg \longrightarrow ZZ} \propto \left| A_{gg \xrightarrow{H} ZZ} + A_{gg \not\xrightarrow{H} ZZ} \right|^2$$
$$= \left| A_{gg \xrightarrow{H} ZZ} \right|^2 + \left| A_{gg \not\xrightarrow{H} ZZ} \right|^2 + 2\,Re \left[A_{gg \xrightarrow{H} ZZ} A^*_{gg \not\xrightarrow{H} ZZ} \right]$$

Interference with decaying Higgs boson

F. Caola, M. Dowling, K. Melnikov, R. Röntsch, L. Tancredi [1605.04610]



Interference with decaying Higgs boson

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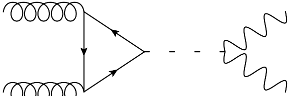
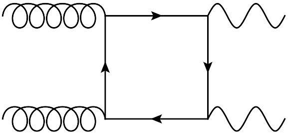


"signal"



"background"

- $gg \to ZZ \to e^+e^-\mu^+\mu^-$
- $150\,GeV < m_{4\ell} < 340\,GeV$

  (below top threshold)
- $60\,GeV < m_{\ell\ell} < 120\,GeV$

  (Z peak)

$\sigma_{\rm LO}^{\rm signal} = 0.043^{+0.012}_{-0.009}$ fb, $\sigma_{\rm NLO}^{\rm signal} = 0.074^{+0.008}_{-0.008}$ fb

$\sigma_{\rm LO}^{\rm bkgd} = 2.90^{+0.77}_{-0.58}$ fb, $\sigma_{\rm NLO}^{\rm bkgd} = 4.49^{+0.34}_{-0.38}$ fb

$\sigma_{\rm LO}^{\rm intf} = -0.154^{+0.031}_{-0.04}$ fb, $\sigma_{\rm NLO}^{\rm intf} = -0.287^{+0.031}_{-0.037}$ fb

$\sigma_{\rm LO}^{\rm full} = 2.79^{+0.74}_{-0.56}$ fb, $\sigma_{\rm NLO}^{\rm full} = 4.27^{+0.32}_{-0.35}$ fb

Z-Boson pair production at the LHC

Standard Model Prediction for pp $\to$ ZZ

G. Heinrich, SJ, S. P. Jones, M. Kerner, J. Pires [1710.06294]

- N-jettiness subtraction scheme

  R. Boughezal et al. [1504.02131], *J. Gaunt et al.* [1505.04794]

- 2 loop virtual amplitude from qqvvamp (massless QCD)

  T. Gehrmann et al. [1503.04812]

- cross check with calculations in q_T-subtraction scheme

  F. Cascioli et al. [1405.2219], *M. Grazzini et al.* [1507.06257]

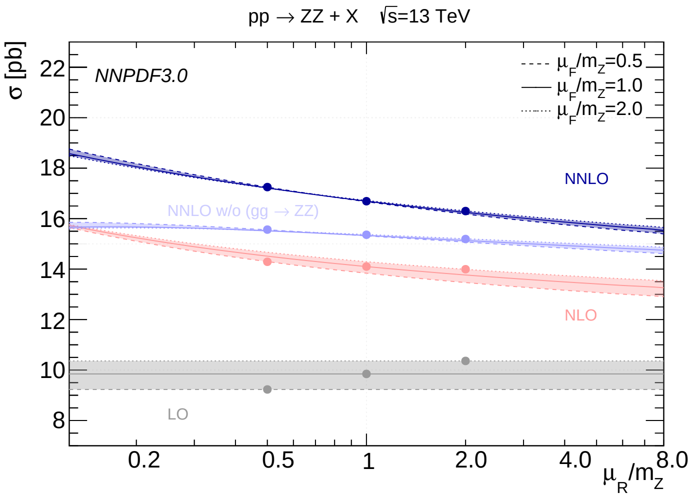
- partial N^{3}LO results in the (anti-)quark-gluon channel

  M. Grazzini et al. [1811.09593]

Outlook: top mass effects in pp $\to$ ZZ

Standard Model Prediction for pp → ZZ

G. Heinrich, SJ, S. P. Jones, M. Kerner, J. Pires [1710.06294]



Standard Model Prediction for pp $\to$ ZZ

- last missing piece for full NNLO(QCD) cross-section: massive top-quark loops in $q\bar{q} \xrightarrow{t} ZZ$
- largest scale uncertainty from: gg $\to$ ZZ (loop-induced)
  - also include 2-loop of gg $\to$ ZZ although formally N^3LO

- calculation similar to $gg \xrightarrow{t} HH$

  S. Borowka et al. [1604.06447]

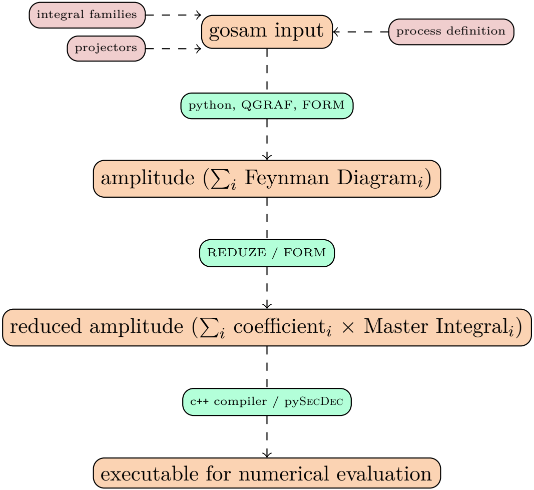
- automated calculation with GoSam-Xloop
  - Larin-Scheme for γ_5 treatment

    S. A. Larin [hep-ph/9302240]

GoSam-Xloop

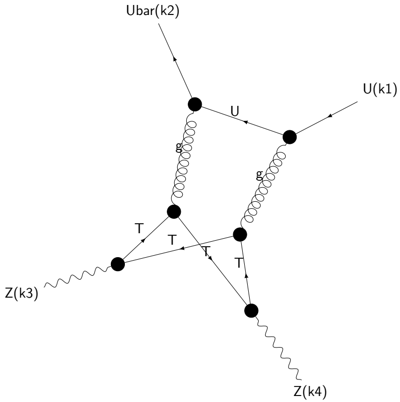
The GoSam-Xloop collaboration

Long Chen
Nicolas Greiner
Gudrun Heinrich
Stephan Jahn
Stephen Jones
Matthias Kerner
et al.



Evaluation of the Master Integrals

- many massive two-loop integrals analytically unknown
- multiple numerical approaches
- automated tools for numerical evaluation available



Evaluation of the Master Integrals
The Sector Decomposition Approach

$$\mathcal{I} \equiv \int d^D k_1 \cdot \ldots \cdot d^D k_L \frac{1}{P_1^{\nu_1} \cdot \ldots \cdot P_N^{\nu_N}}$$

D: dimensionality (typically $4 - 2\epsilon$)

P_i: propagators ($< momentum >^2 \left[- < mass >^2\right] + i\delta$)

Evaluation of the Master Integrals
The Sector Decomposition Approach

$$\mathcal{I} \equiv \int d^D k_1 \cdot \ldots \cdot d^D k_L \frac{1}{P_1^{\nu_1} \cdot \ldots \cdot P_N^{\nu_N}}$$

$$\propto \int_0^1 \prod_{j=1}^{N} dx_j x_j^{\nu_i - 1} \delta\left(1 - \sum_{n=1}^{N} x_n\right) \frac{\mathcal{U}^{N_\nu - (L+1)D/2}}{\mathcal{F}^{N_\nu - LD/2}}$$

D: dimensionality (typically $4 - 2\epsilon$)

P_i: propagators ($< momentum >^2 \left[- < mass >^2\right] + i\delta$)

$N_\nu = \sum_{i=1}^{N} \nu_i$

$\mathcal{U} = \mathcal{U}(\vec{x})$: 1st Symanzik polynomial

$\mathcal{F} = \mathcal{F}(\vec{x}, p_i \cdot p_j, m_i^2)$: 2nd Symanzik polynomial

Evaluation of the Master Integrals
The Sector Decomposition Approach

- loop integral, schematically:

$$\mathcal{I} \equiv \int_0^1 \mathrm{d}x_1 \ldots \int_0^1 \mathrm{d}x_N \prod_{i=1}^{m} f_i\left(\vec{x}\right)^{b_i + c_i \epsilon}$$

  where the f_i are polynomials in $\vec{x}$
- expand **integrand** in the dimensional regulator

$$\mathcal{I} = \int_0^1 \mathrm{d}x_1 \ldots \int_0^1 \mathrm{d}x_N \sum_{k=-2L}^{\infty} \mathcal{I}_k(\vec{x})\epsilon^k$$

- nontrivial due to **overlapping singularities**, e.g. $(x_1 + x_2)^{-2+\epsilon}$
- see also: upcoming talk in the theory journal club

Public Implementations of Sector Decomposition

- sector_decomposition

  C. Bogner, S. Weinzierl [0709.4092]

  - supplemented by: CSectors

    J. Gluza, K. Kajda, T. Riemann, V. Yundin [1010.1667]

- FIESTA 4

  A.V. Smirnov [1511.03614]

- pySecDec

  S. Borowka, G. Heinrich, SJ, S.P. Jones, M. Kerner, J. Schlenk, T.Zirke [1703.09692]

  S. Borowka, G. Heinrich, SJ, S.P. Jones, M. Kerner, J. Schlenk [1811.11720]

Improved Numerical Integration in pySecDec-1.4

S. Borowka, G. Heinrich, SJ, S.P. Jones, M. Kerner, J. Schlenk [1811.11720]

	QMC on GPUs		Vegas	
	rel. acc.	time (s)	rel. acc.	time (s)
`banana 3mass 3L`	$3.8 \cdot 10^{-11}$	15	$1.5 \cdot 10^{-3}$	39
`HZ nonplanar 2L`	$1.3 \cdot 10^{-3}$	24	$5.2 \cdot 10^{-3}$	27
`pentabox fin 2L`	$1.9 \cdot 10^{-4}$	42	$2.6 \cdot 10^{-3}$	139
`elliptic 2L`	$2.0 \cdot 10^{-6}$	9	$3.6 \cdot 10^{-4}$	104
`formfactor 4L`	$4.2 \cdot 10^{-7}$	258	$2.7 \cdot 10^{-4}$	986
`Nbox split b 2L`	$2.5 \cdot 10^{-3}$	60	$1.6 \cdot 10^{-1}$	177
`bubble 6L`	$8.5 \cdot 10^{-7}$	279	$5.7 \cdot 10^{-4}$	199

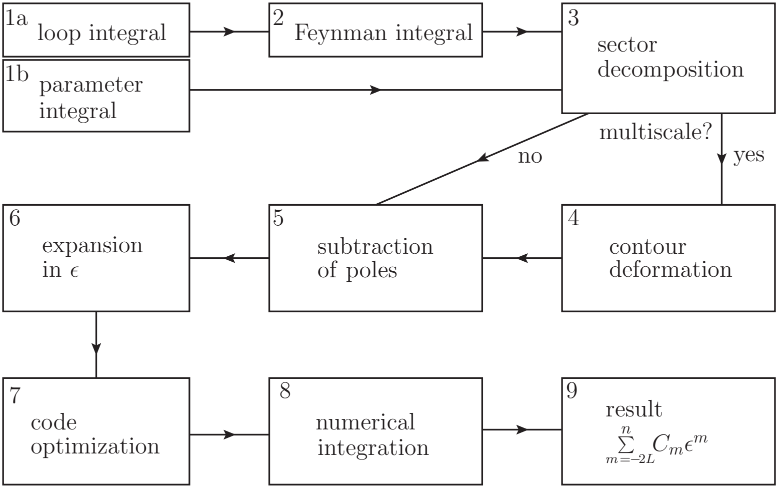
- download: `https://github.com/mppmu/secdec/releases/`
- documentation: `https://secdec.readthedocs.io/`

Conclusion

- electroweak gauge boson production:
  important test of the Standard Model
- mass corrections in gg $\to$ ZZ
  - important for highly energetic Z bosons
    in interference with intermediate Higgs boson production
  - automated computation with numerical methods feasible

BACKUP

Flowchart pySecDec



Loop Integral - Momentum Representation

$$\mathcal{I} = \int d^D k_1 \cdot \ldots \cdot d^D k_L \frac{1}{P_1^{\nu_1} \cdot \ldots \cdot P_N^{\nu_N}}$$

D: dimensionality

L: number of loops

N: number of propagators

P_i: propagators $(< momentum >^2 \; [- < mass >^2] + i\delta)$

ν_i: propagator powers

Loop Integral - Feynman Representation

$$\mathcal{I} = (-1)^{N_\nu} \frac{\Gamma(N_\nu - LD/2)}{\prod_{j=1}^{N} \Gamma(\nu_j)} \int_0^1 \prod_{j=1}^{N} dx_j x_j^{\nu_i - 1} \delta\left(1 - \sum_{n=1}^{N} x_n\right) \frac{\mathcal{U}^{N_\nu - (L+1)D/2}}{\mathcal{F}^{N_\nu - LD/2}}$$

D: dimensionality

L: number of loops

N: number of propagators

ν_i: propagator powers

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$\mathcal{U} = \mathcal{U}(\vec{x})$: 1st Symanzik polynomial

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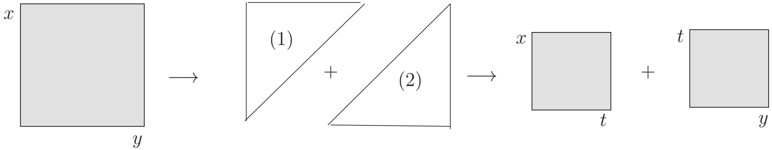
Sector Decomposition

or: Resolution of Overlapping Singularities



$$\int_0^1 dx \int_0^1 dy \ (x+y)^{a+b\epsilon} f(x,y)$$

$$= \int_0^1 dx \int_0^1 dy \ (x+y)^{a+b\epsilon} f(x,y) [\underbrace{\Theta(x-y)}_{(1)} + \underbrace{\Theta(y-x)}_{(2)}]$$

$$= \int_0^1 dx \int_0^1 dt \, x \, x^{a+b\epsilon} (1+t)^{a+b\epsilon} f(x,xt) + \int_0^1 dt \int_0^1 dy \, y \, y^{a+b\epsilon} (t+1)^{a+b\epsilon} f(yt,y)$$

Subtraction of Poles

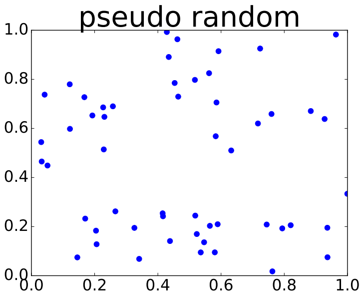
$$\int_0^1 dt\, t^{-1+b\epsilon} g(t)$$

$$= \int_0^1 dt\, t^{-1+b\epsilon} \left(g(0) + g(t) - g(0)\right)$$

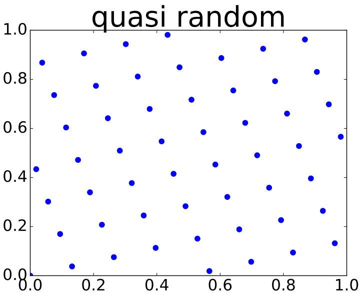
$$= \underbrace{\int_0^1 dt\, t^{-1+b\epsilon} g(0)}_{=\frac{1}{b\epsilon} g(0)} + \underbrace{\int_0^1 dt\, t^{-1+b\epsilon} \left(g(t) - g(0)\right)}_{\text{finite for } \epsilon \to 0 \text{, expand integrand in } \epsilon}$$

Quasi Monte Carlo (QMC)

D. Nuyens et al. (2006), J. Dick et al. (2013), Z. Li et al. [1508.02512], S. Borowka et al. [1811.11720]



integral error $\sim \frac{1}{\sqrt{n}}$



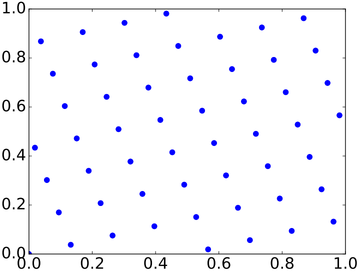
asymptotic integral error $\sim \frac{1}{n}$ or better

with n: number of integrand evaluations

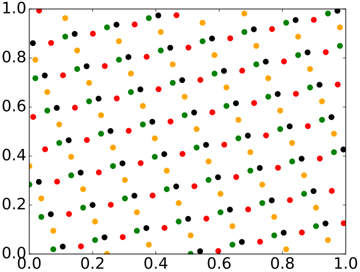
Quasi Monte Carlo (QMC)

D. Nuyens et al. (2006), J. Dick et al. (2013), Z. Li et al. [1508.02512], S. Borowka et al. [1811.11720]

$$I_s[f] \approx \bar{Q}_{s,n,m}[f] \equiv \frac{1}{m}\sum_{k=0}^{m-1} Q_{s,n}^{(k)}[f], \qquad I_s[f] \equiv \int_{[0,1]^s} \mathrm{d}^s x\, f(\mathbf{x}), \qquad Q_{s,n}^{(k)}[f] \equiv \frac{1}{n}\sum_{i=0}^{n-1} f\left(\left\{\frac{i\mathbf{z}}{n} + \mathbf{\Delta}_k\right\}\right)$$



integral error between $\sim \frac{1}{n}$ and $\sim \frac{1}{\sqrt{n}}$



determined from m random shifts

with $\{\cdot\}$ the fractional part, $\mathbf{z}$ the generating vector, $\mathbf{\Delta}_k$ the random shift vector, and f the integrand

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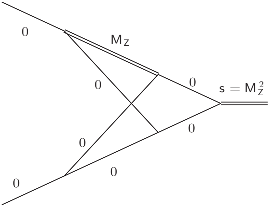
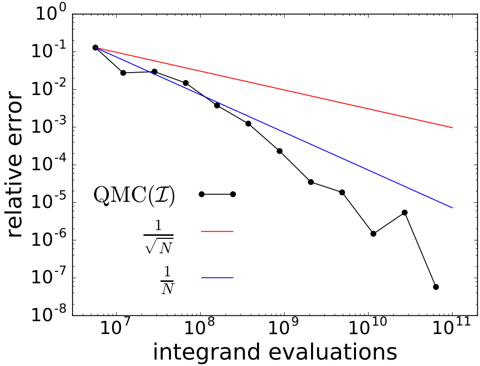
D. Nuyens et al. (2006), J. Dick et al. (2013), Z. Li et al. [1508.02512], S. Borowka et al. [1811.11720]





pySecDec Timings

S. Borowka, G. Heinrich, SJ, S.P. Jones, M. Kerner, J. Schlenk, T.Zirke [1703.09692]

Table 5
Comparison of timings (algebraic, numerical) using pySecDec, SecDec 3 and Fiesta 4.1.

	pySecDec time (s)	SecDec 3 time (s)	Fiesta 4.1 time (s)
triangle2L	(40.5, 9.6)	(56.9, 28.5)	(211.4, 10.8)
triangle3L	(110.1, 0.5)	(131.6, 1.5)	(48.9, 2.5)
elliptic2L_euclidean	(8.2, 0.2)	(4.2, 0.1)	(4.9, 0.04)
elliptic2L_physical	(21.5, 1.8)	(26.9, 4.5)	(115.3, 4.4)
box2L_invprop	(345.7, 2.8)	(150.4, 6.3)	(21.5, 8.8)

Flowchart GoSam