String Theory meets Condensed Matter Physics

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Particle Physics School Colloquium

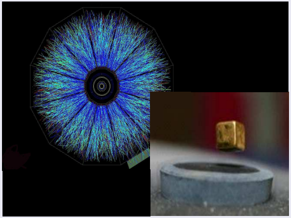
May 7th, 2010

Outline

Motivation

Can we use string theory to study experimental observations?

gauge/gravity
$\longleftrightarrow$
duals



[Uppsala Universitet]

[RHIC]

Model

- $\mathcal{N} = 4$ SYM coupled to $\mathcal{N} = 2$ hypermultiplets
- at finite T and finite density

Results

- Conductivity tensor
- Superconducting state
- emergent Fermi Surfaces

Can we apply string theory to real-world systems?

Quark-Gluon Plasma (QGP)

- observed at RHIC
- behaves as a liquid $\rightarrow$ Hydrodynamics
- strongly coupled
- inputs for hydrodynamic description
  - thermodynamics (equation of state)
  - transport coefficients (shear and bulk viscosity, charge diffusion constant, conductivities)

Question

Can we calculate transport coefficients (e.g. shear viscosity, conductivities) of the QGP?

- Perturbation theory not reliable,
- Lattice Simulations are difficult!

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for the shear viscosity of any strongly coupled, large N gauge theory with a gravity dual

$$\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$$

[Kovtun, Son, Starinets, '03]

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Experimental result

$$\frac{\eta}{s} \approx 0.1 - 0.25$$

[RHIC, '06]

Condensed Matter Systems

Unsolved Problems

- High-T_c superconductors
- Heavy fermion compounds
- Strange metals
- Non-Fermi liquids

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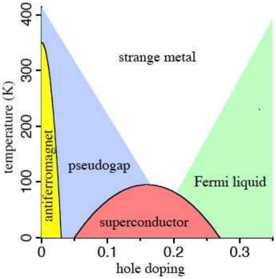

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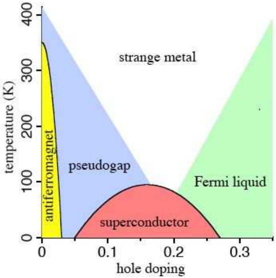

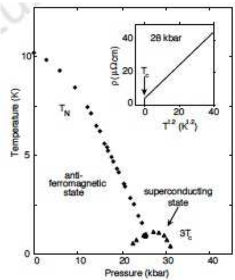

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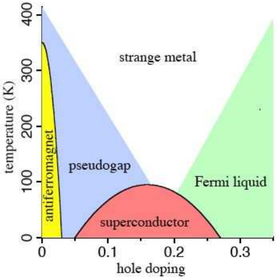

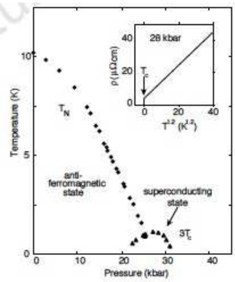

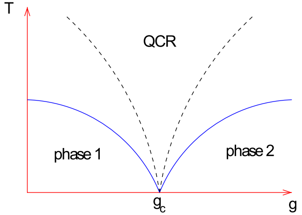

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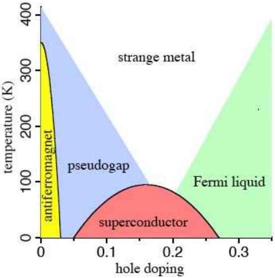
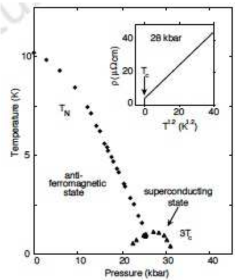
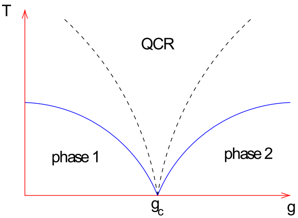




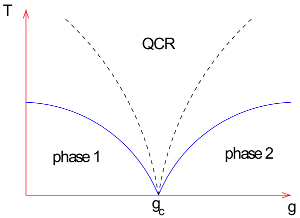


Quantum Phase Transition

- Quantum Phase Transition = Phase Transition at $T = 0$.
- Consider condensed matter systems near quantum critical points.

Quantum Critical Region (QCR)

- temperature is the only relevant scale.
- Quantum critical theory is scale invariant and often strongly coupled $\Rightarrow$ perfect for AdS/CFT!



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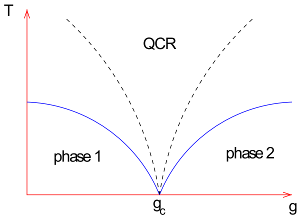

Effective Theories

in QCR-Region:

e.g. O(N) models (Wilson-Fisher fixed point),...

Difficult to find!

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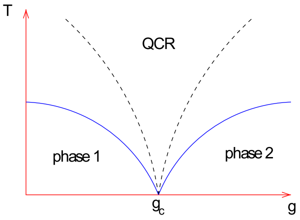

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Scale invariance

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$$t \to \lambda^z t, \qquad \vec{x} \to \lambda \vec{x},$$

where $z > 0$ is the dynamical exponent.

Relativistic case: $z = 1$.

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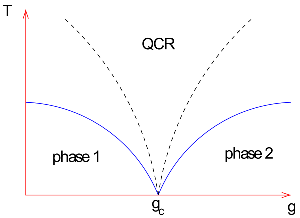

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Use AdS/CFT to describe quantum critical theories!

Gauge/Gravity Duality

AdS/CFT correspondence (as a toy model)

A strongly coupled d-dimensional conformal field theory

is dual to

a gravity theory in asymptotically AdS_{d+1} spacetime

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An explicit string theory realization

4D $\mathcal{N} = 4$ $SU(N_c)$ super Yang-Mills at large N_c and strong coupling

is dual to

type IIB supergravity in asymptotically $AdS_5 \times S^5$ spacetime

May be derived from string theory, in particular from N_c D3-branes

Adding charge carriers

Goal

- Calculate conductivities
- Build superconductors
- Model fermi surfaces

Two different approaches:

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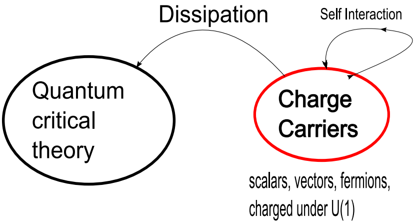

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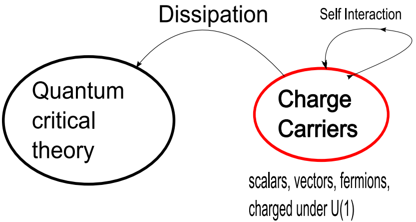

Two different approaches:

Bottom-Up

- Use toy model: gravity
- Add Fermions, scalars, gauge fields by hand
- Charges & masses not fixed $\Rightarrow$ can scan different models!
- Field theory dual not known

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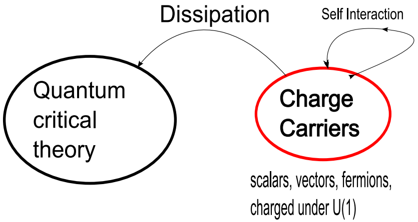

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Top-Down

- Use string theory embedding
- Add D-branes to model fermions, scalars, gauge fields
- Charges & masses fixed
- Dual field theory is known

Results of the Top-Down approach

- Calculation of conductivities
  - DC conductivities: the method [A. Karch, A. O'Bannon, '07]
  - DC conductivities for arbitrary electric and magnetic fields [M.A., H. Ngo, A. O'Bannon, '09]
  - DC & AC conductivities for QCP with $z \neq 1$ (Lifshitz symmetry) [S. Hartnoll, J. Polchinski, E. Silverstein, D. Tong, '09]
  - DC & AC conductivities for QCP with $z = 2$ (Schrödinger symmetry) [M.A., C. Hoyos, A. O'Bannon, J. Wu, '10]
- Holographic p-wave Superconductors [M.A., J. Erdmenger, M. Kaminski, P. Kerner, '08, '09 + many other groups afterwards]
- Holographic fermi surfaces [M.A., J. Erdmenger, M. Kaminski, A. O'Bannon, '10]
  Effective action, Dual Field Theory operators, Fermi Surfaces in p-wave superconductors
- Adding charge carriers to $2+1$-dimensional field theories [M.A., J. Erdmenger, R. Meyer, A. O'Bannon, T. Wrase , '09 + many other groups]

Holographic Setup

- Two types of charge carriers (mass M) with opposite chemical potential μ

  Holographic realization:

  two coincident D5-branes in $AdS - BH_5 \times S^5$ with non-vanishing gauge field $A_t = A_t^3\sigma^3 \in SU(2)$. $\Rightarrow$ breaks $SU(2)$ down to $U(1)_3$.
- Solve the EOMs and calculate the on-shell action $S_{on-shell}$.
- Determine the partition function $Z = \exp(-S_{onshell})$.

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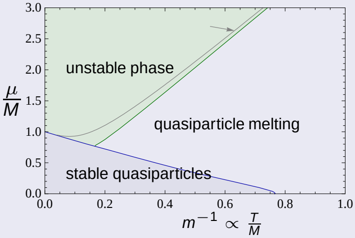
Phase diagram



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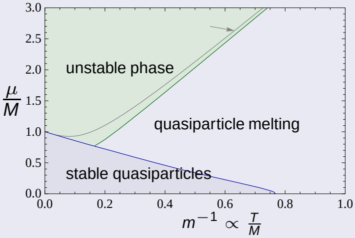
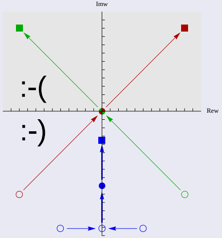


Quasinormal modes



A holographic superconductor

How to cure the instable phase?

Gauge field A_z^1 has to be non-zero,

Set the normalizable mode to zero.

$\Rightarrow \mathcal{J}_z^1$ is not sourced but $\langle \mathcal{J}_z^1 \rangle \neq 0$

Spontaneous breaking of $U(1)$

$\Rightarrow$ Superconductor

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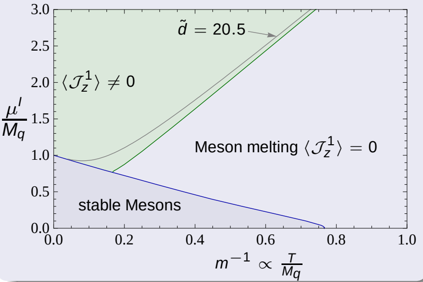

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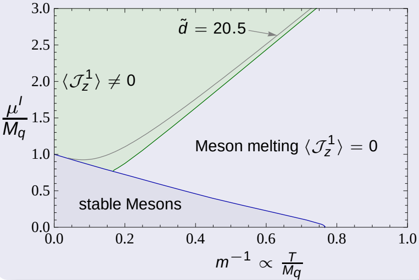
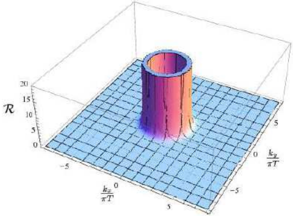


Properties of Phase transition

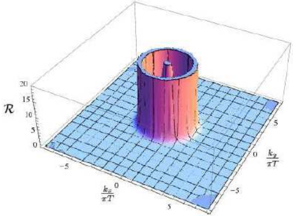
- Order parameter $\mathcal{J}_x^1 \propto \bar{\psi}_u \sigma_3 \psi_d + \bar{\psi}_d \sigma_3 \psi_u + \dots$
- second order phase transition with critical exponent of $1/2$
- Gap in the AC conductivity, Infinite DC conductivity $\Rightarrow$ zero resistance

Fermi Surfaces

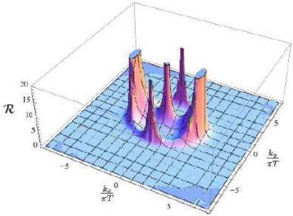
What about fermions in the superconducting state?



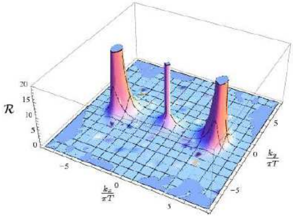
(a) $T = T_c$, $\omega = 0$



(b) $T = 0.91\,T_c$, $\omega = 0$



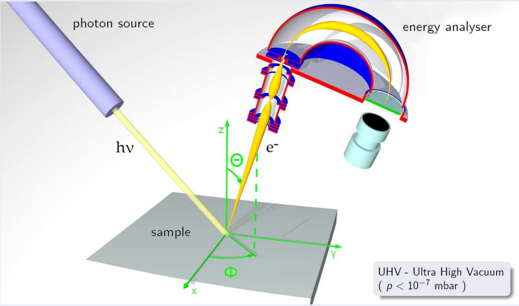
(c) $T = 0.69\,T_c$, $\omega = 0$



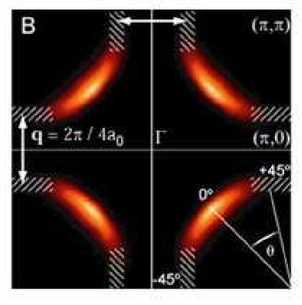
(d) $T = 0.43\,T_c$, $\omega = 0$

ARPES measurements

ARPES experiment



Measurement



Results agree qualitatively:

The Fermi surface collapses to points!

Conclusion

Results & Outlook

- We can study AC & DC conductivities, (p-wave) superconductors and Fermi surfaces in models with an string theory embedding.
- Results qualitatively similar to ARPES, measurements of the conductivity.
- Dual field theory is known explicitly. Compare to a perturbative analysis!
- New insights into High-T_c superconductors and Non-Fermi liquids possible?
- No Universal behavior found so far!

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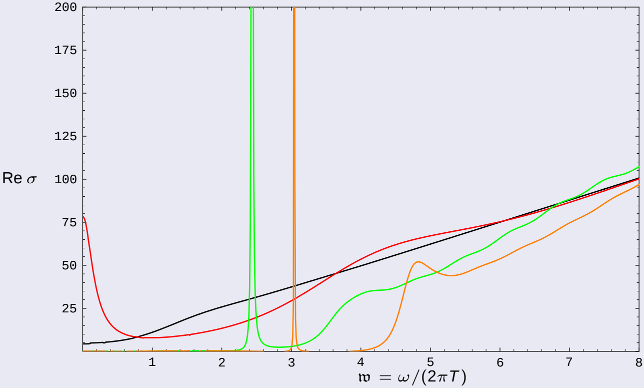
Thank you!

AC Conductivity

conductivity Re $\sigma(\mathfrak{w}) \sim \mathcal{R}/\omega$



for $M_q = 3$: $T/T_c = 10$ black, 1 red, 0.6 green, 0.5 orange

A String Realization of a Holographic superconductor

Can we realize a superconductor? [Gubser, Hartnoll, Herzog, Horowitz, Denef, ... '08, '09]

Superfluid versus Superconductor

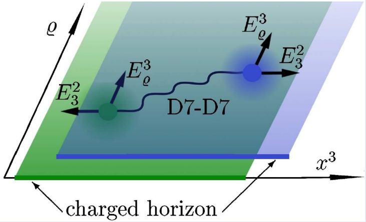
- Superfluid: spontaneous breaking of a global $u(1)$
- Superconductor: spontaneous breaking of a local $u(1)$

Our model as a superconductor

- If we gauge the $u(1)_3$ symmetry, then we can identify
  $u(1)_3$ flavor symmetry $\longleftrightarrow$ $u(1)_{\text{em.}}$
  $\langle \mathcal{J}_z^1 \rangle$ order parameter $\longleftrightarrow$ superconducting condensate
- We have many features of a superconductor
  - infinite DC conductivity, gap in the AC conductivity
  - second order phase transition, critical exponent of 1/2 (mean field)
  - a remnant of the Meissner–Ochsenfeld effect

A String Realization

String Picture



Explanation

- strings spanned from the horizon to the D5–branes *(horizon strings)* induce a charge near the horizon
- System unstable above a critical value of the charge density
- Horizon strings recombine to D5–D5 strings
- D5–D5 strings can propagate into the bulk (balancing the flavorelectric and gravitational forces). Horizon strings cannot propagate into the bulk.
- D5–D5 strings distribute the isospin charge in the bulk and correspond to superconducting condensate.