A SUSY GUT Flavour Model

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in collaboration with Stefan Antusch and Stephen F. King

Outline

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The Flavour Puzzle: Masses

The flavour sector has many parameters with peculiar patterns:

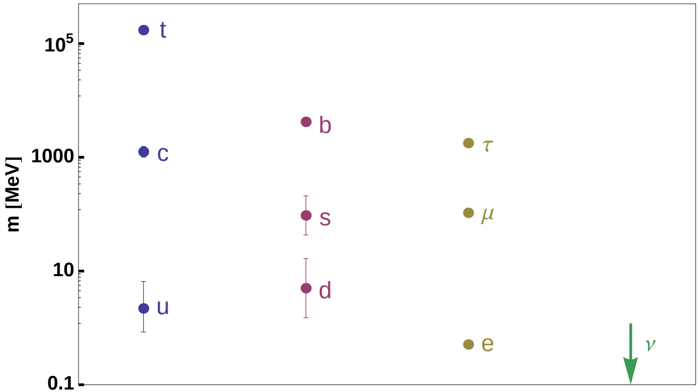


Figure: Pattern of fermion masses. 1σ errors are scaled up by three.

The Flavour Puzzle: Mixing

	Quarks [PDG '08]	Leptons [Gonzalez-Garcia *et al.*, '10]
θ_{12} [°]	12.9	34.5
θ_{23} [°]	2.4	42.3
θ_{13} [°]	0.2	5.7
δ [°]	68.9	?
α_1 [°]	–	?
α_2 [°]	–	?

Table: Best fit values for the fermion mixing parameters.

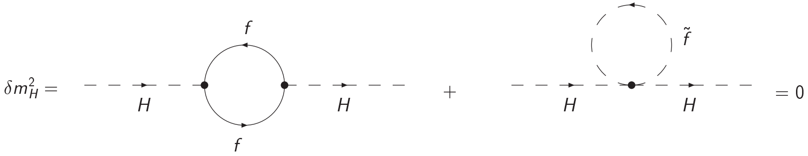
Why is this so?

Outline

What is SUSY?

$$Q|\text{boson}\rangle = |\text{fermion}\rangle \quad \text{resp.} \quad Q|\text{fermion}\rangle = |\text{boson}\rangle$$

What is SUSY good for?



Supersymmetric Gauge Coupling Unification

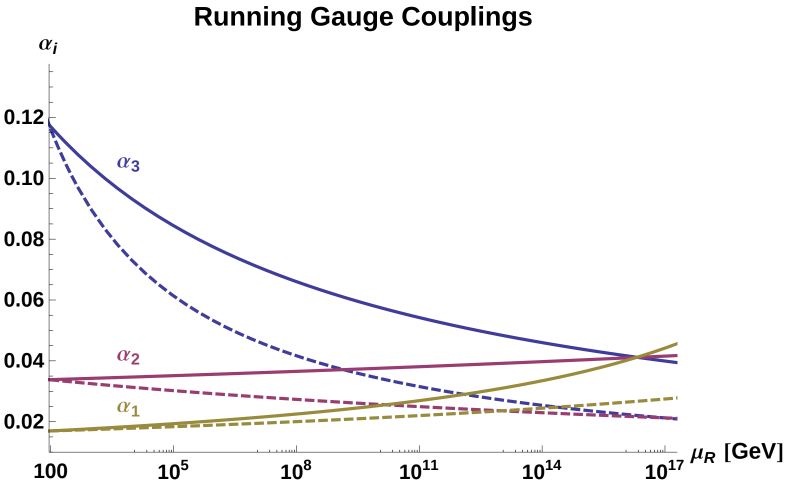


Figure: Running gauge couplings in the SM (dashed) and MSSM (straight).

Grand Unification: What Is It and What Is It Good For?

What is it?

$$SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(5), SO(10), E_6, \ldots$$

What are they good for?

Here: Relations between Yukawa couplings

Outline

Two Examples in $SU(5)$

Consider $F_3 T_3 \langle \overline{H}_5 \rangle$ ($\mathbf{\overline{5}} \times \mathbf{10} \times \mathbf{\overline{5}}$):

$$F_3 T_3 \langle \overline{H}_5 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} b_R^c \\ b_B^c \\ b_G^c \\ \tau \\ -\nu_\tau \end{pmatrix} \begin{pmatrix} 0 & -t_G^c & t_B^c & -t_R & -b_R \\ t_G^c & 0 & -t_R^c & -t_B & -b_B \\ -t_B^c & t_R^c & 0 & -t_G & -b_G \\ t_R & t_B & t_G & 0 & -\tau^c \\ b_R & b_B & b_G & \tau^c & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}$$

$$\xrightarrow{SSB} -\frac{v}{\sqrt{2}} \left(\overline{b} b + \overline{\tau} \tau \right)$$

Similar $F_2 T_2 \langle \overline{H}_{45} \rangle$ gives $y_\mu / y_s = -3$. [Georgi, Jarlskog, 1979]

Another Example

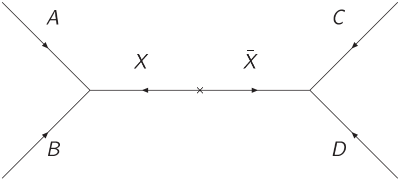


Figure: Supergraph with heavy messenger fields X and $\overline{X}$.

- Take $A = F^2$, $B = \langle H_{24} \rangle$, $C = T^2$, $D = \langle \overline{H}_{45} \rangle$ and $X = \overline{\mathbf{5}}$
- From $\langle H_{24} \rangle$: $y_\mu / y_s = -3/2$
- From $\langle \overline{H}_{45} \rangle$: $y_\mu / y_s = -3$
- In total: $y_\mu / y_s = 9/2$.

GUT Scale Yukawa Couplings from Data

Tools:

- $Y(M_{\text{GUT}}; \widetilde{M})$ calculated via SoftSUSY [Allanach, 2002]
- Observables calculated via micrOMEGAs [Belanger et al., 2008]

Experimental constraints:

- LEP mass bounds for $\chi^{\pm}$, $\tilde{l}$ and $\tilde{\nu}$
- Electroweak precision observables (M_W and $\sin^2 \theta_{\text{eff}}$)
- BR($b \to s\gamma$)
- BR($B_s \to \mu^+\mu^-$)
- Anomalous magnetic moment of the muon

Results for the First and Second Generation

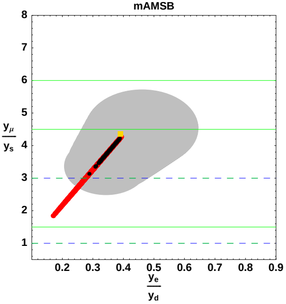
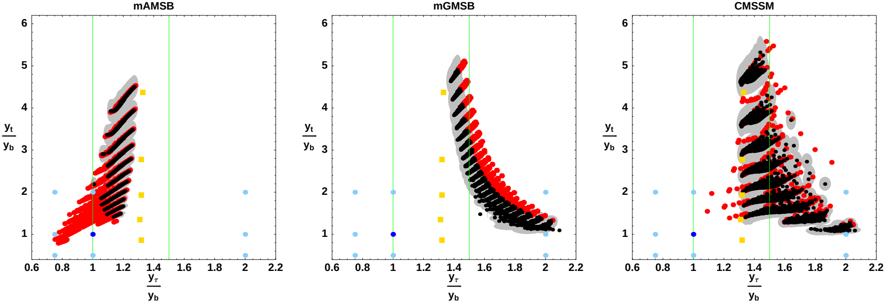


Figure: The (red) black points are the experimentally (excluded) allowed points (may overlap). Grey regions indicate the uncertainties in the quark masses. The green (dashed) lines are $SU(5)$ (PS/$SU(5)$) predictions. Yellow squads are without SUSY threshold corrections. [Antusch, MS '09]

Results for the Third Generation



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Overview of Possible GUT Scale Relations

Prediction	mAMSB	mGMSB	CMSSM
$y_\mu / y_s = 3$ †	✓	×	×
$y_\mu / y_s = 6$ △	×	✓	✓
$y_\mu / y_s = 9/2$ △	✓	✓	✓
$y_t = y_b = y_\tau$	×	×	×
$y_\tau / y_b = 1$	✓	×	×
$y_\tau / y_b = 3/2$ △	×	✓	✓
$2y_t = 2y_b = y_\tau$ ▽	×	✓	✓
$y_t = 2y_b = 2y_\tau$ ▽	✓	×	×

† [Georgi, Jarlskog '79]

△ [Antusch, MS '09]

▽ [Allanach *et al.* '97]

Outline

Field Content and Symmetries [Antusch, King, MS, 2010]

Symmetry:

$$SU(5) \times A_4 \times G_{\text{aux}}$$

Matter:

$$F \equiv (\overline{\mathbf{5}}, \mathbf{3}, \ldots) \quad T_i \equiv (\mathbf{10}, \mathbf{1}, \ldots)$$

Flavons (“Higgs” fields for the family symmetry):

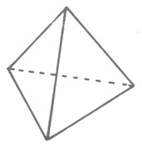
$$\phi_i \equiv (\mathbf{1}, \mathbf{3}, \ldots) \quad \tilde{\phi}_{23} \equiv (\mathbf{24}, \mathbf{3}, \ldots)$$

Higgs fields in $\mathbf{5}, \overline{\mathbf{5}}, \mathbf{15}, \overline{\mathbf{15}}, \mathbf{45}, \overline{\mathbf{45}}$ of $SU(5)$

Some messenger fields...

A Few Words about A_4

A_4 is the symmetry group of a regular tetrahedron.



We need here only the singlet **1** and the real triplet **3** representation:

$$\mathbf{1} \times \mathbf{1} = \mathbf{1} \; ,$$
$$\mathbf{1} \times \mathbf{3} = \mathbf{3} \; ,$$
$$\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3}_s + \mathbf{3}_a \; .$$

Flavon Vacuum Alignment

For the $SU(5)$ singlet flavons we assume the “standard” alignments:

$$\langle \phi_{23} \rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \epsilon_{23} \, , \quad \langle \phi_{123} \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \epsilon_{123} \, , \quad \langle \phi_{3} \rangle \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \epsilon_{3} \, .$$

For the fourth flavon we assume the special ansatz:

$$\langle \tilde{\phi}_{23} \rangle \propto \begin{pmatrix} 0 \\ \mathrm{i} \\ w \end{pmatrix} \tilde{\epsilon}_{23} \, .$$

Effective Yukawa Coupling Matrices

$$Y_u = \begin{pmatrix} 2a_{11}\epsilon_{23}^2 & 0 & a_{13}\epsilon_{23}\epsilon_3 \\ 0 & 3a_{22}\epsilon_{123}^2 + (w^2-1)\tilde{a}_{22}^2\tilde{\epsilon}_{23}^2 & a_{23}\epsilon_{123}\epsilon_3 \\ a_{13}\epsilon_{23}\epsilon_3 & a_{23}\epsilon_{123}\epsilon_3 & a_{33} \end{pmatrix}$$

$$Y_d = \begin{pmatrix} 0 & \epsilon_{23} & -\epsilon_{23} \\ \epsilon_{123} & \epsilon_{123} + \mathrm{i}\,\tilde{\epsilon}_{23} & \epsilon_{123} + w\tilde{\epsilon}_{23} \\ 0 & 0 & \epsilon_3 \end{pmatrix}$$

$$Y_e^T = \begin{pmatrix} 0 & \epsilon_{23} & -\epsilon_{23} \\ \epsilon_{123} & \epsilon_{123} + \frac{9}{2}\,\mathrm{i}\,\tilde{\epsilon}_{23} & \epsilon_{123} + \frac{9}{2}w\tilde{\epsilon}_{23} \\ 0 & 0 & \epsilon_3 \end{pmatrix}$$

Relations between the Dirac CP Phases

We know ϵ_{123}, ϵ_{23} and $\tilde{\epsilon}_{23}$ very precisely from y_e, y_μ and $\theta_{12}^{\mathrm{CKM}}$:

$$\delta_{\mathrm{CKM}}(M_{\mathrm{GUT}}) = \arctan \left| \frac{\tilde{\epsilon}_{23}}{\epsilon_{123}} \right| \approx 69.9^\circ$$

compared to $\delta_{\mathrm{CKM}}^{\mathrm{exp}}(M_{\mathrm{GUT}}) \approx (68.8^{+4.0}_{-2.3})^\circ$!

For the leptonic Dirac CP phase we get $\delta_{\mathrm{MNS}}(M_{\mathrm{GUT}}) \approx 90^\circ$!

Fit Results

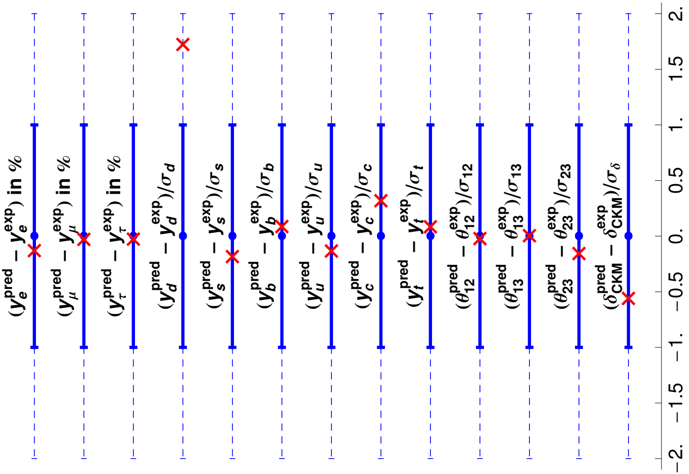


Figure: Deviations of our predictions from low energy experimental data ($\chi^2/\text{dof} \approx 1.6$). The red crosses denote our predictions.

A Few Words about the Seesaw Mechanism

If $M = \begin{pmatrix} M_{LL} & M_{LR} \\ M_{LR} & M_{RR} \end{pmatrix}$ with $M_{LL} \ll M_{LR} \ll M_{RR}$ follows:

$$m_\nu \approx M_{LL} - \frac{M_{LR}^2}{M_{RR}} \text{ and } m_N \approx M_{RR}$$

Neutrino Sector

The neutrino Yukawa couplings:

$$M_{LR} = v_u \begin{pmatrix} 0 & a_{\nu_2}\epsilon_{123} \\ a_{\nu_1}\epsilon_{23} & a_{\nu_2}\epsilon_{123} \\ -a_{\nu_1}\epsilon_{23} & a_{\nu_2}\epsilon_{123} \end{pmatrix}$$

The right-handed neutrino masses:

$$M_{RR} = \begin{pmatrix} 2a_{R_1}\epsilon_{23}^2 & 0 \\ 0 & 3a_{R_2}\epsilon_{123}^2 \end{pmatrix}$$

The type II seesaw contribution:

$$M_{LL} = \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix}$$

Light Neutrino Mass Matrix

$$m_\nu = m_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2 - m_1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3 - m_1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

Quasi-degenerate neutrinos if m_1 is large!

Results for the Neutrino Mixing Parameters

(GUT scale) predictions: [Antusch, King, MS, 2010]

$$\theta_{12}^{\text{MNS}} \approx 35.1^\circ \,, \qquad \delta_{\text{MNS}} \approx 90.0^\circ \,,$$
$$\theta_{13}^{\text{MNS}} \approx 3.3^\circ \,, \qquad \alpha_1 \approx 9.3^\circ \,,$$
$$\theta_{23}^{\text{MNS}} = 45.0^\circ \,, \qquad \alpha_2 = 0^\circ \,.$$

Experimental Data: [Gonzalez-Garcia, Maltoni, Salvado, 2010]

$$\theta_{12}^{\text{MNS}} = (34.5 \pm 1.0)^\circ \,, \qquad \delta_{\text{MNS}} = ? \,,$$
$$\theta_{13}^{\text{MNS}} = (5.7^{+3.0}_{-3.9})^\circ \,, \qquad \alpha_1 = ? \,,$$
$$\theta_{23}^{\text{MNS}} = (42.3^{+5.3}_{-2.8})^\circ \,, \qquad \alpha_2 = ? \,.$$

Results for Neutrinoless Double Beta Decay

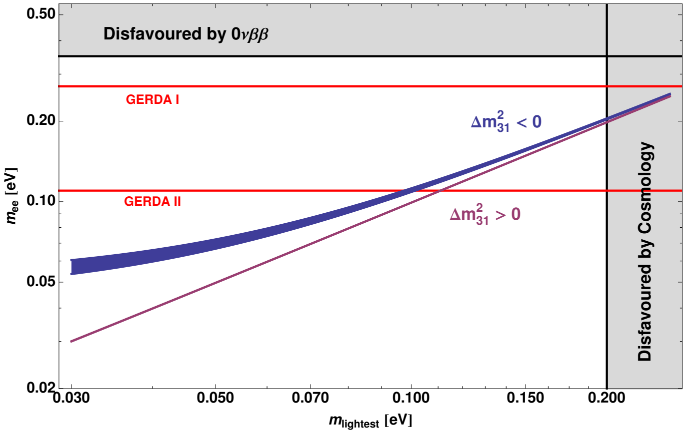


Figure: Effective mass m_{ee} as a function of $m_{lightest}$. [Antusch, King, MS, 2010]

Outline

The Flavour Puzzle

Supersymmetric Grand Unification

GUT Scale Yukawa Couplings

A SUSY GUT Flavour Model

Summary & Conclusions

Summary & Conclusions

New viable GUT relations useful for Model Building:

- $SU(5)$: $y_\mu / y_s = 9/2, 6$ and $y_\tau / y_b = 3/2$
- Pati–Salam: $2y_t = 2y_b = y_\tau$ and $y_t = 2y_b = 2y_\tau$

A SUSY GUT Flavour Model with nice properties:

- First Application of New Ratios $\rightarrow$ $y_\mu / y_s = 9/2$
- Spontaneous CP Violation $\rightarrow$ Phase Predictions
- Approximate Tri-bimaximal Mixing $\rightarrow$ $\theta_{13}^{\text{MNS}} \approx 3.3^\circ$
- Quasi-degenerate Neutrinos $\rightarrow$ Measurable Neutrino Mass Scale

End of the talk

Thanks for your attention!

Experimental Constraints (Numbers)

We have included the following experimental constraints:

- Direct detection: Masses of $\chi^{\pm}$, $\tilde{l}$ and $\tilde{\nu}$ heavier than LEP bounds
- Electroweak precision observables:
  - $M_W = 80.429 \pm 0.039$ GeV
  - $\sin^2 \theta_{eff} = 0.23153 \pm 0.00016$
- B-physics:
  - $BR\,(b \to s\gamma) = \left(3.55^{+0.36}_{-0.37}\right) \times 10^{-4}$
  - $BR\,(B_s \to \mu^+\mu^-) \leq 5.8 \times 10^{-8}$
- Anomalous magnetic moment of the muon: $0 \leq a_\mu \leq 35.9 \times 10^{-10}$

The Full Model [Antusch, King, MS, 2010]

	$SU(5)$	A_4	$\mathbb{Z}_2$	$\mathbb{Z}_3'$	$\mathbb{Z}_2'$	$\mathbb{Z}_3$	$\mathbb{Z}_2^{\mathrm{MP}}$
Chiral Matter							
F	$\mathbf{\bar{5}}$	**3**	+	0	+	0	-
T_1, T_2, T_3	**10, 10, 10**	**1, 1, 1**	+, +, -	0, 1, 0	+, +, +	1, 0, 0	-, -, -
N_1, N_2	**1, 1**	**1, 1**	+, +	0, 1	+, +	1, 0	-, -
Flavons & Higgs Multiplets							
$\phi_{23}, \phi_{123}, \phi_3$	**1**	**3**	+, +, -	0, 2, 0	+, + ,+	2, 0, 0	+, + ,+
$\tilde{\phi}_{23}$	**24**	**3**	+	2	-	0	+
$H_5, \bar{H}_5$	$\mathbf{5}, \mathbf{\bar{5}}$	**1, 1**	+, +	0, 0	+, +	0, 0	+, +
$H_{15}, \bar{H}_{15}$	$\mathbf{15}, \overline{\mathbf{15}}$	**1, 1**	+, +	0, 0	+, +	0, 0	+, +
$H_{45}, \bar{H}_{45}$	$\mathbf{45}, \overline{\mathbf{45}}$	**1, 1**	+, +	2, 1	-, -	0, 0	+, +
Matter-like Messengers							
$A_5, \bar{A}_5$	$\mathbf{5}, \mathbf{\bar{5}}$	**1, 1**	+, +	1, 2	-, -	0, 0	-, -
$A_{10}, \bar{A}_{10}$	$\mathbf{10}, \overline{\mathbf{10}}$	**3, 3**	+, +	0, 0	+, +	0, 0	-, -
A_1	**1**	**3**	+	0	+	0	-
Higgs-like Messengers							
$B, \bar{B}$	$\mathbf{5}, \mathbf{\bar{5}}$	**1, 1**	+, +	1, 2	+, +	0, 0	+, +
$C_1, \bar{C}_1$	**1, 1**	**1, 1**	+, +	0, 0	+, +	2, 1	+, +
$C_2, \bar{C}_2$	**1, 1**	**1, 1**	+, +	2, 1	+, +	0, 0	+, +

Fitted Parameters

Parameter	Value
$2a_{11}\epsilon_{23}^2$ in 10^{-6}	9.62
$3a_{22}\epsilon_{123}^2$ in 10^{-4}	−1.10
$(w^2-1)\tilde{a}_{22}\tilde{\epsilon}_{23}^2$ in 10^{-3}	−1.10
$a_{13}\epsilon_{23}\epsilon_3$ in 10^{-3}	−2.92
$a_{23}\epsilon_{123}\epsilon_3$ in 10^{-2}	3.21
a_{33}	2.44
ϵ_{123} in 10^{-5}	5.88
ϵ_{23} in 10^{-5}	4.30
$\tilde{\epsilon}_{23}$ in 10^{-4}	−1.61
ϵ_3 in 10^{-2}	1.12
w	1.44

Table: The model parameters for $\tan\beta = 1.4$ and $M_{\mathrm{SUSY}} = 500$ GeV from a fit to the experimental data. [Antusch, King, MS, 2010]

Proton Decay

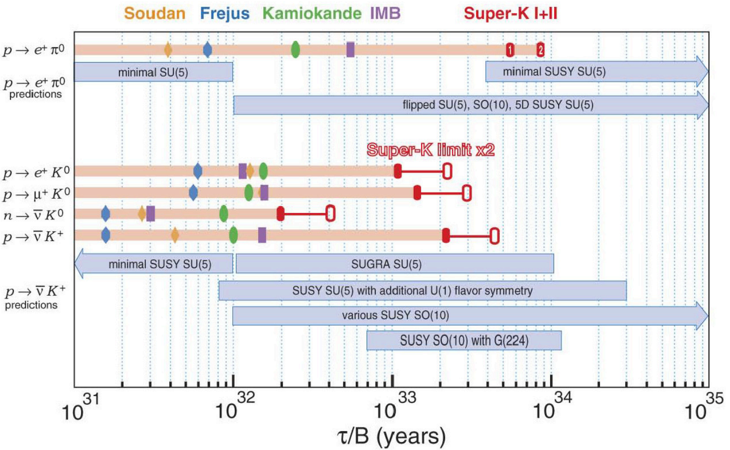


Figure: Proton Decay: Predictions vs. Experiment [Raby *et al.* (DUSEL), '08]