DARK MATTER ASTROPHYSICAL FACTORS FROM STELLAR VELOCITY DISPERSIONS

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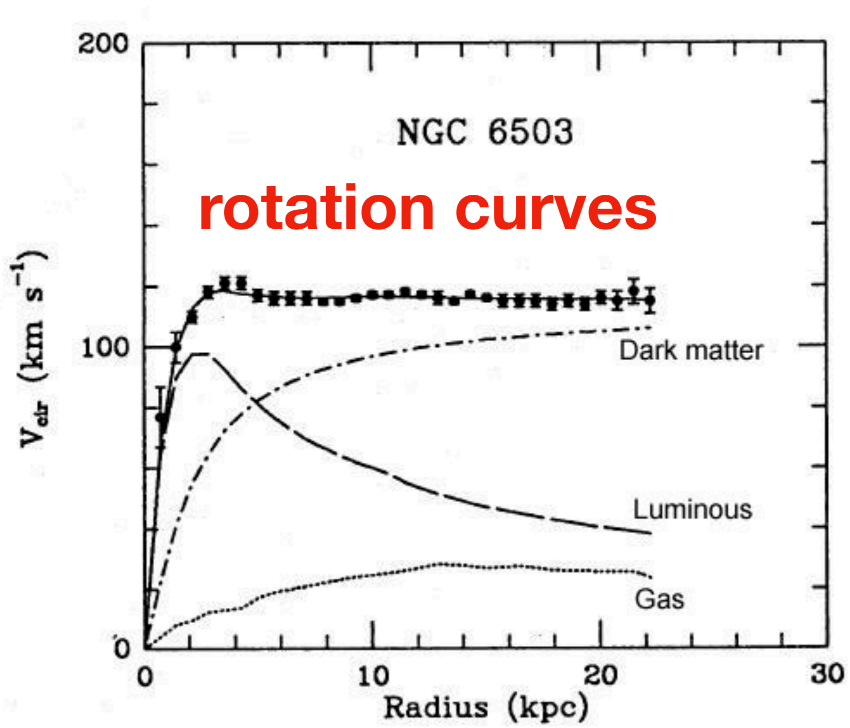
OUTLINE

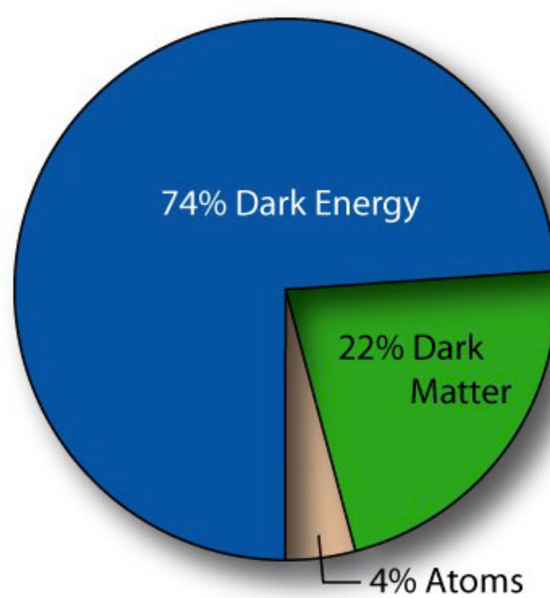
- The quest for dark matter in the Universe
- Dark matter density from rotation curves
- Dark matter density from stellar kinematics
- The case of dwarf spheroidal galaxies
- Summary

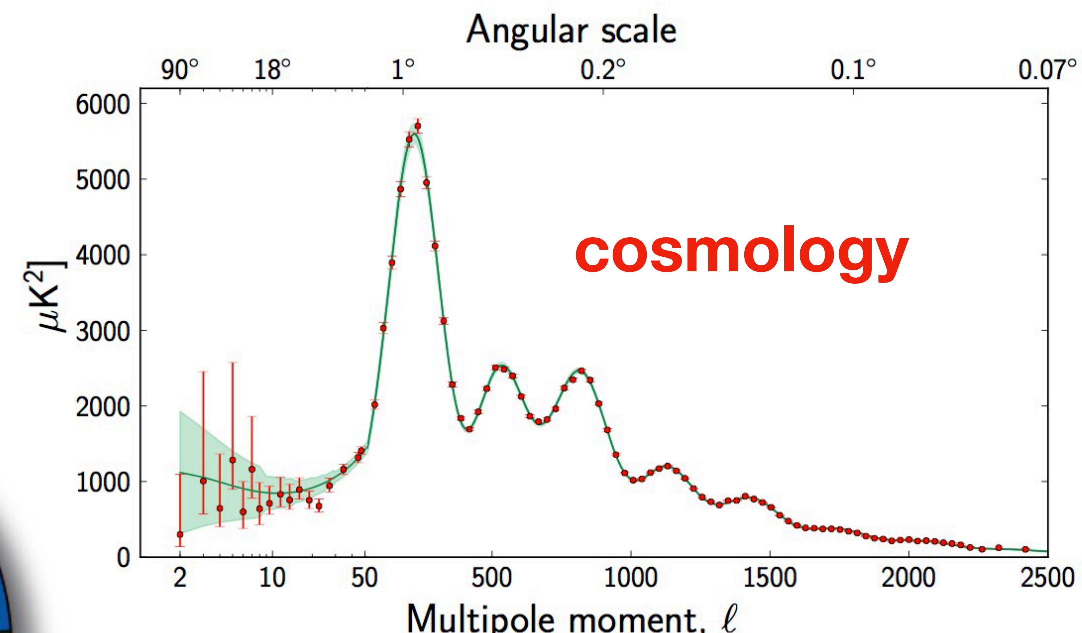
1. The quest for dark matter in the Universe

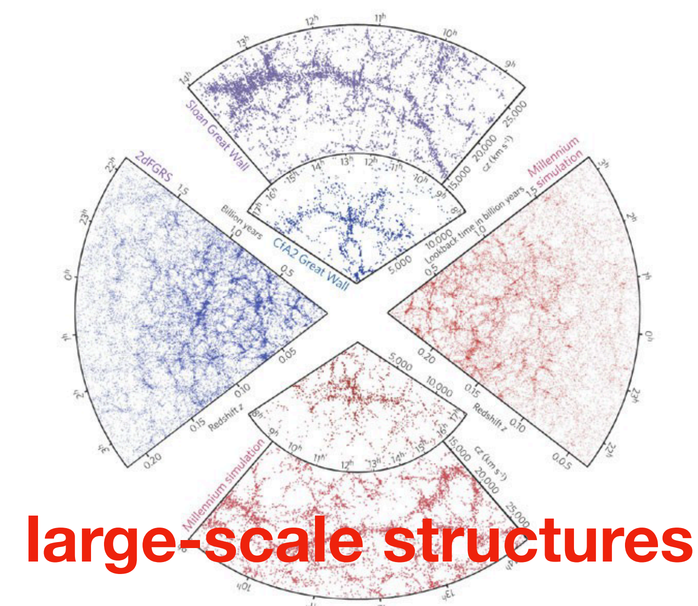
1. The quest for dark matter in the Universe

- **Dark matter** (DM) is the major component of the Universe matter content (~85%, corresponding to ~22% of the Universe total energy content).
- Its existence is only **indirectly** inferred so far from several astrophysical/cosmological observations.





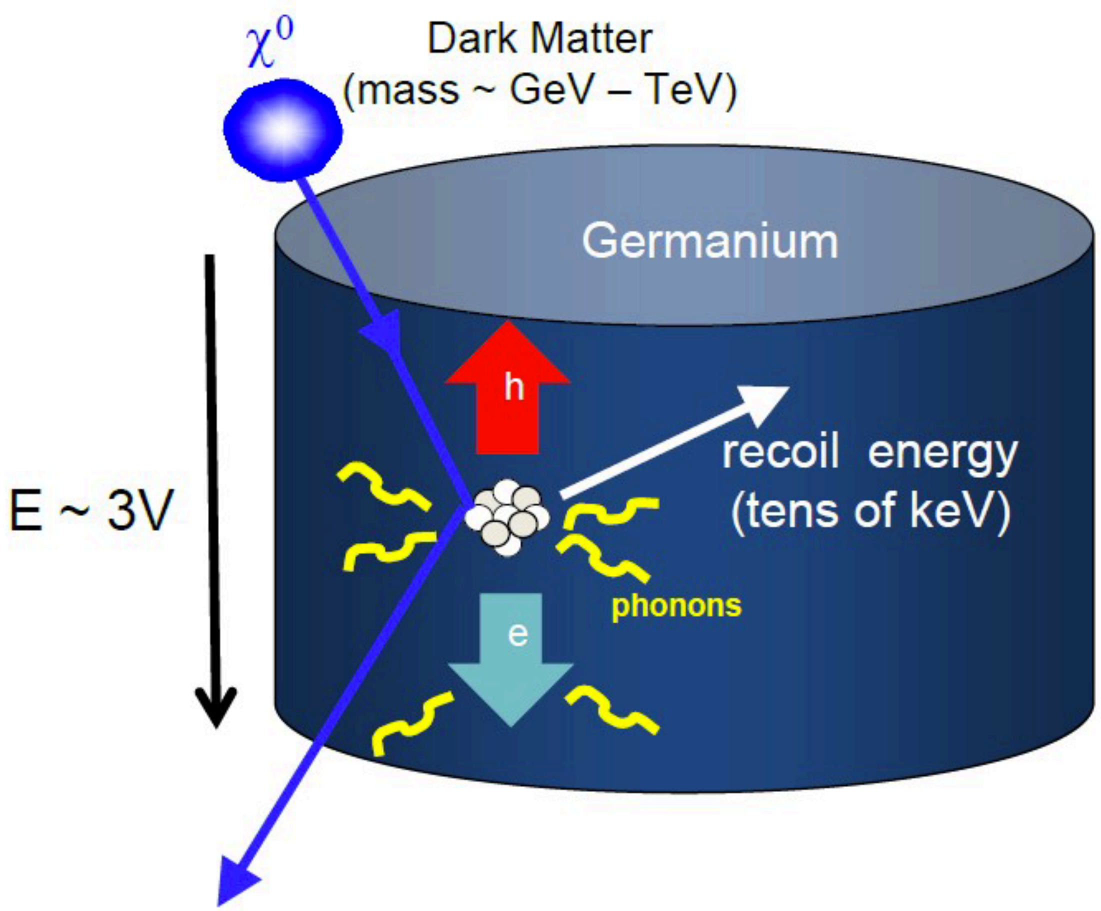




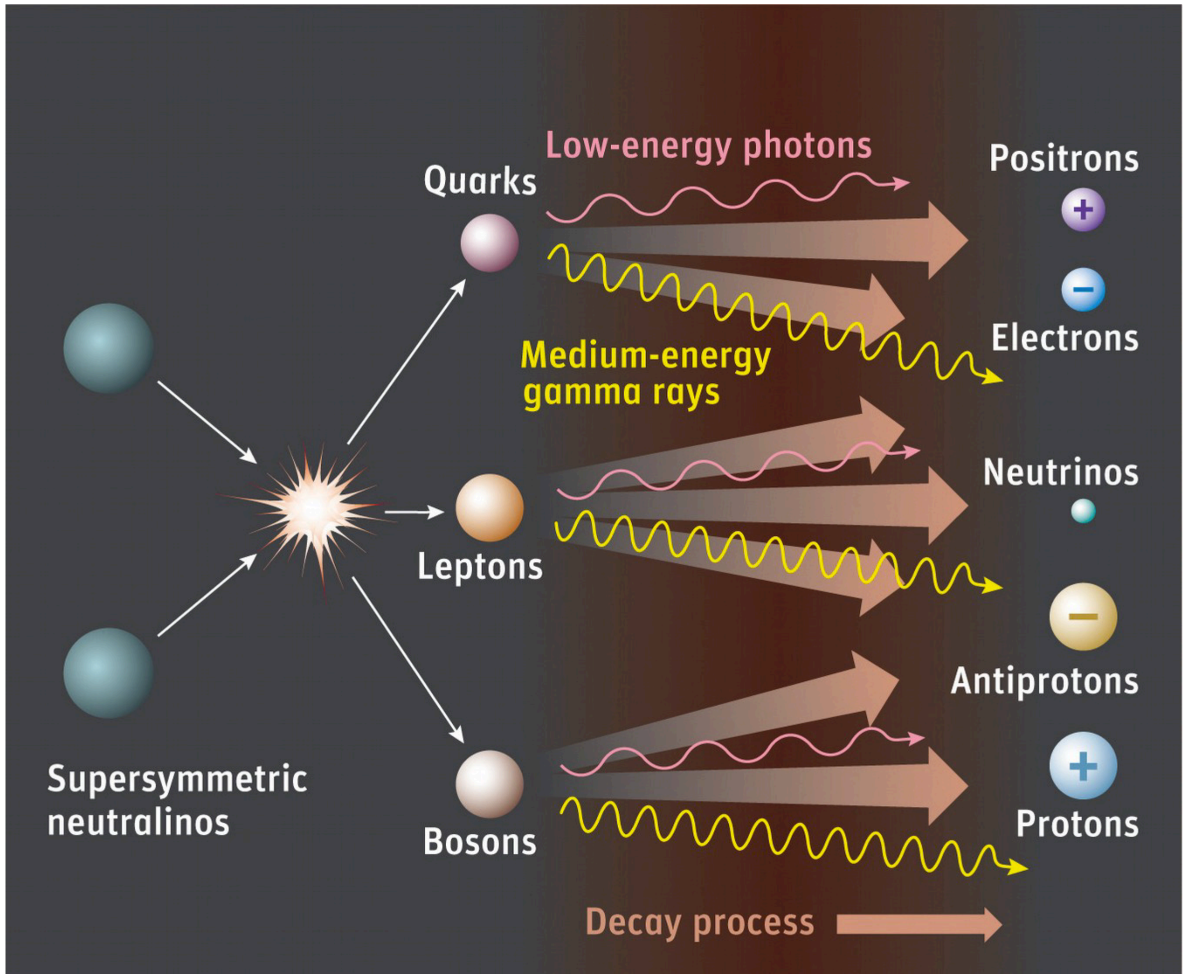
1. The quest for dark matter in the Universe

- DM cross section for interaction with baryonic matter must be extremely small, therefore events of dark-baryonic matter interaction (direct detection) are very rare.
- **Indirect detection** looks instead for production of **γ-rays from DM self-interaction** (annihilation or decay), so it can be attempted with gamma detectors.

DIRECT DETECTION



INDIRECT DETECTION



1. The quest for dark matter in the Universe

- Building up the expected γ-ray flux from e.g. DM annihilation:

differential photon number produced for 1 annihilation event $f_\gamma^{(i)} = \mathrm{BR}_i \frac{dN_\gamma^{(i)}}{dE_\gamma}$

probability of impact for 2 DM particles

interaction probability for the reaction to take place

differential flux for unit line of sight and solid angle

integration along l.o.s. and solid-angle element

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integration along l.o.s. and solid-angle element $\frac{d\Phi_{\mathrm{ann}}}{dE_\gamma} = \frac{\langle \sigma_{\mathrm{ann}} v \rangle}{8\pi m_\chi^2} \sum_i \mathrm{BR}_i \frac{dN_\gamma^{(i)}}{dE_\gamma} J(\Delta\Omega)$

1. The quest for dark matter in the Universe

- What is summarized into $J(\Delta\Omega)$ is exactly what is called **astrophysical factor** (for DM annihilation; e.g., Evans+ 2004, Doro+ 2013):

$$J(\Delta\Omega) = \int_{\Delta\Omega} d\Omega \int_{\mathrm{l.o.s.}} \rho_{\mathrm{DM}}^2(\ell;\Omega) d\ell$$

- In a similar way, the γ-ray flux and astrophysical factor for DM decay $D(\Delta\Omega)$ are defined as:

$$\frac{d\Phi_{\mathrm{dec}}}{dE_\gamma} = \frac{1}{4\pi\tau_{\mathrm{dec}} m_\chi} \sum_i \mathrm{BR}_i \frac{dN_\gamma^{(i)}}{dE_\gamma} D(\Delta\Omega) \qquad D(\Delta\Omega) = \int_{\Delta\Omega} d\Omega \int_{\mathrm{l.o.s.}} \rho_{\mathrm{DM}}(\ell;\Omega) d\ell$$

2. Dark matter density from rotation curves

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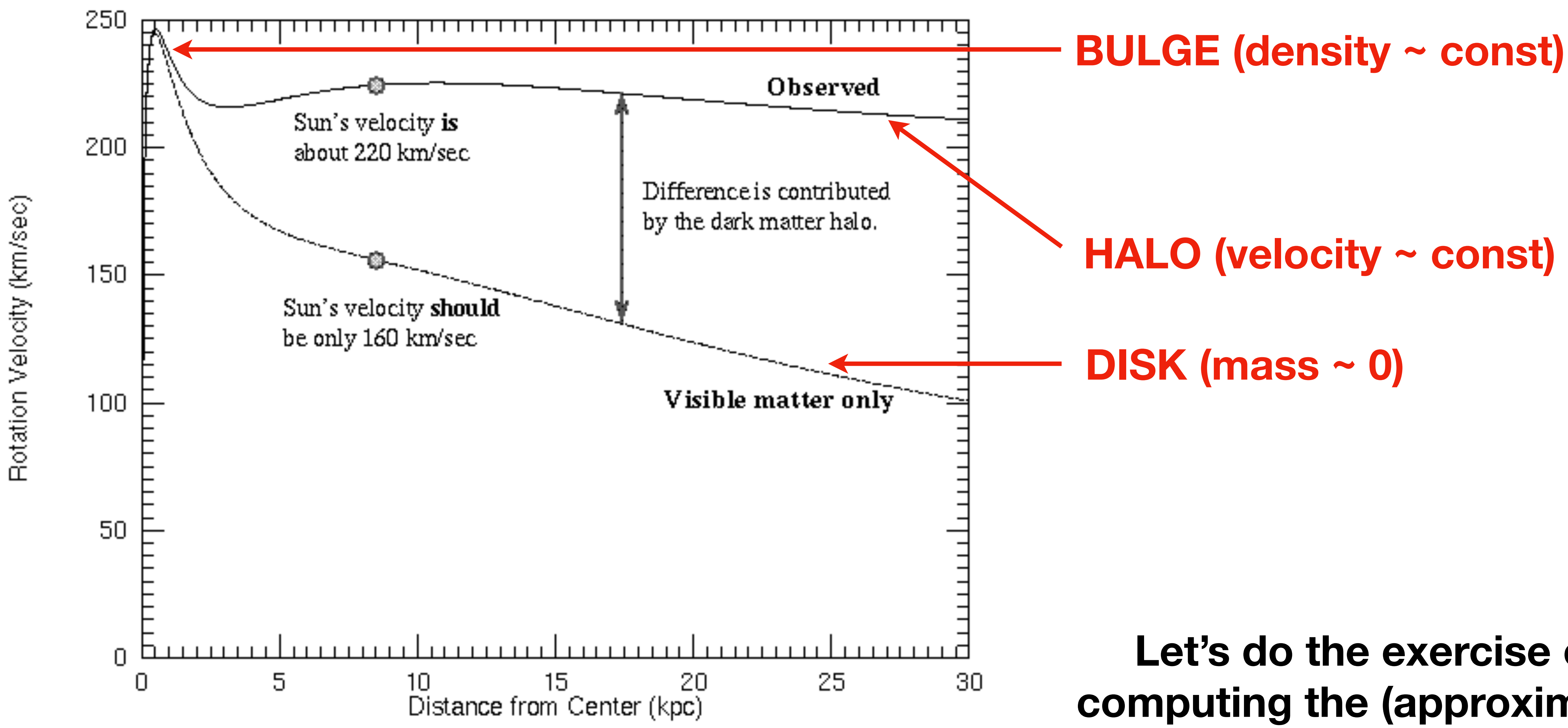
- DM was first introduced in 1933 by F. Zwicky to explain the flattening of the rotation velocity curves of spiral galaxies as observed in e.g. the Milky Way (MW).



The gravity of the visible matter in the Galaxy is not enough to explain the high orbital speeds of stars in the Galaxy. For example, the Sun is moving about 60 km/sec too fast. The part of the rotation curve contributed by the visible matter only is the bottom curve. The discrepancy between the two curves is evidence for a **dark matter halo.**

Let's do the exercise of computing the (approximate) behaviors of such components…

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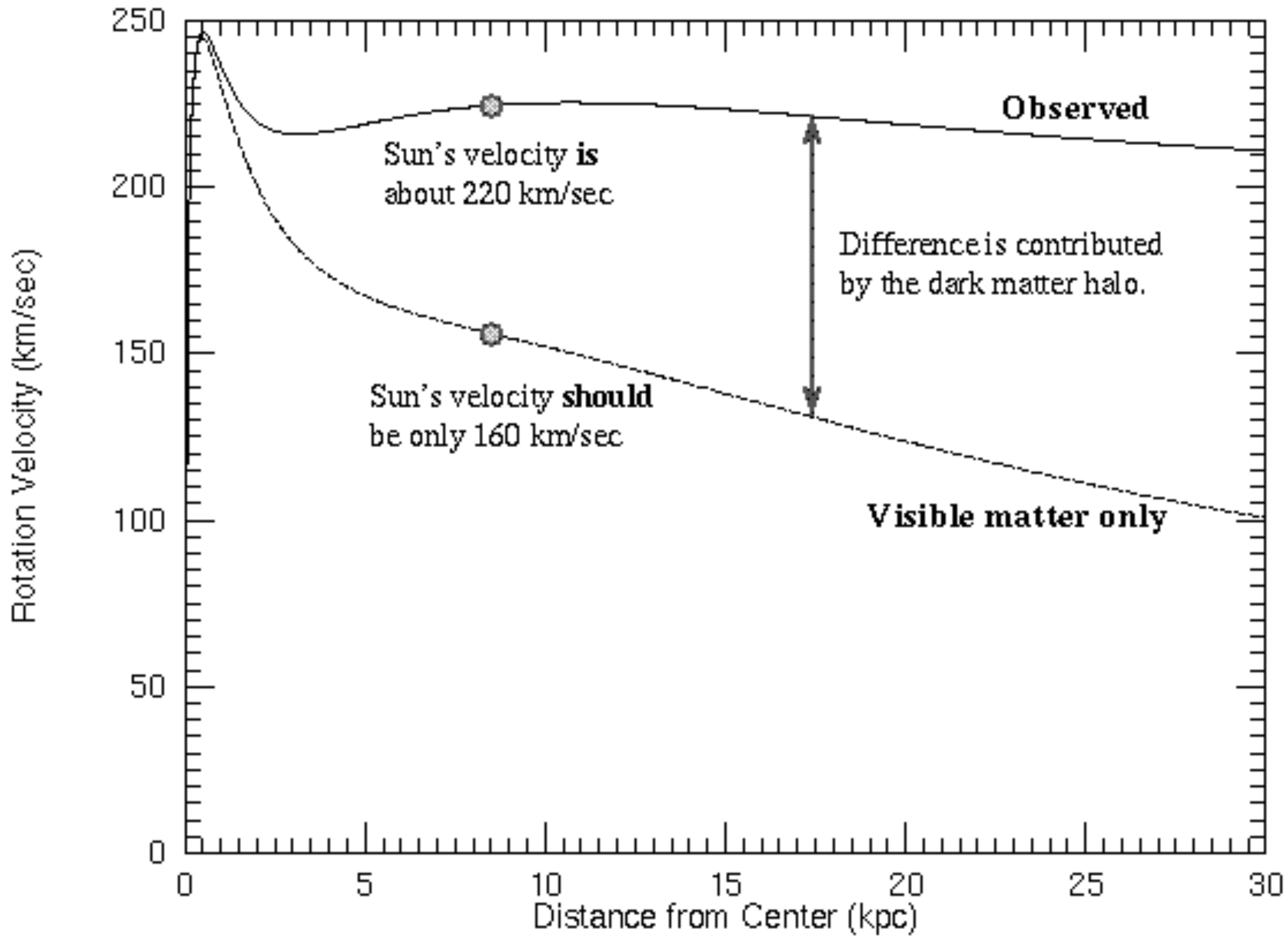

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Poisson equation

$$\nabla^2\Phi = -4\pi G\rho(r)$$

velocity curve

$$v(r) = \sqrt{-r\frac{d\Phi}{dr}}$$

2. Dark matter density from rotation curves

BULGE

$$\rho(r) = \textbf{const} = \rho_0$$

$$\nabla^2\Phi = \frac{1}{r^2}\left[\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right)\right] = -4\pi G\rho_0$$

$$\cancel{r^2}\frac{d\Phi}{dr} - \cancel{r^2\frac{d\Phi}{dr}}\bigg|_{r=0} = -\frac{4}{3}\pi G\rho_0 r^{\cancel{3}}$$

$$v(r) = 2r\sqrt{\frac{\pi}{3}G\rho_0} \propto r$$

DISK

$$M(r) \approx \textbf{const} = M_{\rm bulge}$$

$$\Phi(r) = \frac{GM_{\rm bulge}}{r}$$

$$r\frac{d\Phi}{dr} = -\frac{GM_{\rm bulge}}{r}$$

$$v(r) = \sqrt{\frac{GM_{\rm bulge}}{r}} \propto r^{-1/2}$$

HALO

$$v(r) \approx \textbf{const} = v_\infty$$

$$\Phi(r) = \Phi_s - v_\infty^2 \ln\left(\frac{r}{r_s}\right)$$

$$r\frac{d\Phi}{dr} = -v_\infty^2$$

$$\frac{1}{r^2}\left[\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right)\right] = -\left(\frac{v_\infty}{r}\right)^2 = -4\pi G\rho(r)$$

$$\rho(r) = \frac{v_\infty^2}{4\pi G r^2} \propto r^{-2}$$

2. Dark matter density from rotation curves

- Actually, rotation curves are not exactly flat (presence of bumps, decrease at very large radii) => DM density profile not following the r^2 relation!

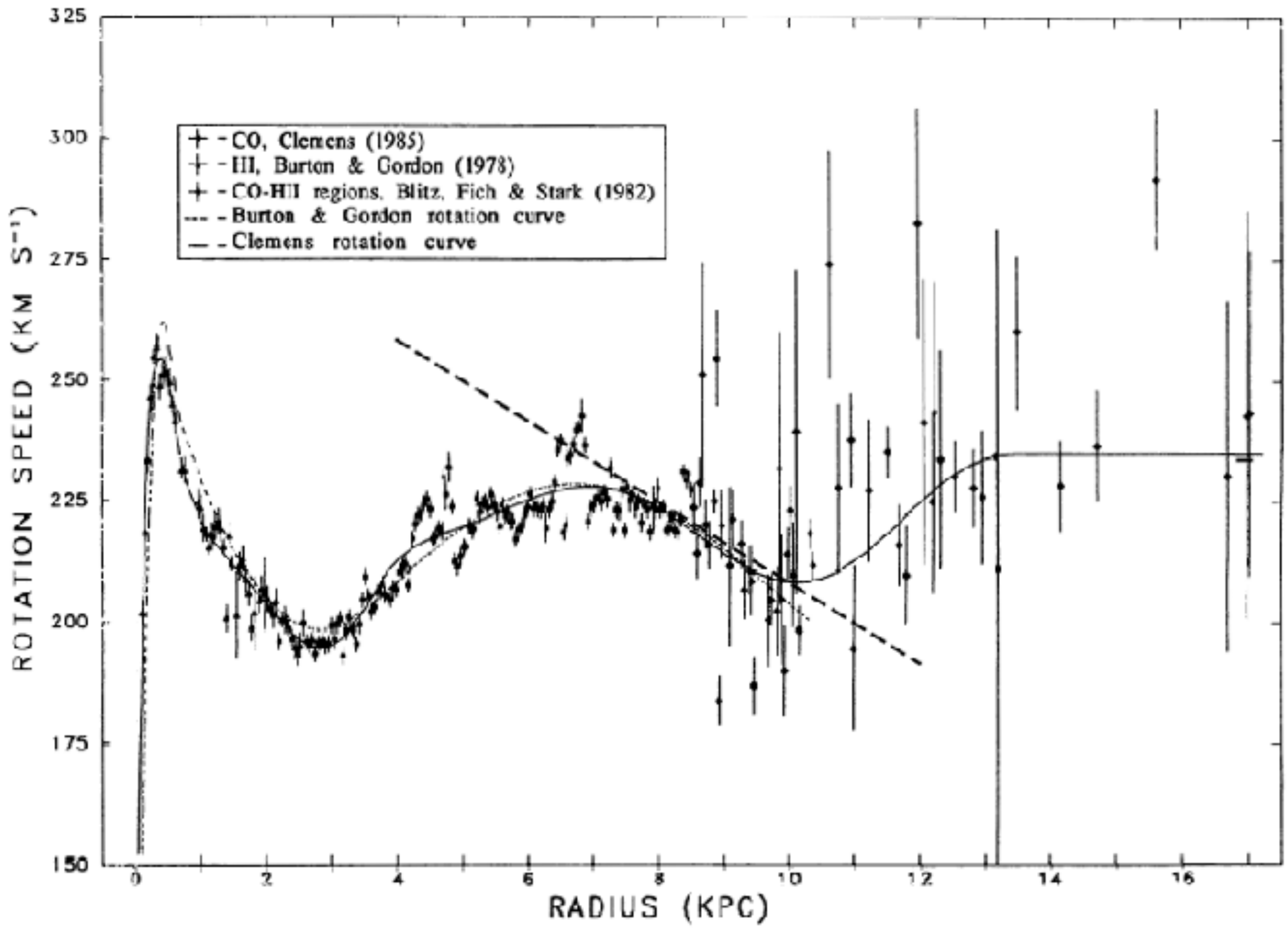


Figure 3 Comparison between several rotation curves obtained by the method of terminal velocity in CO and H I emissions inside the solar radius, and by CO complexes associated with H II regions of known distances (from Clemens 1985). The IAU (1985) galactic constants have been adopted: $R_0 = 8.5$ kpc, $V_0 = 220$ km s^{-1}.

2. Dark matter density from rotation curves

- Alternative profile shapes to the r^2 one have been proposed in the literature to better explain matter-density and rotation-curve features found in observations and cosmological simulations:

Einasto (1965)

$$\rho(r) = \rho_s e^{-\frac{2}{\alpha}\left[\left(r/r_s\right)^{\alpha} - 1\right]}$$

Burkert (1995)

$$\rho(r) = \frac{\rho_s}{\left(1 + r/r_s\right)\left[1 + \left(r/r_s\right)^2\right]}$$

Navarro, Frenk & White (1996)

$$\rho(r) = \frac{\rho_s}{\left(r/r_s\right)\left(1 + r/r_s\right)^2}$$

Zhao (1996) & Hernquist (1990)

$$\rho(r) = \frac{\rho_s}{\left(r/r_s\right)^{\gamma}\left[1 + \left(r/r_s\right)^{\alpha}\right]^{\frac{\beta - \gamma}{\alpha}}}$$

2. Dark matter density from rotation curves

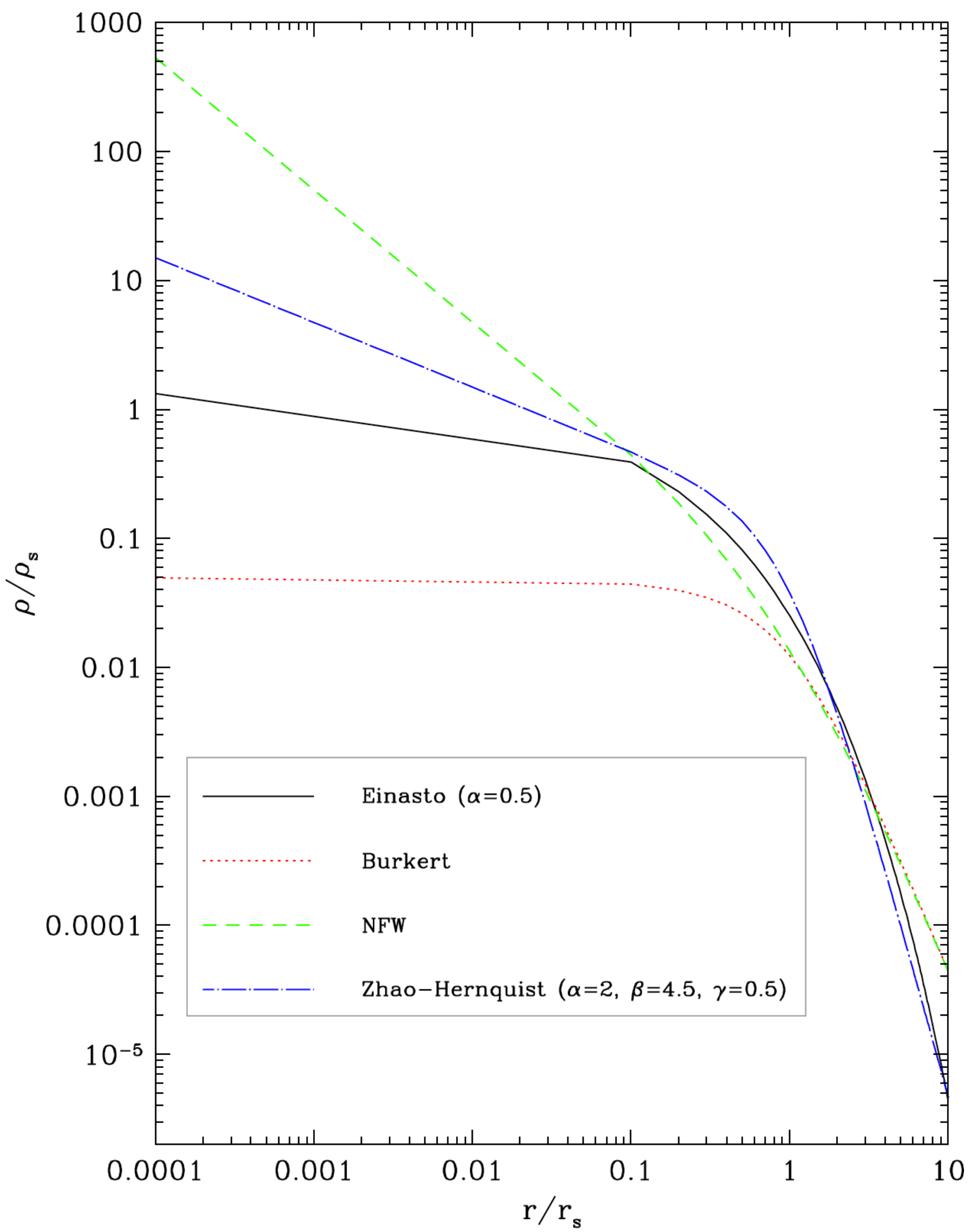
- Alternative profile shapes to the r^2 one have been proposed in the literature to better explain matter-density and rotation-curve features found in observations and cosmological simulations:

Einasto (1965) **CUSPY**

Burkert (1995) **CORED**

Navarro, Frenk & White (1996) **CUSPY**

Zhao (1996) & Hernquist (1990) **–**



3. Dark matter density from stellar kinematics

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- Rotation curves of spiral galaxies are usually derived from mostly measurements of neutral/ionized gas clouds.

- For other types of galaxies (e.g., ellipticals), there are two major problems:
  - **no or little rotational support** (i.e., quasi-radial orbits);
  - **no gas** to measure the rotation velocity.

- Need to change paradigm:
  - change equations;
  - change velocity tracer (gas => stars).

Jeans analysis!

3. Dark matter density from stellar kinematics

- Assumptions:
  - collisionless system (stars do not interact with each other)
  - steady state
  - spherical symmetry <= not so essential
  - negligible rotational support

- The law to be adopted is the **second-order development of the Jeans equation** (e.g., Binney & Tremaine 2008):

$$\frac{1}{n_*}\left[\frac{d}{dr}\left(n_*\overline{v_r^2}\right)\right] + \frac{2}{r}\beta_{\rm ani}(r)\overline{v_r^2} = -\frac{G\left[M_{\rm DM}(r) + M_*(r)\right]}{r^2}$$

$$n_* = n_*(r) \qquad \beta_{\rm ani}(r) = 1 - \frac{\overline{v_\theta^2}}{\overline{v_r^2}} \qquad M_*(r) \approx 0$$

luminosity profiles | **velocity anisotropy** | **DM domination (if verified!)**

3. Dark matter density from stellar kinematics

- NOT easy to derive it from scratch!

Liouville equation

$$\frac{\partial f^{(6N)}}{\partial t} + \left\langle \nabla f^{(6N)}, \mathbf{w}^{(6N)} \right\rangle = 0$$

$$\mathbf{w} = \left(\mathbf{v}, -\nabla \phi_{\rm tot}\right)$$

no collisions →

collisionless Boltzmann equation

$$\begin{cases} \dfrac{\partial f}{\partial t} + \left\langle \mathbf{v}, \dfrac{\partial f}{\partial \mathbf{r}} \right\rangle - \left\langle \nabla \phi_{\rm tot}, \dfrac{\partial f}{\partial \mathbf{v}} \right\rangle = 0 \\ \phi_{\rm tot} = \phi + \phi_{\rm ext} \\ \nabla^2 \phi = -4\pi G \displaystyle\int_{\mathbb{R}^3} f d^3\mathbf{v} \end{cases}$$

steady state ↓

Jeans equation (any symmetry)

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial}{\partial x_i}\left(\rho \overline{v}_i\right) = 0 \\ \dfrac{\partial}{\partial t}\left(\rho \overline{v}_i\right) + \dfrac{\partial}{\partial x_j}\left(\rho \overline{v_i v_j}\right) + \rho \dfrac{\partial \phi}{\partial x_i} = 0 \end{cases}$$

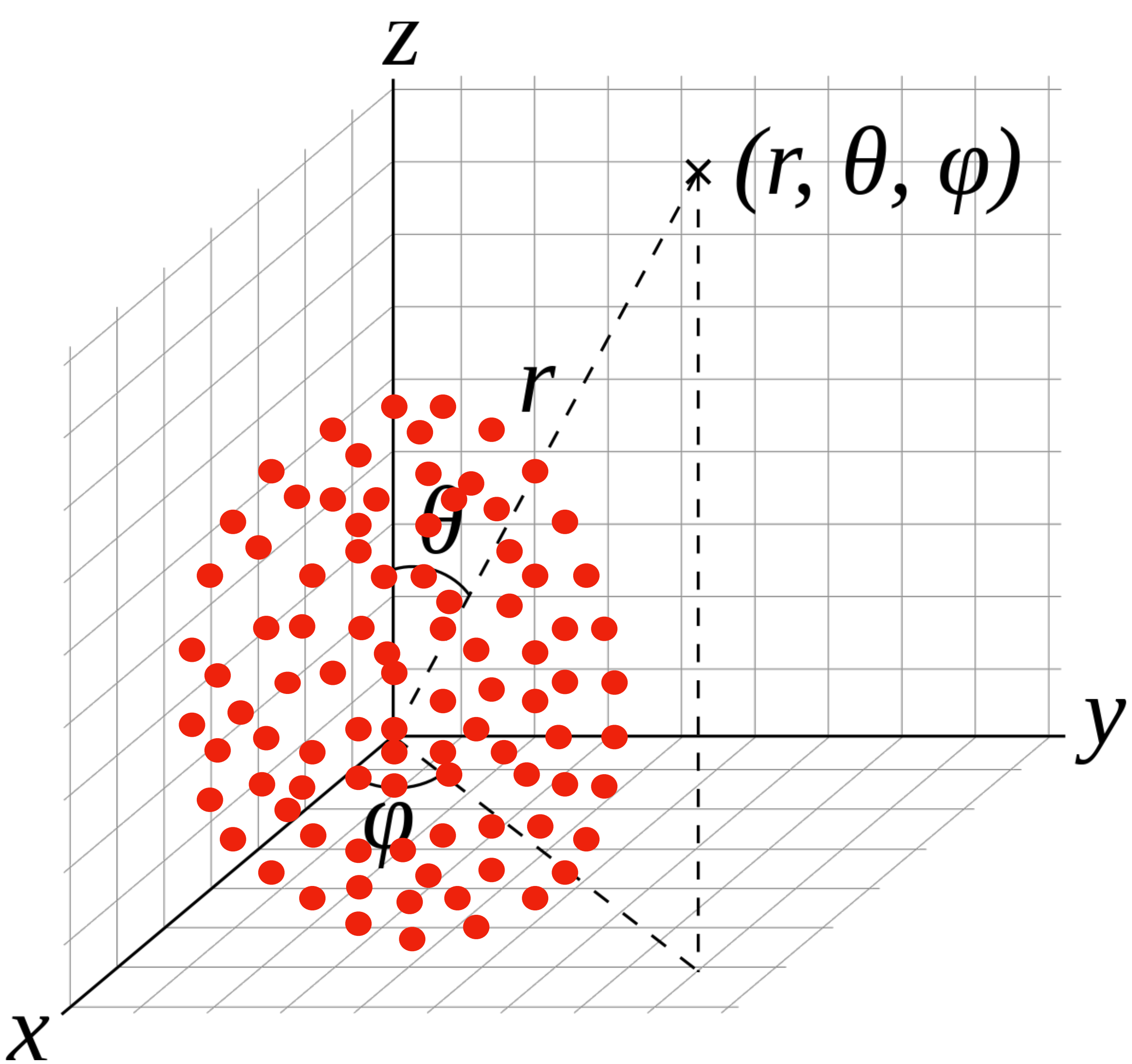
← spher. symmetry, no rot. support

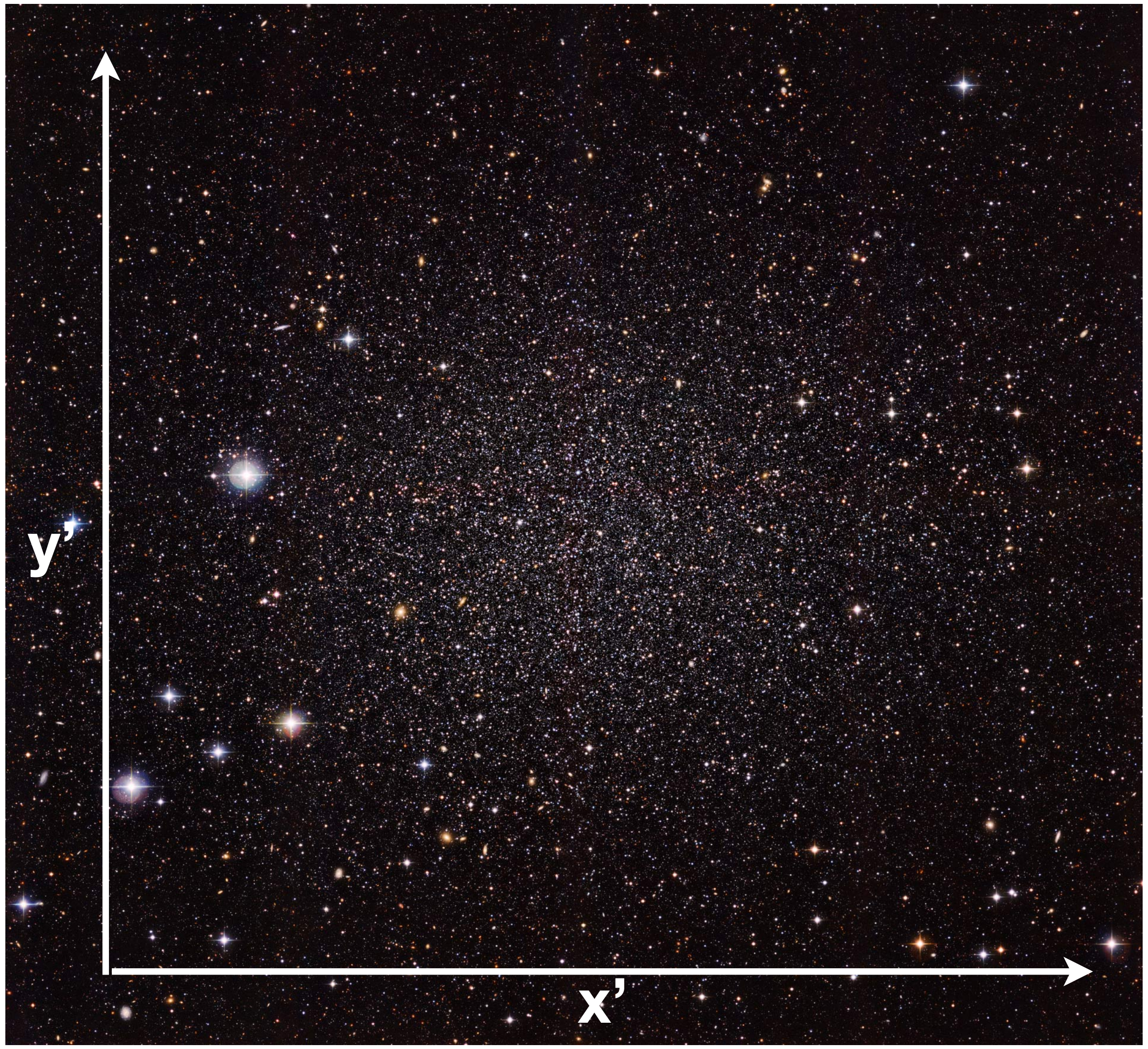
spherical Jeans equation

$$\frac{1}{n_*}\left[\frac{d}{dr}\left(n_* \overline{v_r^2}\right)\right] + \frac{2}{r}\beta_{\rm ani}(r)\overline{v_r^2} = -\frac{G\left[M_{\rm DM}(r) + M_*(r)\right]}{r^2}$$

3. Dark matter density from stellar kinematics

- First caveat: we are dealing with **projected quantities** (2D instead of 3D)!

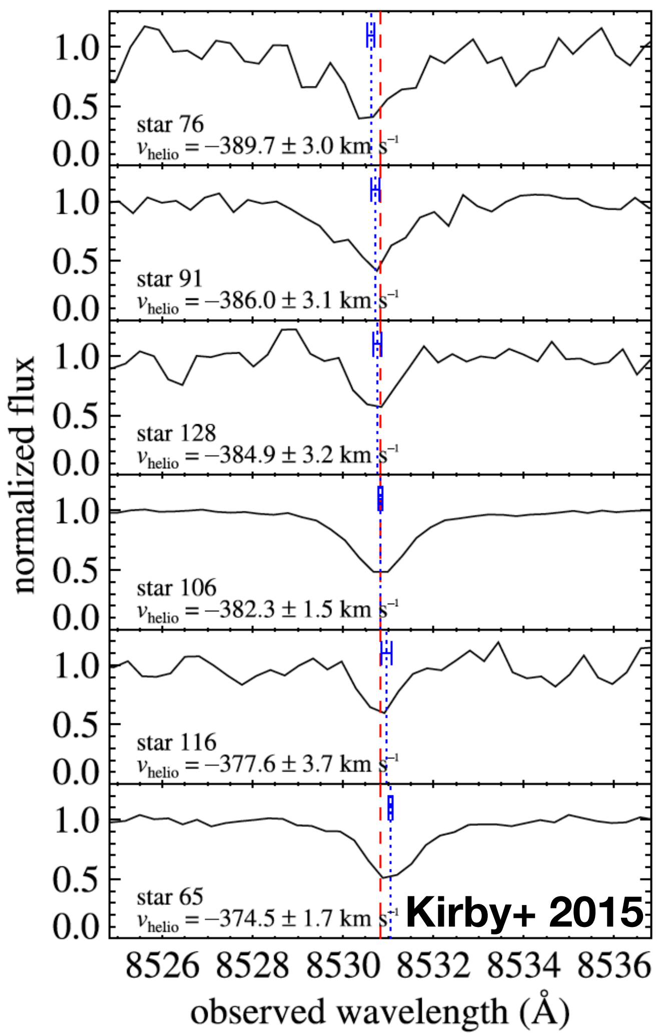


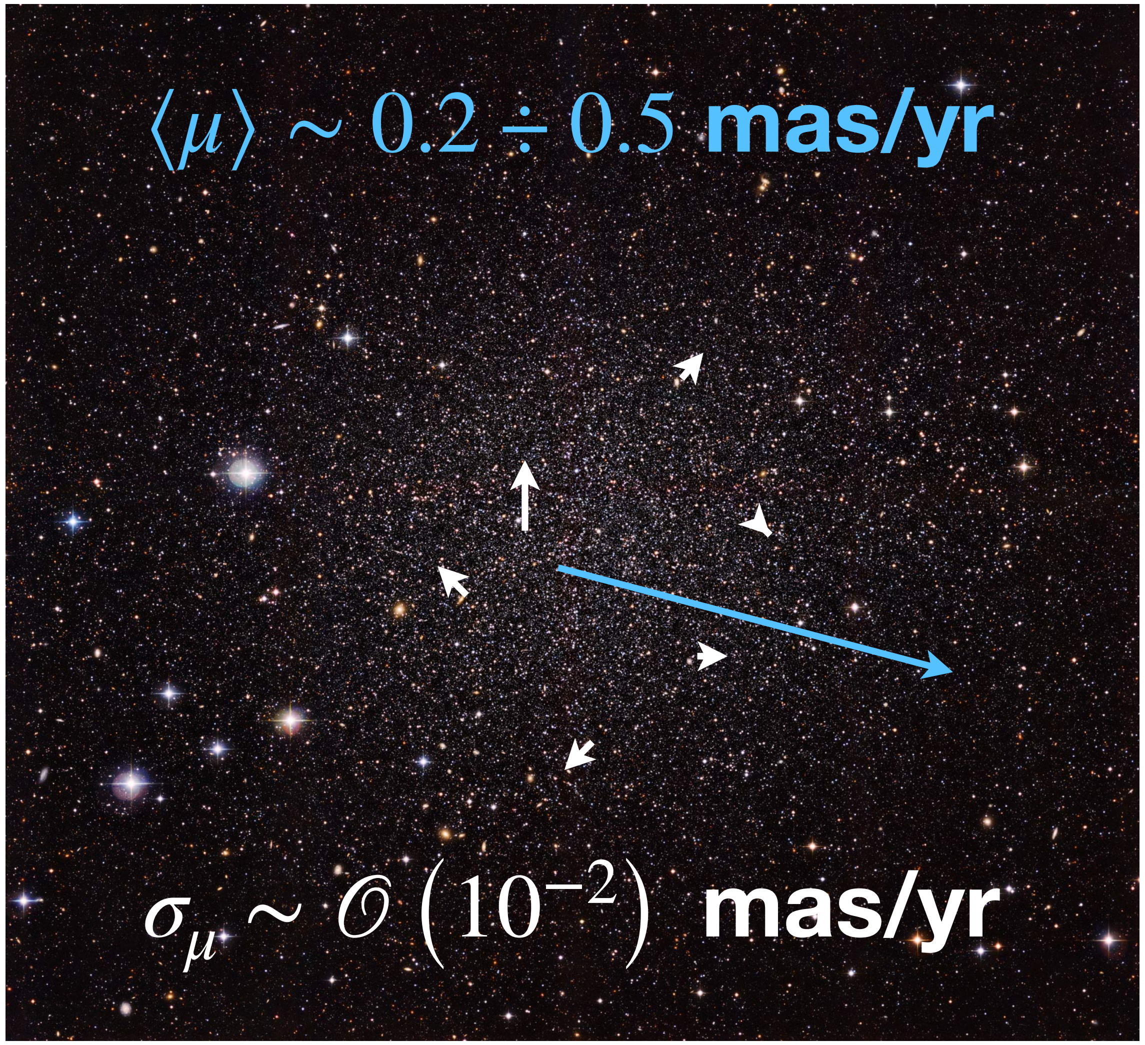


Instead of seeing this… …we actually see this!

3. Dark matter density from stellar kinematics

- Second caveat:
  - radial velocities of member stars are easy to measure from ground-based optical spectroscopy;
  - **tangential velocities are only obtainable from repeated photographs taken at years of time distance.**





3. Dark matter density from stellar kinematics

- Necessity of working with 2D observables and parametrizations of 3D quantities:

2D observable: surface luminosity

$$\Sigma_*(R) = 2\int_R^{+\infty} \frac{n_*(r)r}{\sqrt{r^2 - R^2}}dr$$

2D observable: velocity dispersion

$$\sigma^2_{\text{proj}}(R) = \frac{2}{\Sigma_*(R)}\int_R^{+\infty}\left[1 - \beta_{\text{ani}}(r)\left(\frac{R}{r}\right)^2\right]\frac{n_*(r)\bar{v}_r^2 r}{\sqrt{r^2 - R^2}}dr$$

Parametrization: DM density profile

$$\rho^{\text{Ein}}_{\text{DM}}(r) = \rho_s e^{-\frac{2}{\alpha}\left[\left(\frac{r}{r_s}\right)^{\alpha} - 1\right]}$$

$$\rho^{\text{ZH}}_{\text{DM}}(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)^{\gamma}\left[1 + \left(\frac{r}{r_s}\right)^{\alpha}\right]^{\frac{\beta-\gamma}{\alpha}}}$$

Parametrization: light profile

$$n_*(r) = \frac{n_s^*}{\left(\frac{r}{r_s^*}\right)^{\gamma_*}\left[1 + \left(\frac{r}{r_s^*}\right)^{\alpha_*}\right]^{\frac{\beta_* - \gamma_*}{\alpha_*}}}$$

Parametrization: velocity anisotropy profile

$$\beta^{\text{BvH}}_{\text{ani}}(r) = \frac{\beta_0 + \beta_\infty\left(r/r_a\right)^{\eta}}{1 + \left(r/r_a\right)^{\eta}}$$

3. Dark matter density from stellar kinematics

- Example: fitting the surface luminosity profile.

$$n_*(r) = \frac{n_s^*}{\left(\frac{r}{r_s^*}\right)^{\gamma_*}\left[1+\left(\frac{r}{r_s^*}\right)^{\alpha_*}\right]^{\frac{\beta_*-\gamma_*}{\alpha_*}}}$$

$$\Sigma_*(R) = 2\int_R^{+\infty} \frac{n_*(r)r}{\sqrt{r^2-R^2}}dr$$

Table 1: New surface brightness parameters for the dSph analyzed with CLUMPY.

Name	Ref.	ρ_s^* (10^5 $L_\odot$ kpc^{-3})	r_s^* (kpc)	α^*	β^*	γ^*	χ^2	$N_{d.o.f.}$
Car	[1]	23.8 ± 11.6	0.250 ± 0.021	3.8	3.8	0.0		123
CBe	[2]	3.70 ± 1.70	0.092 ± 0.008	3.8	3.8	1.4		6
CVn I	[3]	2.15 ± 0.50	0.568 ± 0.033	3.2	3.8	0.2		5
Dra I	[1]	466 ± 136	0.170 ± 0.016	2.8	3.8	0.2		123
Ret II	[4]	157 ± 23.6	0.023 ± 0.003	3.2	3.8	0.4		14
Scl	[1]	152 ± 70.0	0.250 ± 0.017	3.4	3.8	0.2		123
Seg 1	[3]	13.1 ± 11.4	0.034 ± 0.007	3.6	3.8	0.8		5
Tri II	[5]	45.4 ± 23.5	0.020 ± 0.002	3.6	3.2	0.2		34
Tuc II	[4]	2.50 ± 0.691	0.141 ± 0.032	3.4	3.8	0.0		11
UMa II	[2]	6.21 ± 3.43	0.035 ± 0.004	2.4	1.6	2.6		8
UMi	[1]	40.3 ± 18.6	0.183 ± 0.031	3.4	3.6	0.2		123

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[4]Bechtol, K., Drlica-Wagner, A., Balbinot, E., et al. 2015, ApJ, 807, 50
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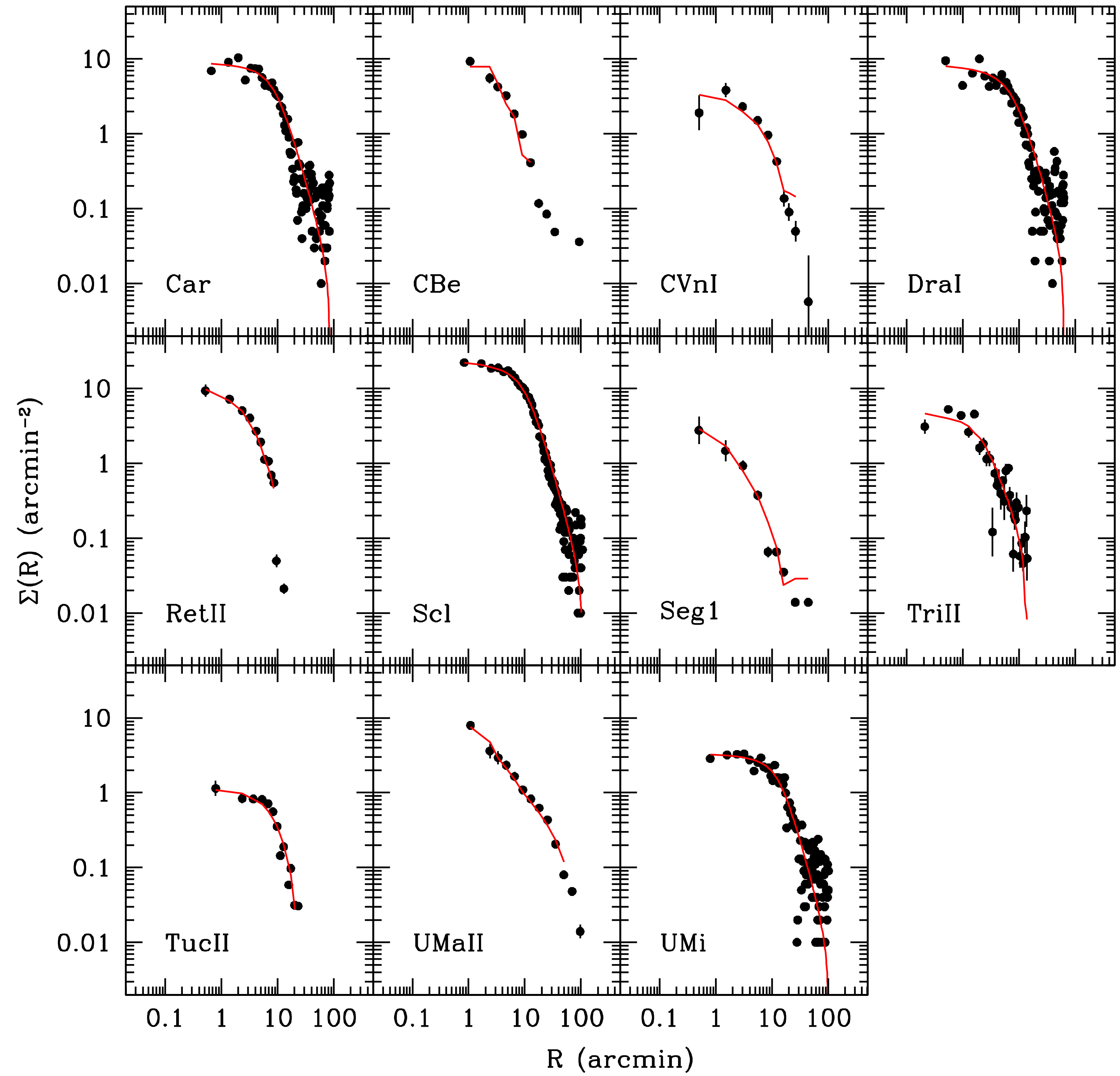

3. Dark matter density from stellar kinematics

- Example: fitting the surface luminosity profile.

$$n_*(r) = \frac{n_s^*}{\left(\frac{r}{r_s^*}\right)^{\gamma_*}\left[1+\left(\frac{r}{r_s^*}\right)^{\alpha_*}\right]^{\frac{\beta_*-\gamma_*}{\alpha_*}}}$$

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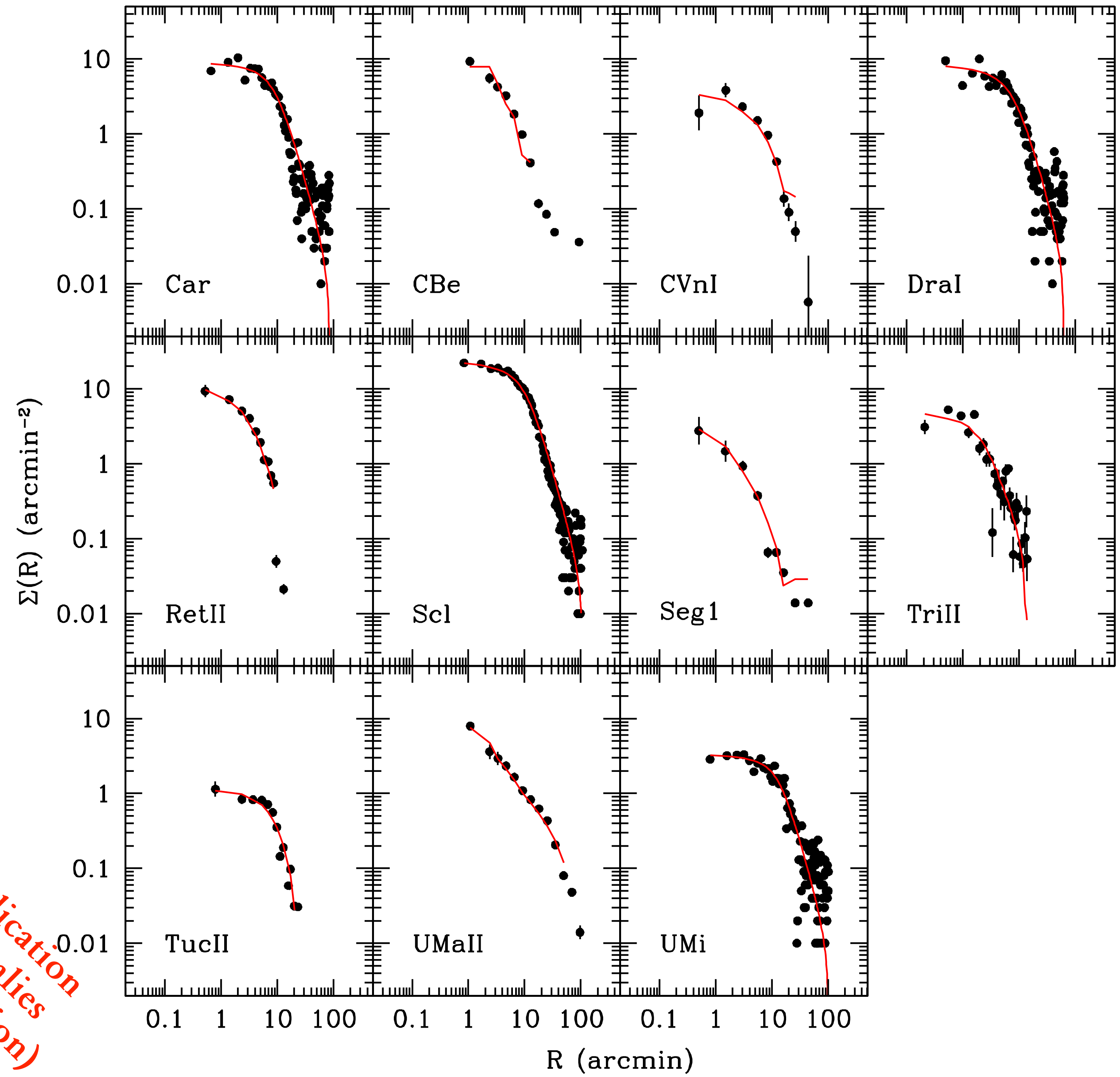
Possible indication of halo anomalies (e.g. tidal disruption)



3. Dark matter density from stellar kinematics

- A note on parametrizations:

  - DM densities from rotation curves/simulations

  - Light profile from surface luminosity fitting

  - Velocity anisotropy as most general solution of (Baes & van Hese 2007, simpler choices exist):

$$\rho_{\mathrm{DM}}(r) = \tilde{\rho}_{\mathrm{DM}}\left[\psi(r), r\right] = f(\phi)g(r) \Rightarrow \beta_{\mathrm{ani}}(r) = -\frac{1}{2}\left(\frac{d\ln g}{d\ln r}\right)$$

4. The case of dwarf spheroidal galaxies

4. The case of dwarf spheroidal galaxies

- Dwarf spheroidal galaxies (dSphs) are satellites of the MW and other Local Group galaxies that exhibit virial masses much higher than what expected from their stellar luminosities.
- Possible reason: **extreme DM domination!**

$$2\langle \mathcal{T} \rangle + \langle U \rangle = 0$$

$$3 m_* \sigma_r^2 = \frac{G M_{\rm tot} m_*}{R}$$

$$M_{\rm tot} = \frac{3 R \sigma_r^2}{G}$$

$$m_* \approx 1\ \mathbf{M}_\odot \rightarrow M_* \approx N\ \mathbf{M}_\odot \Rightarrow L_{\rm tot} \approx N\ \mathbf{L}_\odot$$

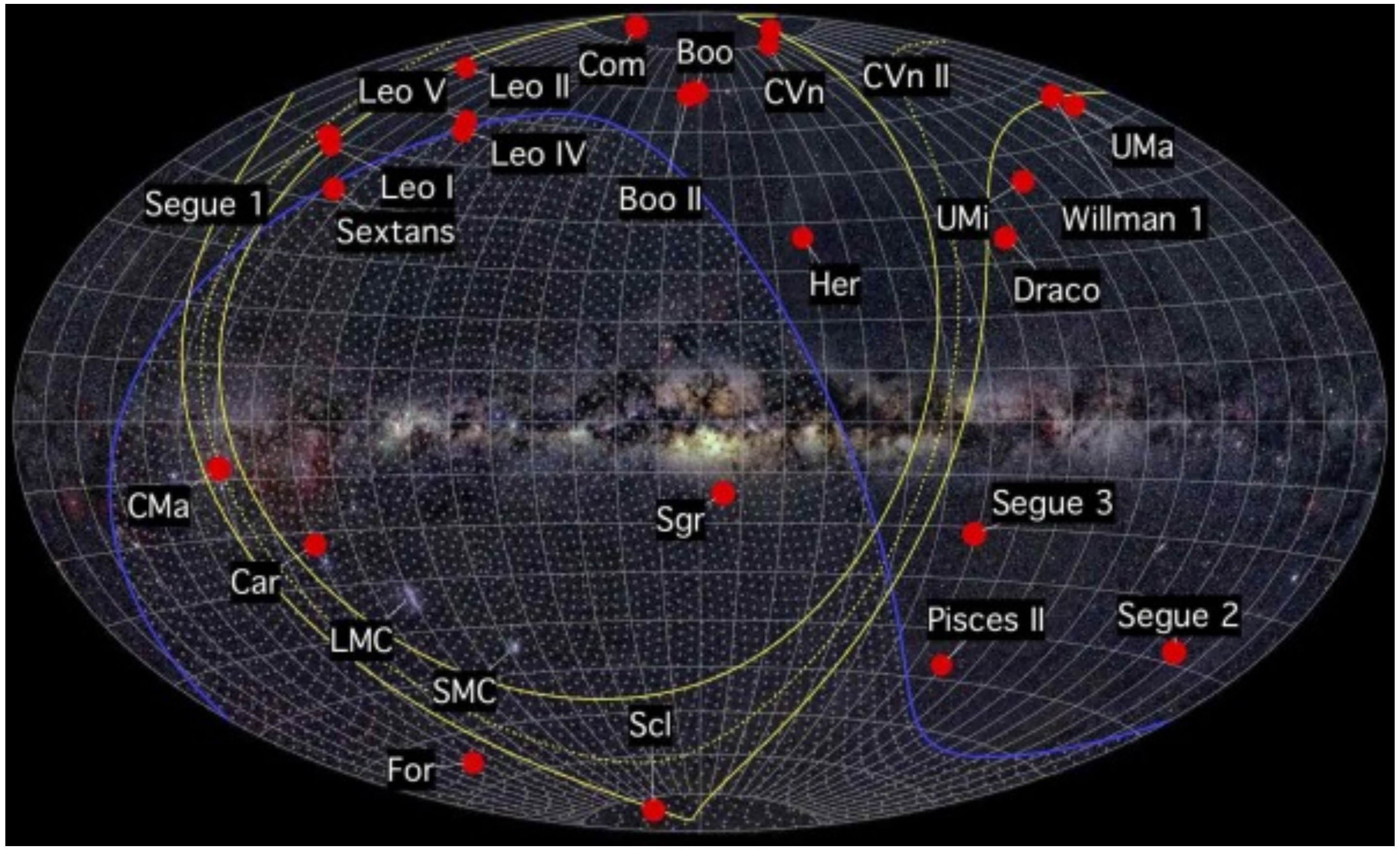
expected | measured

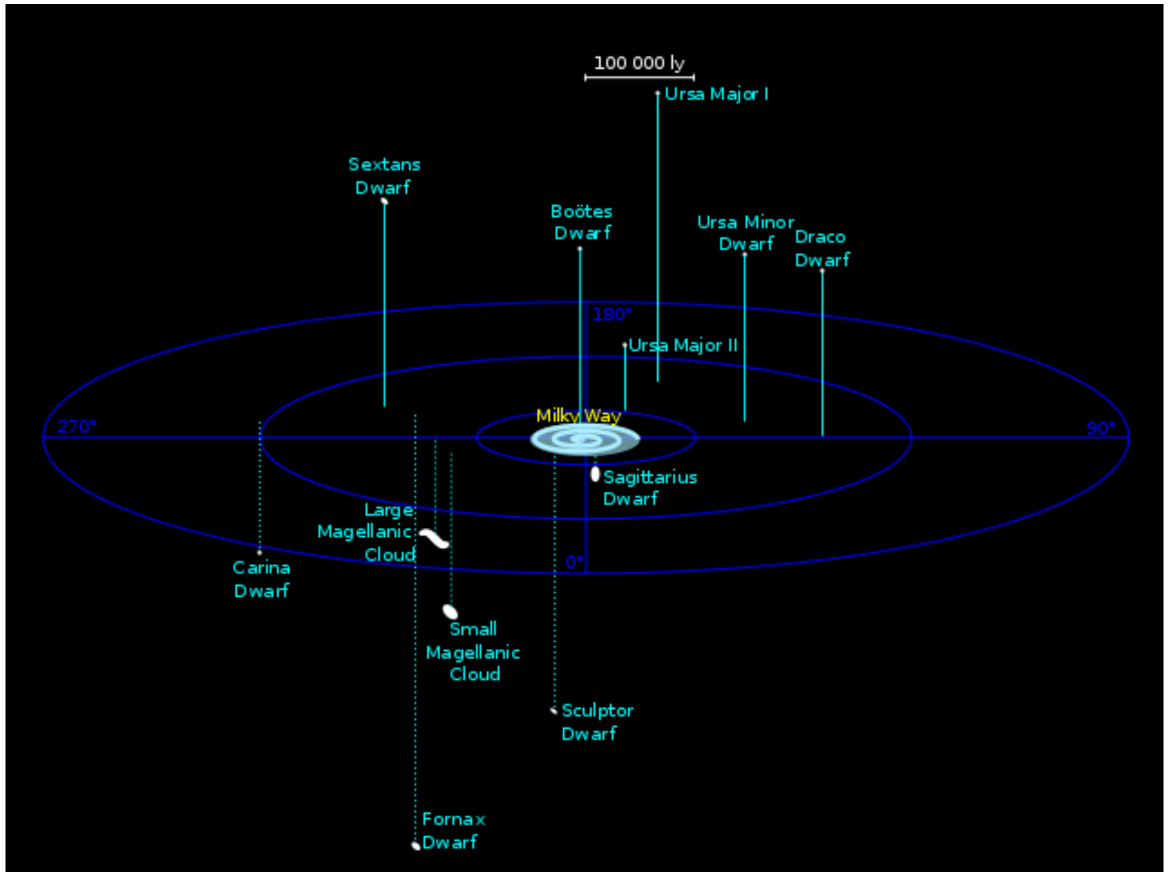
$$\left(\frac{M_{\rm tot}}{L_{\rm tot}}\right)_{\rm theo} = \frac{M_*}{L_{\rm tot}} \approx 1 \quad \Bigg| \quad 10 \lesssim \left(\frac{M_{\rm tot}}{L_{\rm tot}}\right)_{\rm meas} \lesssim 1000$$

4. The case of dwarf spheroidal galaxies

- Several dSphs known around the MW (+ many more being discovered now thanks to performance improvements of telescope technologies).

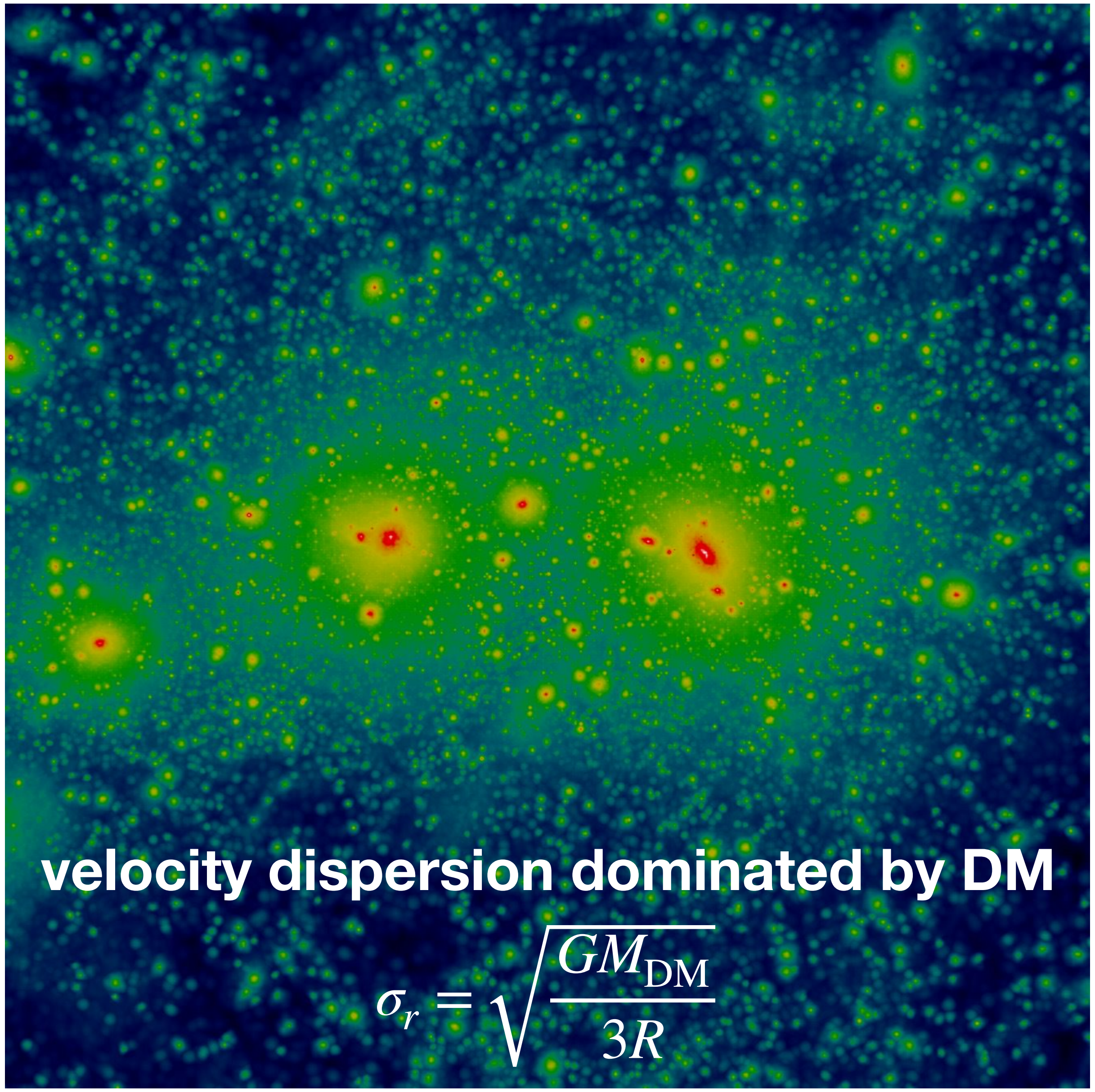
- Two main categories: **classical dSphs** ($\mathcal{O}(100)$ to $\mathcal{O}(1000)$ member stars) and **ultra-faint dSphs** (less than $\mathcal{O}(10)$ to less than $\mathcal{O}(100)$ member stars).

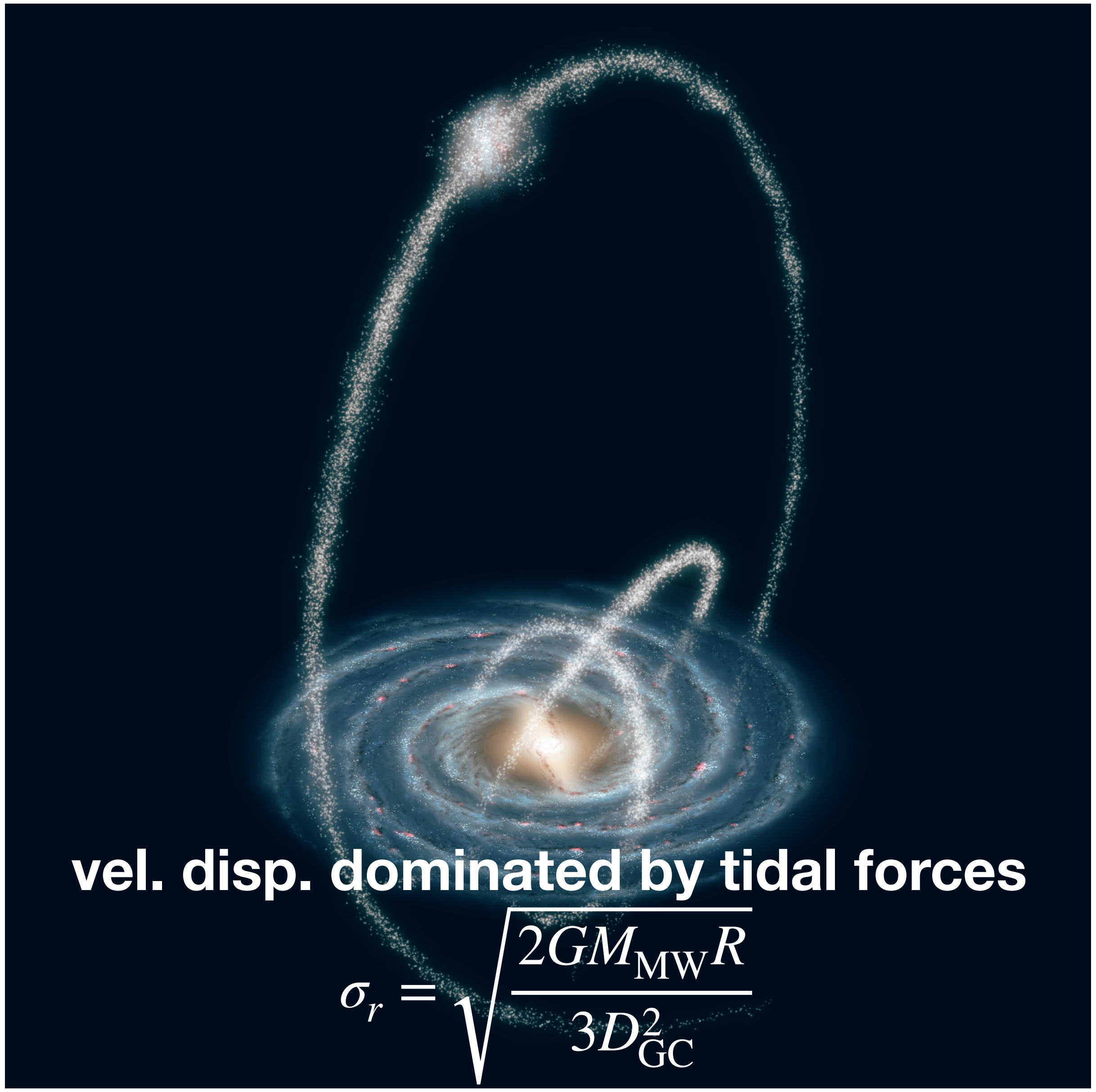




4. The case of dwarf spheroidal galaxies

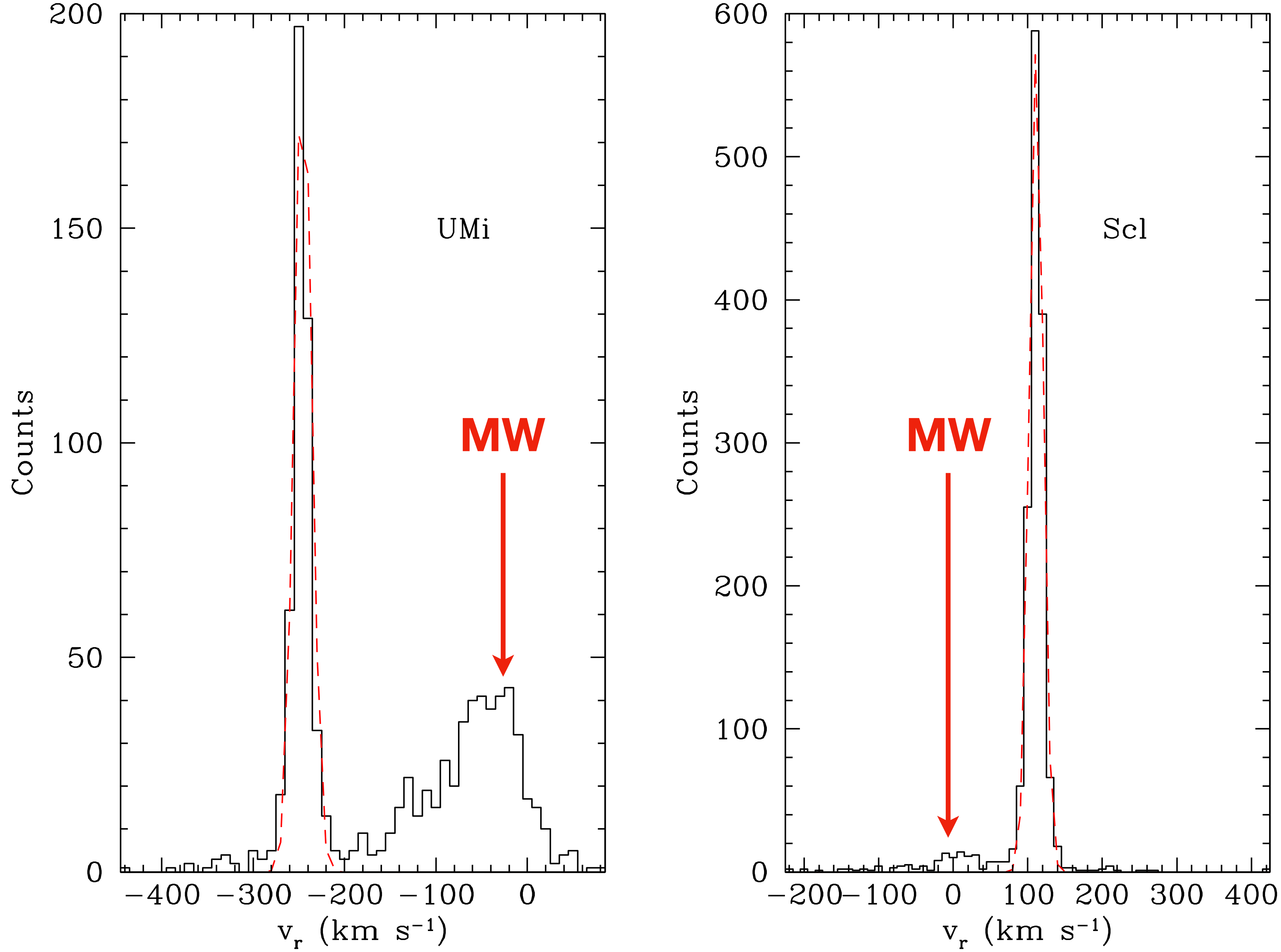
- Third caveat: **uncertain origin of dSphs** (mini-halos of cosmological origin vs. remnants of tidally disrupted bigger galaxies)!





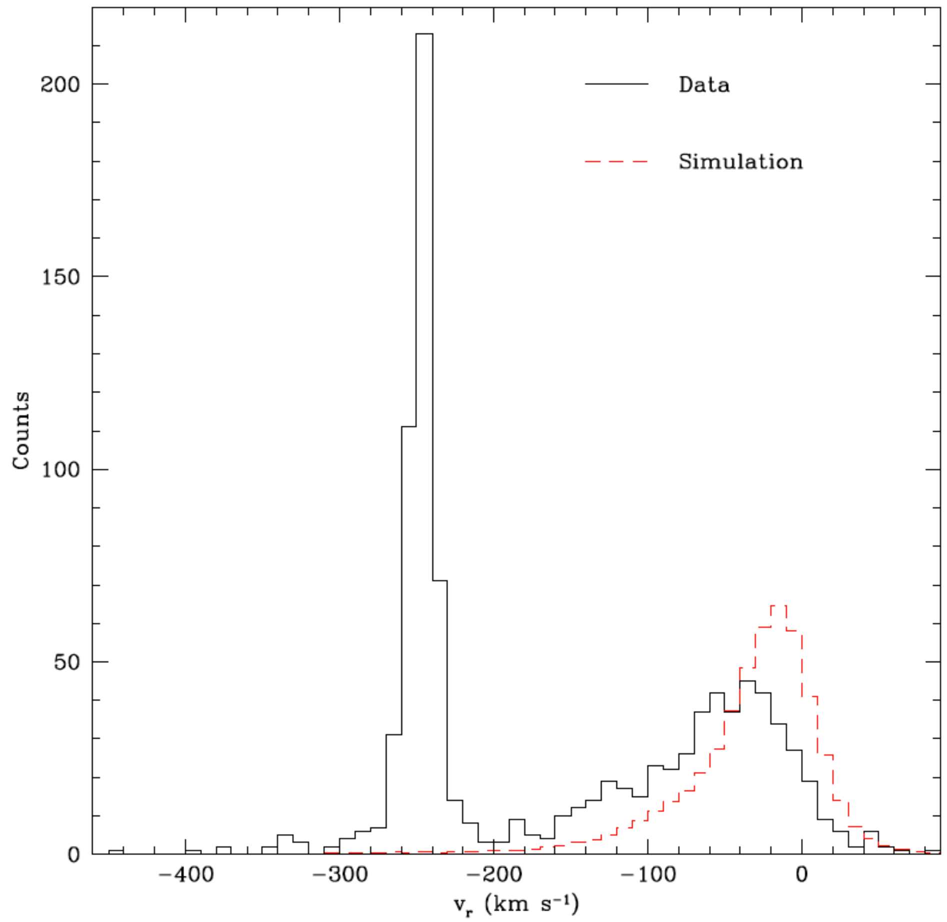
4. The case of dwarf spheroidal galaxies

- Fourth caveat: due to viewing dSphs through MW disk + halo, we see them in the background of Galactic stellar populations!



4. The case of dwarf spheroidal galaxies

- Necessity of removing (statistically speaking) the foreground MW stars from the dSph data sample:
  - go to http://model.obs-besancon.fr/ to retrieve a simulated distribution of foreground contaminants;
  - compare them to the data using an **expectation maximization (EM) algorithm** (e.g., Walker+ 2009).



$$p_{\rm mem}\left(v_i, W_i\right) = \frac{\exp\left\{-\frac{1}{2}\left[\frac{\left(v_i - \langle v_{\rm mem}\rangle\right)^2}{\sigma(v)^2_{\rm mem} + \sigma(v)^2_i} + \frac{\left(W_i - \langle W_{\rm mem}\rangle\right)^2}{\sigma(W)^2_{\rm mem} + \sigma(W)^2_i}\right]\right\}}{2\pi\sqrt{\left[\sigma(v)^2_{\rm mem} + \sigma(v)^2_i\right]\left[\sigma(W)^2_{\rm mem} + \sigma(W)^2_i\right]}}$$

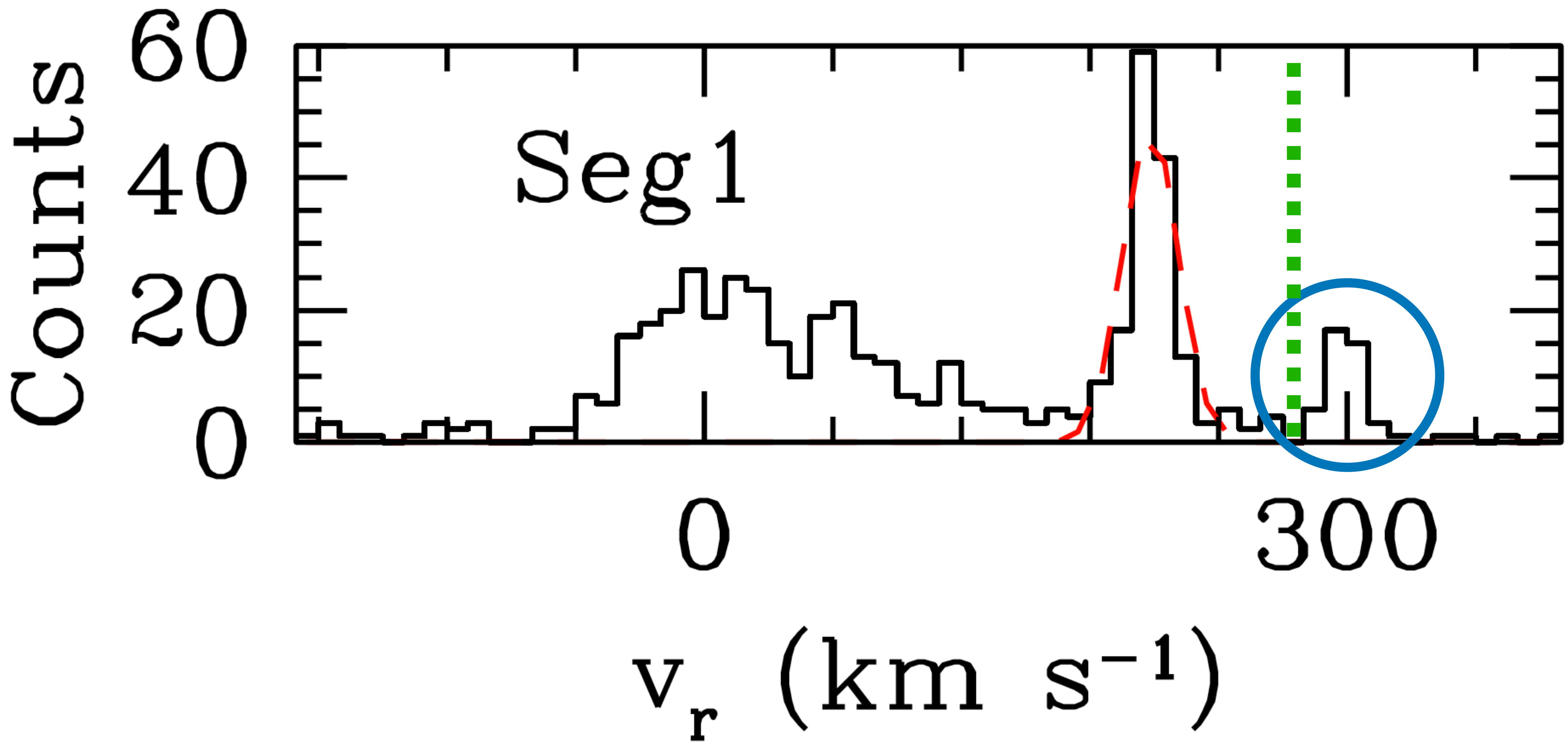
$$p_{\rm Bes}\left(v_i\right) = \frac{1}{N_{\rm Bes}\sigma_{\rm Bes}\sqrt{2\pi}}\sum_{i=1}^{N_{\rm Bes}}\exp\left\{-\frac{\left[v^{(i)}_{\rm Bes} - v_i\right]^2}{2\sigma^2_{\rm Bes}}\right\}$$

$$p_{\rm non}\left(v_i, W_i\right) = \frac{p_{\rm Bes}\left(v_i\right)}{\sqrt{2\pi\left[\sigma(W)^2_{\rm non} + \sigma(W)^2_i\right]}}\exp\left\{-\frac{\left(W_i - \langle W_{\rm non}\rangle\right)^2}{2\left[\sigma(W)^2_{\rm non} + \sigma(W)^2_i\right]}\right\}$$

Implement the above equations into an iterative algorithm and run…

4. The case of dwarf spheroidal galaxies

- Is this enough for all dSphs? **NO!** Fifth caveat!
- E.g. the case of Seg1: high-velocity foreground stellar population contaminating the sample (need to remove it by hand with static velocity cuts).



4. The case of dwarf spheroidal galaxies

- Final product: membership probability for each observed star in the dSph field to appropriately weigh the corresponding position and radial velocity in the Jeans analysis.
- Usually, only stars with $p_{\text{mem}} > 0.95$ are classified as members.

R[kpc]	dR[kpc]	V[km/s_or_km^2/s^2]	dV[km/s_or_km^2/s^2]	MembershipProba	RA[deg]	Dec[deg]
0.009898	0.001650	-247.800000	1.000000	1.000000	227.283210	67.231694
0.016446	0.002741	-256.300000	2.300000	1.000000	227.301000	67.224444
0.022344	0.003724	-246.500000	0.900000	1.000000	227.293500	67.242250
0.030086	0.005014	-245.200000	0.700000	1.000000	227.272670	67.248250
0.036702	0.006117	-241.500000	1.100000	1.000000	227.250750	67.227667
0.043062	0.007177	-248.100000	1.700000	1.000000	227.308420	67.188417
0.044791	0.007465	-251.100000	1.500000	1.000000	227.286920	67.265250
0.046904	0.007817	-237.400000	1.000000	1.000000	227.291960	67.178194
0.047053	0.007842	-239.700000	1.200000	1.000000	227.290000	67.177806
0.052193	0.008699	-267.200000	0.500000	1.000000	227.273460	67.270889
0.052251	0.008708	-237.800000	1.300000	1.000000	227.315830	67.262056
0.052286	0.008714	-255.500000	0.800000	1.000000	227.303250	67.269139
0.053317	0.008886	-245.500000	1.300000	1.000000	227.277960	67.172139
0.054500	0.009083	-252.100000	0.500000	1.000000	227.306870	67.269917
0.055537	0.009256	-251.800000	0.600000	1.000000	227.233250	67.212944
0.056221	0.009370	-261.200000	1.800000	1.000000	227.332130	67.248972

4. The case of dwarf spheroidal galaxies

- Final array of parameters to be fitted (up to now):
  - 2-5 pars for light profile;
  - 2-5 pars for DM density profile;
  - 1-4 pars for velocity anisotropy profile.

- **Total of 5-14 free parameters!** May be reduced by e.g. fixing the light profile (availability of very good survey data for each dSph) => **3-9 free parameters** to be fitted in order to solve the Jeans equation.

- Last step: set appropriate parameter priors to avoid divergences.

DM profile	Parameter	Prior	Added condition	
'Zhao' equation (8)	$\log_{10}(\rho_s/\mathrm{M_\odot\,kpc^{-3}})$	[5, 13]	–	
	$\log_{10}(r_s/\mathrm{kpc})$	[− 3, 1]	$r_s \geq r_s^*$	(Section 4.1)
	α	[0.5, 3]	–	
	β	[3, 7]	–	
	γ	[0, 1.5]	$\gamma \leq 1$	(Section 5)
'Einasto' equation (9)	$\log_{10}(\rho_{-2}/\mathrm{M_\odot\,kpc^{-3}})$	[5, 13]	–	
	$\log_{10}(\mathrm{r}_{-2}/\mathrm{kpc})$	[−3, 1]	$r_{-2} \geq r_s^*$	(Section 4.1)
	α	[0.05, 1]	$\alpha \geq 0.12$	(Section 5)

Anisotropy profile	Parameter	Prior
'Cst' equation (16)	β_0	[− 9, 1]
'Osipkov–Merritt' equation (17)	$\log_{10}(r_a)$	[− 3, 1]
'Baes and van Hese' equation (18)	β_0	[− 9, 1]
	β_∞	[− 9, 1]
	$\log_{10}(r_a)$	[− 3, 1]
	η	[0.1, 4]

Bonnivard+ 2015a

4. The case of dwarf spheroidal galaxies

- The final recipe to compute *J*/*D* from velocity dispersion:

  - choose an appropriate code to be fed with your input data (e.g. **CLUMPY**, see talk by Moritz!);
  - select your analysis type (binned analysis in case of >100 member stars available, unbinned analysis otherwise);
  - select your input data type corresponding to the adopted priors (shape of light profile, shape of DM density, shape of velocity anisotropy);
  - select your fitting method (χ^2 minimization, bootstrap, MCMC) and run a significant number of extractions.

NOTE: CLUMPY is not able a.t.m. to perform triaxial analyses, so one must circularize radii from dSph centroid and account for further ~0.4 dex in the J/D uncertainty (sixth caveat)!

$$R_* = a_* \sqrt{1 - e}$$

$$r_e = \sqrt{\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2}$$

4. The case of dwarf spheroidal galaxies

- Assuming that everything works fine, you end up with a **posterior distribution** of your best-fit parameters which you can use to compute your J/D-factor profiles!

Name	long.(deg)	lat.(deg)	l(kpc)	Ndata	Npar	Rmax
Sculptor	2.875334e+02	-8.315680e+01	8.600000e+01	1352	7	5.000000e+02

rhos(Msol/kpc3)	rs(kpc)	Rvir(kpc)	profile	alpha	beta	gamma	lightprofile	L(Lsol)	rL(kpc)	alpha*	beta*	gamma*	anisoprofile	beta0	betainf	raniso(kpc)	eta	chi2
2.508742E+07	5.201636E-01	3.980339E+00	kEINASTO	2.941071E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-1.119524E+00	-2.610201E+00	8.813193E+00	3.546839E-01	-3.698610E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
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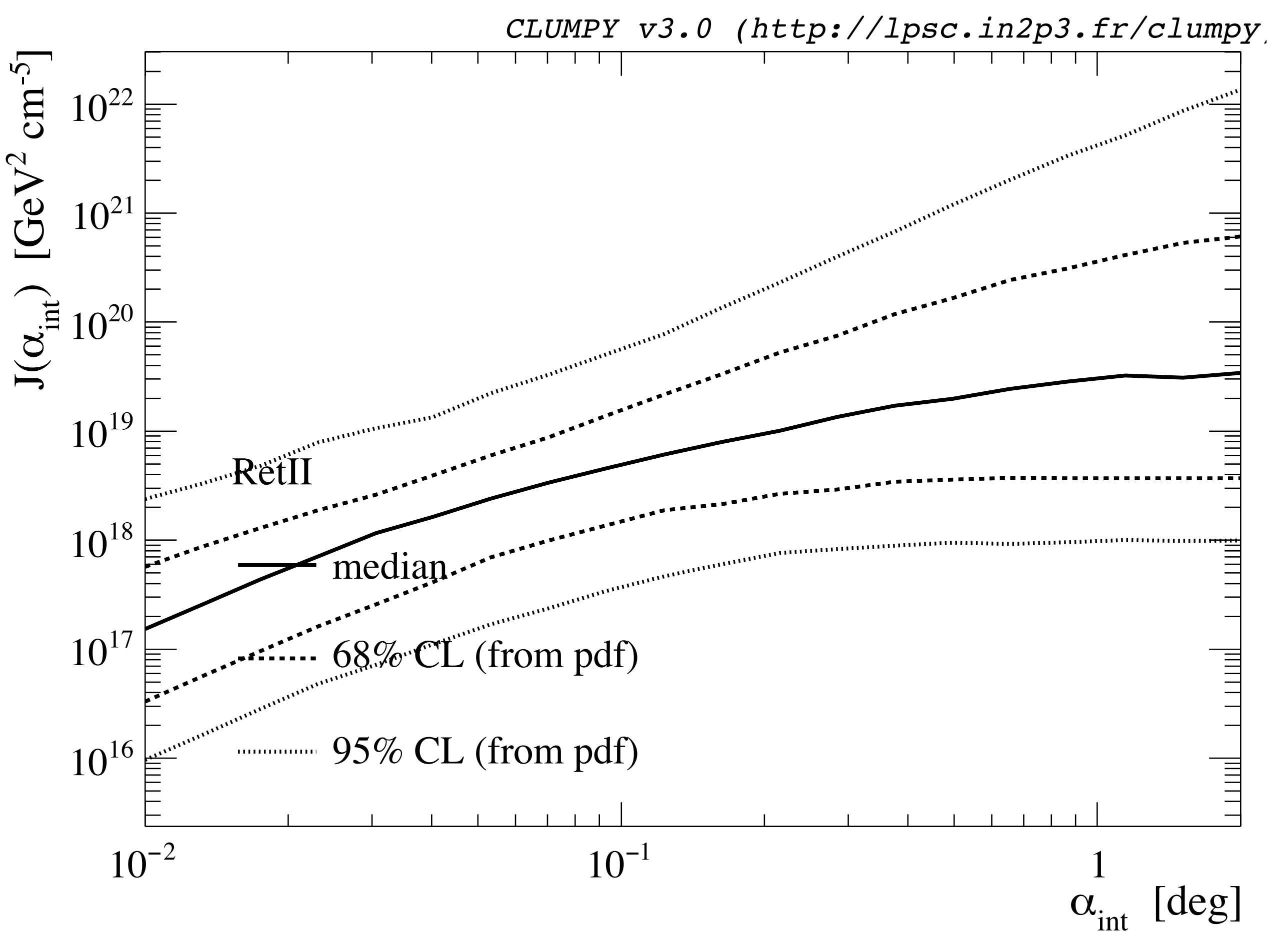
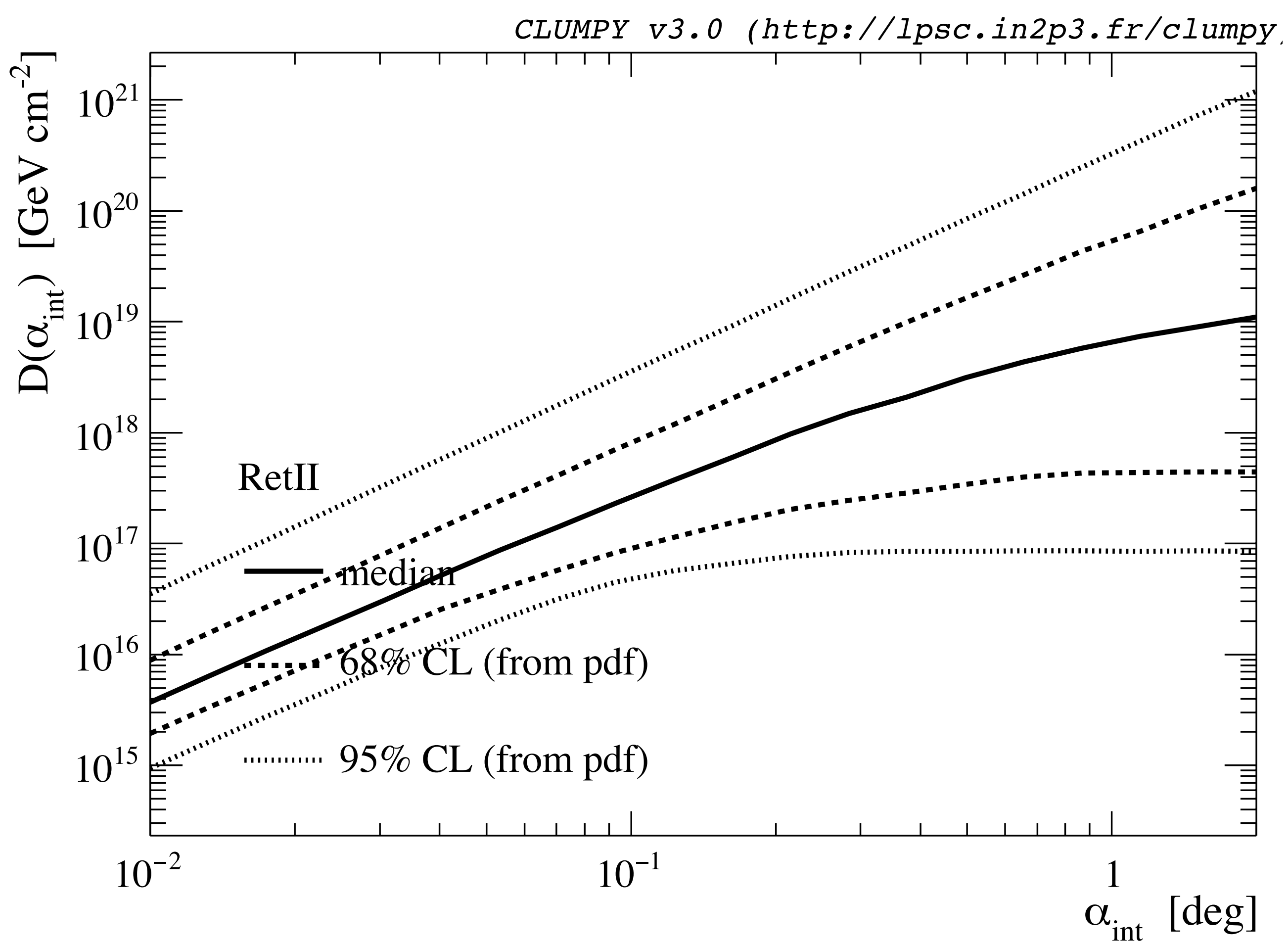
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
2.273206E+07	6.904886E-01	4.050722E+00	kEINASTO	8.020583E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.174676E+00	-3.174558E-01	3.126429E-03	1.300202E+00	-3.699685E+03
3.597555E+07	4.382141E-01	3.229199E+00	kEINASTO	5.973305E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-1.182630E+00	-7.991006E-01	1.206131E-02	1.507694E+00	-3.700747E+03
3.597555E+07	4.382141E-01	3.229199E+00	kEINASTO	5.973305E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-1.182630E+00	-7.991006E-01	1.206131E-02	1.507694E+00	-3.700747E+03
3.023840E+07	4.552988E-01	3.933889E+00	kEINASTO	2.301160E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-6.949423E-02	-3.199261E+00	8.332613E-01	8.946716E-01	-3.698257E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
6.036310E+06	1.124903E+00	5.070496E+00	kEINASTO	1.883500E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.517894E-01	1.119312E-01	1.763531E-01	4.584356E-01	-3.698640E+03
3.380930E+06	1.662231E+00	5.791025E+00	kEINASTO	1.677794E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	9.278384E-01	-4.712070E+00	1.289013E+00	2.472279E-01	-3.701322E+03
3.700656E+06	1.526684E+00	5.517636E+00	kEINASTO	1.833705E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-5.461117E-01	-1.627384E-01	9.000341E-01	7.240001E-01	-3.697057E+03
3.744262E+07	4.074615E-01	3.530895E+00	kEINASTO	3.308829E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.690050E+00	-1.275272E+00	2.103918E-03	3.675757E+00	-3.698399E+03
2.745295E+07	4.565711E-01	3.963694E+00	kEINASTO	1.799342E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.597568E+00	-2.172356E+00	1.049639E-02	3.903059E+00	-3.699453E+03
7.357019E+06	1.011708E+00	5.022643E+00	kEINASTO	1.715716E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.298504E+00	-5.211906E-01	1.583903E-03	3.989680E+00	-3.698710E+03
1.049863E+07	8.818633E-01	4.602114E+00	kEINASTO	3.756864E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-8.263103E+00	-1.514993E-01	1.041265E-03	3.755006E+00	-3.698708E+03
3.839074E+06	1.274408E+00	4.770299E+00	kEINASTO	1.350538E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-5.486738E+00	-7.300297E-01	3.608500E-03	2.202090E+00	-3.701835E+03
3.097150E+07	5.260085E-01	3.793859E+00	kEINASTO	5.302938E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.512230E+00	-4.063986E-01	1.468648E-03	2.555749E+00	-3.700469E+03
3.097150E+07	5.260085E-01	3.793859E+00	kEINASTO	5.302938E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.512230E+00	-4.063986E-01	1.468648E-03	2.555749E+00	-3.700469E+03
2.197898E+07	5.710484E-01	3.982659E+00	kEINASTO	3.609562E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-6.979384E+00	-8.081901E-01	6.437573E-03	2.920421E+00	-3.696913E+03
2.250062E+07	4.858238E-01	4.095365E+00	kEINASTO	1.207093E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-7.758659E+00	-2.784973E+00	7.099699E-02	3.275920E+00	-3.702465E+03
1.995867E+07	6.355029E-01	4.332832E+00	kEINASTO	3.378093E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-6.925366E-02	-5.549010E+00	2.891260E+00	7.880674E-01	-3.697379E+03
8.207837E+06	9.544456E-01	5.083619E+00	kEINASTO	1.331557E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-3.914280E-01	-5.044711E+00	9.263965E-01	8.853404E-01	-3.698840E+03
1.380981E+07	7.181385E-01	4.554933E+00	kEINASTO	2.180020E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-1.096706E+00	-5.055826E+00	8.536376E+00	8.096233E-01	-3.697702E+03
9.984212E+06	8.926913E-01	4.858481E+00	kEINASTO	2.480674E-01	3.000000E+00	1.000000E+00	kZHAO3D	1.520000E+07	2.500000E-01	3.400000E+00	3.800000E+00	2.000000E-01	kBAES	-2.727667E-03	-3.609323E+00	2.098274E+00	5.998308E-01	-3.697315E+03

4. The case of dwarf spheroidal galaxies

- Assuming that everything works fine, you end up with a **posterior distribution** of your best-fit parameters which you can use to compute your J/D-factor profiles!





4. The case of dwarf spheroidal galaxies

- Is this method stable? It depends on how large the stellar sample is (seventh caveat). . .

- Example: the J-factor profile as a function of the instrument integration angle for the ultra-faint dSph Tri II (same stellar sample before and after revision of measured velocities).

ID	R (pc)	v_r (km s^{-1})	δv_r (km s^{-1})	Data set
1	1.9	−381.4	1.3	K&M
2	5.0	−380.7	2.4	K&M
3	8.5	−382.1	2.1	K&M
4	10.2	−384.9	3.2	K
5	10.3	−383.1	4.9	M
6	10.7	−389.0	2.3	K&M
7	11.2	−373.8	1.4	K&M
8	19.4	−387.0	3.8	M
9	21.2	−401.4	6.6	M
10	30.3	−362.8	5.6	M
11	31.4	−397.1	7.8	M
12	32.7	−404.7	5.1	M
13	36.8	−387.1	7.7	M
14	80.4	−375.8	3.1	M

ID (K15a)	ID (M16)	R.A. (J2000)	Decl. (J2000)	Radius (arcmin)	$(g_{P1})_0$ (mag)	δg_{P1} (mag)	$(i_{P1})_0$ (mag)	δi_{P1} (mag)	Masks	S/N[a] (Å^{-1})	v_{helio} (km s^{-1})	$\sigma(v)$	Member?
···	22	02 13 12.69	+36 08 49.4	2.11	20.71	0.09	20.34	0.09	cdefg	44.5	−380.6 ± 3.0	0.6	Y
128	···	02 13 14.24	+36 09 51.1	1.06	19.93	0.03	19.41	0.03	bdeg	25.0	−383.3 ± 1.8	0.4	Y
116	21	02 13 15.96	+36 10 15.8	0.53	20.38	0.02	19.92	0.02	bcdeg	26.4	−381.4 ± 3.2	0.9	Y
106	40	02 13 16.55	+36 10 45.8	0.19	17.34	0.01	16.58	0.01	bdef	219.5	−381.6 ± 1.6	0.4	Y
91	20	02 13 19.32	+36 11 33.3	0.93	20.33	0.03	19.79	0.03	bcfg	29.7	−380.1 ± 4.9	1.7	Y
76	23	02 13 20.61	+36 09 46.5	1.12	20.83	0.06	20.53	0.06	bcg	17.0	−385.2 ± 4.2	1.3	Y
···	27	02 13 21.35	+36 08 29.1	2.36	21.30	0.07	21.27	0.07	cd	20.0	−376.8 ± 11.7	1.6	Y
65	46	02 13 21.54	+36 09 57.4	1.11	19.03	0.01	18.42	0.01	bdfg	81.8	−381.0 ± 5.9	3.0	Y
···	24	02 13 22.00	+36 10 25.9	0.97	21.22	0.07	21.14	0.07	d	17.8	−370.4 ± 17.1	···	Y
···	26	02 13 24.83	+36 10 21.8	1.54	21.40	0.11	21.17	0.11	c	19.9	−375.6 ± 11.2	···	Y
···	9	02 13 27.33	+36 13 30.5	3.45	21.25	0.10	21.05	0.10	d	17.6	−387.6 ± 7.7	···	Y
···	29	02 13 30.95	+36 11 56.0	3.00	21.96	0.20	21.68	0.20	c	14.0	−386.2 ± 4.7	···	Y
···	31	02 13 52.66	+36 13 24.1	7.61	20.63	0.03	20.12	0.03	cdg	42.8	−377.1 ± 2.7	0.9	Y
···	25	02 13 17.14	+36 07 14.1	3.47	21.15	0.05	21.07	0.05	cd	21.3	···	···	?[b]

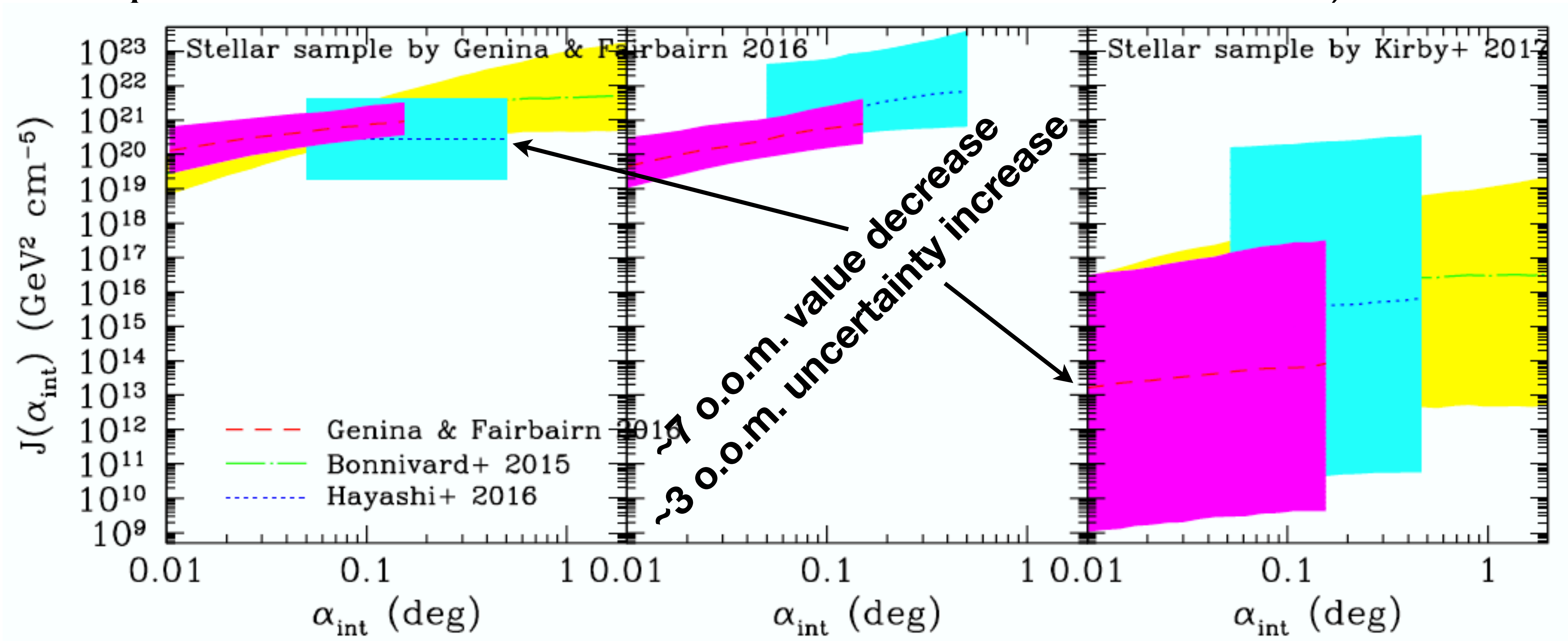
binary star

total sample dimension: 14 stars
revised sample dimension: 13 stars

4. The case of dwarf spheroidal galaxies

- Is this method stable? It depends on how large the stellar sample is (seventh caveat). . .

- Example: the J-factor profile as a function of the instrument integration angle for the ultra-faint dSph Tri II (same stellar sample before and after revision of measured velocities).



5. Summary

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- The problem of estimating the γ-ray signal from DM self-interaction in halos deprived of gaseous velocity tracers can be solved through the Jeans analysis of stellar kinematics;

- the calculation of DM astrophysical factors from stellar kinematics data can be achieved through computational methods (e.g., bootstrap or MCMC);

- although such methods produce a statistically significant output (J/D posterior distributions, 1-2σ uncertainties), they are subject to several caveats and not stable in case of low quantity/quality of input data => need to **implement new features to solve some caveats** (e.g., triaxiality; Hayashi+ 2016) and **collect better and more abundant kinematic data for the most promising targets**.