Jet Function with a Jet Algorithm

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Outline

- Factorizing Jet Cross Sections
- Defining Jet Function with a Jet Algorithm
- Zero-bin Scaling for Jet Algorithms
- Results for Sterman-Weinberg Jet Function

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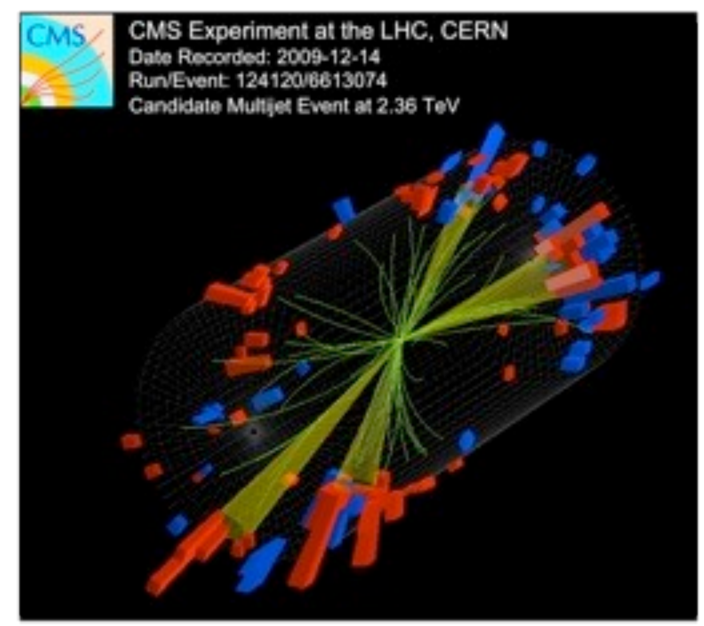


Previous SCET Work on Jet Algorithms

- Many papers on hemisphere jets
- Factorization with generic jet algorithm — Bauer, Hornig, Tackmann (2008)
- Resumming phase space logs for Sterman-Weinberg jets — Trott (2006)
- Phase space for JADE, SW, kT jets — Cheung, Luke, Zuberi (2009)

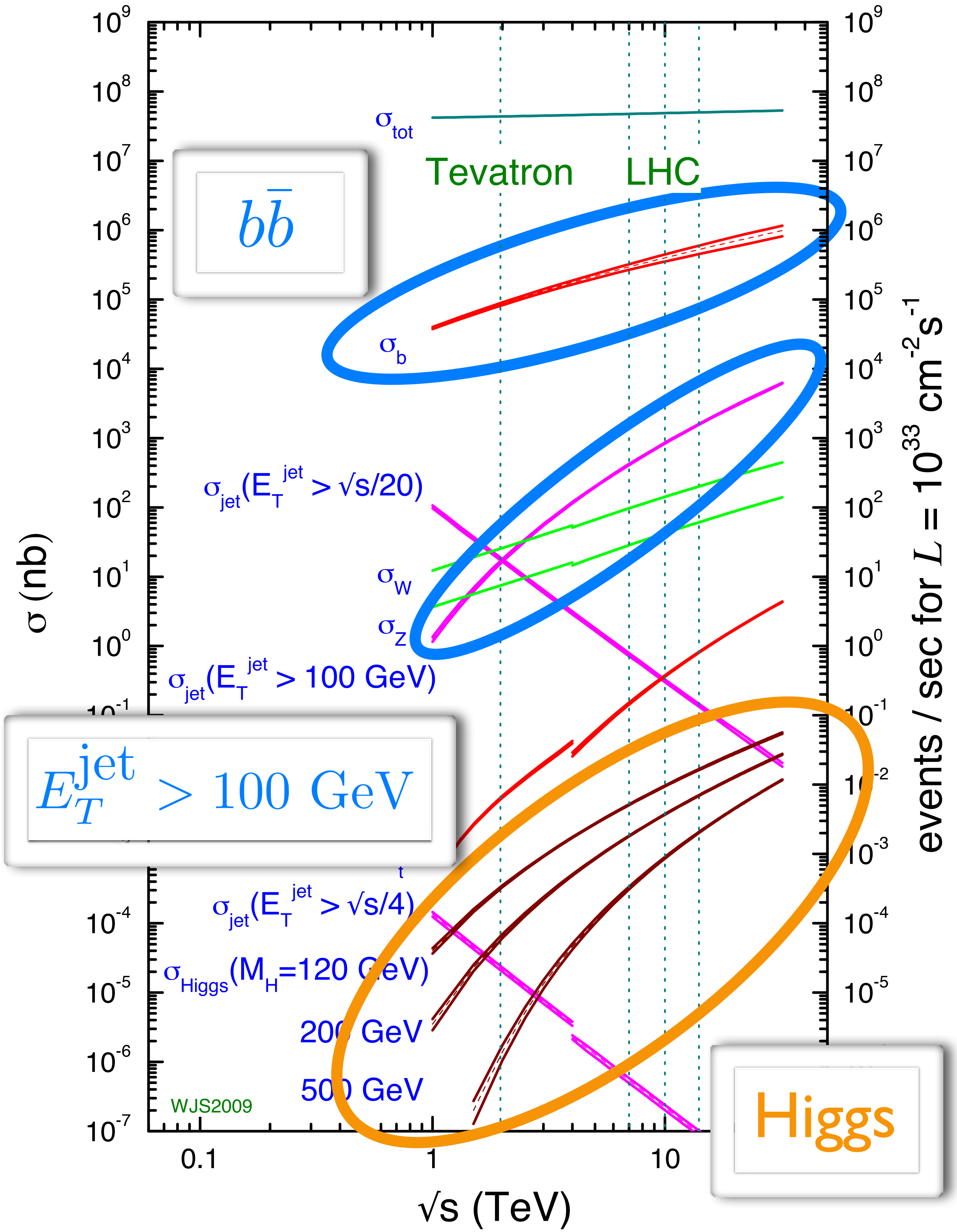
This Talk - Jet Function

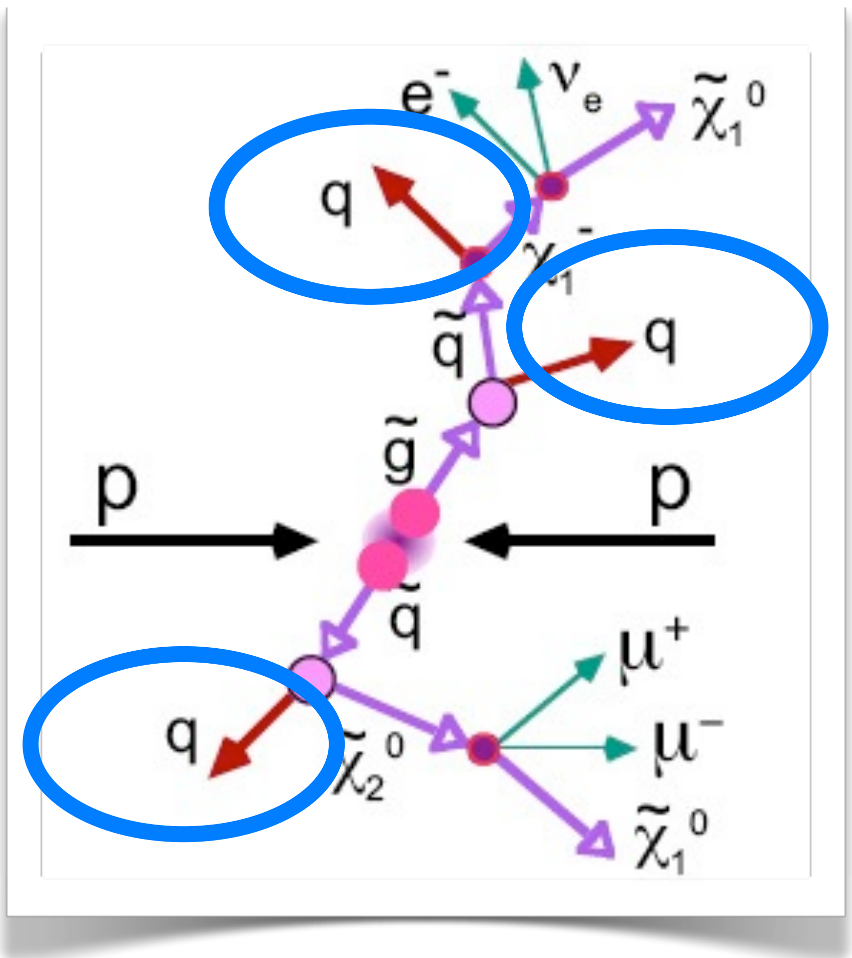
Ellis, Hornig, Lee, Vermilion, Walsh 0912.0262, 1001.0014

T.J. 0912.5509

Why Study Jets?

- Large jet cross sections at the LHC and Tevatron
- Many BSM signatures have multiple jets



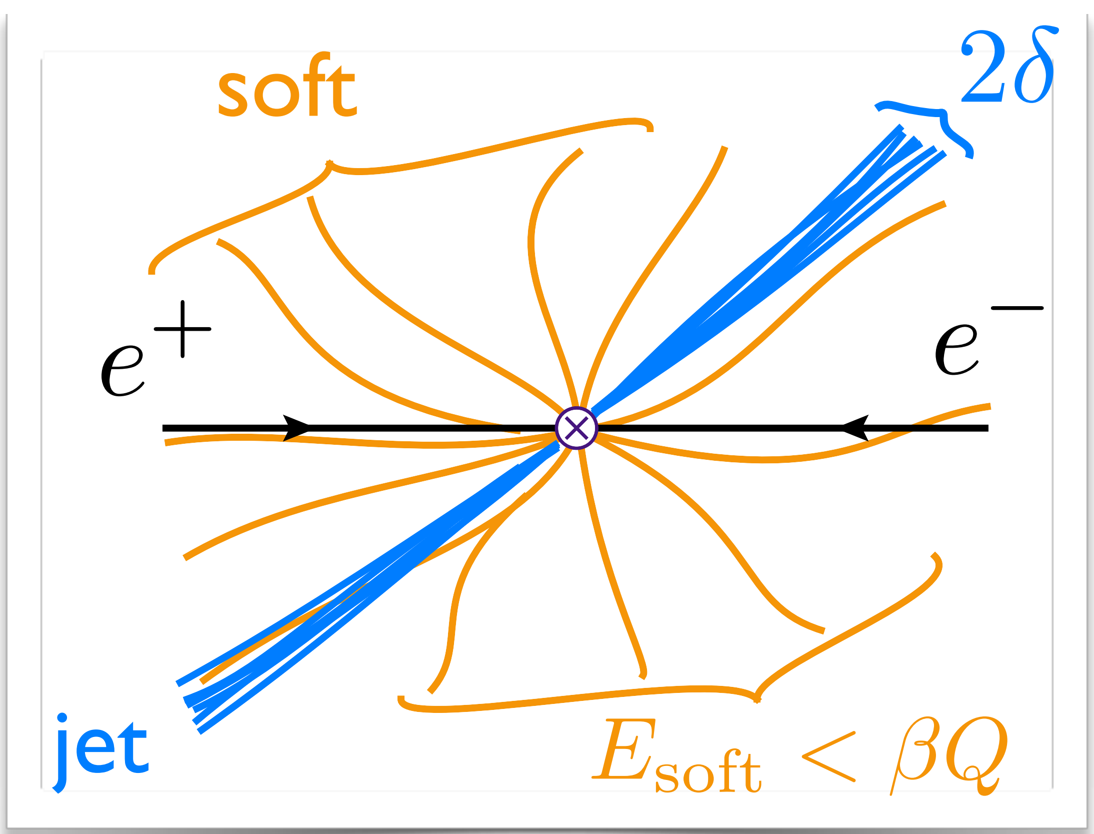


SUSY decay chain

Factorized Jet Cross Section with a Jet Algorithm

- Factorize different scales into
  - Hard function H: Q
  - Jet functions J: m_J
  - Soft function S: m_J^2/Q

- Factorization theorem specialized to Sterman-Weinberg (SW) dijets, measured jet mass:



$$\frac{d\sigma^{\mathrm{SW}(\delta,\beta)}}{ds\, d\bar{s}} = \sigma^{(0)} H(Q) \int dl^+ d\bar{l}^- \times J_n^{\mathrm{SW}}(s - Ql^+) J_{\bar{n}}^{\mathrm{SW}}(\bar{s} - Q\bar{l}^-) \times S^{\mathrm{SW}}(l^+, \bar{l}^-)$$

$$s = \Big(\sum_{i\in \mathrm{jet}} p_i\Big)^2$$

Ellis, Hornig, Lee, Vermilion, Walsh (2010)

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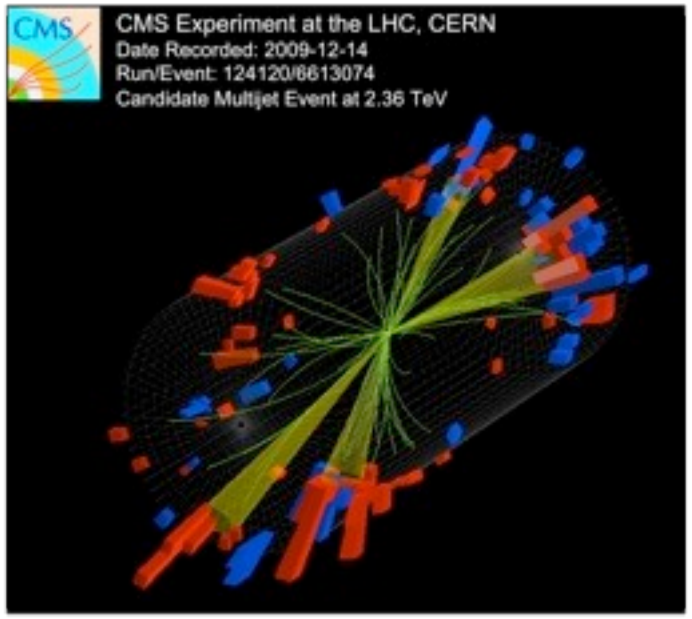
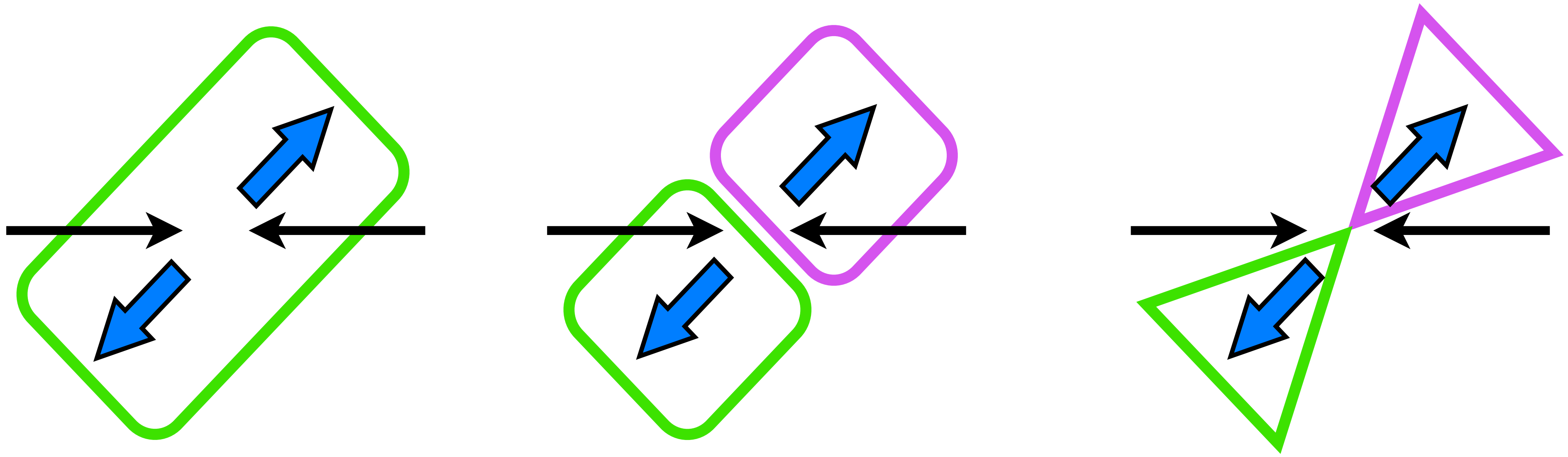


What is Included in a Jet Functions?

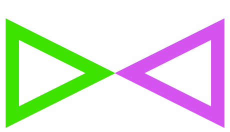


- We want to study jets in increasing detail

 Sum over everything - no jet function

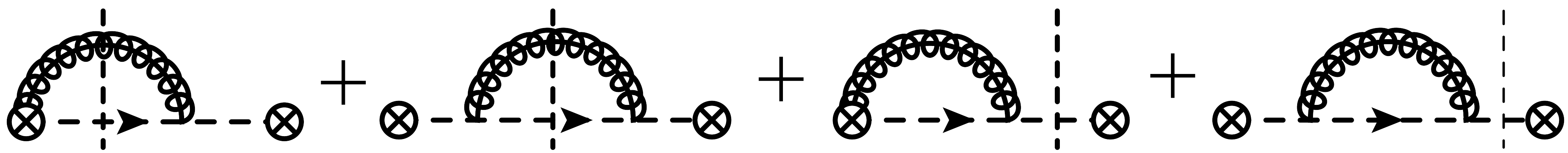
 Hemisphere jets - inclusive jet function

$$J_n^{(\mathrm{inc})} \sim \mathrm{Disc} \int d^4x\, e^{ir\cdot x} \langle 0|\mathrm{T}\{\bar{\chi}(0)\chi(x)\}|0\rangle, \quad \chi_n \equiv W_n^\dagger \xi_n$$

 Realistic jet algorithm - algorithm dependent jet function

$$J_n^{F(a_i)} \equiv J_n^{(\mathrm{inc})} + \Delta J_n^{F(a_i)}$$

Define the Jet Function

$J_n^{(\mathrm{inc})} \sim$ 

$$\sim \mathrm{Disc} \int d^4x\, e^{ir\cdot x} \langle 0 | \mathrm{T}\{\bar{\chi}(0)\chi(x)\} | 0 \rangle \propto \sum_X \int d^4x\, e^{ir\cdot x} \langle 0 | \chi(x) | X \rangle \langle X | \bar{\chi}(0) | 0 \rangle$$

Optical theorem

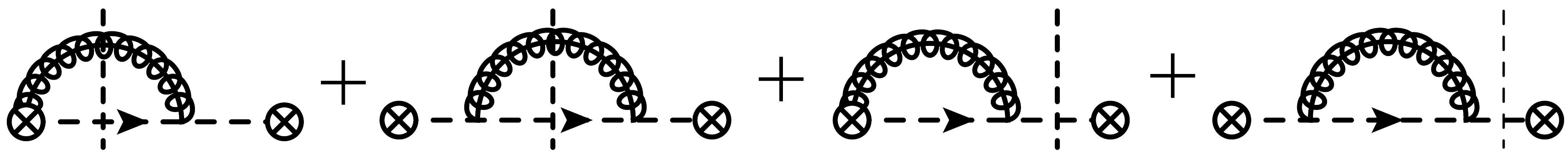
$$\chi_n \equiv W_n^\dagger \xi_n$$

$$\frac{1}{p^2 + i0} \to -2\pi i \delta\left(p^2\right) \theta(p^0)$$

Cutkosky cutting rules

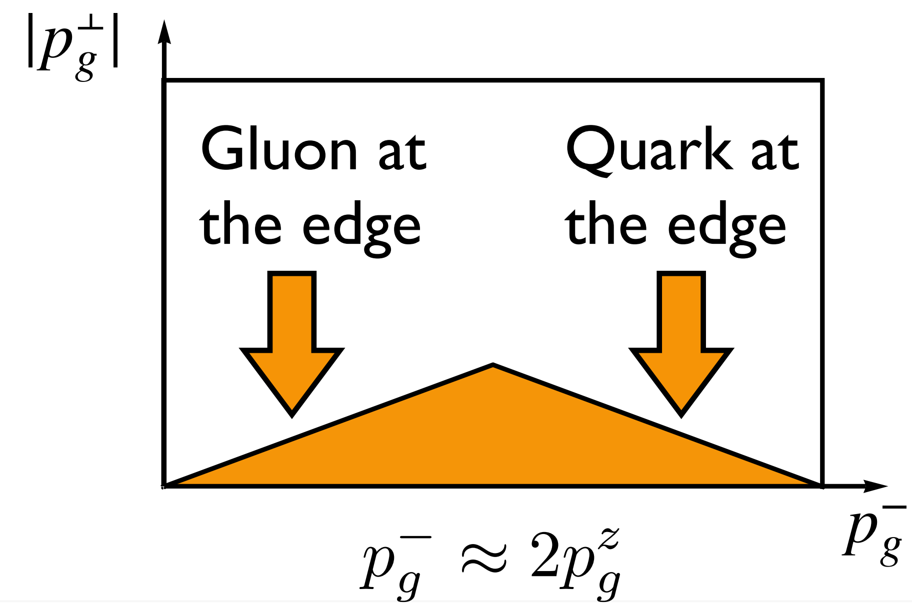
Define the Jet Function

$J_n^F \sim$



$$\sim \sum_X \int d^4x\, e^{ir\cdot x} \langle 0|\chi(x)|X\rangle \, F(p_j) \, \langle X|\bar{\chi}(0)|0\rangle$$

$$F_{\rm cone} = \theta\left(\tan\delta - \frac{|p_g^\perp|}{|p_g^z|}\right)\theta\left(\tan\delta - \frac{|p_q^\perp|}{|p_q^z|}\right)$$



$$J_n^F \sim \mathrm{Disc}_F \int d^4x\, e^{ir\cdot x} \langle 0|\mathrm{T}\{\bar{\chi}(0)\chi(x)\}|0\rangle$$

Hornig, Lee, Ovanesyan (2009)

$$\prod_i \frac{1}{p_i^2 + i0} \to F(p_j) \prod_i (-2\pi i)\delta\left(p_i^2\right)\theta(p_i^0)$$

Modified Cutkosky cutting rules

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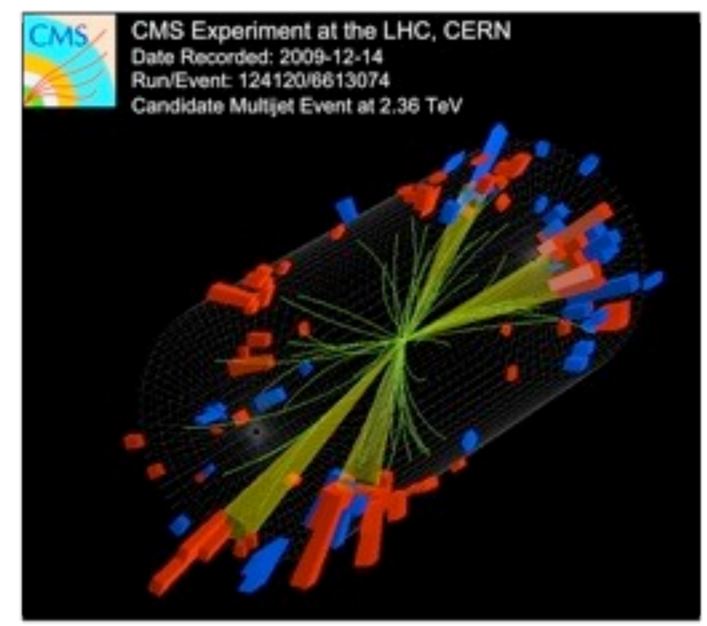


Subtracting the Zero-bin

Manohar, Stewart (2007)

- Split collinear momenta into large "label" and small "residual" components

$$p_n = \tilde{p}_n + k_n, \qquad \tilde{p}_n \gg k_n \Rightarrow \tilde{p}_n \neq 0$$

$$\mathcal{L}_n = \sum_{\boxed{p_n, p'_n \neq 0}} e^{-i(p_n - p'_n)\cdot x}\, \bar{\xi}_{n,p'_n}(x)\, in\cdot\partial \frac{\not{\bar{n}}}{2} \xi_{n,p_n}(x) + \ldots$$

- Sums are inconvenient, we want integrals

$$\sum_{\tilde{p}_n \neq 0} \int \mathrm{d}k_n\, \mathcal{M}(\tilde{p}_n + k_n) = \underbrace{\int \mathrm{d}\tilde{p}_n\, \mathcal{M}(\tilde{p}_n)}_{\text{naive collinear}} - \underbrace{\int \mathrm{d}\tilde{p}_n\, \mathcal{M}^{\tilde{p}_n \to 0}(\tilde{p}_n)}_{\text{zero-bin}}$$

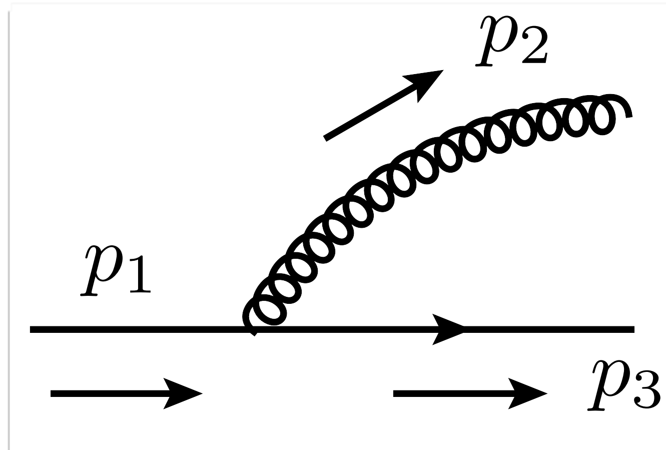
- The physics of the zero-bin region is in the soft function. Must remove double counting to avoid unphysical singularities

- Because of phase space cuts from a nontrivial jet algorithm, zero-bin is not scaleless but gives an explicit contribution

Finding the Zero-bin Contribution

- Use independent momenta
  - Delta functions conserve momentum
  - Works for phase space or loop integrals

- Zero-bin scaling only for vertex rules
  - $\delta(p_1^\perp - p_2^\perp - p_3^\perp) \to \delta(p_1^\perp - p_3^\perp)$
  - Momentum integrals convey information to rest of the diagram



$$\delta(p_1 - p_2 - p_3)$$

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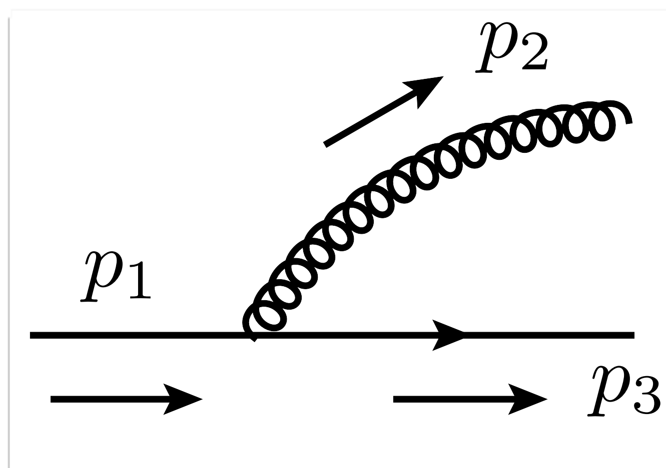

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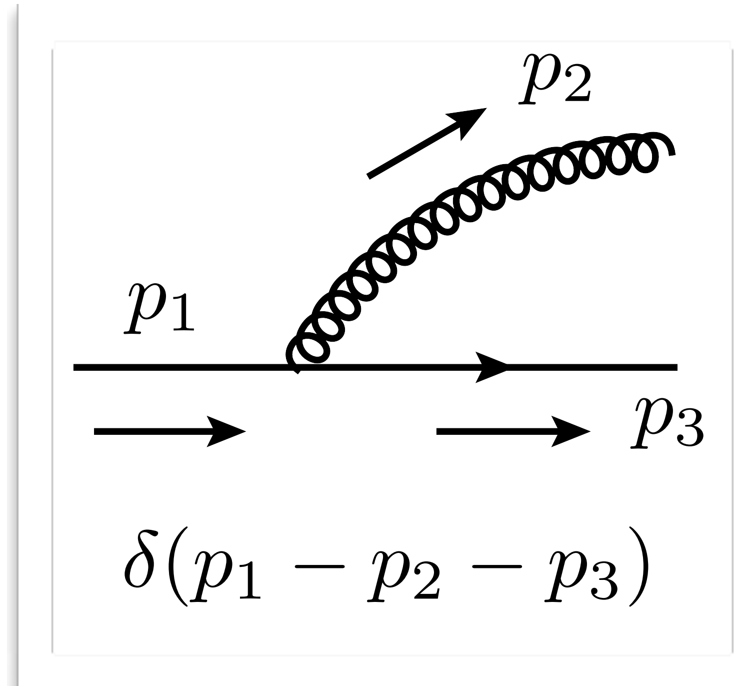
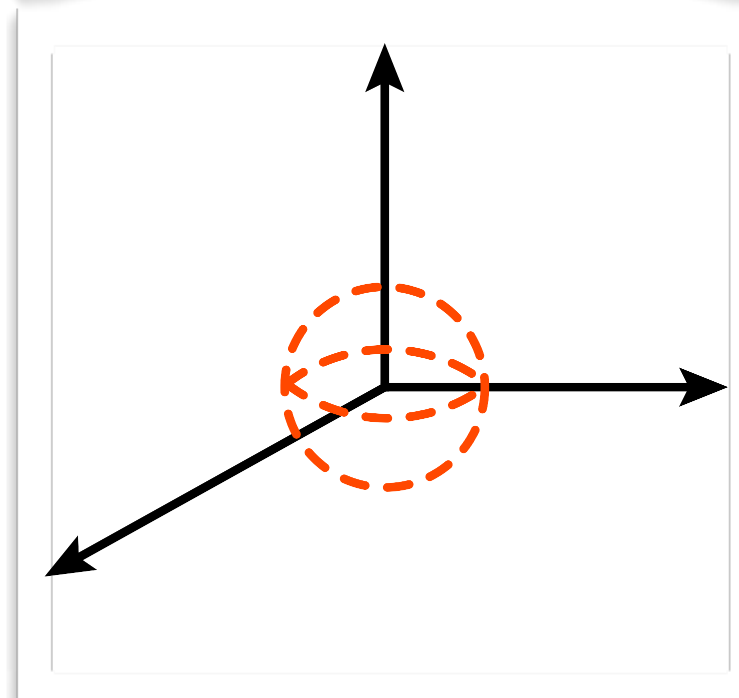


- Zero-bin scaling only for vertex rules
  - $$\underbrace{\delta(p_1^{\perp} - p_2^{\perp} - p_3^{\perp})}_{\mathcal{M}} \rightarrow \underbrace{\delta(p_1^{\perp} - p_3^{\perp})}_{\mathcal{M}^{p_2 \rightarrow 0}}$$
  - Momentum integrals convey information to rest of the diagram
- Schematic representation of phase space with zero-bin region



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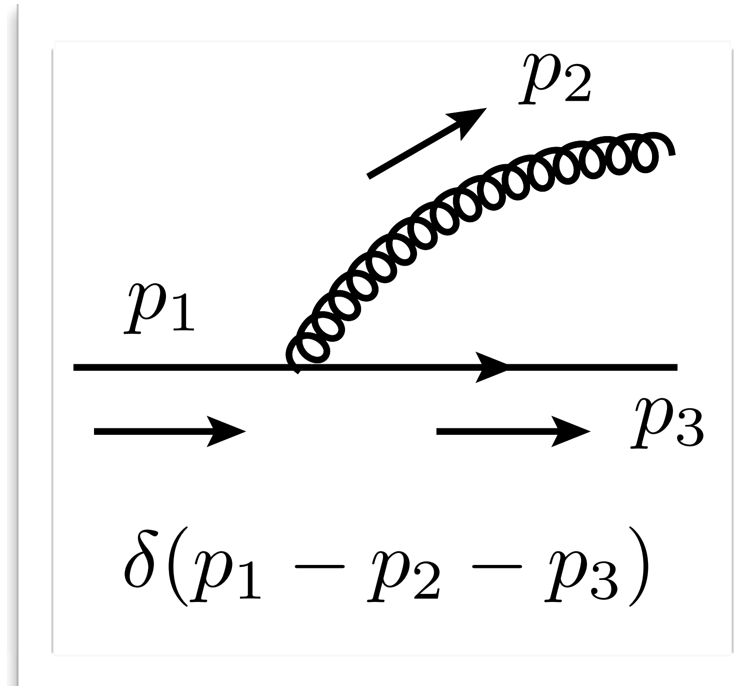
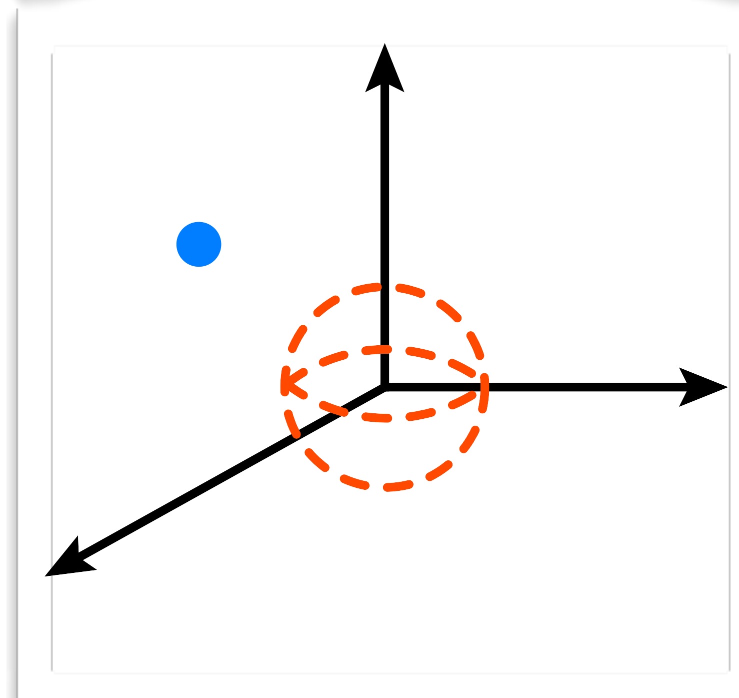


- Schematic representation of phase space with zero-bin region

- Fully differential x-section is a point

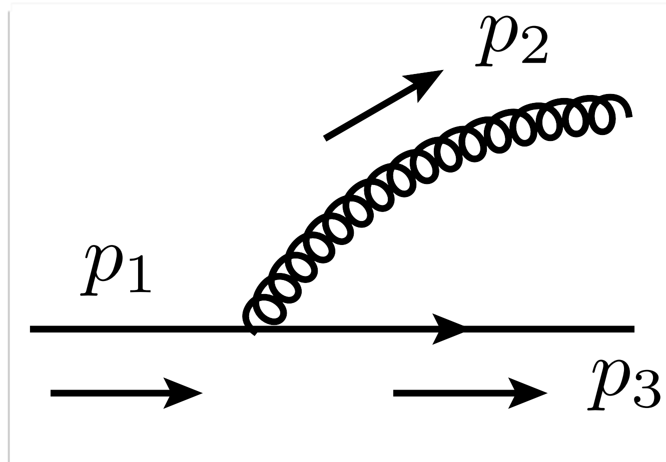
  ⇒ no zero-bin subtraction

  (cf. plus-function with nonzero argument)

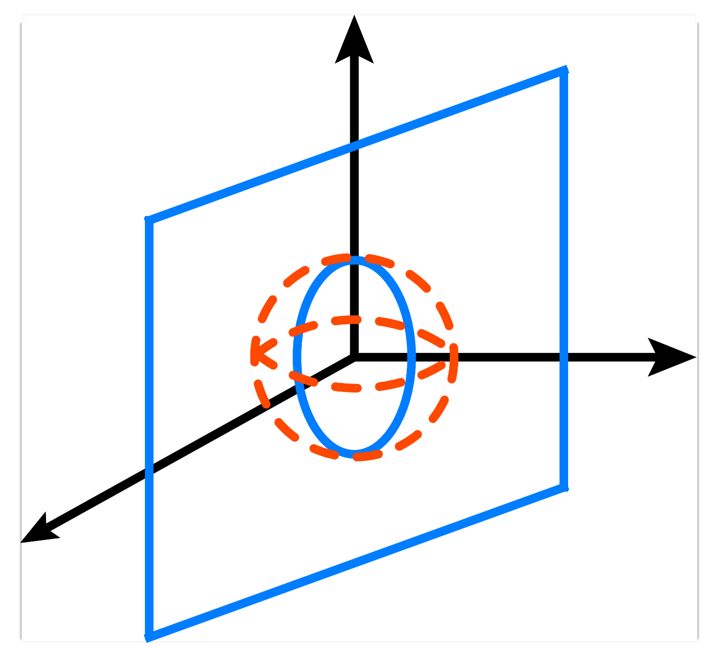


Finding the Zero-bin Contribution

- Measurement and/or algorithm limit integration to hypersurface which intersects the zero-bin region



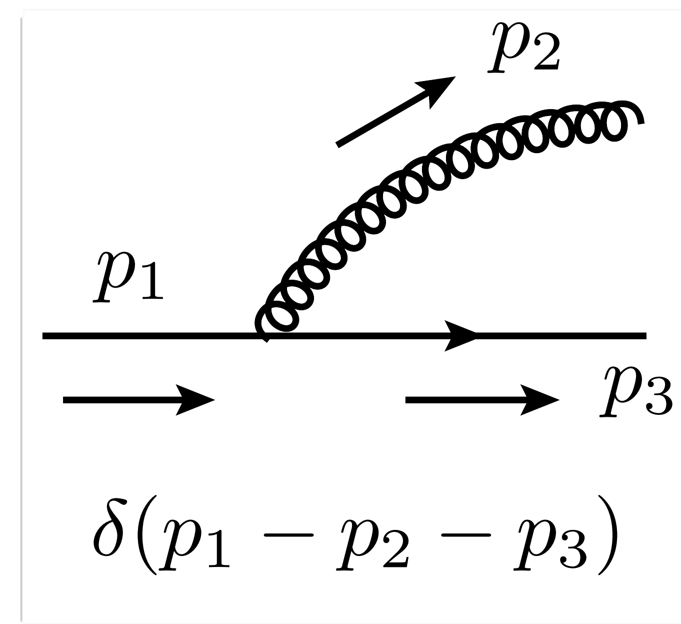
$$\delta(p_1 - p_2 - p_3)$$

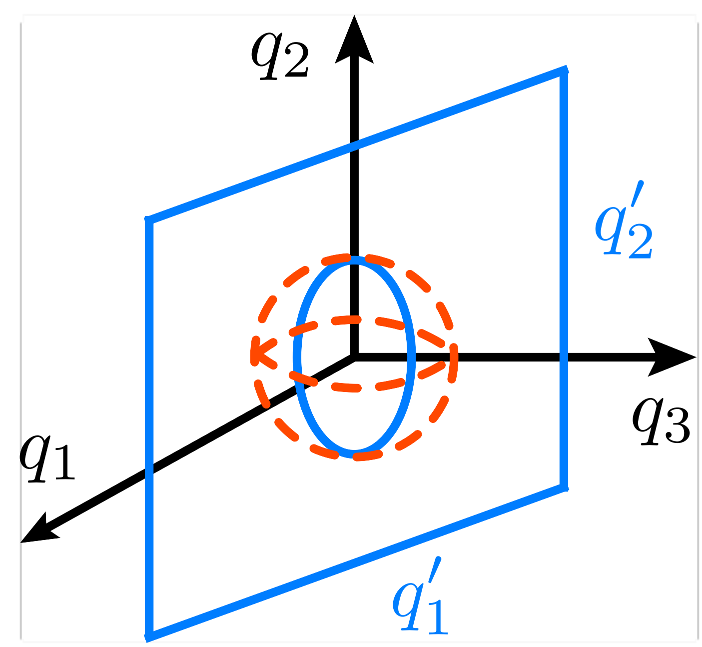


Finding the Zero-bin Contribution

- Measurement and/or algorithm limit integration to hypersurface which intersects the zero-bin region

$$\int \prod_{i=1}^{N} dq_i \prod_{k=1}^{M} \delta(s_k - \hat{s}_k)\ \mathcal{M} = \int \prod_{i=1}^{N-M} dq_i'\ \mathcal{M}$$





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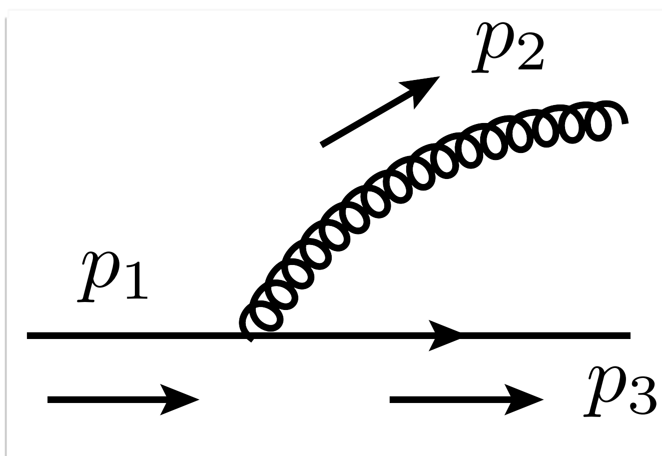
$$\int \prod_{i=1}^{N} dq_i \prod_{k=1}^{M} \delta(s_k - \hat{s}_k)\ \mathcal{M} = \int \prod_{i=1}^{N-M} dq'_i\ \mathcal{M}$$

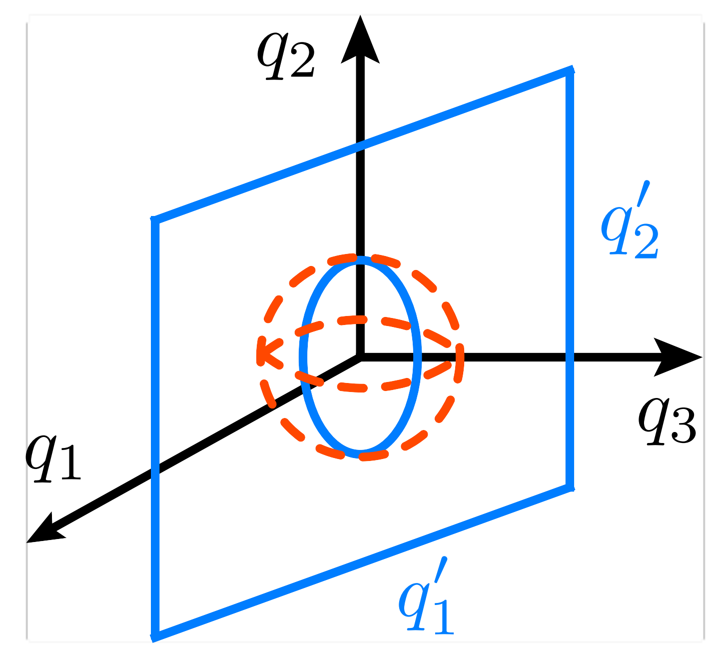
coordinate choice

Our case:

$$\underbrace{\delta(p_1^{\perp})\delta(p_1^{\perp} - p_2^{\perp} - p_3^{\perp})}_{\mathcal{M}} \rightarrow \underbrace{\delta(p_1^{\perp})\delta(p_3^{\perp})}_{\mathcal{M}^{p_2 \rightarrow 0}}$$



$$\delta(p_1 - p_2 - p_3)$$



Finding the Zero-bin Contribution

- Measurement and/or algorithm limit integration to hypersurface which intersects the zero-bin region

$$\int \prod_{i=1}^{N} dq_i \prod_{k=1}^{M} \boxed{\underbrace{\delta(s_k - \hat{s}_k)}_{\sim F}} \, \mathcal{M} = \int \prod_{i=1}^{N-M} dq_i' \, \mathcal{M}$$

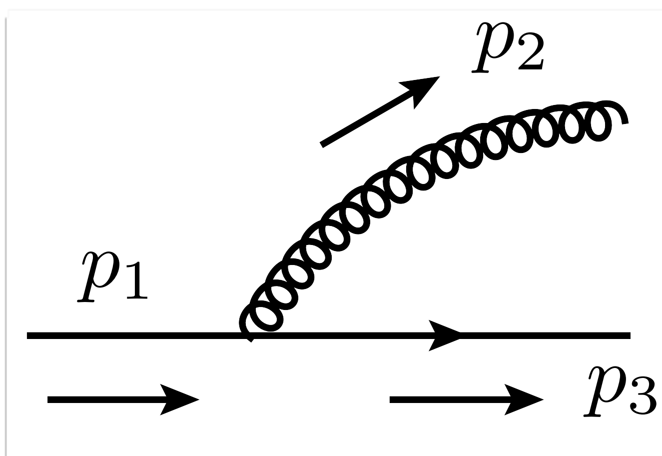
coordinate choice

Our case:

$$\underbrace{\delta(p_1^\perp)\delta(p_1^\perp - p_2^\perp - p_3^\perp)}_{\mathcal{M}} \to \underbrace{\delta(p_1^\perp)\delta(p_3^\perp)}_{\mathcal{M}^{p_2 \to 0}}$$

$$\underbrace{\theta\left(|p_2^z|\,\delta - |p_2^\perp|\right)\theta\left(|p_3^z|\,\delta - |p_3^\perp|\right)}_{F_{\text{cone}}} \underbrace{\delta(p_2^\perp + p_3^\perp)}_{\mathcal{M}}$$

$$\to \underbrace{\theta\left(|p_2^z|\,\delta - |p_2^\perp|\right)}_{F_{0,\text{cone}}} \underbrace{\delta(p_3^\perp)}_{\mathcal{M}^{p_2 \to 0}}$$



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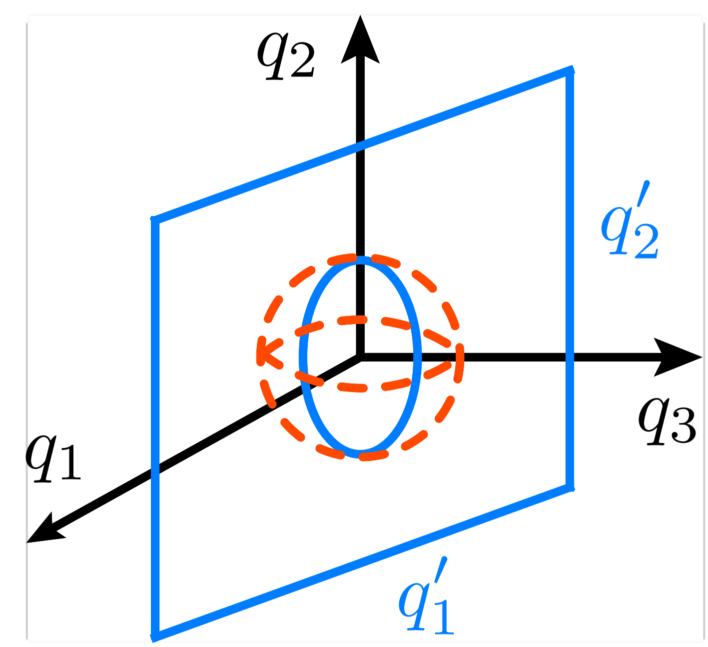
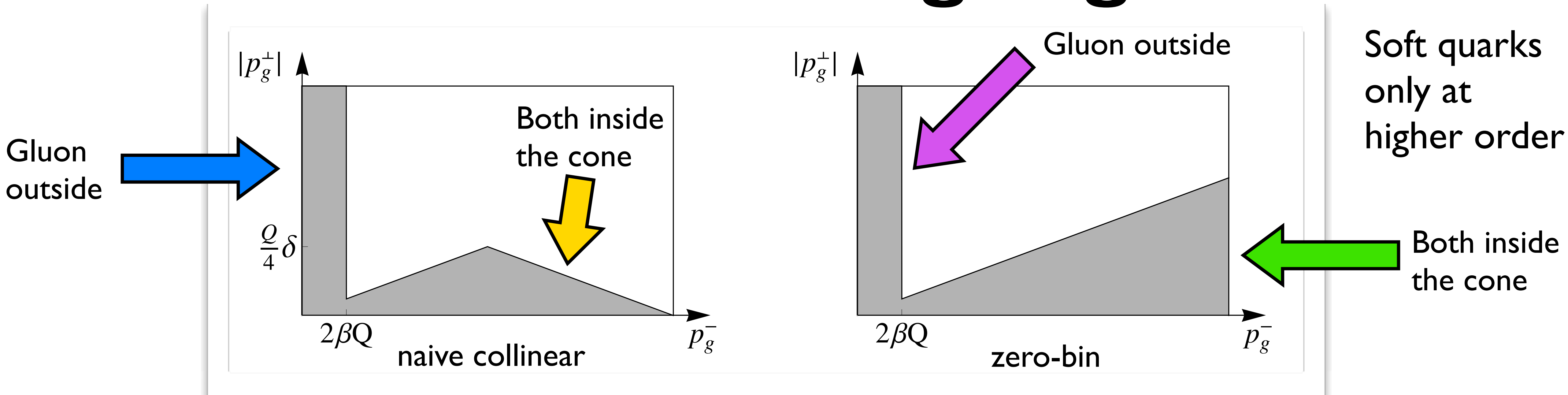


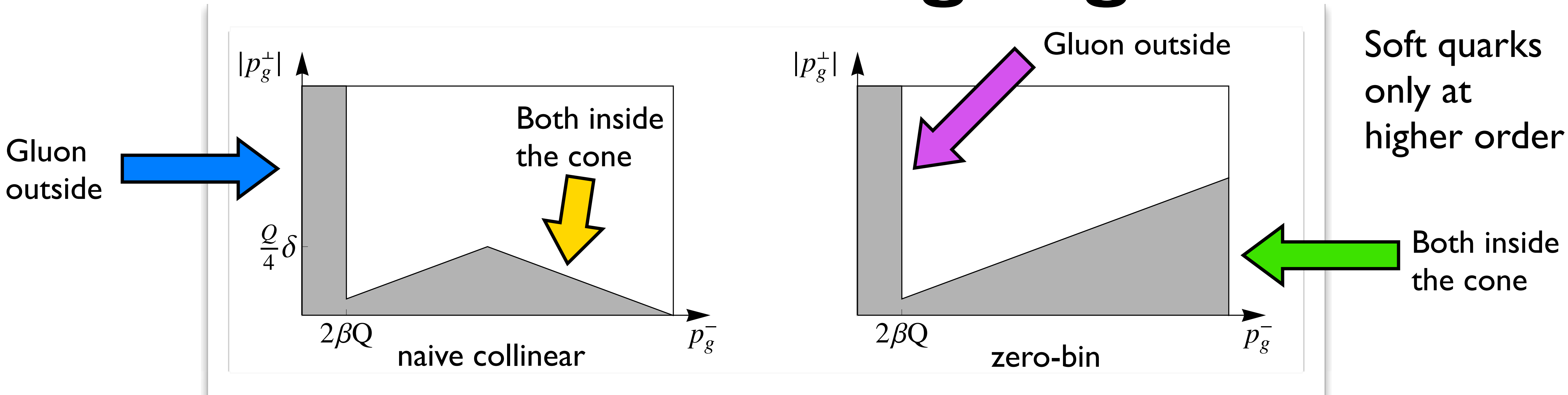
Sterman-Weinberg Algorithm



Soft quarks only at higher order

$$F_{\rm SW} = \boxed{\theta\left(\frac{|p_g^\perp|}{|p_g^z|} - \tan\delta\right)\theta\left(\beta Q - p_g^0\right)} + \boxed{\theta\left(\tan\delta - \frac{|p_g^\perp|}{|p_g^z|}\right)\theta\left(\tan\delta - \frac{|p_q^\perp|}{|p_q^z|}\right)}$$

Sterman-Weinberg Algorithm



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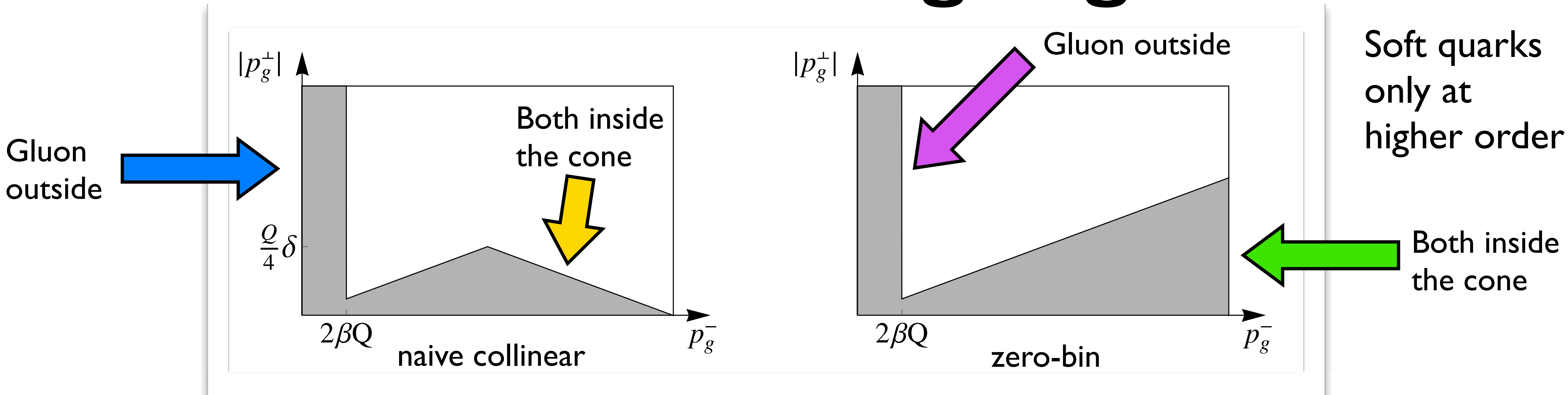
$$= \theta\left(2\beta - \frac{p_g^-}{Q}\right) + \boxed{\theta\left(\delta^2 - \frac{4s}{Q^2}\right)}\,\theta\left(\boxed{\frac{p_g^-}{Q}} - \frac{4s}{4s+Q^2\delta^2}\right)\theta\left(\frac{Q^2\delta^2}{4s+Q^2\delta^2} - \frac{p_g^-}{Q}\right) + \mathcal{O}(\lambda, \delta, \beta)$$

Sterman-Weinberg Algorithm



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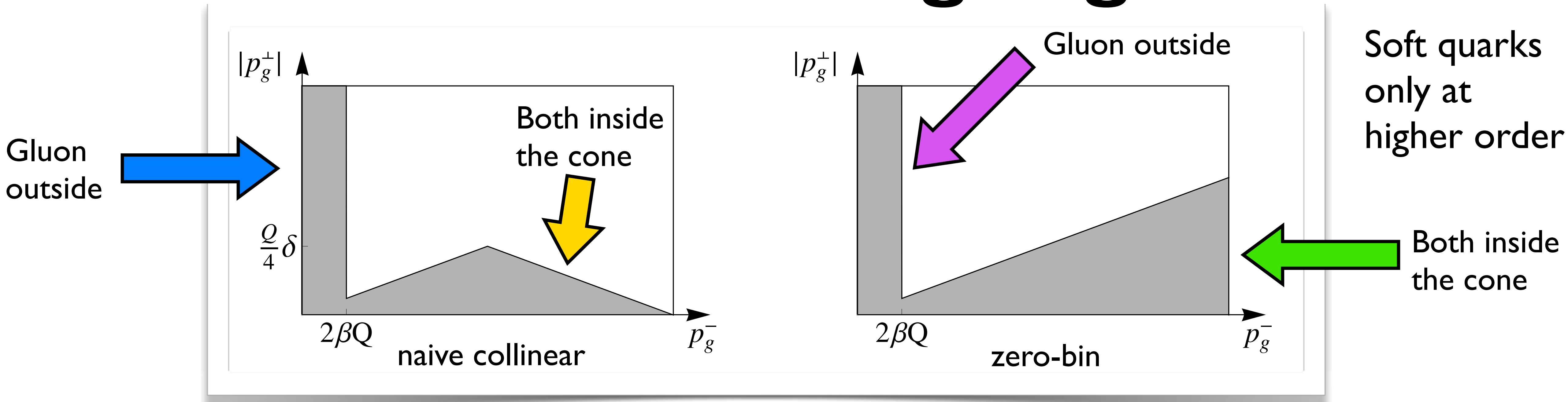
$$F_{0,\rm SW} = \boxed{\theta\left(\frac{|p_g^\perp|}{|p_g^z|} - \tan\delta\right)\theta(\beta Q - p_g^0)} + \boxed{\theta\left(\tan\delta - \frac{|p_g^\perp|}{|p_g^z|}\right)}$$

Sterman-Weinberg Algorithm



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$$= \theta\left(2\beta - \frac{p_g^-}{Q}\right) + \theta\left(\boxed{\frac{p_g^-}{Q}} - \frac{4s}{Q^2\delta^2}\right) + \mathcal{O}(\lambda, \delta, \beta)$$

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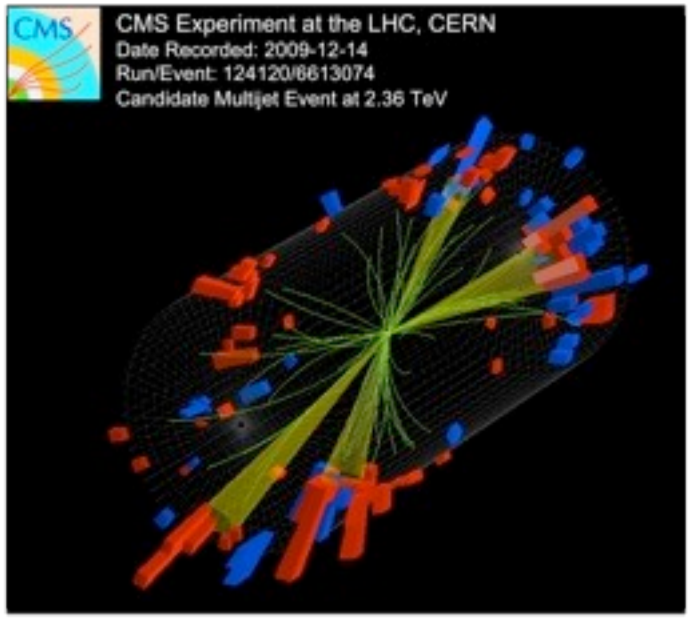


Combine Naive and Zero-bin Contributions

- The difference gives the algorithm contribution

$$J_n^{\mathrm{SW}}(s) \equiv J_n^{(\mathrm{inc})}(s) + \Delta J_n^{\mathrm{SW}}(s), \qquad \Delta J_n^{\mathrm{SW}}(s) = \Delta \tilde{J}_n^{\mathrm{SW}} - \Delta J_{n0}^{\mathrm{SW}}$$

- Both have the same pole structure

$$\Delta \tilde{J}_n^{\mathrm{SW}}(s) = \frac{\alpha_s C_F}{4\pi} A(\epsilon)\delta(s)\left\{\frac{2}{\epsilon^2} - \frac{4}{\epsilon}\left[\ln 2\beta + \frac{1}{2}\ln\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right]\right\} + \mathcal{O}(\epsilon^0), \quad A(\epsilon) = 1 + \mathcal{O}(\epsilon^2)$$

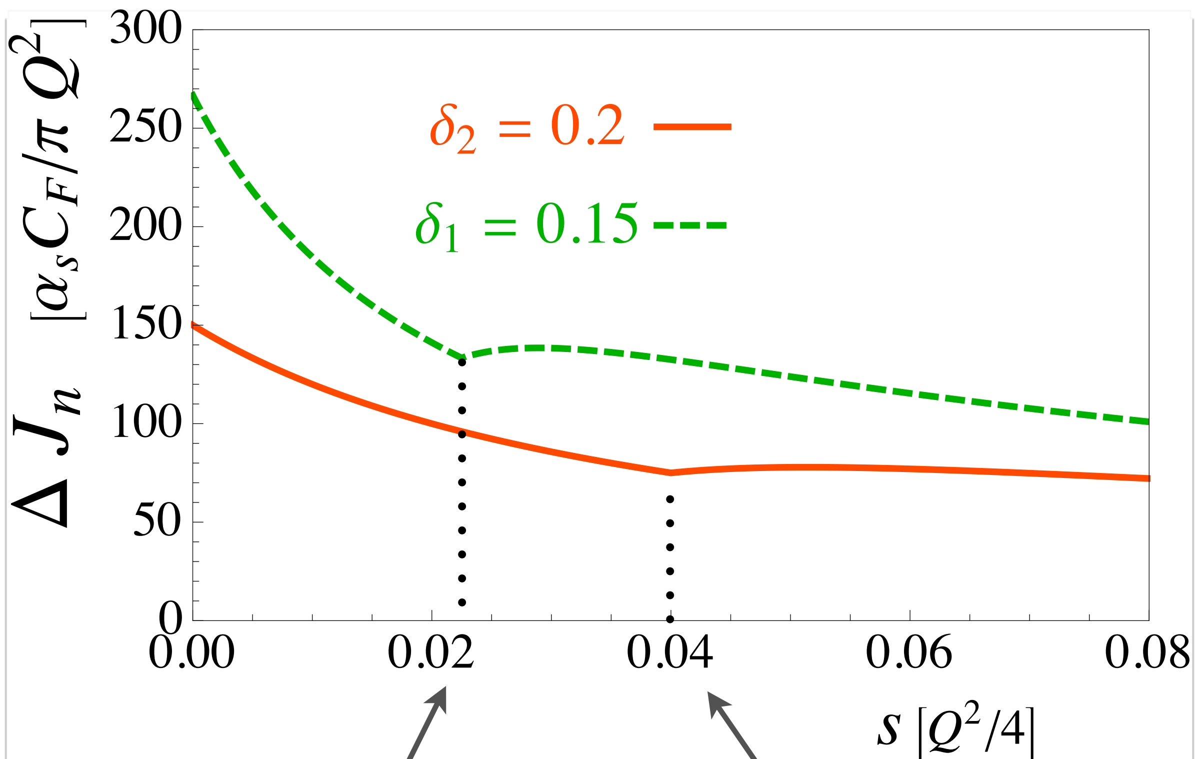
$$\Delta J_{n0}^{\mathrm{SW}}(s) = \frac{\alpha_s C_F}{4\pi} A(\epsilon)\delta(s)\left\{\frac{2}{\epsilon^2} - \frac{4}{\epsilon}\left[\ln 2\beta + \frac{1}{2}\ln\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right]\right\} + \mathcal{O}(\epsilon^0)$$

- Algorithm modifies finite part, anomalous dimension unchanged

$$\Delta J_n^{\mathrm{SW}}(s) = \frac{\alpha_s C_F}{4\pi}\theta\left(\delta^2 - \frac{4s}{Q^2}\right)\boxed{\frac{24}{4s + Q^2\delta^2}} + \frac{\alpha_s C_F}{4\pi}\theta\left(\frac{4s}{Q^2} - \delta^2\right)\boxed{\left[\frac{3}{s} + \frac{4}{s}\ln\left(\frac{4s}{Q^2\delta^2}\right)\right]}$$

Plot Renormalized Jet Function

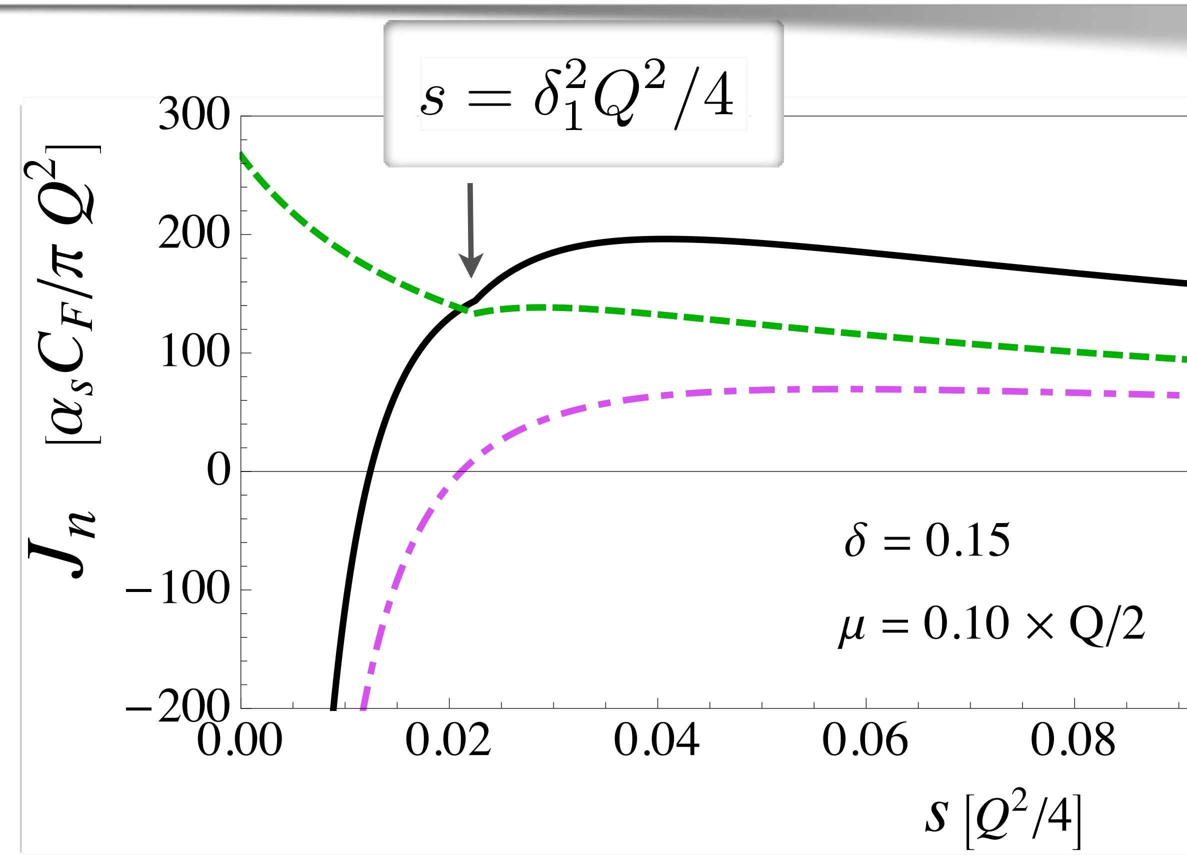
$$\Delta J_n^{\rm SW}(s) = \frac{\alpha_s C_F}{4\pi}\theta\left(\delta^2 - \frac{4s}{Q^2}\right)\boxed{\frac{24}{4s+Q^2\delta^2}} + \frac{\alpha_s C_F}{4\pi}\theta\left(\frac{4s}{Q^2} - \delta^2\right)\boxed{\left[\frac{3}{s} + \frac{4}{s}\ln\left(\frac{4s}{Q^2\delta^2}\right)\right]}$$



$s = \delta_1^2 Q^2/4$

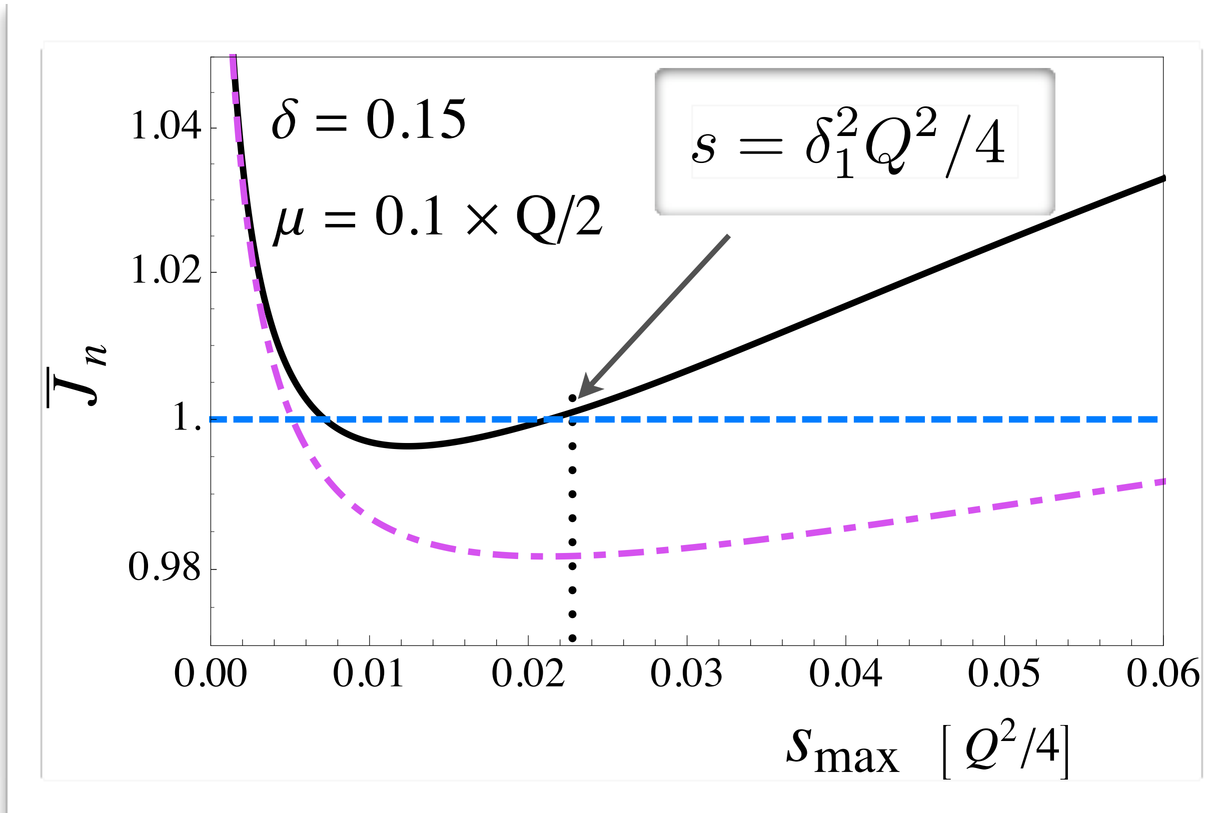
$s = \delta_2^2 Q^2/4$

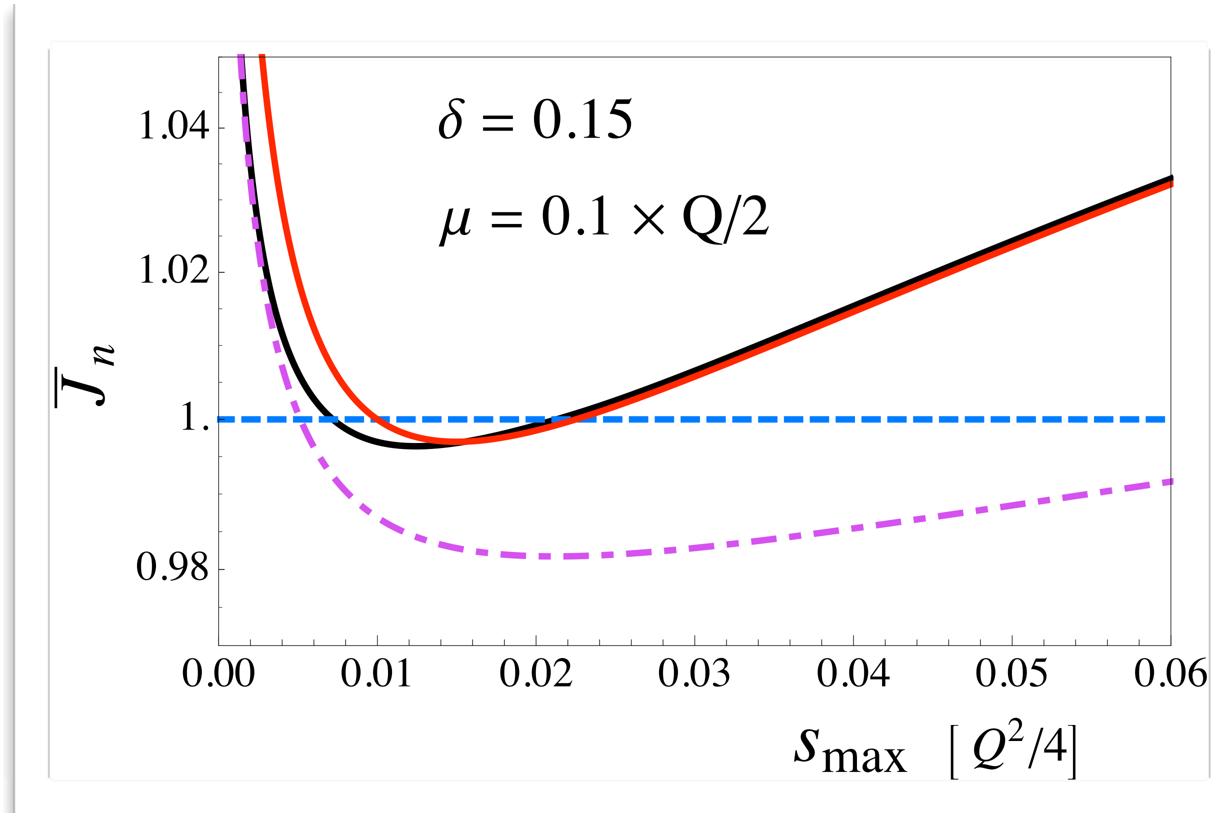
- Compare algorithm contributions
  - $\delta_1 = 0.15$
  - $\delta_2 = 0.20$



- Compare jet functions
  - Algorithm contribution
  - NLO inclusive jet function
  - NLO SW jet function

Plot Integrated Jet Function





- Compare
  - Tree level
  - NLO inclusive jet function
  - NLO SW jet function
  - $\propto \ln\left(s_{\max}/\mu^2\right) \ln\left(4 s_{\max}/Q^2\delta^2\right)$

$$\bar{J}_{n,\mathrm{ren}}^{\mathrm{SW}(s_{\max})}(\mu) \equiv \int_0^{s_{\max}} \mathrm{d}s\, J_{n,\mathrm{ren}}^{\mathrm{SW}}(s,\mu)$$

Conclusions

- Developed a general procedure to calculate jet function $J_n^F(s)$ with a jet algorithm *F*
  - Discussed how to apply zero-bin scaling to phase space constraints
- Jet function with the Sterman-Weinberg algorithm $J_n^{\mathrm{SW}}(s)$
  - Anomalous dimension same with and without algorithm
  - Finite pieces change the inclusive jet function
- Next steps:
  - Extend the factorization theorem by Ellis et al. $(\delta \sim 1)$ to our power counting for the jet size, $(\delta \sim \lambda)$
  - Apply jet algorithm in the presence of beam functions

Backup slides

Full Algorithm Contributions

$$\Delta\tilde{J}_n^{\mathrm{SW}}(s) = \frac{\alpha_s C_F}{4\pi} A(\epsilon)\theta\left(\delta^2 - \frac{4s}{Q^2}\right)\left[\delta(s)\left\{\frac{2}{\epsilon^2} - \frac{4}{\epsilon}\left(\ln 2\beta + \frac{1}{2}\ln\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right) + 2\ln^2 2\beta - \ln^2\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right\}\right.$$
$$\left. + \frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+\left\{\ln 2\beta + \ln\left(\frac{Q^2\delta^2}{4\mu^2}\right) + \frac{6s}{4s + Q^2\delta^2}\right\} - \frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)\ln\left(s/\mu^2\right)}{s}\right]_+\right]$$
$$+ \frac{\alpha_s C_F}{4\pi}\theta\left(\frac{4s}{Q^2} - \delta^2\right)\frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+\left(\frac{3}{4} + \ln 2\beta\right).$$

$$\Delta J_{n0}^{\mathrm{SW}}(s) = \frac{\alpha_s C_F}{4\pi} A(\epsilon)\theta\left(\delta^2 - \frac{4s}{Q^2}\right)\left[\delta(s)\left\{\frac{2}{\epsilon^2} - \frac{4}{\epsilon}\left(\ln 2\beta + \frac{1}{2}\ln\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right) + 2\ln^2 2\beta - \ln^2\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right\}\right.$$
$$\left. + \frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+\left\{\ln 2\beta + \ln\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right\} - \frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)\ln\left(s/\mu^2\right)}{s}\right]_+\right]$$
$$+ \frac{\alpha_s C_F}{4\pi}\theta\left(\frac{4s}{Q^2} - \delta^2\right)\left[\frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+\left\{\ln 2\beta + \ln\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right\} - \frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)\ln\left(s/\mu^2\right)}{s}\right]_+\right].$$

$$\Delta J_n^{\mathrm{SW}}(s) = \frac{\alpha_s C_F}{4\pi}\theta\left(\delta^2 - \frac{4s}{Q^2}\right)\frac{24}{4s + Q^2\delta^2} + \frac{\alpha_s C_F}{4\pi}\theta\left(\frac{4s}{Q^2} - \delta^2\right)\left[\frac{3}{s} + \frac{4}{s}\ln\left(\frac{4s}{Q^2\delta^2}\right)\right]$$

Integrated Jet Function

$$\bar{J}_{n,\mathrm{ren}}^{\mathrm{SW(s_{max})}}(\mu) \equiv \int_0^{s_{\max}} \mathrm{d}s\, J_{n,\mathrm{ren}}^{\mathrm{SW}}(s,\mu)$$

$$= 1 + \theta\left(\delta^2 - \frac{4s_{\max}}{Q^2}\right)\frac{\alpha_s C_F}{4\pi}\left[7 - \pi^2 - 3\ln\left(\frac{s_{\max}}{\mu^2}\right) + 2\ln^2\left(\frac{s_{\max}}{\mu^2}\right) + 6\ln\left(\frac{4s_{\max}+Q^2\delta^2}{Q^2\delta^2}\right)\right]$$

$$+ \theta\left(\frac{4s_{\max}}{Q^2} - \delta^2\right)\frac{\alpha_s C_F}{4\pi}\left[7 - \pi^2 - 3\ln\left(\frac{Q^2\delta^2}{4\mu^2}\right) + 2\ln^2\left(\frac{Q^2\delta^2}{4\mu^2}\right) + 6\ln 2 + 4\ln\left(\frac{s_{\max}}{\mu^2}\right)\ln\left(\frac{4s_{\max}}{Q^2\delta^2}\right)\right]$$

Kinematics of Constraint Function

$$p_g^+ = -\frac{\left(p_g^\perp\right)^2}{p_g^-}, \qquad \left(p_g^\perp\right)^2 = -\frac{p_g^-(Q - p_g^-)s}{Q^2}$$

$$p_g^+ = \frac{s}{Q}, \qquad \left(p_g^\perp\right)^2 = -p_g^- p_g^+ = -\frac{p_g^- s}{Q}, \qquad p_q^- = Q, \qquad p_q^+ = 0, \qquad p_q^\perp = 0$$

Integrals for Jet Function

$$\Delta\tilde{J}_n^F(s) = \frac{\alpha_s C_F}{4\pi} A(\epsilon) \frac{1}{\mu^2} \left(\frac{\mu^2}{s}\right)^{1+\epsilon} \int_0^1 dy\ \frac{1}{y^\epsilon} (1-y)^{-\epsilon} \left(\frac{4(1-y)}{y} + y(d-2)\right) (F(a_i, y) - 1),$$

$$\Delta J_{n0}^F(s) = \frac{\alpha_s C_F}{4\pi} A(\epsilon) \frac{1}{\mu^2} \left(\frac{\mu^2}{s}\right)^{1+\epsilon} \int_0^\infty dy\ \frac{4}{y^{1+\epsilon}} (F_0(a_i, y) - 1),$$