# Exclusive decays of $\chi_{bJ}$ and $\eta_b$ into two charmed mesons

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in collaboration with R. Azevedo and B. Long; Phys. Rev. D **80** 074026 (2009)

# Motivation

Decay rates $\chi_{bJ} \to DD$ and $\eta_b \to DD^* + \text{c.c.}$

- many different scales in the problem
  $2m_b \gg m_c \sim m_b w \gg \Lambda_{QCD} \sim m_b w^2$
- large mass scale ($m_c$) in highly energetic final state

$$e^+e^- \to t\bar{t} \text{ at } Q \gg m_t$$

S. Fleming et al, Phys. Rev. D **77**:074010 (2008);
Phys. Rev. D **77**:114003 (2008).

$$\Upsilon \to J/\psi + X \text{ at } x \to 1$$

X. Liu, Phys. Lett. B **685**:151 (2010).

$$B \to \chi_{cJ} K$$

M. Beneke and L. Vernazza, Nucl. Phys. B **811**:155 (2009);

- **inclusive** production $\chi_{bJ} \to c\bar{c} + X$

| State | $\mathcal{B}(\chi_{bJ}(nP) \to D^0 X)$ |
|---|---|
| $\chi_{b0}(1P)$ | $5.6 \pm 3.6 \pm 0.5\%$ |
| $\chi_{b1}(1P)$ | $12.6 \pm 1.9 \pm 1.1\%$ |
| $\chi_{b2}(1P)$ | $5.4 \pm 1.9 \pm 0.5\%$ |

CLEO, Phys. Rev. D **78**:092007 (2008).

- $\eta_b \to DD^*$ possible channel for $\eta_b$ discovery at the Tevatron . . .

*BABAR*-PUB-08/029
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arXiv:0807.1086 [hep-ex]

# Observation of the bottomonium ground state in the decay $\Upsilon(3S) \to \gamma\, \eta_b$

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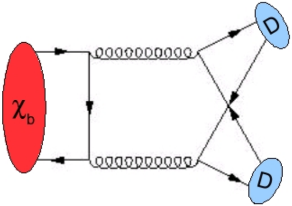
# Scales and Effective Theories

**bottomonium**

- decay $2m_b$,
- structure $m_b w$, $m_b w^2$




***D* meson**

- mass $m_c$

$$p_c^\mu = m_c v^\mu + k^\mu$$
$$p_l^\mu \sim k^\mu$$

- *D* meson rest frame
  $v^\mu = (1, \mathbf{0})$

  $k^\mu = \Lambda_{QCD}$

- bottomonium rest frame
  $v^\mu = 2m_b/m_c(\lambda^2, \lambda^0, \lambda)$

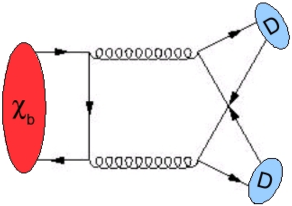
  $k^\mu = 2m_b\Lambda_{QCD}/m_c(\lambda^2, \lambda^0, \lambda)$

$$\lambda = m_c/2m_b$$

# Scales and Effective Theories

**bottomonium**

- decay $2m_b$,
- structure $m_b w$, $m_b w^2$




**boosted $D$ meson**

- mass $m_c$

$$p_c^\mu = m_c v^\mu + k^\mu$$
$$p_l^\mu \sim k^\mu$$

- $D$ meson rest frame
  $v^\mu = (1, \mathbf{0})$

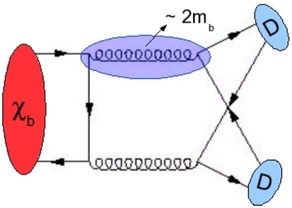
  $k^\mu = \Lambda_{QCD}$

- bottomonium rest frame
  $v^\mu = 2m_b/m_c(\lambda^2, \lambda^0, \lambda)$

  $k^\mu = 2m_b\Lambda_{QCD}/m_c(\lambda^2, \lambda^0, \lambda)$

$$\lambda = m_c/2m_b$$

# Scales and Effective Field Theories



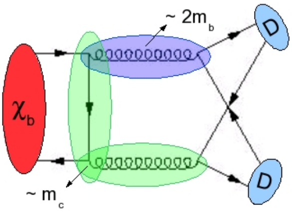

- integrate out $2m_b$
- $m_b w$, $m_c$ still dynamical

**EFT I: NRQCD & SCET**

| | NRQCD | field | momentum | SCET | field | momentum |
|---|---|---|---|---|---|---|
| quark | $b, \bar{b}$ | $\psi_b,\ \chi_{\bar{b}}$ | $(m_b w^2, m_b w)$ | $c, \bar{c}$ | $\xi^c_{\bar{n}},\ \ \xi^{\bar{c}}_n$ | $2m_b(\lambda^2, 1, \lambda)$ |
| gluon | potential | $A^\mu$ | $(m_b w^2, m_b w)$ | collinear | $A^\mu_{\bar{n}},\ \ A^\mu_n$ | $2m_b(\lambda^2, 1, \lambda)$ |
| | soft | $A^\mu$ | $(m_b w, m_b w)$ | soft | $A^\mu_s$ | $2m_b(\lambda, \lambda, \lambda)$ |
| | usoft | $A^\mu$ | $(m_b w^2, m_b w^2)$ | usoft | $A^\mu_{us}$ | $2m_b(\lambda^2, \lambda^2, \lambda^2)$ |

$m_b w \sim 2m_b\lambda \sim m_c \gg \Lambda_{QCD}$

# Scales and Effective Field Theories




- integrate out $2m_b$
- $m_b w$, $m_c$ still dynamical

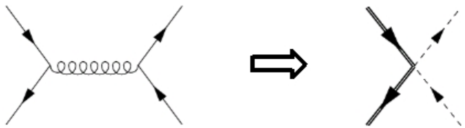
**EFT I: NRQCD & SCET**

- integrate out $m_c$ and $m_b w$ (the soft modes)

**EFT II: pNRQCD & bHQET**

| | pNRQCD | field | momentum | bHQET | field | momentum |
|---|---|---|---|---|---|---|
| quark | $b, \bar{b}$ | $\psi_b,\ \chi_{\bar{b}}$ | $(m_b w^2, m_b w)$ | $c, \bar{c}$ | $h^c_{\bar{n}},\ \ h^{\bar{c}}_n$ | $Q\,(\lambda^2, 1, \lambda)$ |
| | | | | $u, d$ | $\xi_{\bar{n}},\ \ \xi_n$ | $Q\,(\lambda^2, 1, \lambda)$ |
| gluon | usoft | $A^\mu$ | $(m_b w^2, m_b w^2)$ | usoft | $A^\mu_{us}$ | $Q(\lambda, \lambda, \lambda)$ |
| | | | | ucollinear | $A^\mu_{\bar{n}},\ \ A^\mu_n$ | $Q\,(\lambda^2, 1, \lambda)$ |

$$Q = 2m_b \Lambda_{QCD}/m_c$$



$$iJ_{\mathrm{QCD}} = iC(\mu)J_{\mathrm{EFT_I}}(\mu) \ .$$

**Matching**

- At leading order in the EFT and tree level in $\alpha_s$ :

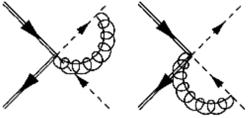
$$J_{\mathrm{EFT_I}} = \chi_{\bar{b}}^{\dagger}\sigma_{\perp}^{\mu}\, t^a \psi_b \ \bar{\chi}_{\bar{n}}^{c}\, S_{\bar{n}}^{\dagger}\, \gamma_{\mu\,\perp} t^a \, S_n \, \chi_n^{\bar{c}} \quad \text{and} \quad C(\mu = 2m_b) = \frac{\alpha_s(2m_b)\pi}{m_b^2} \ .$$

**and Running**

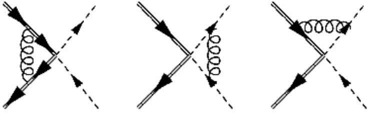
- $\mu$-dependence driven by RGE in NRQCD + SCET

$$J_{\mathrm{QCD}} = C(\mu)J_{\mathrm{EFT_I}}(\mu) = C(\mu_b = 2m_b)\left(\frac{2m_b}{\sqrt{n\cdot p_c \bar{n}\cdot p_{\bar{c}}}}\right)^{g} \exp U(2m_b, m_c) J_{\mathrm{EFT_I}}(\mu = m_c) \ ,$$

# Running in EFT I



collinear loops



usoft loops

$$\gamma_{EFT_I} = -2 \left\{ \gamma(\alpha_s) + \Gamma_{\rm cusp}(\alpha_s) \ln\left( \frac{\mu}{\sqrt{n\cdot p_c\, \bar{n}\cdot p_{\bar{c}}}} \right) \right\}$$

- resum LL Sudakov logarithms $\alpha_s^n \ln^{n+1} m_c/2m_b$
- resum NLL $\alpha_s^n \ln^n m_c/2m_b$

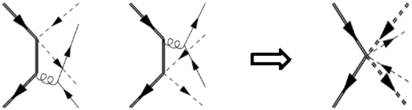
one loop $\Gamma_{\rm cusp}(\alpha_s) = \frac{\alpha_s}{4\pi} 4C_F$ ✓

one loop $\gamma(\alpha_s) = \frac{\alpha_s}{4\pi}\left(3C_F + N_c + i\frac{\pi}{N_c}\right)$

two loop $\Gamma_{\rm cusp}(\alpha_s)$, ✓

- *numerically* LL and NLL approximately equal

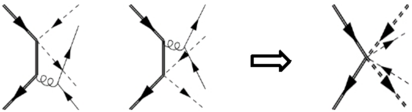
# Effective Field Theory II.



$$iM_{EFT_I} = C(\mu)\ T_{AB}(\omega,\bar{\omega},\mu,\mu',\ {}^{2S+1}L_J)\otimes F^2(\mu')\,\langle D_A D_B|\mathcal{O}_{AB}^{{}^{2S+1}L_J}\,(\omega,\bar{\omega};\mu')\,|\,\bar{b}b({}^{2S+1}L_J)\rangle$$

$$A,B\in\{P,V_L,V_T\}$$

# Effective Field Theory II.



$$i\mathcal{M}_{EFT_I} = C(\mu)\; T_{AB}(\omega,\bar{\omega},\mu,\mu',\, {}^{2S+1}L_J) \otimes F^2(\mu')\, \langle D_A D_B | \mathcal{O}_{AB}^{{}^{2S+1}L_J}\,(\omega,\bar{\omega};\mu')\,|\,\bar{b}b({}^{2S+1}L_J)\rangle$$

$$A, B \in \{P, V_L, V_T\}$$

- P wave

$$T_{AA}(\omega,\bar{\omega},\mu,\mu' = m_c,\, {}^3P_J) = \frac{C_F}{N_c^2}\frac{4\pi\alpha_s(\mu')}{m_b}\frac{1}{\omega+\bar{\omega}} \qquad A \in \{P, V_L\},\; J = 0, 2$$

$$T_{V_T V_T}(\omega,\bar{\omega},\mu,\mu' = m_c,\, {}^3P_2) = \frac{C_F}{N_c^2}\frac{4\pi\alpha_s(\mu')}{m_b}\frac{1}{\omega+\bar{\omega}}$$

- S wave

$$T_{PV_L}(\omega,\bar{\omega},\mu,\mu' = m_c,\, {}^1S_0) = \frac{C_F}{N_c^2}\frac{4\pi\alpha_s(\mu')}{m_b}\frac{1}{2}\frac{\omega-\bar{\omega}}{\omega+\bar{\omega}}$$

- non trivial dependence on $\omega$ and $\bar{\omega}$ at tree level

# Non-perturbative matrix elements

$$F^2(\mu')\,\mathcal{O}_{PP}^{{}^3P_J}(\omega,\bar{\omega},\mu') = \chi_b^\dagger\,\mathbf{p}_b\cdot\sigma_\perp\psi_b\;\bar{\mathcal{H}}_{\bar{n}}^c\,\frac{\not{n}}{2}\gamma^5\,\delta\left(-\bar{\omega}-n\cdot\mathcal{P}\right)\chi_{\bar{n}}^{\bar{l}}\;\bar{\chi}_n^l\delta\left(\omega-\bar{n}\cdot\mathcal{P}^\dagger\right)\;\frac{\not{\bar{n}}}{2}\gamma^5\,\mathcal{H}_n^{\bar{c}}\,,$$

- usoft decoupling: $\mathcal{H}_n^{\bar{c}} \to Y_n\mathcal{H}_n^{\bar{c}}$, $\bar{\chi}_n^l \to \bar{\chi}_n^l Y_n^\dagger$

# Non-perturbative matrix elements

$$\langle PP|F^2(\mu')\,\mathcal{O}^{^3P_J}_{PP}(\omega,\bar\omega,\mu')|^3P_0\rangle$$
$$= \langle 0|\chi^\dagger_{\bar b}\,\mathbf{p}_b\cdot\sigma_\perp\psi_b|^3P_0\rangle\;\langle P|\bar{\mathcal{H}}^c_{\bar n}\,\frac{\not n}{2}\gamma^5\,\delta\,(-\bar\omega-n\cdot\mathcal{P})\,\chi^{\bar l}_{\bar n}|0\rangle\;\langle P|\bar\chi^l_n\delta\left(\omega-\bar n\cdot\mathcal{P}^\dagger\right)\,\frac{\not{\bar n}}{2}\gamma^5\,\mathcal{H}^{\bar c}_n|0\rangle\;,$$

- usoft decoupling: $\mathcal{H}^{\bar c}_n \to Y_n\mathcal{H}^{\bar c}_n$, $\bar\chi^l_n \to \bar\chi^l_n Y^\dagger_n$
- factorization of initial and final state:

$$|^3P_0\rangle = |0\rangle_n\,|0\rangle_{\bar n}|^3P_0\rangle_{\rm us}\,,\qquad \langle PP| =_n \langle P|\,_{\bar n}\langle P|\,_{\rm us}\langle 0|$$

# Non-perturbative matrix elements

$$\langle PP|F^2(\mu')\,\mathcal{O}^{^3P_J}_{PP}(\omega,\bar{\omega},\mu')|^3P_0\rangle$$

$$= \langle 0|\chi^\dagger_{\bar{b}}\,\mathbf{p}_b\cdot\sigma_\perp\psi_b|^3P_0\rangle\;\langle P|\bar{\mathcal{H}}^c_{\bar{n}}\,\frac{\not{n}}{2}\gamma^5\,\delta\left(-\bar{\omega}-n\cdot\mathcal{P}\right)\chi^{\bar{l}}_{\bar{n}}|0\rangle\;\langle P|\bar{\chi}^l_n\delta\left(\omega-\bar{n}\cdot\mathcal{P}^\dagger\right)\,\frac{\not{\bar{n}}}{2}\gamma^5\,\mathcal{H}^{\bar{c}}_n|0\rangle\;,$$

- usoft decoupling: $\mathcal{H}^{\bar{c}}_n\to Y_n\mathcal{H}^{\bar{c}}_n$, $\bar{\chi}^l_n\to\bar{\chi}^l_n Y^\dagger_n$
- factorization of initial and final state:

$$|^3P_0\rangle = |0\rangle_n\,|0\rangle_{\bar{n}}|^3P_0\rangle_{\rm us}\,,\qquad \langle PP| =_n \langle P|\,_{\bar{n}}\langle P|\,_{\rm us}\langle 0|$$

- quarkonium wavefunctions

$$\langle 0|\chi^\dagger_{\bar{b}}\mathbf{p}_b\cdot\sigma_\perp\psi_b|\chi_{b0}\rangle = \frac{2}{\sqrt{3}}\sqrt{\frac{3N_c}{2\pi}}R'_{\chi_{b0}}(0,\mu'),$$

and for S-wave

$$\langle 0|\chi^\dagger_{\bar{b}}\psi_b|\eta_b\rangle = \sqrt{\frac{N_c}{2\pi}}R_{\eta_b}(0,\mu').$$

# D-meson distribution amplitudes

$$\langle PP|F^2(\mu')\,\mathcal{O}^{^3P_J}_{PP}(\omega,\bar{\omega},\mu')|^3P_0\rangle$$
$$= \langle 0|\chi^\dagger_{\bar{b}}\,\mathbf{p}_b\cdot\sigma_\perp\psi_b|^3P_0\rangle\,\langle P|\bar{\mathcal{H}}^c_{\bar{n}}\,\frac{\not{\bar{n}}}{2}\gamma^5\,\delta\left(-\bar{\omega}-n\cdot\mathcal{P}\right)\chi^{\bar{l}}_{\bar{n}}|0\rangle\,\langle P|\bar{\chi}^l_n\delta\left(\omega-\bar{n}\cdot\mathcal{P}^\dagger\right)\,\frac{\not{\bar{n}}}{2}\gamma^5\,\mathcal{H}^{\bar{c}}_n|0\rangle\,,$$

- *D*-meson light-cone distribution amplitudes (DA)

$$\langle P|\bar{\chi}^l_n\,\frac{\not{\bar{n}}}{2}\gamma^5\,\delta\left(\omega-\bar{n}\cdot\mathcal{P}^\dagger\right)\mathcal{H}^{\bar{c}}_n|0\rangle = iF_P(\mu')\frac{\bar{n}\cdot v}{2}\phi_P(\omega,\mu')$$

and for vector mesons

$$\langle V_L|\bar{\chi}^l_n\,\frac{\not{\bar{n}}}{2}\,\delta\left(\omega-\bar{n}\cdot\mathcal{P}^\dagger\right)\mathcal{H}^{\bar{c}}_n|0\rangle = F_{V_L}(\mu')\frac{\bar{n}\cdot v}{2}\phi_{V_L}(\omega,\mu')$$

$$\langle V_T|\bar{\chi}^l_n\,\frac{\not{\bar{n}}}{2}\gamma^\mu_\perp\,\delta\left(\omega-\bar{n}\cdot\mathcal{P}^\dagger\right)\mathcal{H}^{\bar{c}}_n|0\rangle = F_{V_T}(\mu')\frac{\bar{n}\cdot v}{2}\varepsilon^\mu_\perp\,\phi_{V_T}(\omega,\mu')$$

heavy quark limit $F_A(\mu'=m_c)=f_D\sqrt{m_D}$,
$f_D = 205.8\pm 8.5\pm 2.5$ MeV

# Factorization of the decay rate

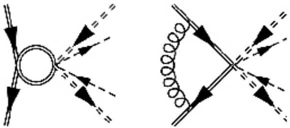
- P wave

$$\Gamma\left(\chi_{b0}\to AA\right)=\frac{4}{3}\frac{m_D^2\sqrt{m_{\chi_{b0}}^2-4m_D^2}}{8\pi m_{\chi_{b0}}}\frac{3N_c}{2\pi}\,|C\left(\mu\right)|^2\,|R'_{\chi_{b0}}(0,\mu')|^2$$
$$\left[F^2(\mu')\frac{n\cdot v'}{2}\frac{\bar{n}\cdot v}{2}\int\frac{d\omega}{\omega}\frac{d\bar{\omega}}{\bar{\omega}}T\left(\omega,\bar{\omega},\mu,\mu';{}^3P_J\right)\phi_A(\bar{\omega},\mu')\phi_A(\omega,\mu')\right]^2$$
$$A\in\{P,V_L\}$$

- analogous expressions for $\chi_{b2}\to PP, V_LV_L$ or $V_TV_T$.
- S wave

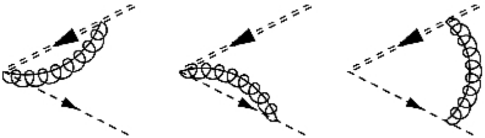
$$\Gamma\left(\eta_b\to PV_L+\text{c.c.}\right)=\frac{m_D^2\sqrt{m_{\eta_b}^2-4m_D^2}}{8\pi m_{\eta_b}}\frac{N_c}{2\pi}|C\left(\mu\right)|^2\;|R_{\eta_b}(0,\mu')|^2\,\frac{1}{2}\left[F^2(\mu')\frac{n\cdot v'}{2}\frac{\bar{n}\cdot v}{2}\right.$$
$$\left.\int\frac{d\omega}{\omega}\frac{d\bar{\omega}}{\bar{\omega}}T\left(\omega,\bar{\omega},\mu,\mu';{}^1S_0\right)\left(\phi_{V_L}(\bar{\omega},\mu')\phi_P(\omega,\mu')-\phi_{V_L}(\omega,\mu')\phi_P(\bar{\omega},\mu')\right)\right]^2.$$

# Running in EFT II



- pNRQCD graphs do not contribute to NLL running

# Running in EFT II



- pNRQCD graphs do not contribute to NLL running
- from bHQET graphs, convolution running

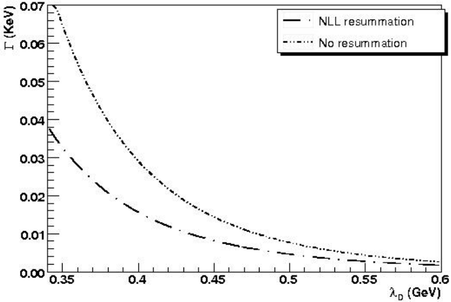
$$\gamma_{EFT_{II}} = 2\gamma_F \delta(\omega - \omega')\delta(\bar{\omega} - \bar{\omega}') + \gamma_{\mathcal{O}}(\omega, \omega'; \bar{\omega}, \bar{\omega}'; \mu')$$

- $\gamma_F$ governs the running of D-meson decay constant
- $\gamma_{\mathcal{O}}$ running of the D-meson DA

B. Lange and M. Neubert, Phys. Rev. Lett. **91**:102001 (2003).

- analytical (though gory) solution for the evolved matching coefficients.

# Results. $\Gamma(\chi_{b0} \to DD)$

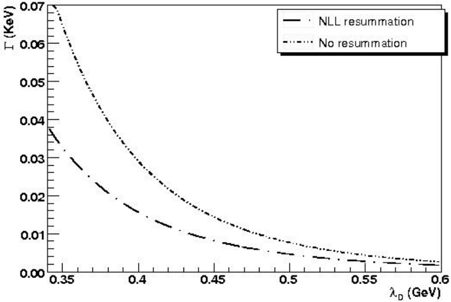



$\lambda_B = 0.460 \pm 0.110$ GeV

**Non perturbative parameters**

- $|R'_{\chi_{bJ}}(0)|^2 = 2.3$ GeV$^5$ lattice
- D-meson
  $\phi_D(\omega, \mu' \sim 1\,\mathrm{GeV}) = \frac{\omega}{(\bar{n}\cdot v)^2 \lambda_D^2} e^{-\frac{\omega}{\bar{n}\cdot v \lambda_D}}$
- choice of $\lambda_D$ inspired by B-physics!

# Results. $\Gamma(\chi_{b0} \to DD)$




$\lambda_B = 0.460 \pm 0.110$ GeV

**Non perturbative parameters**

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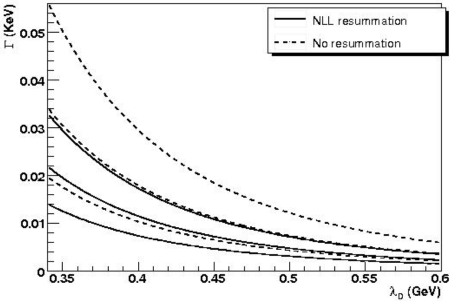


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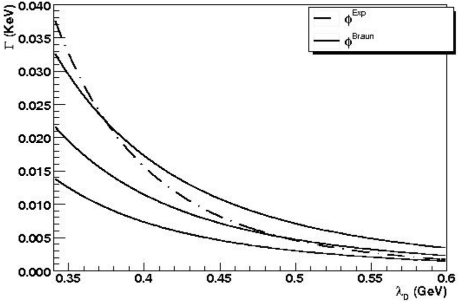


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- two DAs in rough agreement

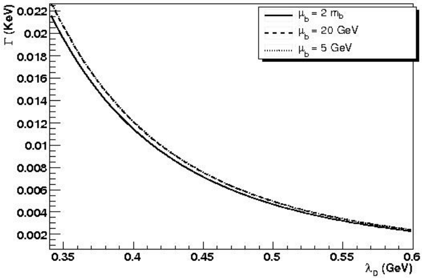
# Results. $\Gamma(\chi_{b0} \to DD)$

**Uncertainties**

- non-perturbative corrections: $\Lambda_{QCD}/m_c \sim 30\%$

perturbative corrections

- QCD onto EFT I matching: $\alpha_s(2m_b) \sim 10\%$
- EFT I onto EFT II matching: $\alpha_s(m_c) \sim 30\%$

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One loop EFT I to EFT II matching for stable result!

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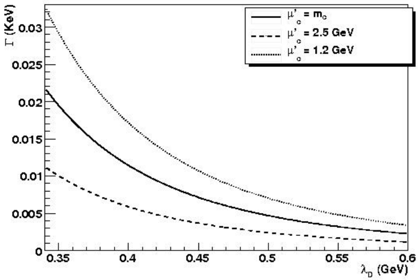


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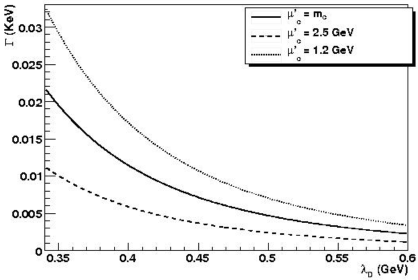


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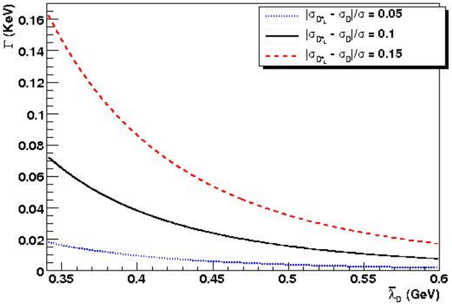


$$\bar{\lambda}_D = \frac{1}{2}(\lambda_D + \lambda_{D^*_L})$$
$$\delta = \frac{1}{2}\frac{\lambda_D - \lambda_{D^*_L}}{\bar{\lambda}_D}$$
$$\sigma = 2\sigma_D$$

**Non perturbative parameters**

- bottomonium
  $|R_{\eta_b}(0)|^2 = 6.92 \pm 0.38\,\text{GeV}^3$
  $\Upsilon$ decay
- D-meson $\phi_D = \phi_D^{\text{Braun}}$, $\phi_{D^*} = \phi_{D^*}^{\text{Braun}}$

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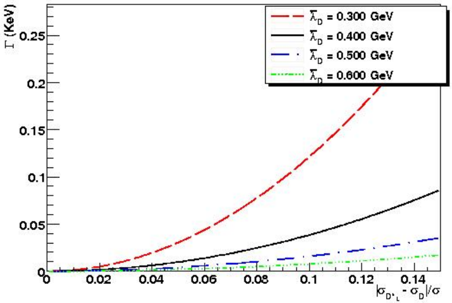


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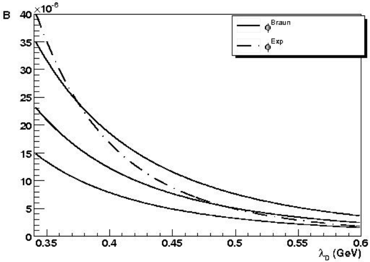
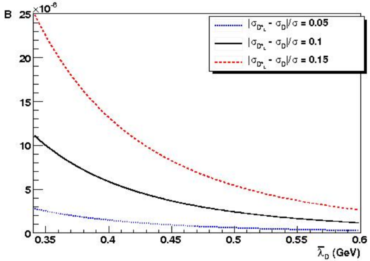
- strong dependence on DA parameters $\sim \bar{\lambda}_D^{-4}$
- vanishes in spin-symmetry limit $\phi_D = \phi_{D^*}$
- strong dependence on the functional form

# Branching Ratios




- $\mathcal{B}(\chi_{b0} \to PP)$ in the range $4 \cdot 10^{-6} - 4 \cdot 10^{-5}$
- $\mathcal{B}(\eta_b \to PV_L + \text{c.c.})$ in the same range
- too small in this range for $\lambda_D$ . . .

. . . but hope if $\lambda_D$ in the $0.25 - 0.35$ GeV range

- $\eta_b \to DD^*$ *does not* dominate $\eta_b \to c\bar{c} + X$

# Conclusions

EFT approach:

- allows factorization of dynamics of different scales; $2m_b$, $m_c$, $\Lambda_{QCD}$.
- can be extended to $C$-odd bottomonium decays, $\Upsilon \to DD$, $\Upsilon \to D^* D^*$.
- and to power-suppressed processes, $\eta_b \to D^* D^*$, $\chi_{b2} \to DD^* +$ c.c.

Exclusive, charmed decays of bottomonium

- good candidates for extraction of bottomonium and $D$-meson parameters

*strong dependence of DA paramaters*

**but**

- quite large theoretical error

include perturbative and non-perturbative corrections!

- rather small branching ratio

# Backup Slides

# Running Factors in EFT I

$$U(\mu_0,\mu) = -2\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)} \left\{ \gamma(\alpha) + \Gamma_{\rm cusp}(\alpha) \int_{\alpha(\mu_0)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')} \right\} \,,$$

$$g(\mu_0,\mu) = -2\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)} \Gamma_{\rm cusp}(\alpha) \,.$$

Explicitly, at NLL

$$U(\mu_b,\mu) = \frac{2\pi\Gamma^{(0)}_{\rm cusp}}{\beta_0^2} \left[ \frac{r-1-r\ln r}{\alpha_s(\mu)} + \frac{\beta_0\gamma^{(0)}_{\rm Re}}{2\pi\Gamma^{(0)}_{\rm cusp}} \ln r + \left( \frac{\Gamma^{(1)}_{\rm cusp}}{\Gamma^{(0)}_{\rm cusp}} - \frac{\beta_1}{\beta_0} \right) \frac{1-r+\ln r}{4\pi} + \frac{\beta_1}{8\pi\beta_0} \ln^2 r \right] + \frac{\gamma^{(0)}_{\rm Im}}{\beta_0} \ln r \,,$$

and

$$g(\mu_b,\mu) = \frac{\Gamma^{(0)}_{\rm cusp}}{\beta_0} \left[ \ln r + \left( \frac{\Gamma^{(1)}_{\rm cusp}}{\Gamma^{(0)}_{\rm cusp}} - \frac{\beta_1}{\beta_0} \right) \frac{\alpha_s(\mu_b)}{4\pi} (r-1) \right] \,,$$

where $r = \alpha_s(\mu)/\alpha_s(\mu_b)$.

# Running Factors in EFT II

The anomalous dimension in EFT is

$$\gamma_{\mathrm{EFT_{II}}}(\omega,\omega';\bar\omega,\bar\omega';\mu') = 2\gamma_F\,\delta\left(\omega-\omega'\right)\delta\left(\bar\omega-\bar\omega'\right) + \gamma_{\mathcal{O}}(\omega,\omega';\bar\omega,\bar\omega';\mu')\ ,$$

with

$$\begin{aligned}
&\gamma_{\mathcal{O}}(\omega,\omega';\bar\omega,\bar\omega';\mu') \\
&= \frac{\alpha_s}{4\pi}\,4C_F\delta\left(\omega-\omega'\right)\delta\left(\bar\omega-\bar\omega'\right)\left[-1+\ln\left(\frac{\mu' n\cdot v'}{\bar\omega'}\right)+\ln\left(\frac{\mu'\bar n\cdot v}{\omega'}\right)\right] \\
&- \frac{\alpha_s}{4\pi}4C_F\delta\left(\omega-\omega'\right)\left[\theta\left(\bar\omega-\bar\omega'\right)\left(\frac{1}{\bar\omega-\bar\omega'}\right)_+ + \theta\left(\bar\omega'-\bar\omega\right)\theta\left(\bar\omega\right)\frac{\bar\omega}{\bar\omega'}\left(\frac{1}{\bar\omega'-\bar\omega}\right)_+\right] \\
&- \frac{\alpha_s}{4\pi}4C_F\delta\left(\bar\omega-\bar\omega'\right)\left[\theta\left(\omega-\omega'\right)\left(\frac{1}{\omega-\omega'}\right)_+ + \theta\left(\omega'-\omega\right)\theta\left(\omega\right)\frac{\omega}{\omega'}\left(\frac{1}{\omega'-\omega}\right)_+\right]\ .
\end{aligned}$$

# Running factors in EFT II

$$F^2(\mu')\,T(\omega,\bar\omega,\mu,\mu';{}^3P_J) = F^2(\mu'_c)\,\frac{C_F}{N_c^2}\,\frac{4\pi\alpha_s(\mu'_c)}{m_b}\,\exp\left[V(\mu'_c,\mu')\right]\,\left(\frac{{\mu'_c}^2\bar n\cdot v n\cdot v'}{\omega\,\bar\omega}\right)^g$$

$$\frac{\theta(\bar\omega-\omega)}{\bar\omega}\left\{\frac{\Gamma(1+g)\Gamma(2+g)}{\Gamma(1-g)\Gamma(-g)}\left[1-\ln\frac{\omega}{\bar\omega}+\psi(1-g)-\psi(-g)+\psi(1+g)-\psi(2+g)\right]\right.$$

$$+\frac{1}{2}\,\frac{\omega}{\bar\omega}\,\frac{\Gamma(g+2)\Gamma(g+3)}{\Gamma(1-g)\Gamma(2-g)}\,{}_4F_3\left(1,1,g+2,g+3;3,1-g,2-g;-\frac{\omega}{\bar\omega}\right)$$

$$\left.-\left(\frac{\omega}{\bar\omega}\right)^{1+g}4\cos(g\pi)\,\frac{\Gamma(2+2g)^2}{g+2}\,{}_3F_2\left(g+1,2g+2,2g+3;2,g+3;-\frac{\omega}{\bar\omega}\right)\right\}+(\omega\to\bar\omega)\ ,$$

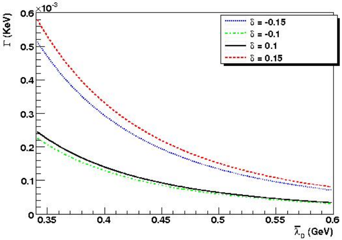
- at NLL

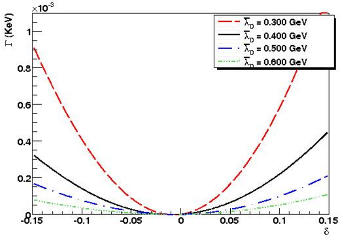
$$g(\mu'_0,\mu') = -\frac{\Gamma^{(0)}_{\rm cusp}}{2\beta_0}\left\{\ln r+\left(\frac{\Gamma^{(1)}_{\rm cusp}}{\Gamma^{(0)}_{\rm cusp}}-\frac{\beta_1}{\beta_0}\right)\frac{\alpha_s(\mu'_0)}{4\pi}(r-1)\right\}\ ,$$

$$V(\mu'_0,\mu') = -\,\Gamma^{(0)}_{\rm cusp}\frac{2\pi}{\beta_0^2}\left\{\frac{r-1-r\ln r}{\alpha_s(\mu')}+\left(\frac{\Gamma^{(1)}_{\rm cusp}}{\Gamma^{(0)}_{\rm cusp}}-\frac{\beta_1}{\beta_0}\right)\frac{1-r+\ln r}{4\pi}+\frac{\beta_1}{8\pi\beta_0}\ln^2 r\right\}$$

$$+\,\frac{C_F}{\beta_0}\,(2-8\gamma_E)\ln r\ ,$$

# $\Gamma(\eta_b \to DD^*)$ with exponential DA




- one or two orders smaller than the result with $\phi_D^{\text{Braun}}$