Threshold Resummation at Hadron Colliders

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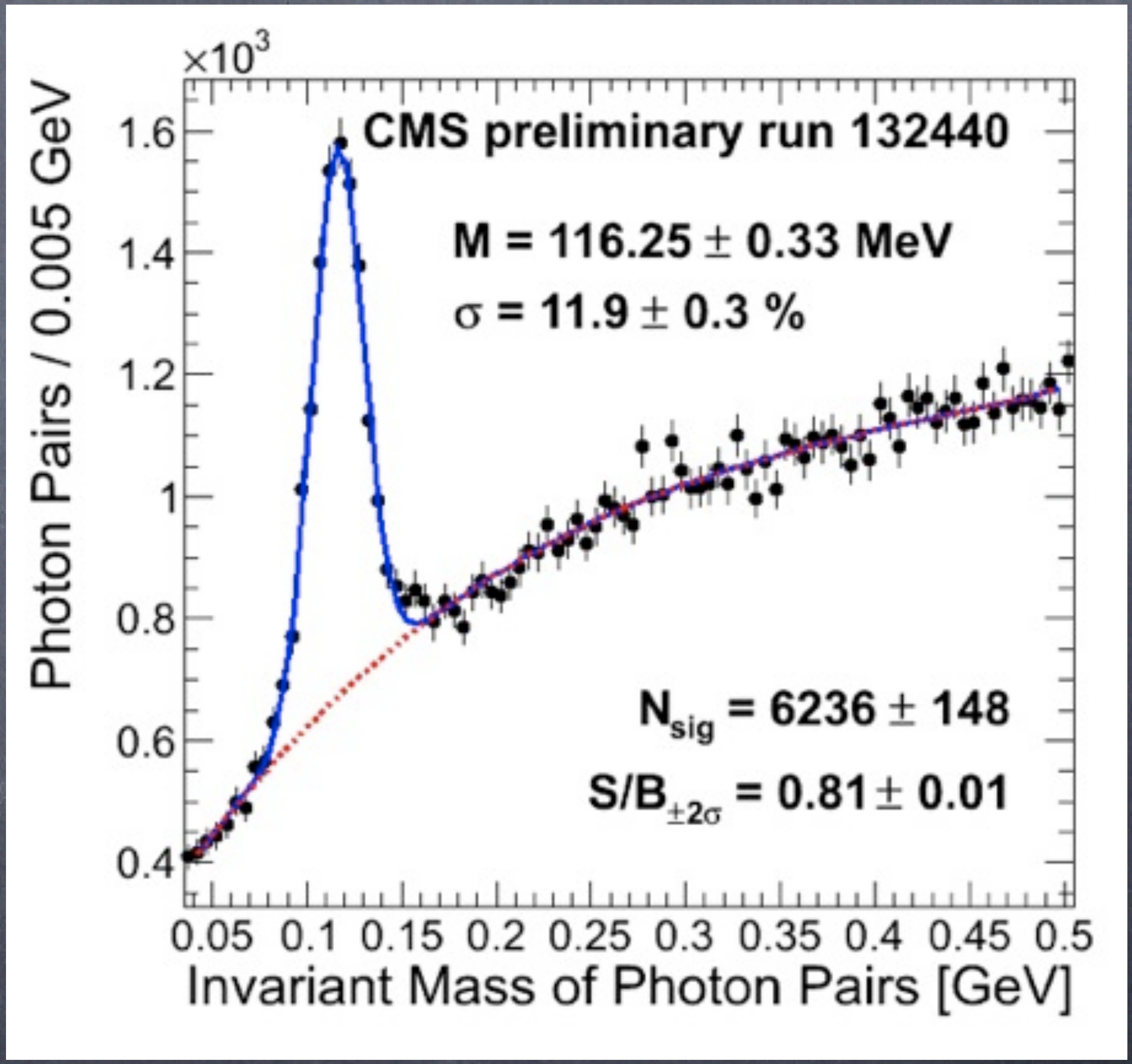
Based on work with A. Hornig and C. Bauer
arxiv:1002.1307
arxiv:1005.????

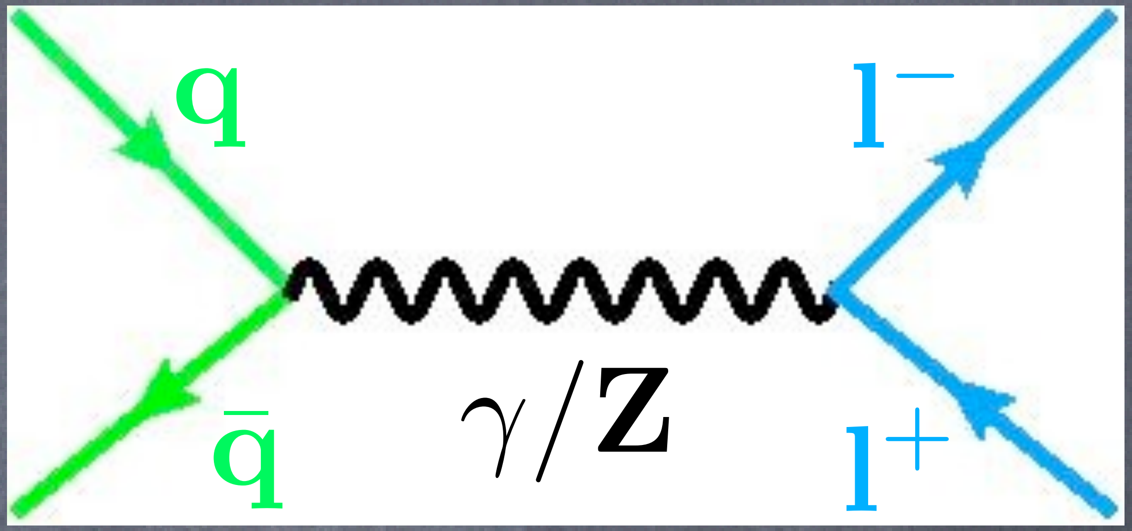
Motivations

- New physics
  - LHC era is here!
- Multiple jet backgrounds
  - need jet algorithm
  - new soft function



Previous Work

- Drell-Yan
  - 0 jet final state



Idilbi, Ji '05
Becher, Neubert, Xu '08

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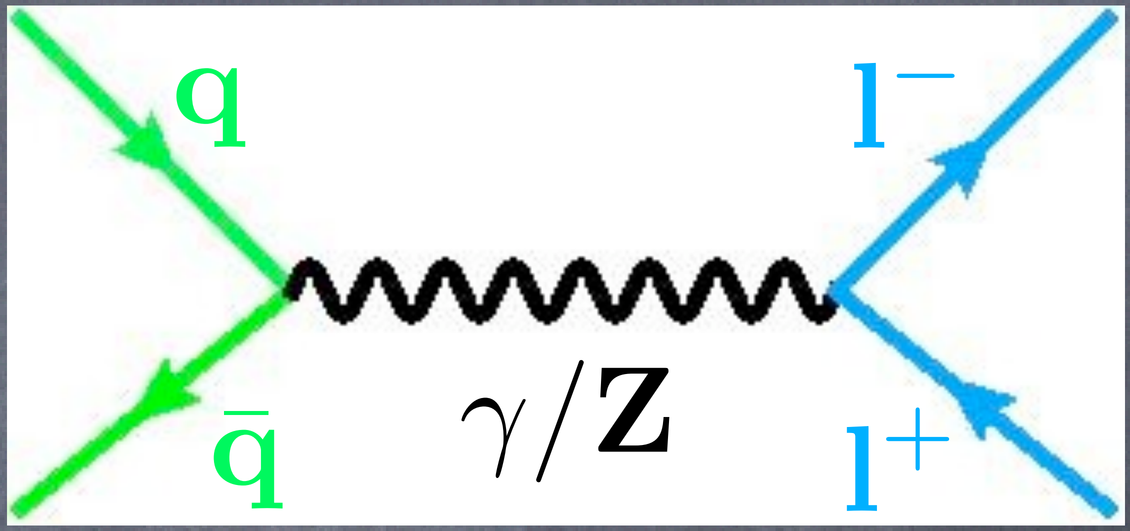
- Drell-Yan
  - 0 jet final state
  - time-like soft function



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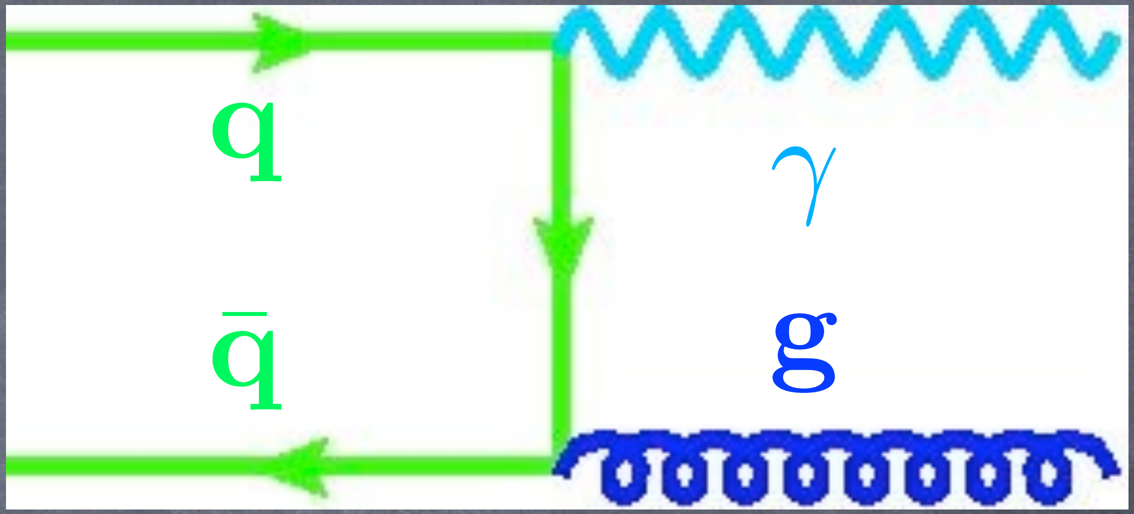
Becher, Neubert, Xu '08

$$W_{DY}(\omega, \mu_f) = \int \frac{dx^0}{4\pi} e^{i\omega x^0/2} \hat{W}_{DY}(x^0, \vec{x} = 0, \mu_f)$$

time-like projection

Previous Work

- Prompt photon ($B \to X_s \gamma$)
  - 1 inclusive jet



Becher, Schwartz '09

Previous Work

- Prompt photon ($B \to X_s \gamma$)
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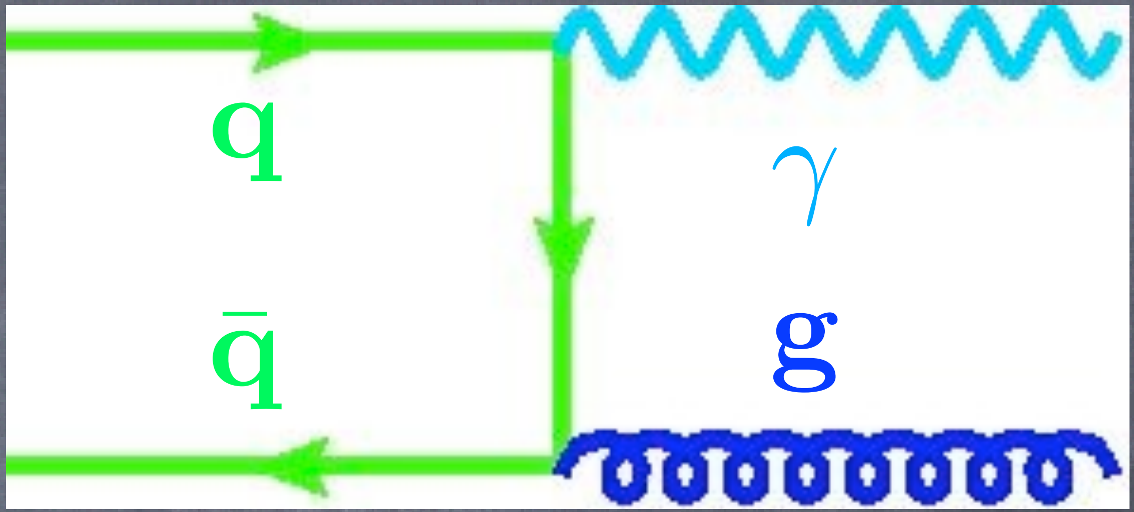
  - null soft function



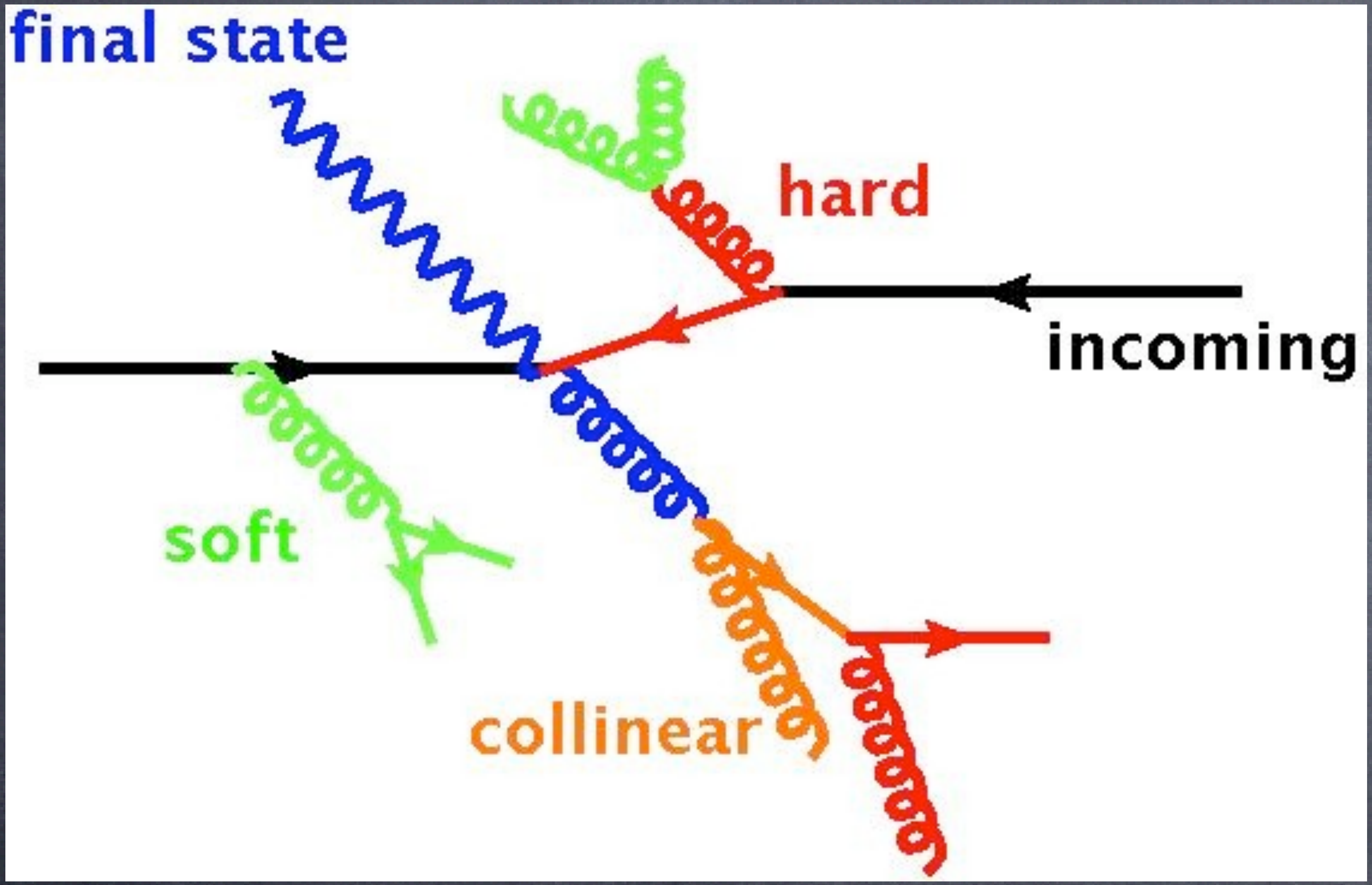
Becher, Schwartz '09

$$\frac{1}{N_c}\langle 0|\mathrm{Tr}\,\bar{\mathbf{T}}\left[\left(Y_1^\dagger Y_J t^a Y_J^\dagger Y_2\right)(x_-)\right]\mathbf{T}\left[\left(Y_2^\dagger Y_J t^a Y_J^\dagger Y_1\right)(0)\right]|0\rangle =$$

$$\int_0^\infty dk_+ e^{-ik_+(\bar{n}_J\cdot x)/2} S_{q\bar{q}}(k_+)$$

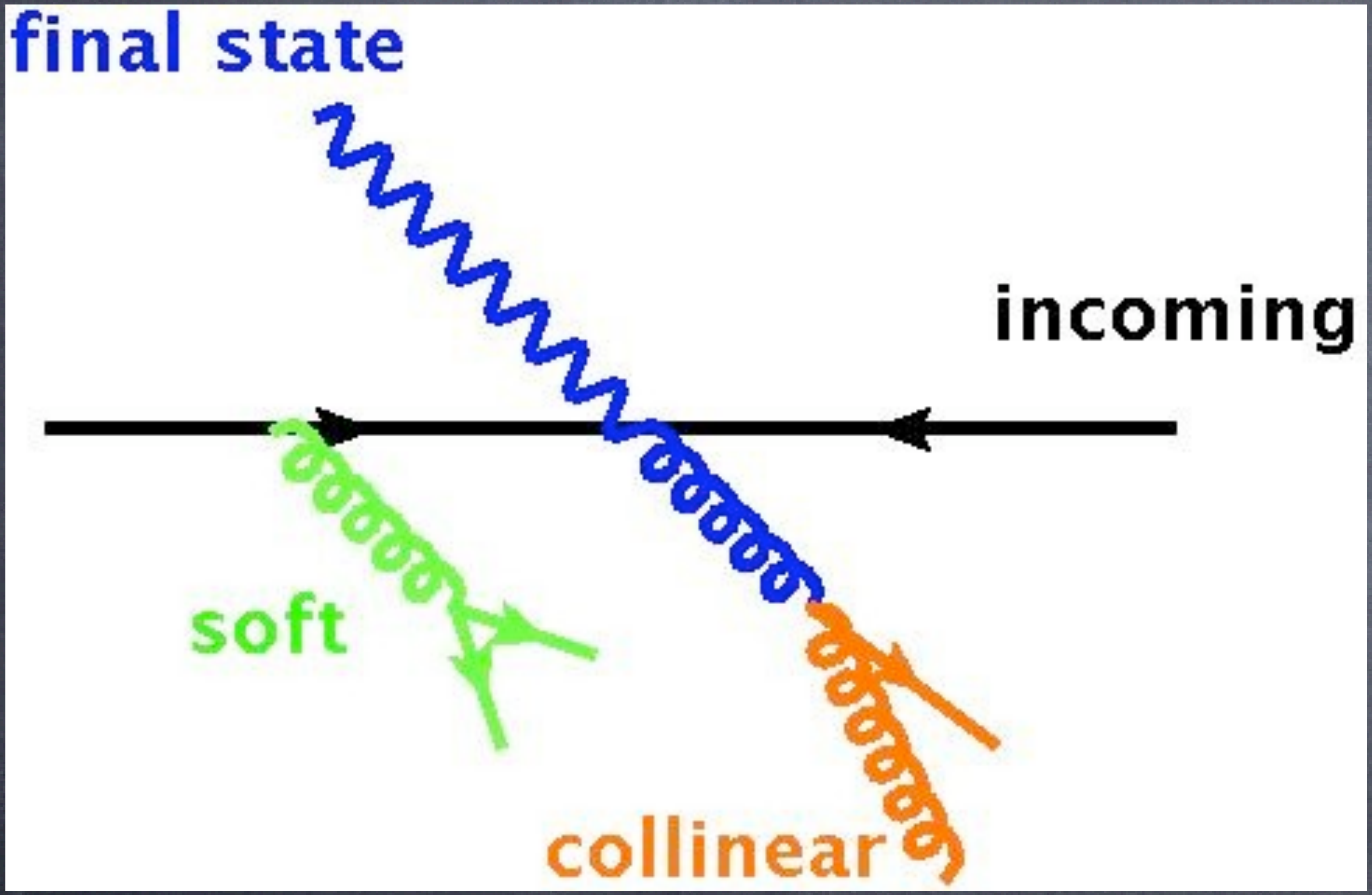
null projection

Prompt Photon



- $pp \to \gamma + X$
  - differential in η and p_T of photon

Prompt Photon



- Expand around $p_T \sim E_{CM}/2$
- Only soft and collinear radiation kinematically allowed
- Implicitly demand jet

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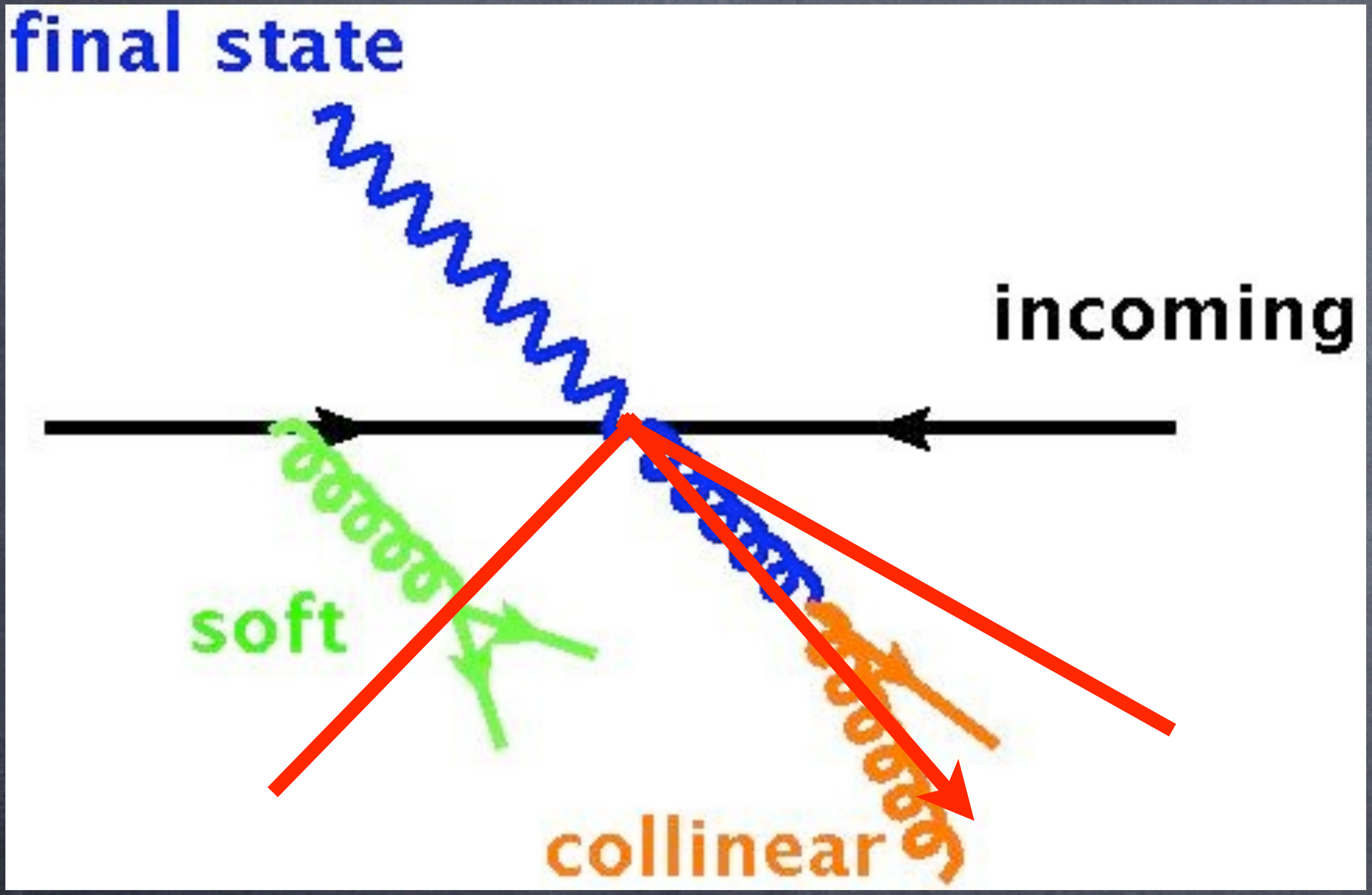

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- All QCD radiation grouped into jet

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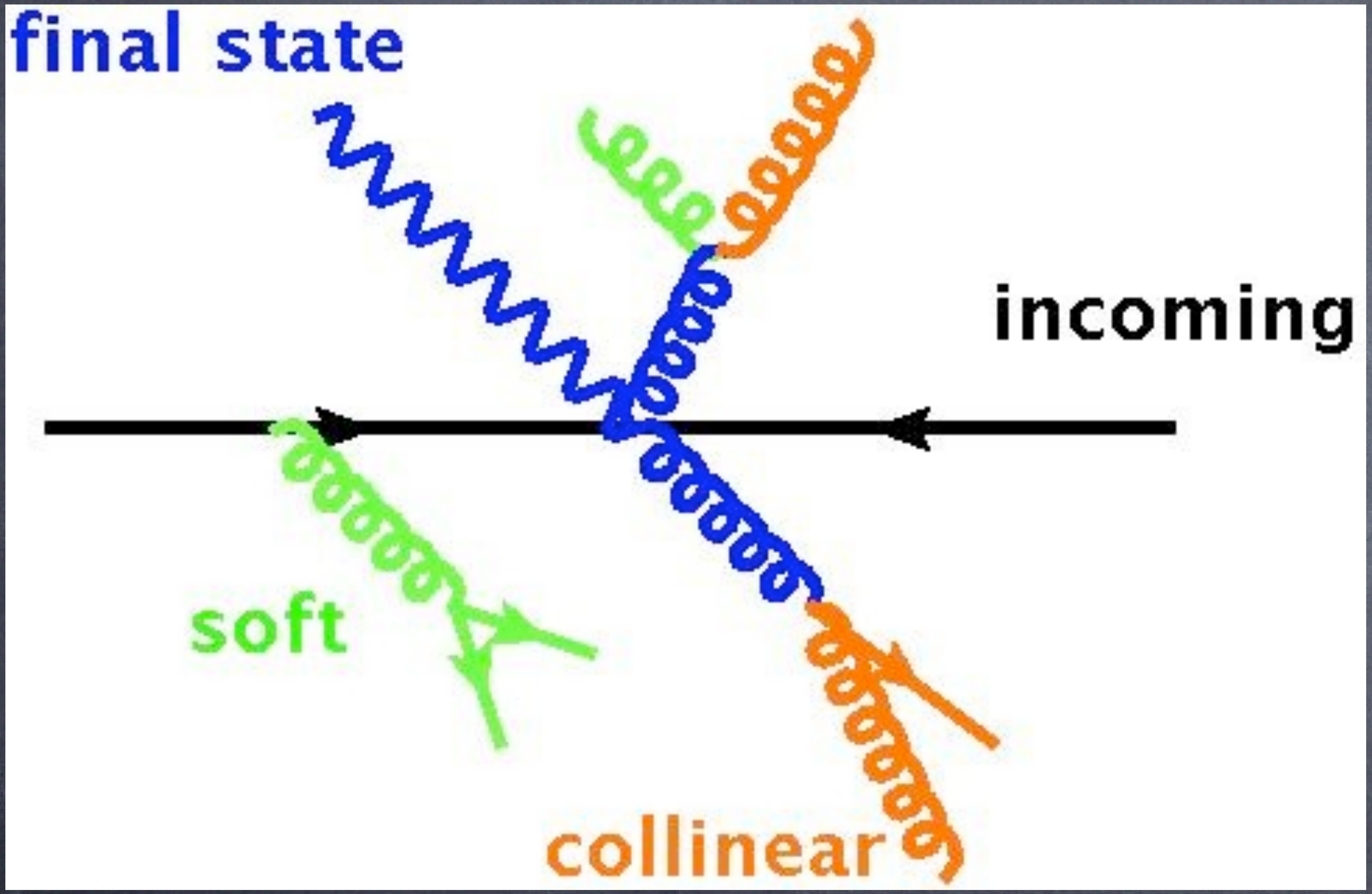

- Cannot group all QCD radiation together
- Cannot implicitly demand multiple jets

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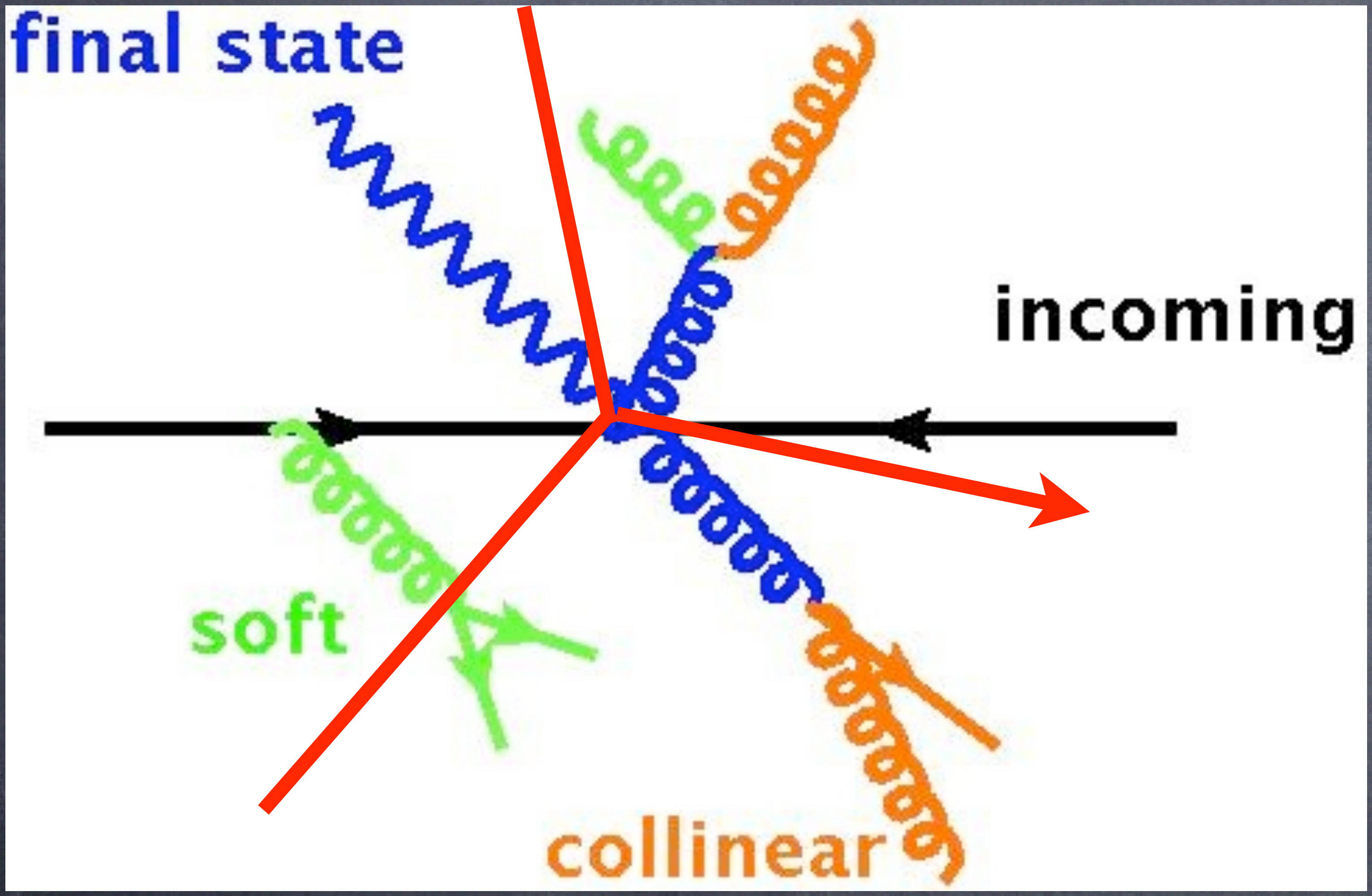

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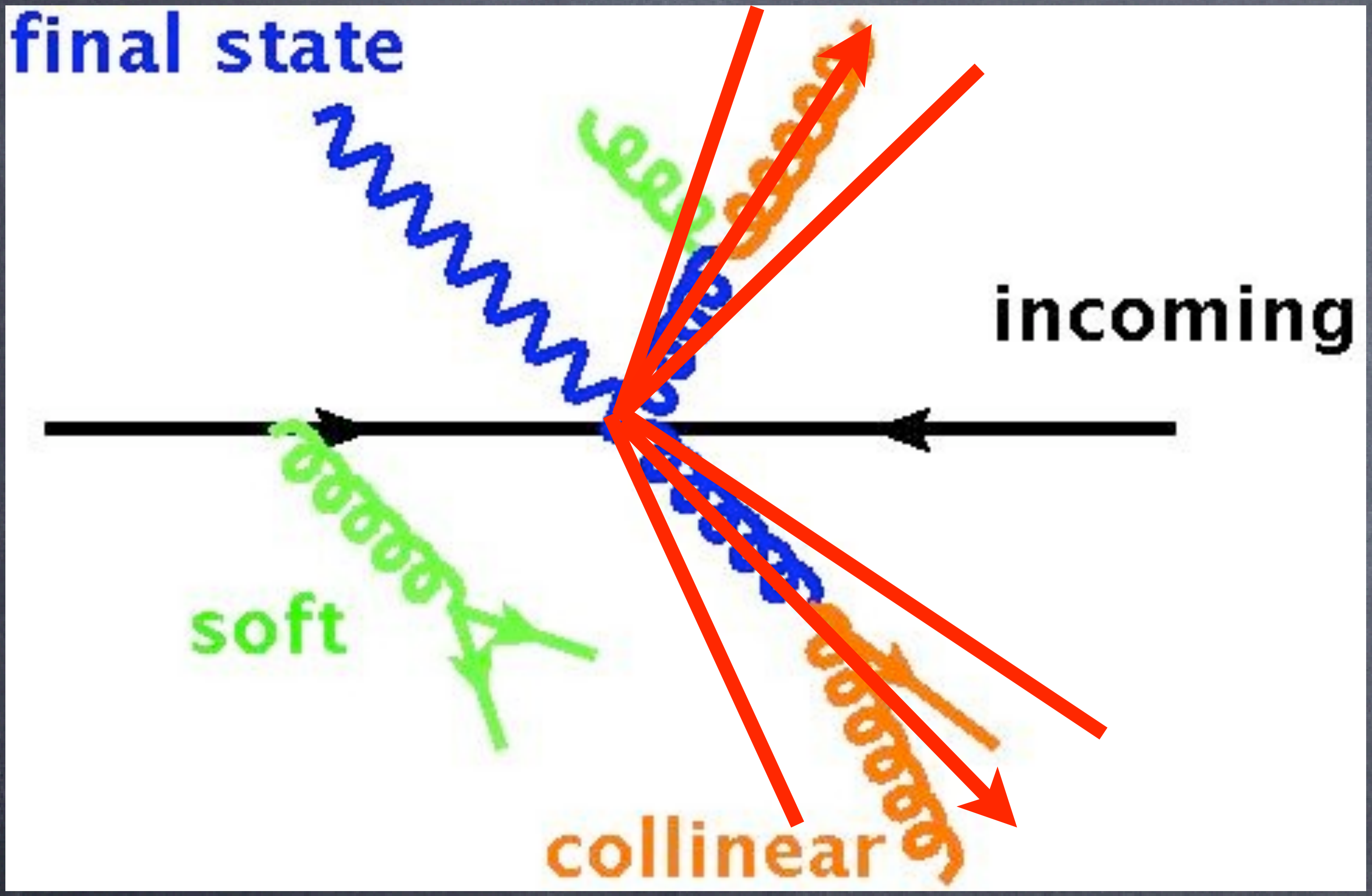
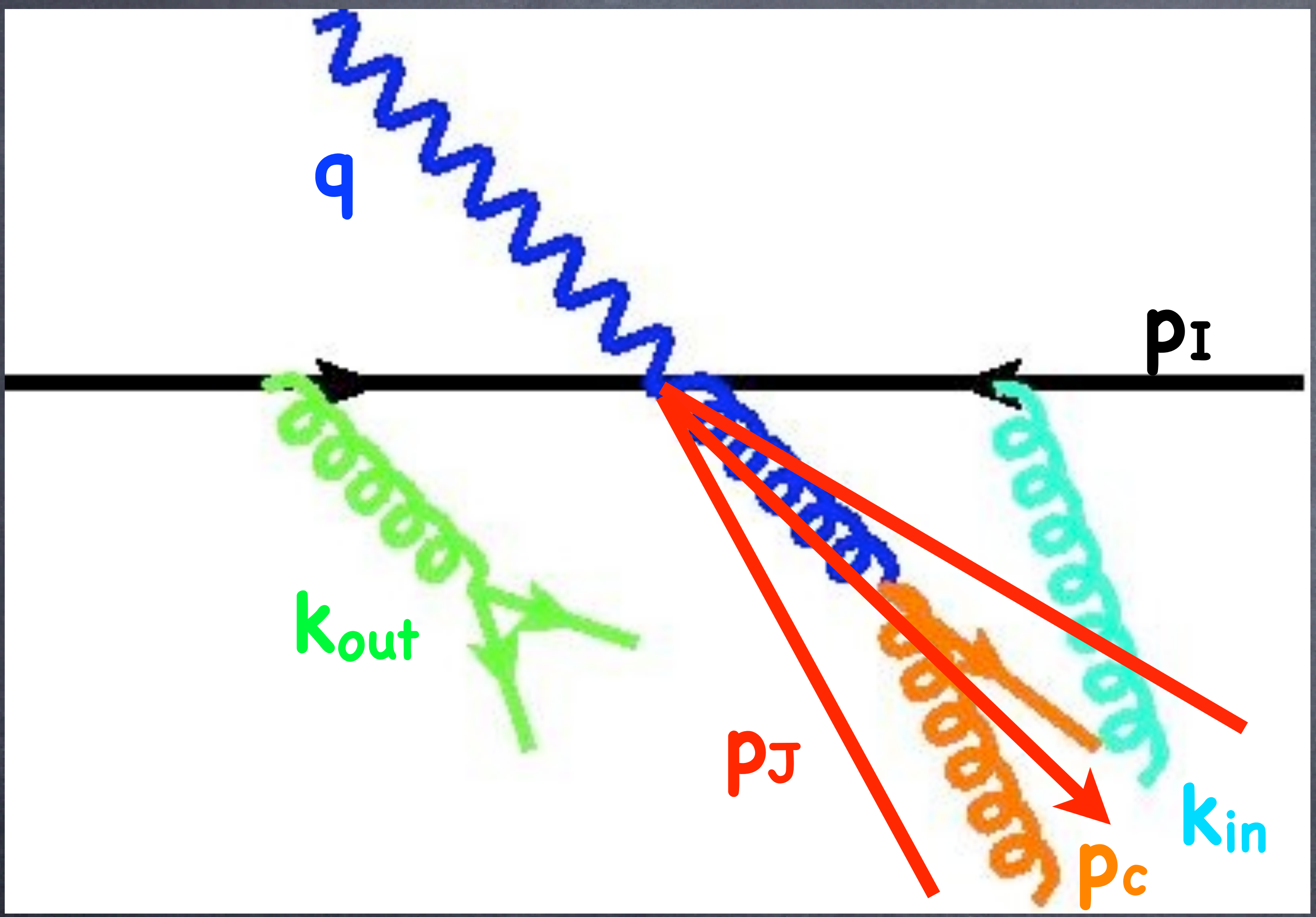


- Cannot group all QCD radiation together
- Cannot implicitly demand multiple jets
- Need jet algorithm

Prompt Photon



- For simplicity, only 1 jet

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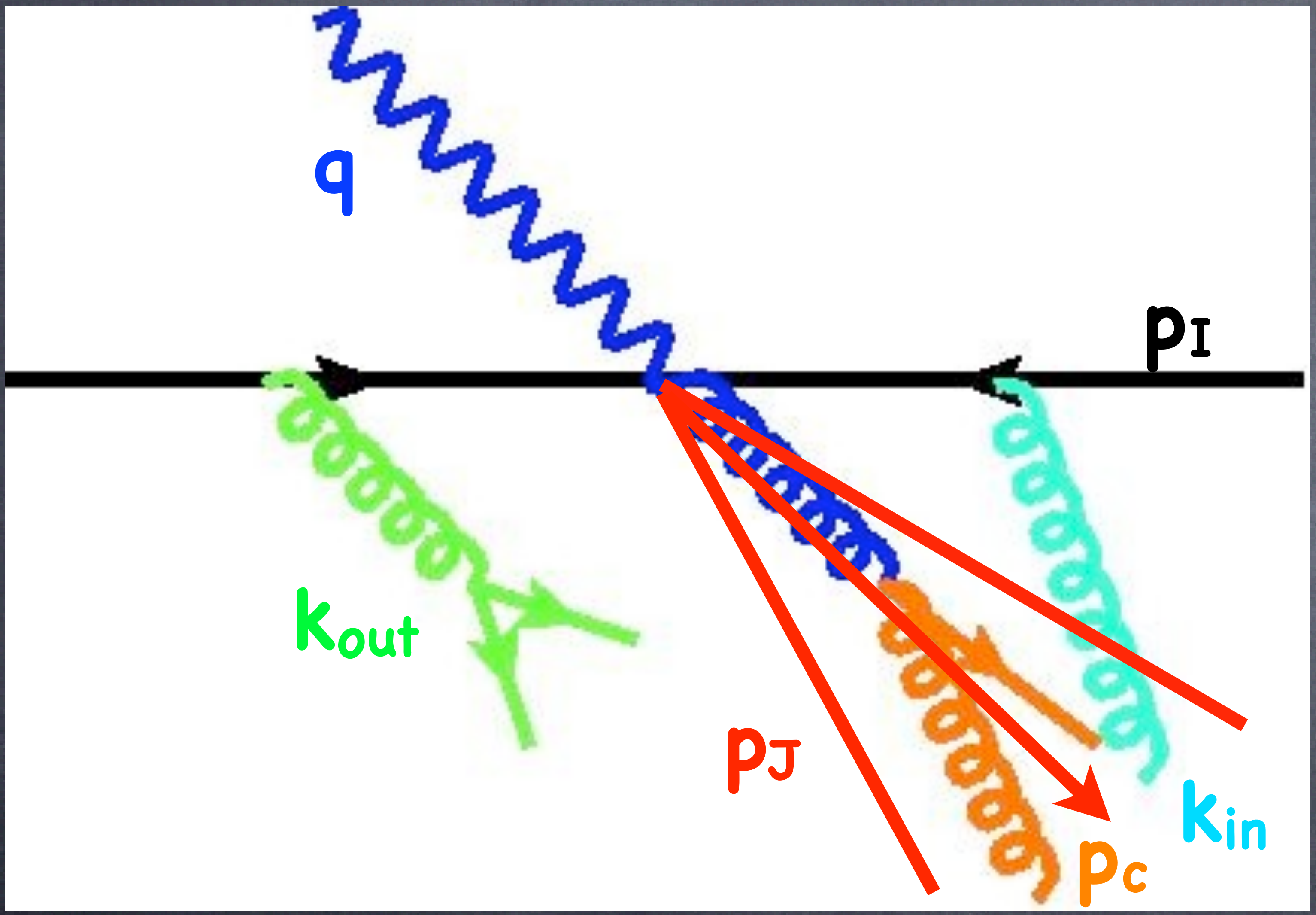

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$$p_c + k_{in} = p_J + (p_c + k_{in})^+ v$$

$$v = (1, \vec{0})$$

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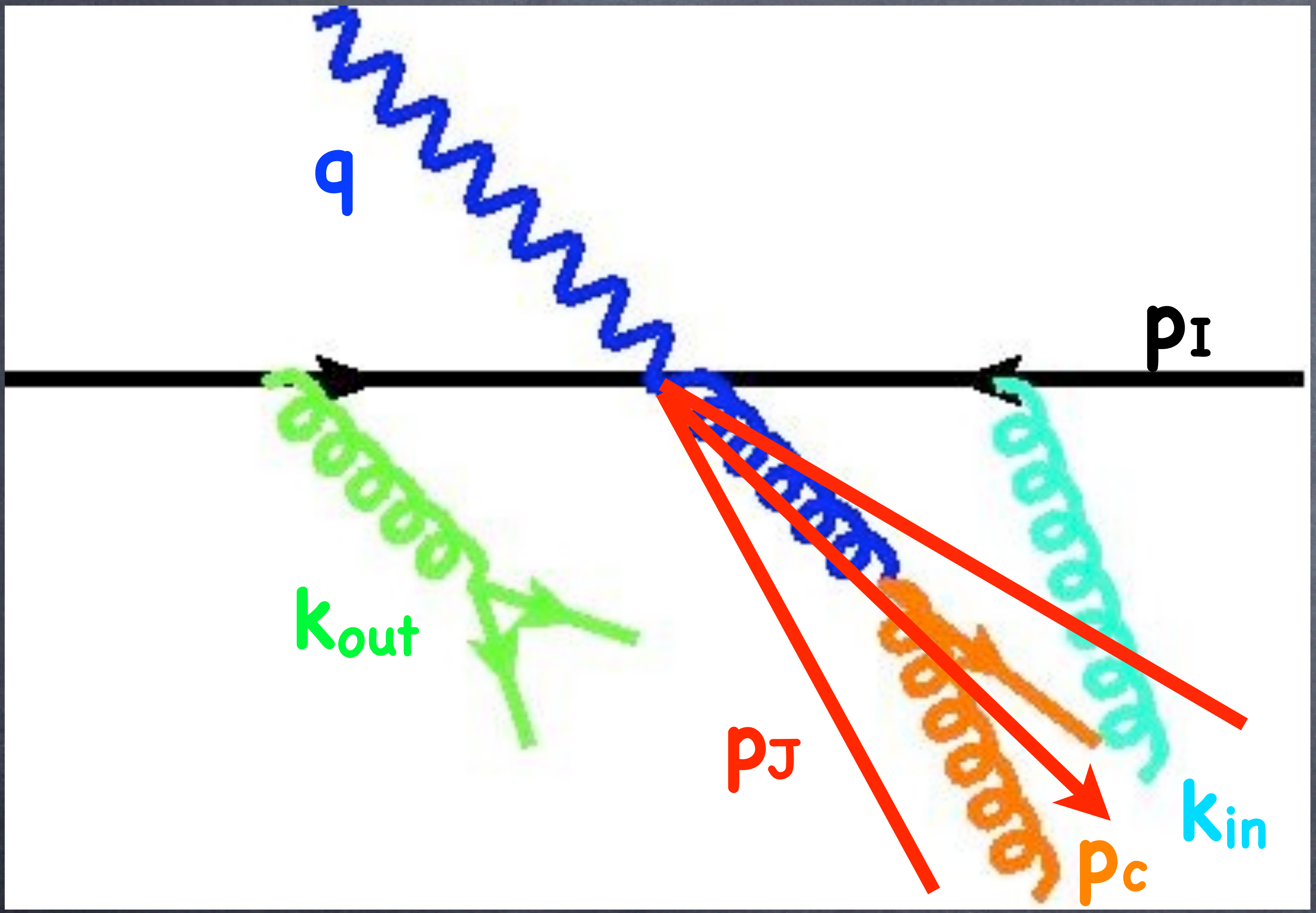

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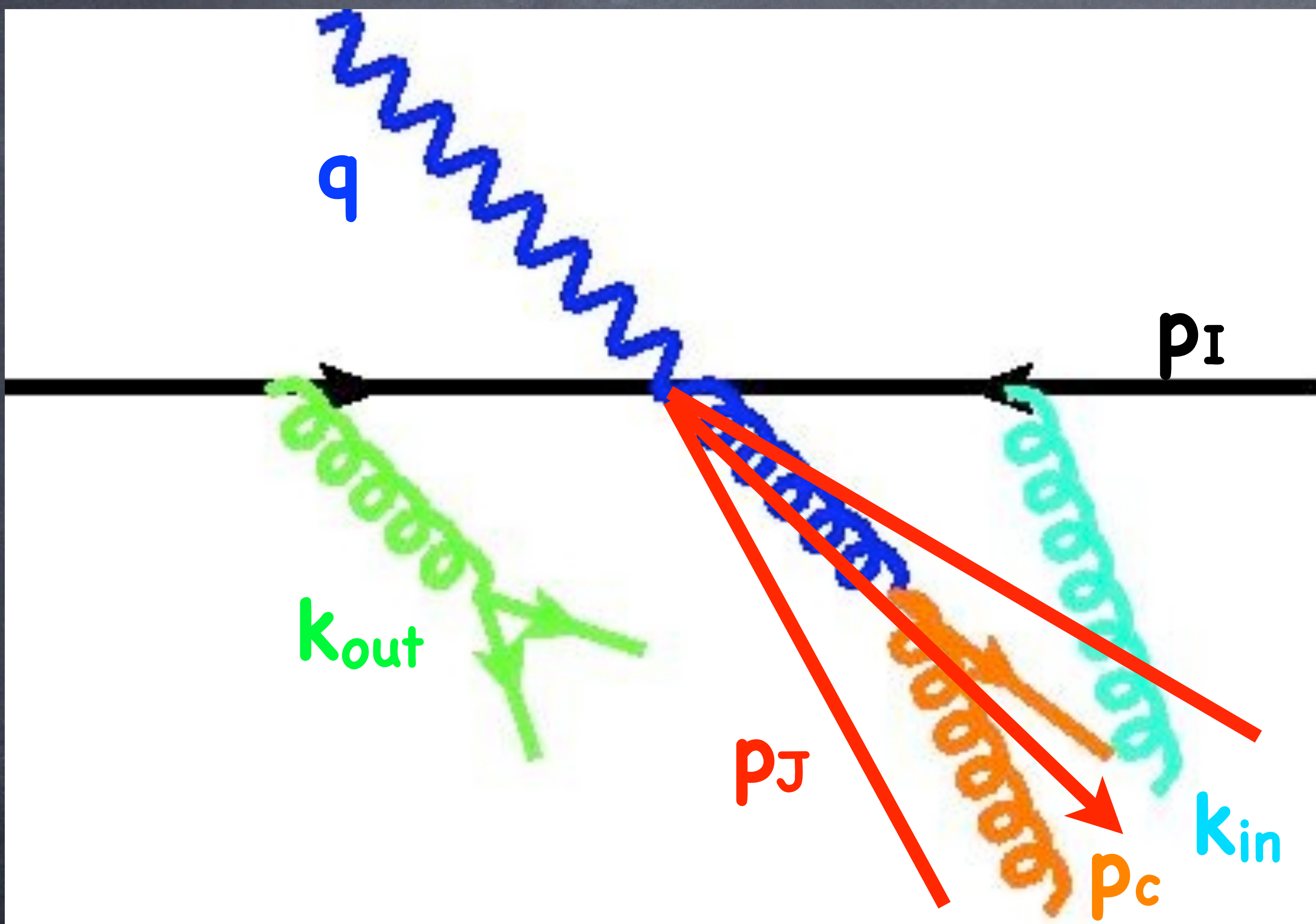

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$$\tau = \frac{\hat{s}_{min}}{E_{cm}^2} = \frac{(q+p_J)^2}{E_{cm}^2}$$

$$z = \frac{\hat{s}_{min}}{\hat{s}} = \frac{(q+p_J)^2}{p_I^2}$$

$$\tau \leq z \leq 1$$

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Calculating 1-z

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time-like projection

Factorization Theorem

$$d\sigma \propto \int dz f_1(x_1) f_2(x_2) H(p_I, q, p_J) J_R(p_c^+) S_R(k^0_{out}, k^+_{in})$$

$$\times \, \delta \left(1 - z - \frac{p_c^+ + k_{in}^+}{\Lambda^+} - \frac{k^0_{out}}{\Lambda_{out}} \right)$$

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$$J_R \sim \langle 0|B\hat{\theta}_R B|0\rangle$$

Soft Function

At one loop, all soft momentum either in or out

$$S_R(k^0_{out}, k^+_{in}) \propto \delta(k^0_{out})\delta(k^+_{in}) + S_{out}(k^0_{out})\delta(k^+_{in}) + S_{in}(k^+_{in})\delta(k^0_{out})$$

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$$S_{out}(k^0_{out}) = -\sum_{\langle i,j \rangle} \mathbf{T}_i \cdot \mathbf{T}_j 2g^2\mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{n_i \cdot n_j}{(n_i \cdot k)(n_j \cdot k)}$$

$$\times 2\pi\delta(k^2)\delta\left(\frac{p_I \cdot k}{|p_I|} - k^0_{out}\right)\left[1 - \sum_{k \in \{jets\}} \hat{\theta}^R_k\right]$$

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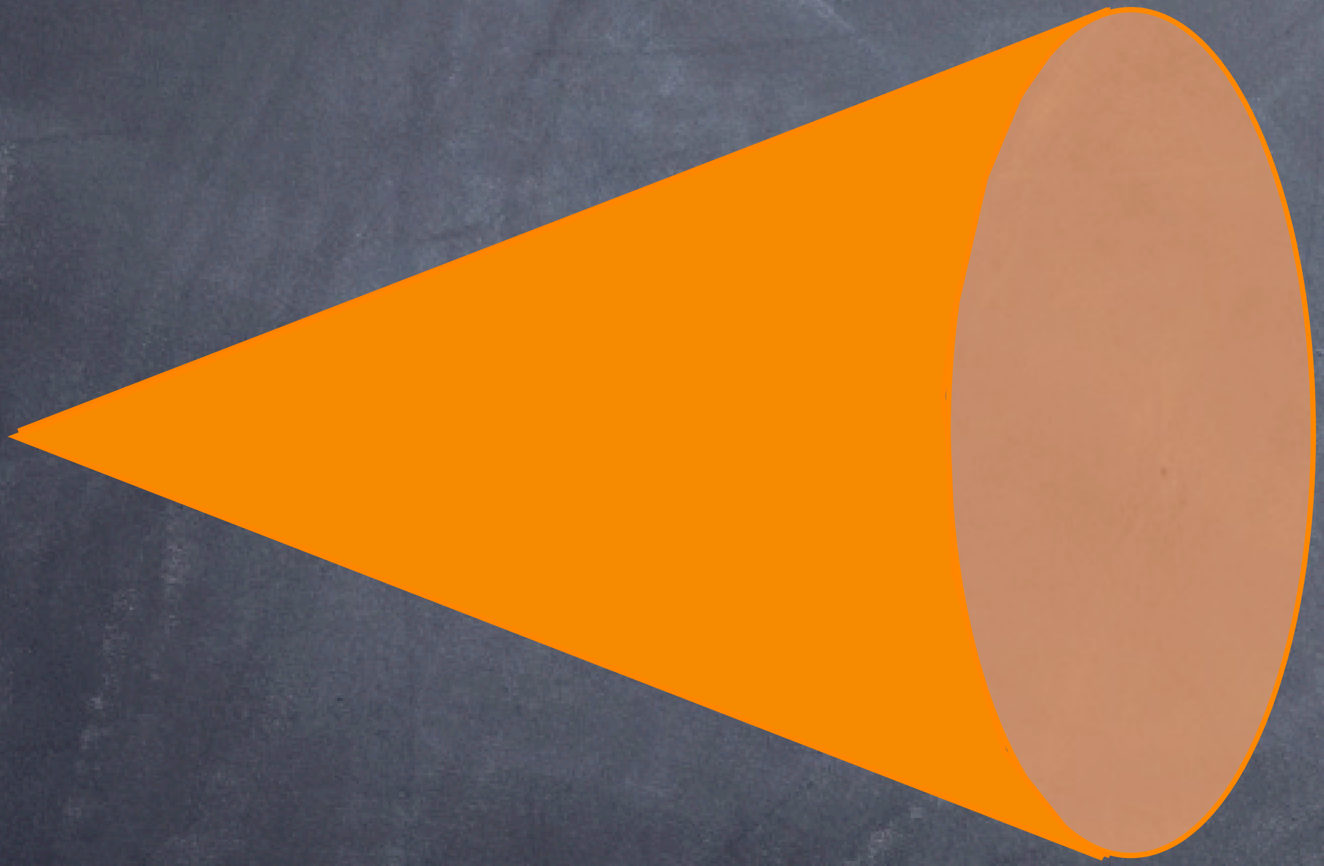
Frame invariant for θ^R invariant

Soft Function



- Calculated using polar angle algorithm
- η-φ algorithm frame invariant
- Same for small R

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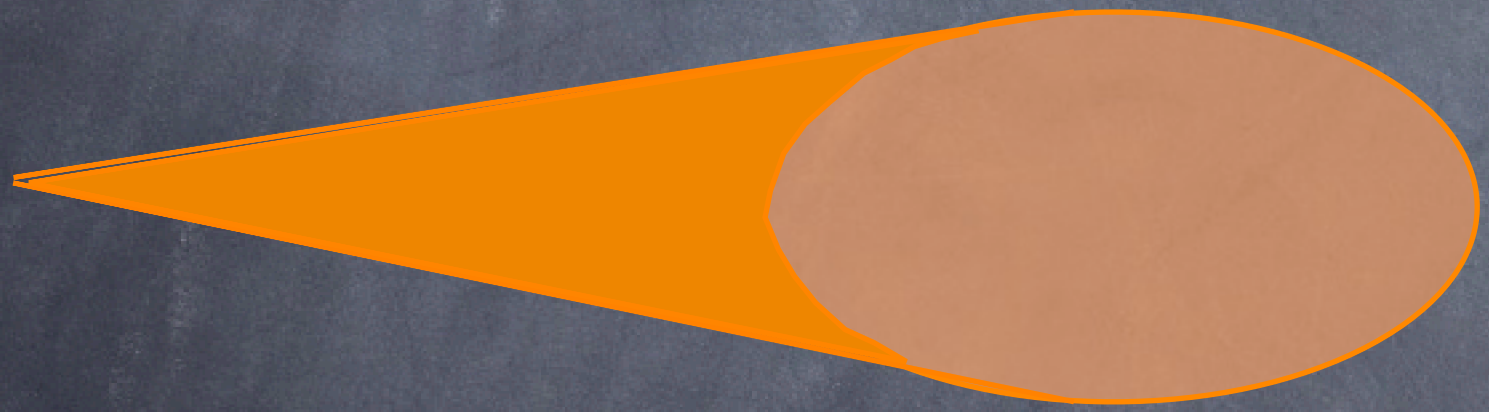

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Anomalous dimension independent of algorithm

Consistency

$$\frac{d\sigma}{d\ln\mu} \sim \gamma_{f_1} + \gamma_{f_2} + \gamma_H + \gamma_J + \gamma_{S_{out}} + \gamma_{S_{out}} = 0$$

- Jet anom. dim. independent of R
- Total soft anom. dim. independent of R
- Endpoint DGLAP running suppresses mixing

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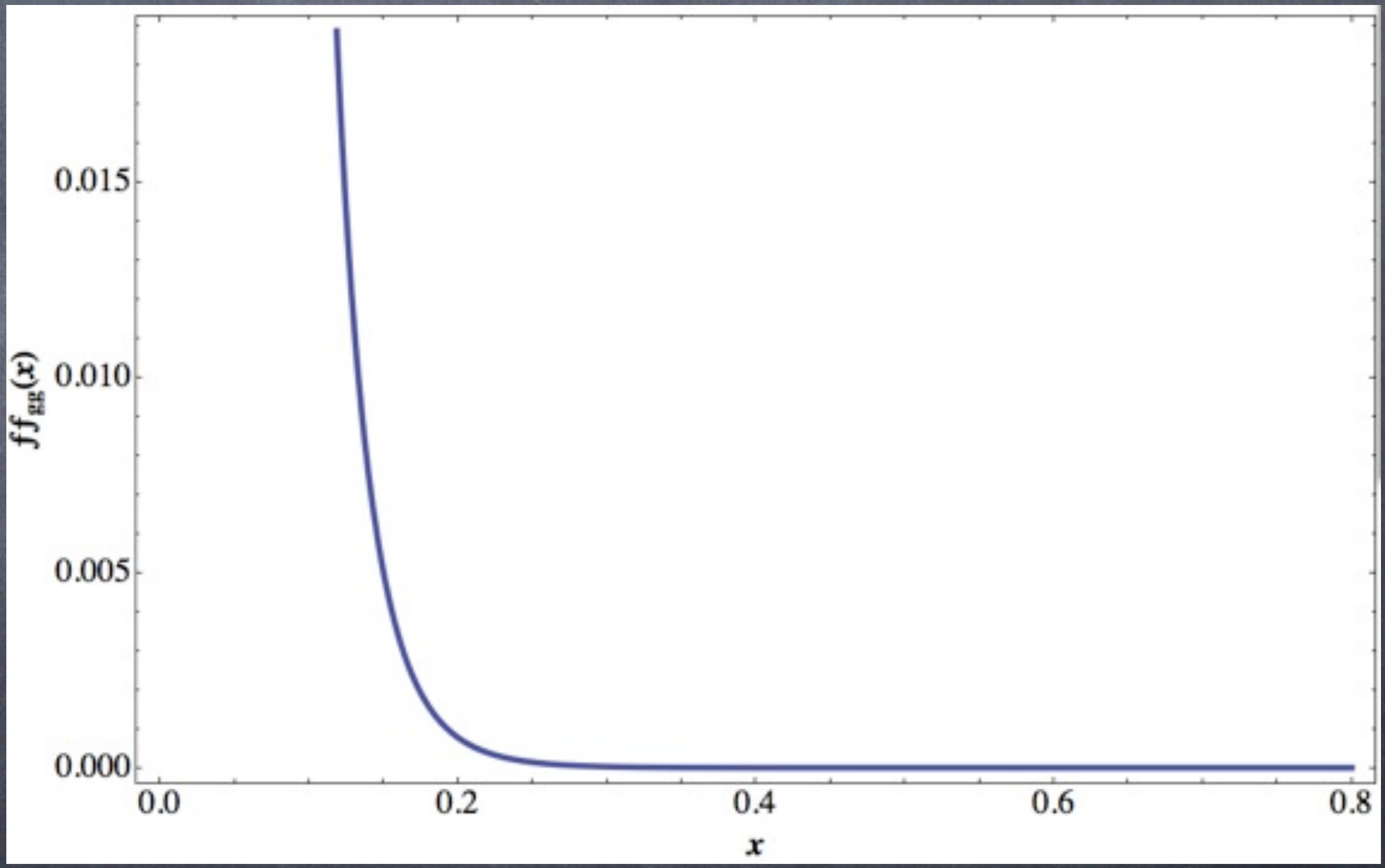
Factorization consistent for 1-z << 1

Moving Away From Threshold

Partonic Threshold

Sterman et al. '88
Catani et al. '98

- Steep parton luminosities dominated by 1-z << 1
- Factorization valid for 1-z << 1
- Reason for 1-z << 1 is irrelevant



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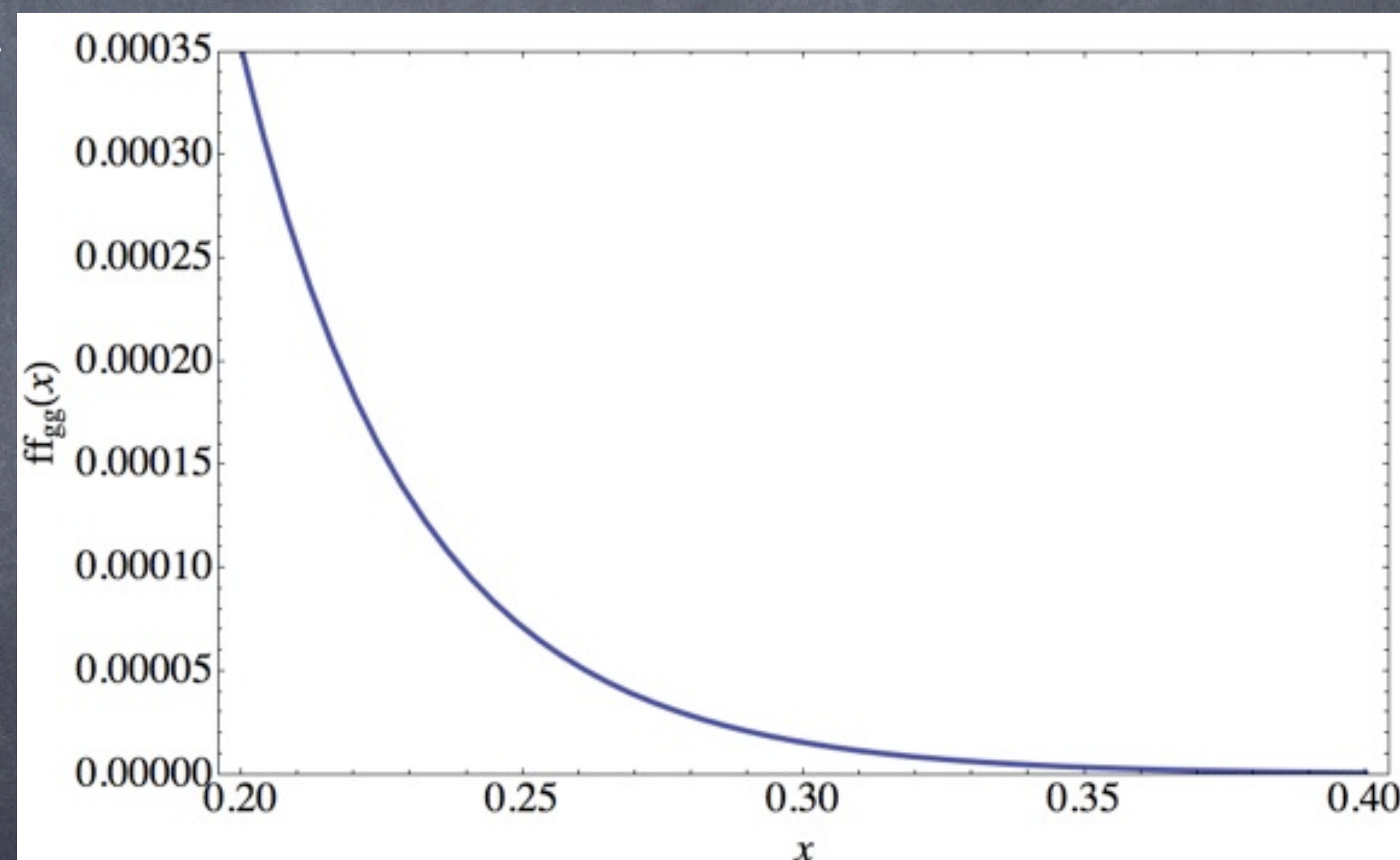

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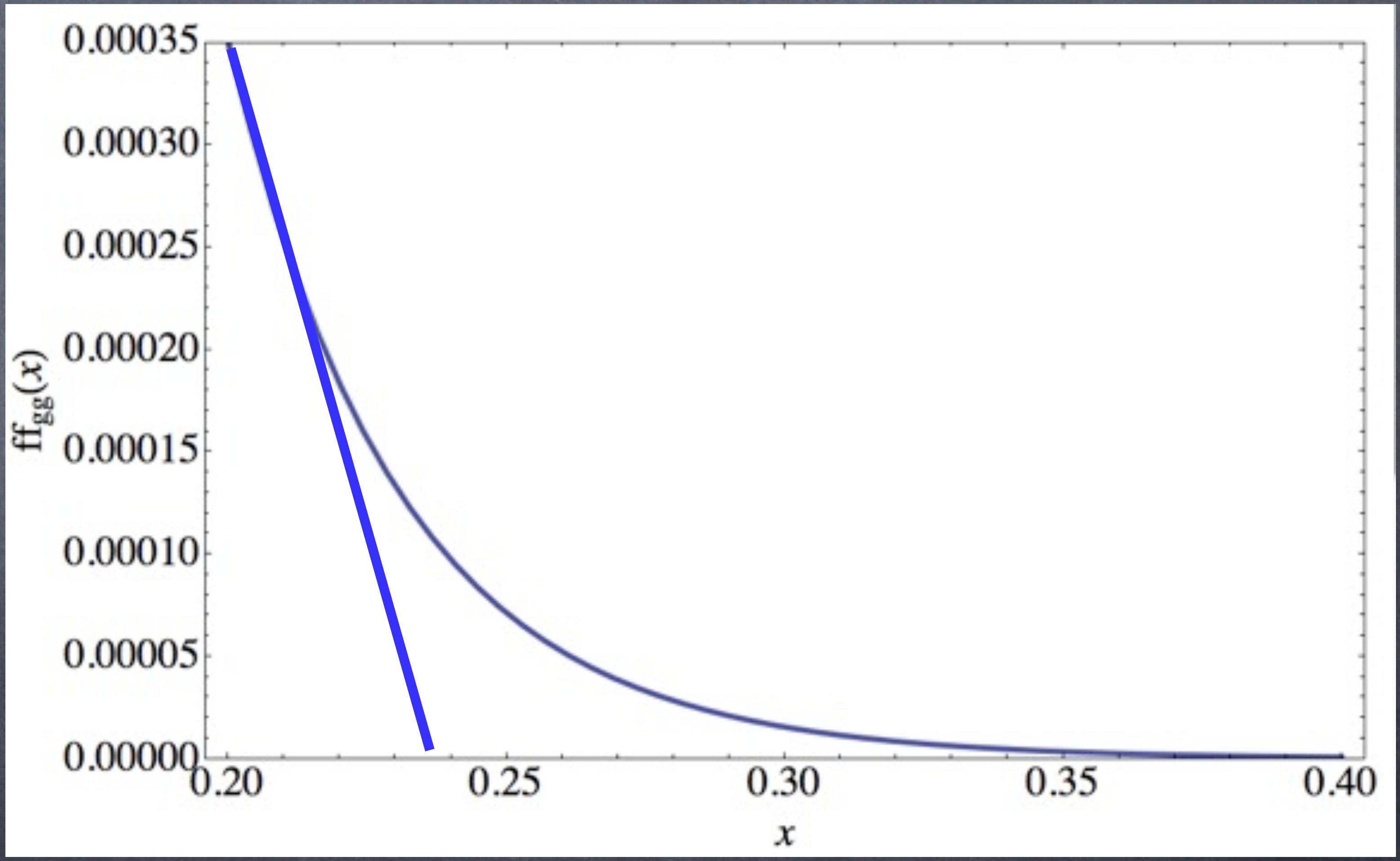

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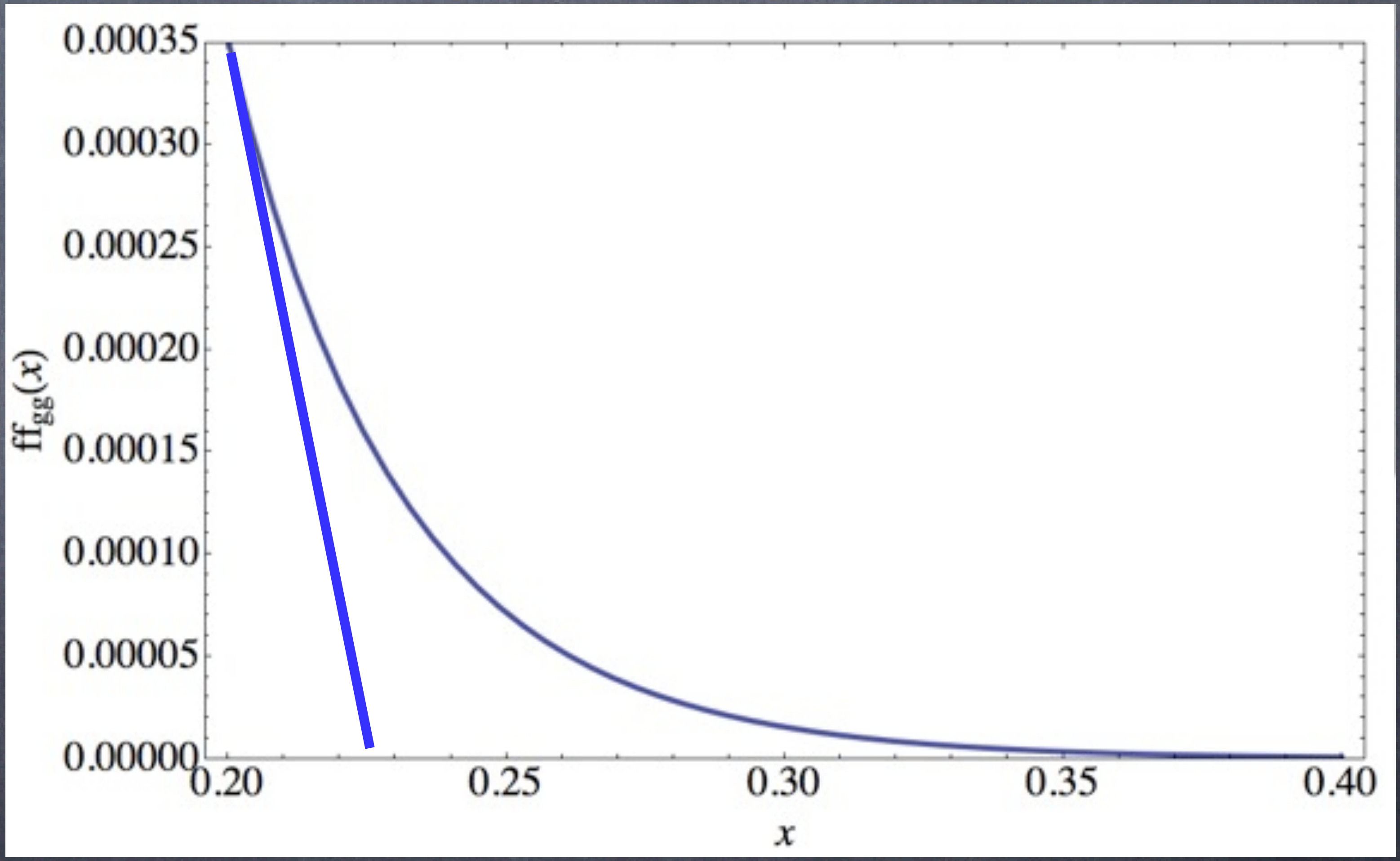

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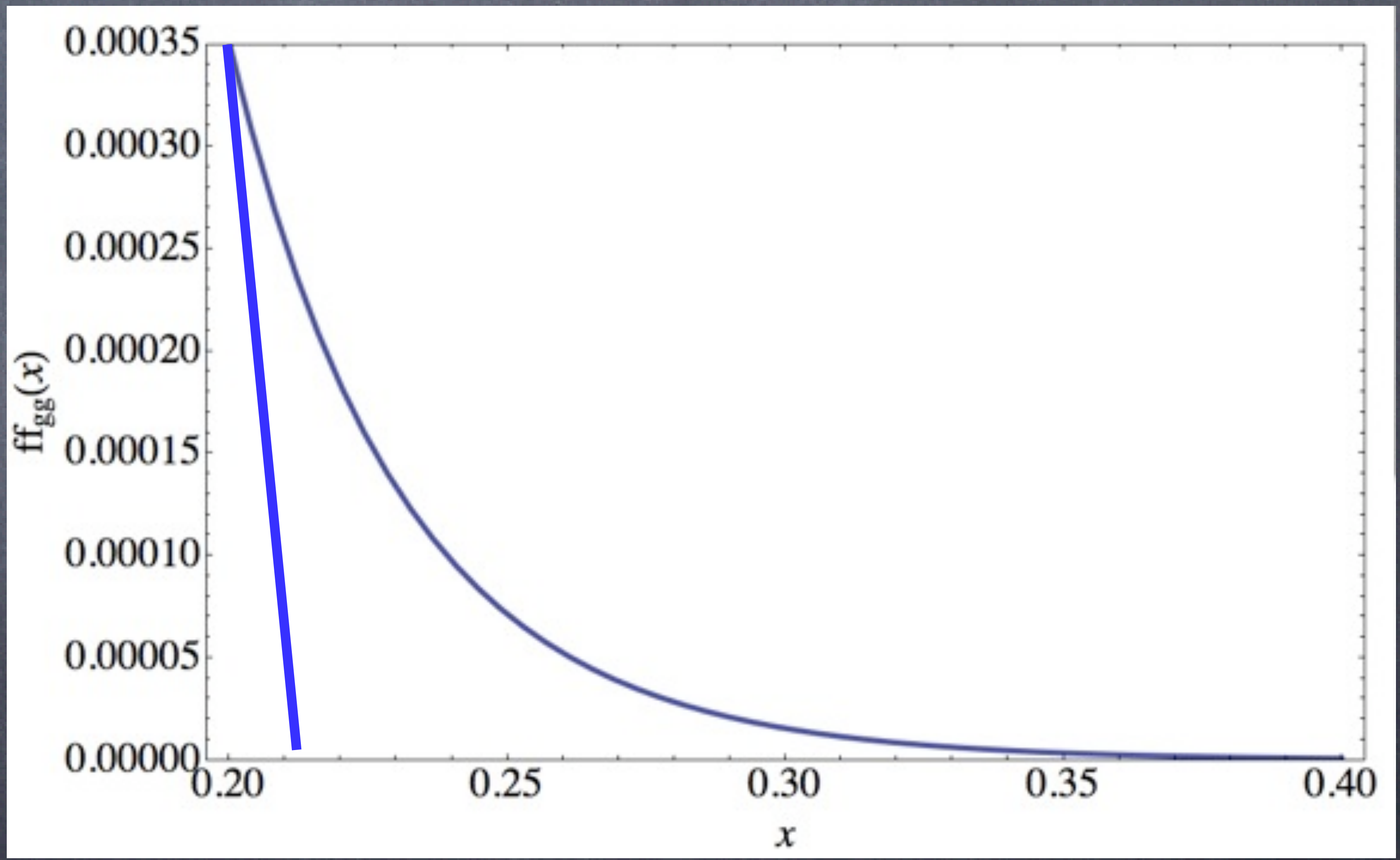
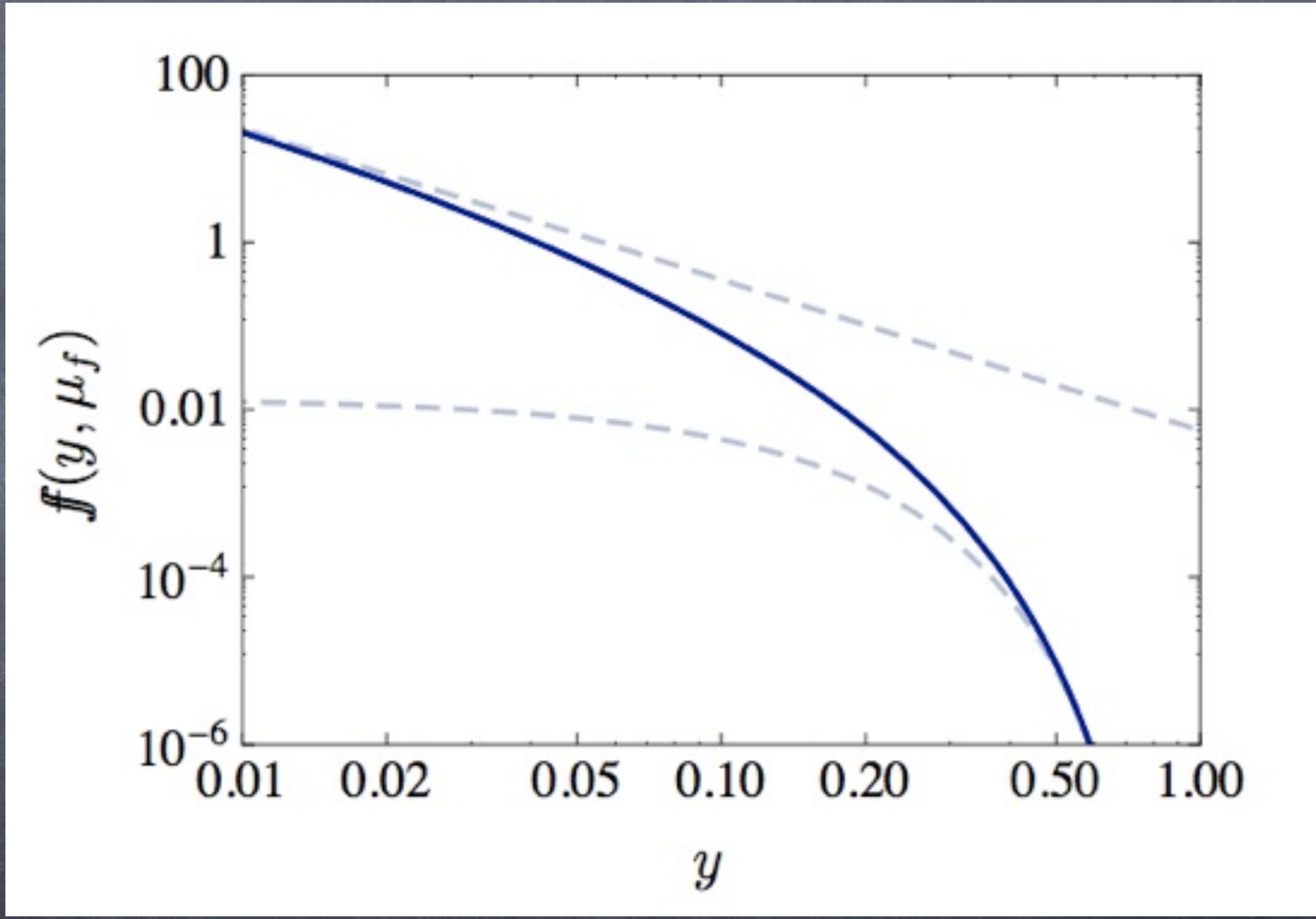


Partonic Threshold

- Becher, Neubert, Xu '08
- ff(x)~x^{-a} for x~0
- ff(x)~$(1-x)^b$ for x~1
- a bounded by integrability

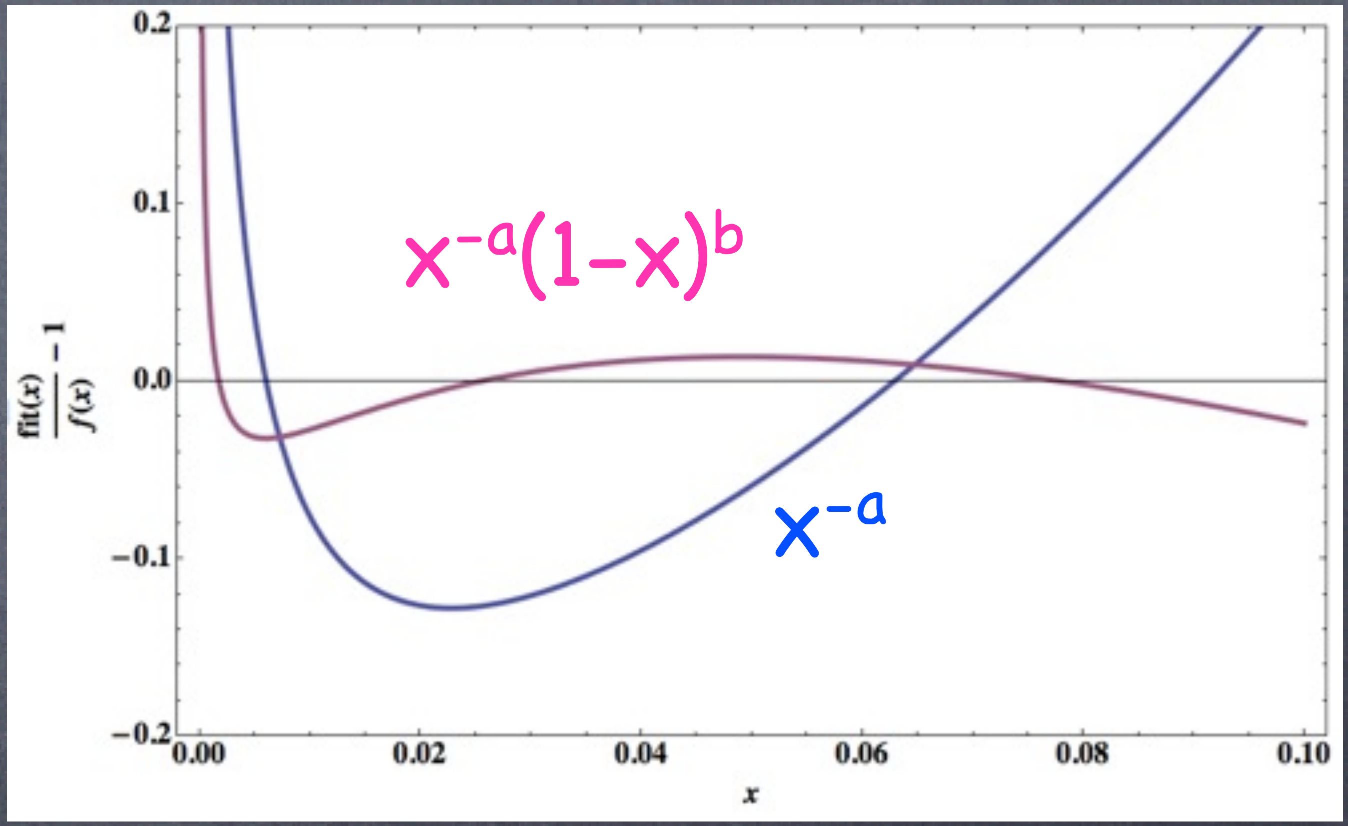


Our Model

- PDFs as convolutions

$$f_i(x) = \int_x^1 dz \Big[\delta(1-z)\delta_{ij} + \frac{\alpha_s}{\pi}\ln\frac{\mu}{\mu_0}P_{ij}(z)\Big] f_j^{mod}\left(\frac{x}{z}\right)$$

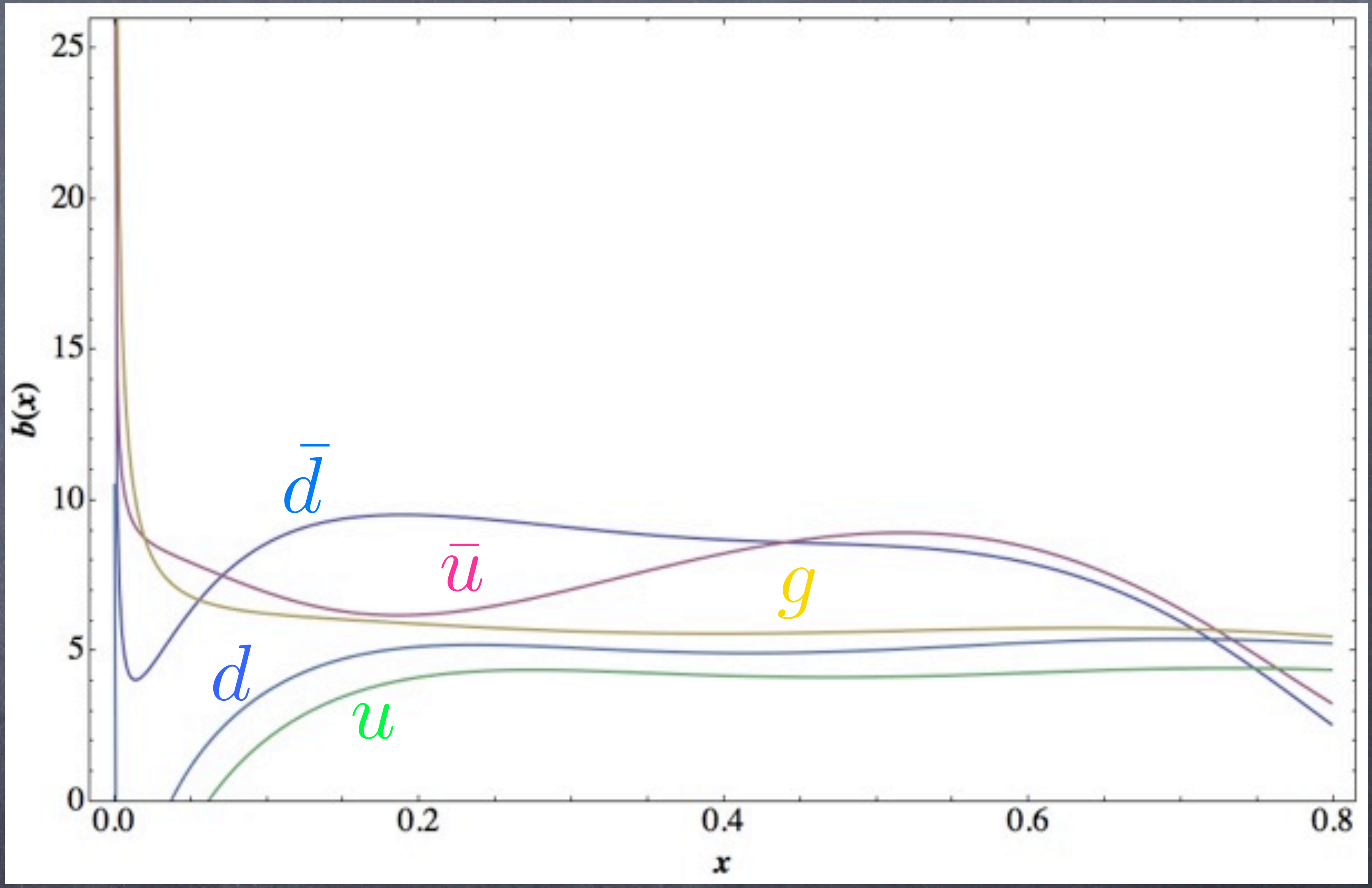
$$f_i^{mod} = x^{-a_i}(1-x)^{b_i}$$



- AP running satisfied
- Expand around b→∞

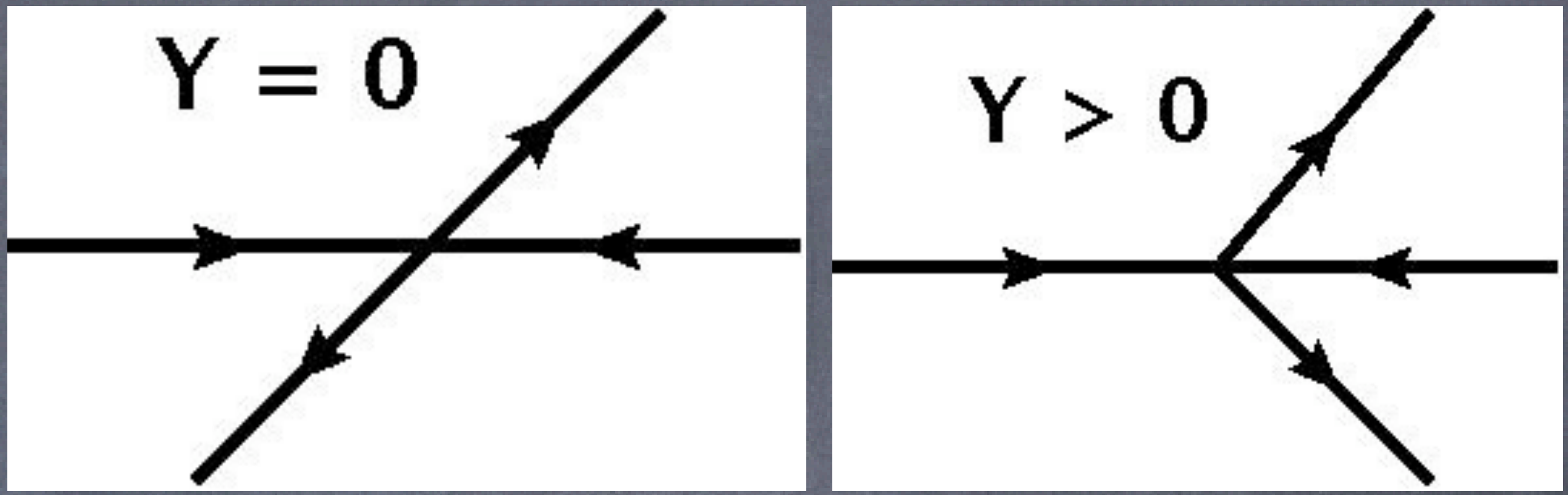
Our Model

- Fit a and b in segments using 2 or 3 parameters
- For 10 TeV LHC, x~0.01 gives 100 GeV final state (b_g~30)



Total Rapidity

$$Y = \frac{1}{2}\ln\frac{x_1}{x_2}$$



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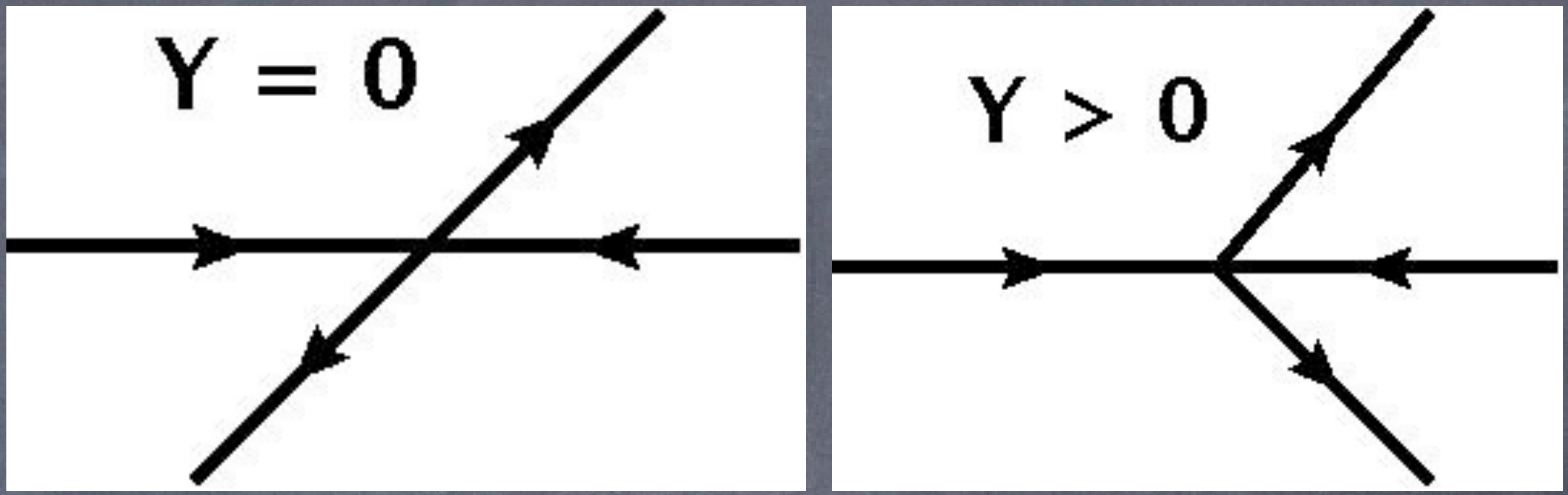

$$f_1^{mod}(x_1) f_2^{mod}(x_2) \sim$$

$$\left(1 - \sqrt{\frac{\tau}{z}} e^{Y}\right)^{b_1} \left(1 - \sqrt{\frac{\tau}{z}} e^{-Y}\right)^{b_2}$$

$$\frac{d}{dY} f_1^{mod}(x_1) f_2^{mod}(x_2) = 0$$

$$\Rightarrow Y = Y_{dom}$$

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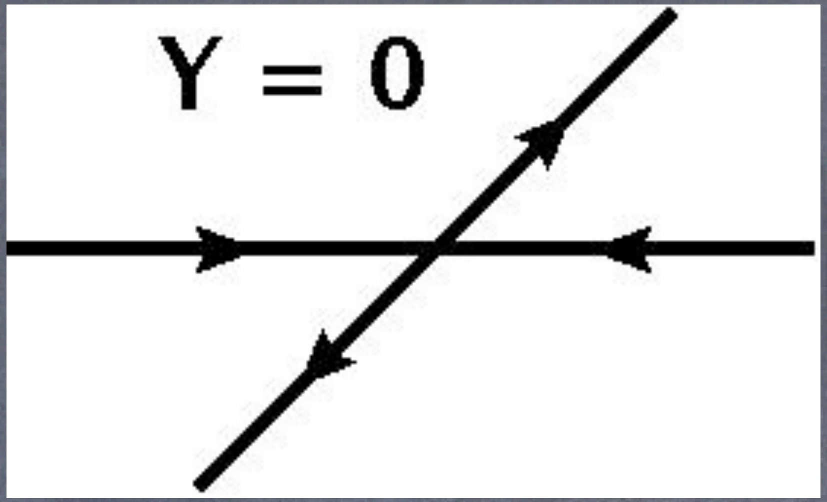

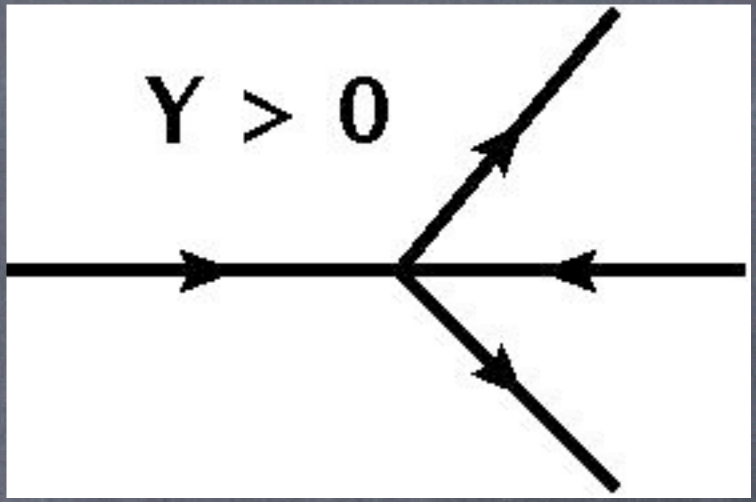

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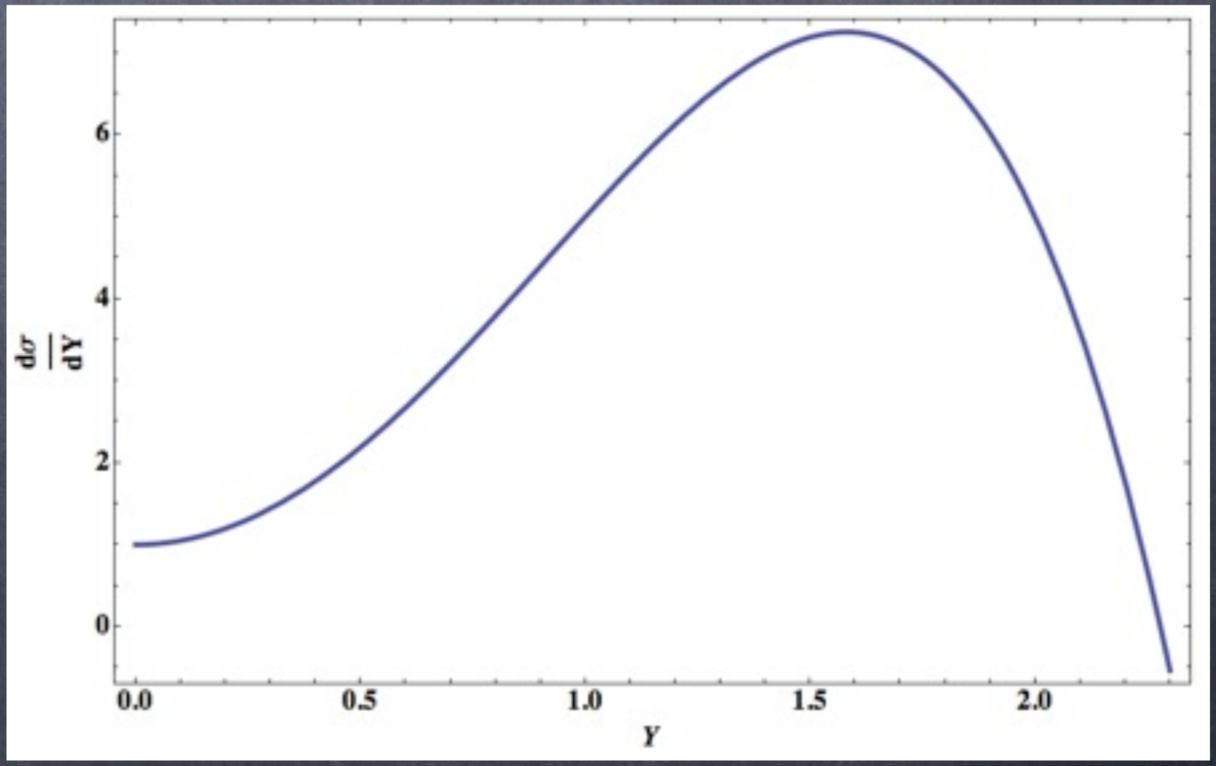

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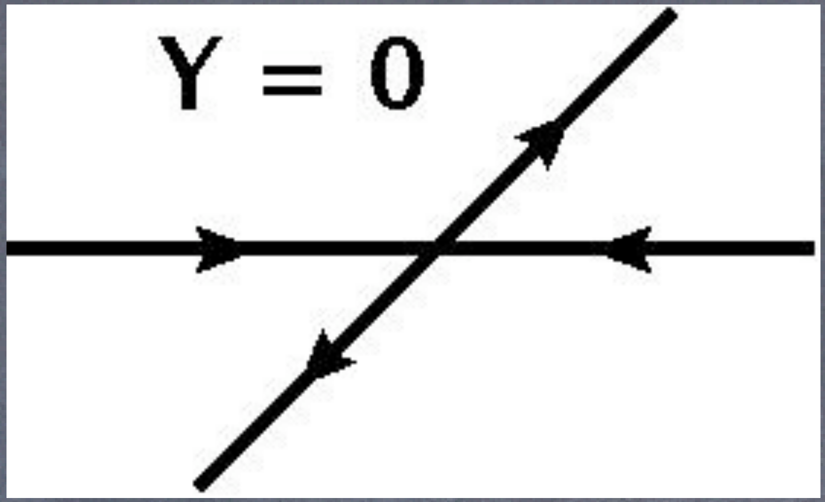

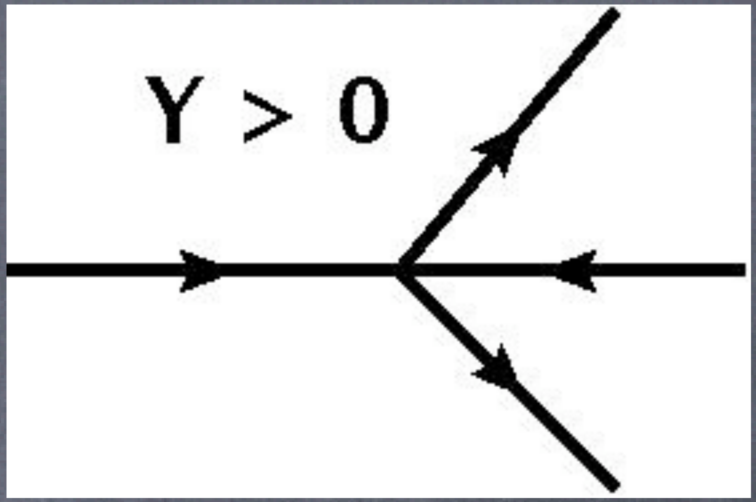

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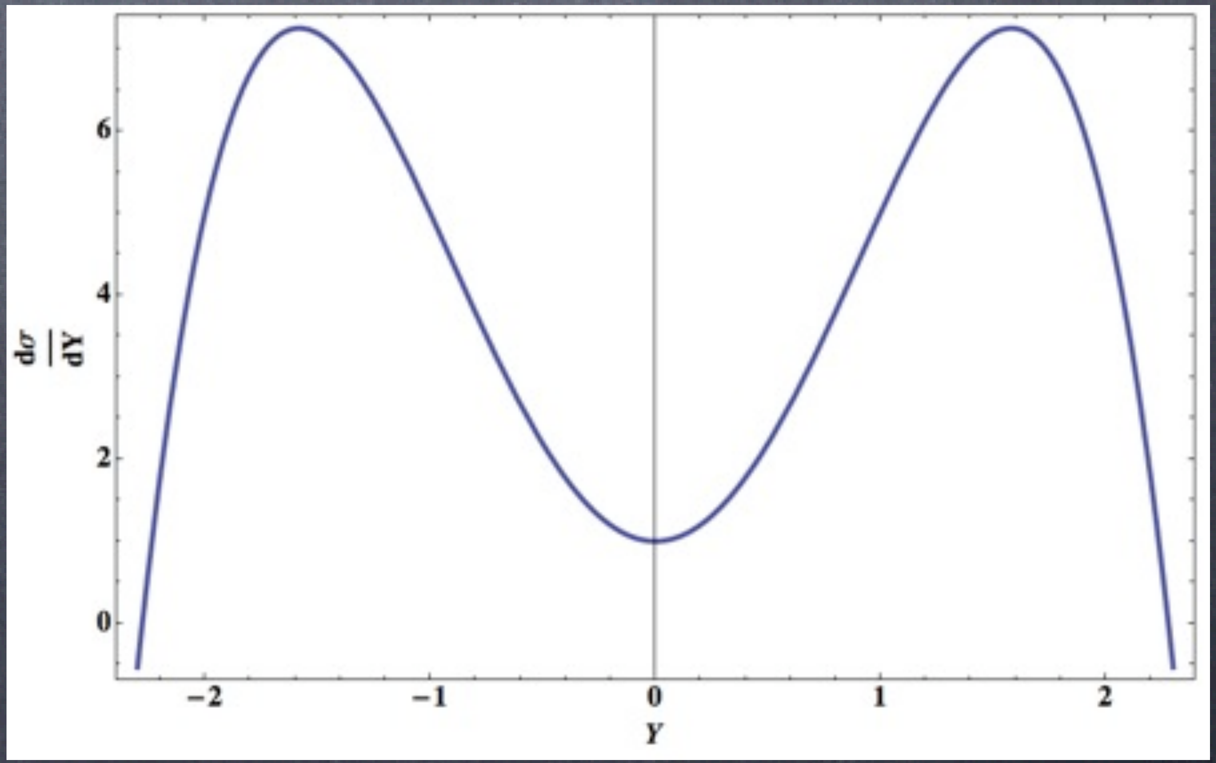

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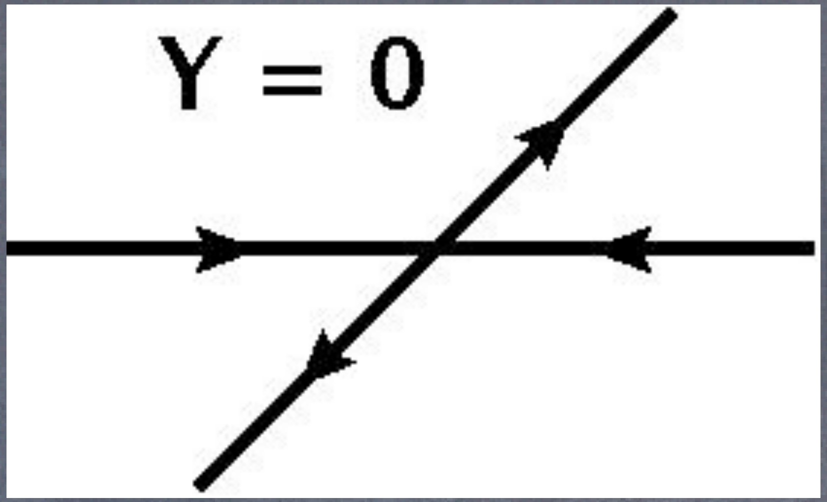

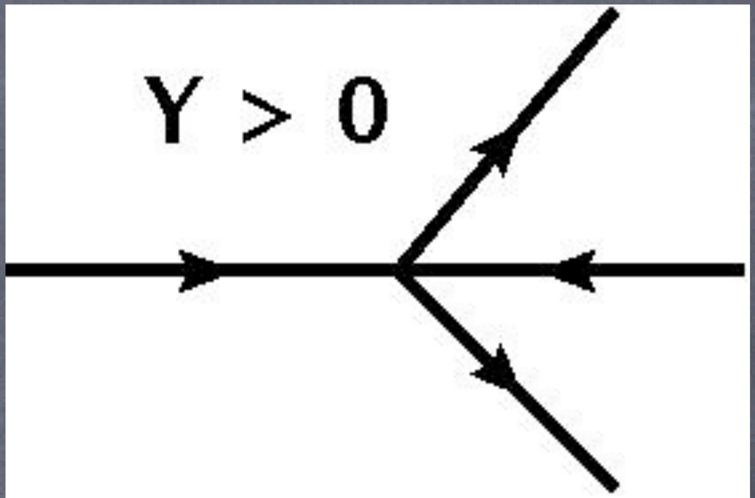

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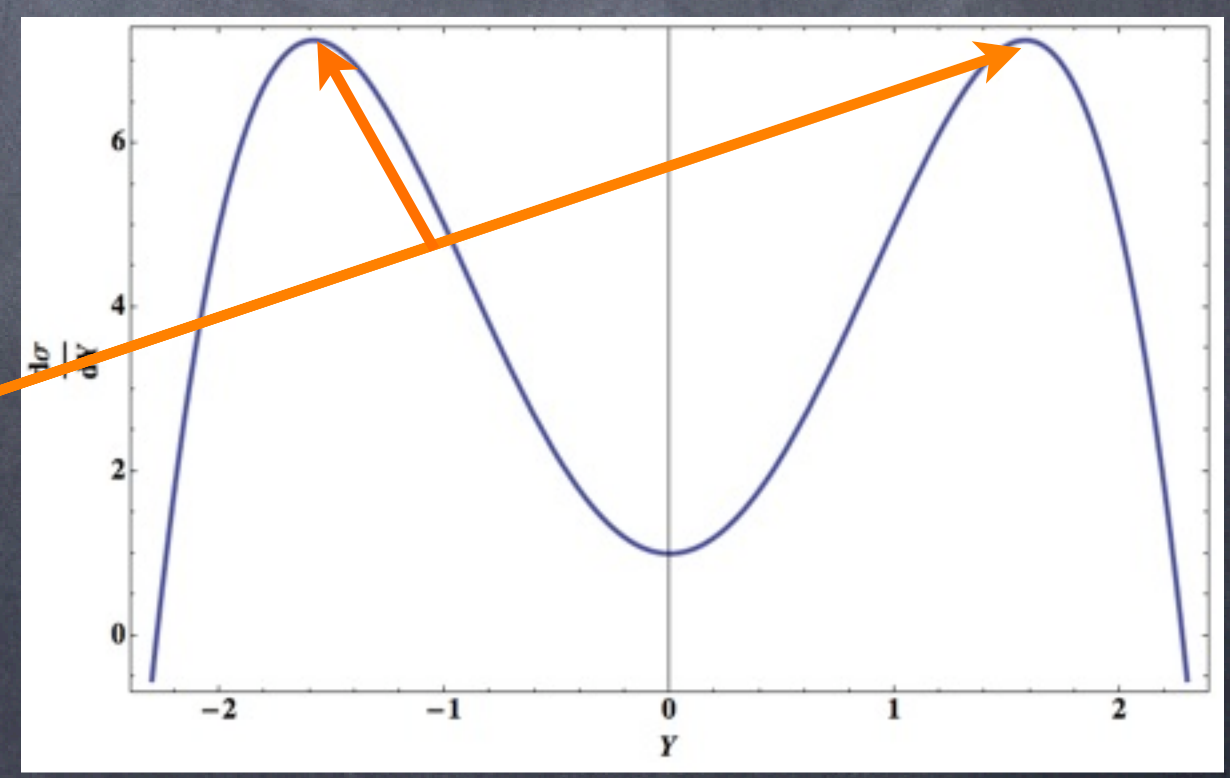

Consistency

- Steepness gives endpoint AP equations
- Jet and soft anomalous dimensions convolved with model functions
- Factorization consistent for $Y = Y_{dom}$
- Power corrections ~ 1/b at worst

Factorization consistent away from threshold!

Future Work

- Understand power corrections (PDFs vs parton luminosities)
- Differences between hadronic and partonic endpoint
- Compare Y_{dom} predictions to data
- Include jet observables

Conclusions

- Factorization of generic N-jet cross section at hadronic endpoint
- Generic soft function includes time-like and null dependence
- Steep luminosities allow for factorization away from endpoint
- Partonic threshold factorization consistent to leading order in steepness