Precision calculations for top-quark pair production at hadron colliders

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Outline

- motivation
- higher-order QCD corrections to differential cross section in partonic threshold region
- applications and numerical results

Tevatron has produced thousands of $t\bar{t}$ pairs, LHC will produce millions

Observables:

- total cross section (m_t, etc)
- $t\bar{t}$ invariant mass distribution (m_t, new physics through resonances)
- FB asymmetry (new physics, tension in CDF measurement)

can calculate all three from $d\sigma/dM_{t\bar{t}}d\cos\theta$

FACTORIZATION FOR $pp(p\bar{p}) \rightarrow t\bar{t} + X$

Factorization:

$$\frac{d\sigma}{dM_{t\bar{t}}\,d\cos\theta} = \sum_{i,j=q,\bar{q},g} ff_{ij} \otimes \frac{d\hat{\sigma}_{ij}}{dM_{t\bar{t}}\,d\cos\theta}$$

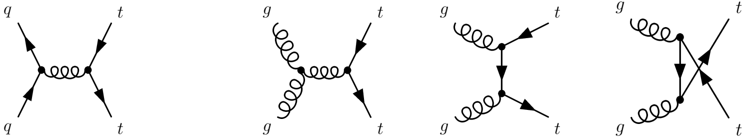
Parton luminosity:

$$ff_{ij}(y,\mu_f) = \int_y^1 \frac{dx}{x}\, f_{i/N_1}(x,\mu_f)\, f_{j/N_2}(y/x,\mu_f)$$

- extract PDFs from experiment
- calculate $d\hat{\sigma}_{ij}$ in perturbative QCD

FEYNMAN DIAGRAMS FOR $d\hat{\sigma}_{ij}$

Born level:



- $q\bar{q}$ dominant at Tevatron ($\sim$ 90% of cross section)
- gg dominant at LHC ($\sim$ 75% of cross section at 7 TeV)

Higher-order corrections:

- virtual corrections and real emission
- $(qg, \bar{q}g) \to t\bar{t}X$ (numerically small)

Status of QCD calculations

NLO completely known

- total cross section (Nason, Dawson, Ellis '88, Czakon, Mitov '08)
- differential distributions with Monte Carlo programs (e.g. MCFM)

pert. errors are larger than PDF errors with NLO calculations

⇒ motivates higher-order calculations

NNLO very difficult, many partial results (long list of authors)

today will focus on higher-orders in partonic threshold region

Ahrens, Ferroglia, Neubert, BP, Yang ('09, '10)

The partonic threshold region

Kinematic variables: ($\hat{s} = x_1 x_2 s$)

$$M^2 = (p_t + p_{\bar{t}})^2, \qquad \tau = \frac{M^2}{s}, \qquad z = \frac{M^2}{\hat{s}}, \qquad \beta_t = \sqrt{1 - \frac{4m_t^2}{M^2}}$$

Threshold limits:

- pair invariant mass threshold: $z \to 1$, $\beta_t = \mathcal{O}(1)$
  (final state X soft, but top quarks not produced at rest)
- production threshold: $\beta_t \to 0$
  (final state X soft, top quarks produced at rest)

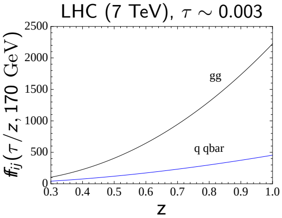
Calculations in threshold limits much simpler, since real emission is soft
(soft gluons couple through Wilson lines, phase-space basically two body)

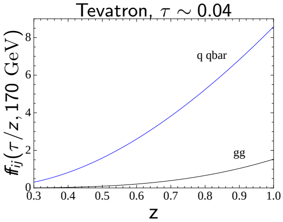
Relevance of partonic threshold region

$$\frac{d^2\sigma}{dMd\cos\theta} = \frac{8\pi\beta_t}{3sM}\sum_{i,j}\int_\tau^1 \frac{dz}{z}\, ff_{ij}(\tau/z,\mu_f)\, C_{ij}(z,M,m_t,\cos\theta,\mu_f)$$

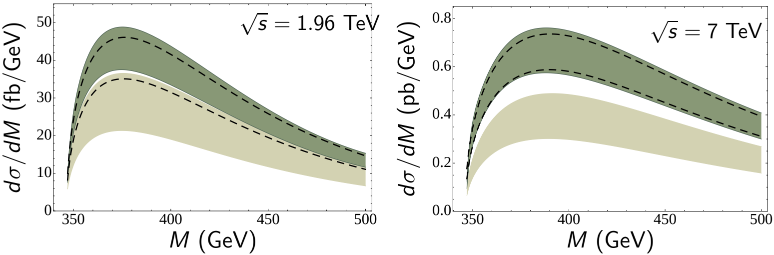
Partonic threshold region $z \to 1$ important if:

- $\tau \to 1$ (high invariant mass)
- $ff_{ij}(\tau/z,\mu)$ fall off very fast for larger τ/z





Invariant mass distribution at low M



- green band = exact fixed order at NLO ($\mu_f = 200$, 800 GeV)
- dashed lines = leading terms for $z \to 1$ at NLO ($\mu_f = 200$, 800 GeV)

Good agreement shows dominance of threshold corrections even at low M

Factorization in the threshold region

Momentum scales at $z \to 1$:

$$\hat{s}, M^2, m_t^2 \gg \hat{s}(1-z)^2 \gg \Lambda_{\rm QCD}^2$$

<u>Factorization:</u>

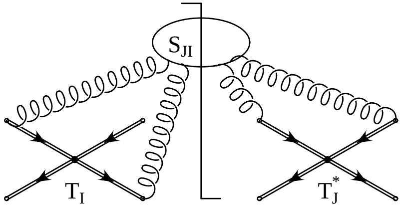
$$C_{ij}(z,M,m_t,\cos\theta,\mu_f) = \mathrm{Tr}\left[\mathbf{H}_{ij}(M,m_t,\cos\theta,\mu_f)\,\mathbf{S}_{ij}(\sqrt{\hat{s}}(1-z),M,m_t,\cos\theta,\mu_f)\right] + \mathcal{O}(1-z)$$

Kidonakis, Sterman ('97)

- $\mathbf{H}_{ij}$ are hard matrices (related to virtual corrections)
- $\mathbf{S}_{ij}$ are soft matrices (related to real emission in soft limit $z \to 1$)
  Soft corrections involve $\delta(1-z)$ or

$$\alpha_s^n \left[\frac{\ln^m(1-z)}{1-z}\right]_+ ; \quad m = 0, \cdots, 2n-1$$

Factorization in SCET (quark channel)



(b)

$$T_I \sim C_I(M, m_t, \dots) \times \bar{\chi}_{\bar{n}}^{a_2}\, \chi_n^{a_1}\; \bar{h}_{v_3}^{a_3} h_{v_4}^{a_4} \times c_I^{\{a\}}$$

$$\left(c_1^{q\bar{q}}\right)_{\{a\}} = \delta_{a_1 a_2}\delta_{a_3 a_4}\,, \quad \left(c_2^{q\bar{q}}\right)_{\{a\}} = t^c_{a_2 a_1} t^c_{a_3 a_4}\,,$$

$$d\hat{\sigma} \sim \sum_{I,J} C_I S_{IJ} C_J^* \times |\langle t\bar{t}|\bar{\chi}_{\bar{n}}\, \chi_n\; \bar{h}_{v_3} h_{v_4}|q\bar{q}\rangle|^2 \equiv \mathrm{Tr}[\mathbf{H}\,\mathbf{S}]$$

Resummation = RG evolution in SCET

Resummed hard function (recall $H_{IJ} \sim C_I C_J^*$)

$$\mathbf{H}(M, m_t, \cos\theta, \mu) = \mathbf{U}(M, m_t, \cos\theta, \mu_h, \mu)\, \mathbf{H}(M, m_t, \cos\theta, \mu_h)\, \mathbf{U}^\dagger(M, m_t, \cos\theta, \mu_h, \mu)$$

$$\mathbf{U}(M, m_t, \cos\theta, \mu_h, \mu) = \mathcal{P} \exp \int_{\mu_h}^{\mu} \frac{d\mu'}{\mu'}\, \mathbf{\Gamma}_H(M, m_t, \cos\theta, \mu')$$

Resummed soft function ($\tilde{\mathbf{s}}$ is Laplace transform)

$$\mathbf{S}(\omega, M, m_t, \cos\theta, \mu_f) = \exp\left[-4S(\mu_s, \mu_f) + 4a_{\gamma^\phi}(\mu_s, \mu_f)\right]$$

$$\times\, \mathbf{u}^\dagger(M, m_t, \cos\theta, \mu_f, \mu_s)\, \tilde{\mathbf{s}}(\partial_\eta, M, m_t, \cos\theta, \mu_s)\, \mathbf{u}(M, m_t, \cos\theta, \mu_f, \mu_s)\, \frac{1}{\omega}\left(\frac{\omega}{\mu_s}\right)^{2\eta} \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)}$$

- should choose $\mu_h \sim M$
- μ_s chosen to minimize correction from soft function

$$\mu_s = \frac{M(1-\tau)}{(a + b\tau^{1/4})^c} \approx \frac{M}{4} \dots \frac{M}{10}$$

Resummed coefficient

$$C(z,M,m_t,\cos\theta,\mu_f) = \exp\left[4a_{\gamma^\phi}(\mu_s,\mu_f)\right]$$
$$\times\,\mathrm{Tr}\Bigg[\mathbf{U}(M,m_t,\cos\theta,\mu_h,\mu_s)\,\mathbf{H}(M,m_t,\cos\theta,\mu_h)\,\mathbf{U}^\dagger(M,m_t,\cos\theta,\mu_h,\mu_s)$$
$$\times\,\tilde{\mathbf{s}}\left(\ln\frac{M^2}{\mu_s^2}+\partial_\eta,M,m_t,\cos\theta,\mu_s\right)\Bigg]\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\,\frac{z^{-\eta}}{(1-z)^{1-2\eta}}$$

RG-improved PT	log accuracy	Γ_{cusp}	γ^h, γ^ϕ	$\mathbf{H}$, $\tilde{\mathbf{s}}$
LO	NLL	2-loop	1-loop	tree-level
NLO	NNLL	3-loop	2-loop	1-loop

All pieces for first NNLL calculation now known

Ahrens, Ferroglia, Neubert, BP, Yang ('10)

Matching with fixed order: the NLO+NNLL approximation

So far considered only leading terms in $z \to 1$ limit. At NLO, can add on effects of hard real emission by evaluating

$$d\sigma^{\text{NLO+NNLL}} \equiv d\sigma^{\text{NNLL}}\Big|_{\mu_h,\mu_s,\mu_f} + d\sigma^{\text{NLO, subleading}}\Big|_{\mu_f}$$

$$\equiv d\sigma^{\text{NNLL}}\Big|_{\mu_h,\mu_s,\mu_f} + \left(d\sigma^{\text{NLO}}\Big|_{\mu_f} - d\sigma^{\text{NLO, leading}}\Big|_{\mu_f}\right)$$

- NLO+NNLL reduces to NLO if $\mu_h = \mu_s = \mu_f$, otherwise resums logs

Approximate NNLO results

$$C^{(\mathrm{NNLO})}(z,M,m_t,\cos\theta,\mu) = D_3\left[\frac{\ln^3(1-z)}{1-z}\right]_+ + D_2\left[\frac{\ln^2(1-z)}{1-z}\right]_+ + D_1\left[\frac{\ln(1-z)}{1-z}\right]_+ + D_0\left[\frac{1}{1-z}\right]_+ + C_0\,\delta(1-z) + R(z)$$

- RG equations determine $D_3 \ldots D_0$ (through soft logs $\ln\sqrt{\hat{s}}(1-z)/\mu_s$) , scale-dependent part of C_0

  Ahrens, Ferroglia, Neubert, BP, Yang '09

- scale-independent part of C_0 not determined (requires virtual+soft corrections at NNLO)

Applications

- $d\sigma/dM$
- total cross section
- forward-backward asymmetry

Invariant mass distribution: fixed-order vs. resummed (MSTW2008NNLO PDFs, $m_t = 173.1$ GeV)

Resummed

$\mu_h = \mu_f = M$

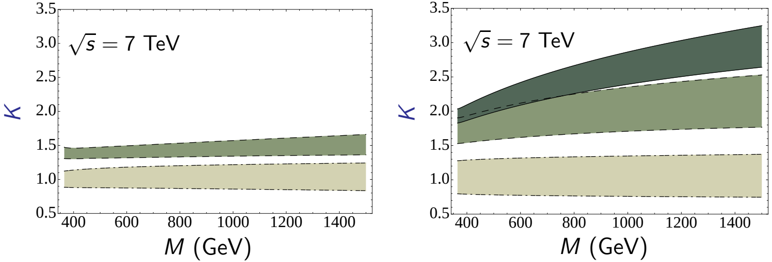
$\mu_s = \frac{M(1-\tau)}{(a+b\tau^{1/4})^c} \sim \frac{M}{5}$

Fixed-order

$\mu_f = M$

$K = (d\sigma/dM)/(d\sigma^{\mathrm{NLL,\,def}}/dM)$

$K = (d\sigma/dM)/(d\sigma^{\mathrm{LO,\,def}}/dM)$



grey= NLL (LO), light-green = NNLL (NLO), dark-green = approx NNLO

Total cross section

$$\sigma(s, m_t) = \int_{2m_t}^{\sqrt{s}} dM \frac{d\sigma}{dM}$$

Default scale choices

- $\mu_h = M$, $\mu_s \sim \frac{M}{5}$, $\mu_f = 400$ GeV

	Tevatron	LHC (7 TeV)
$\sigma_{\text{NLO, leading}}$	$5.34^{+0.73\,+0.28}_{-0.73\,-0.21}$	$127^{+14\,+6}_{-15\,-7}$
σ_{NLO}	$5.64^{+0.73\,+0.30}_{-0.75\,-0.22}$	$126^{+19\,+7}_{-18\,-7}$
$\sigma_{\text{NLO+NNLL}}$	$6.30^{+0.19\,+0.31}_{-0.19\,-0.23}$	$149^{+7\,+8}_{-7\,-8}$

- first error from scale variations, second PDF (MSTW2008NNLO at 90% CL)
- perturbative errors smaller than PDF at NLO+NNLL

Forward-backward asymmetry at Tevatron

$$\Delta\sigma_{\rm FB} \equiv \int_{2m_t}^{\sqrt{s}} dM \left[\int_0^1 d\cos\theta \frac{d^2\sigma^{p\bar{p}\to t\bar{t}X}}{dMd\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d^2\sigma^{p\bar{p}\to t\bar{t}X}}{dMd\cos\theta} \right]$$

$$A^t_{\rm FB} = \Delta\sigma_{\rm FB}/\sigma$$

parton frame	$m_t/2 < \mu_f < 2m_t$	
	$\Delta\sigma$ (pb)	$A^t_{\rm FB}$ (%)
NLO	$0.38^{+0.20}_{-0.12}$	$7.3^{+0.7}_{-0.6}$
NLO+NNLL	$0.45^{+0.08}_{-0.07}$	$7.3^{+1.1}_{-0.7}$

- higher-order effects small due to cancellations in ratio
- results in lab frame are $\sim 30\%$ lower than in parton frame
- CDF $A^t_{\rm FB}(\rm EXP) = 19.3 \pm 6.9\%$ (lab frame)

Summary

- heavy-quark production important process at hadron colliders
- need to go beyond NLO to get under PDF uncertainties
- presented results valid to NLO+NNLL order, arguably best predictions available at present
- full NNLO results obviously the ultimate solution...

Backup slides

Approximate cross section at NNLO

Two options:

- integrate the approximate NNLO invariant mass distribution numerically

$$\sigma(s, m_t) = \int_{2m_t}^{\sqrt{s}} dM \frac{d\sigma}{dM} \approx \int_{2m_t}^{\sqrt{s}} dM \frac{d\sigma}{dM}\bigg|_{z\to 1}$$

- expand total partonic cross section around $\hat{s} \sim 4m_t^2$ ($\beta \to 0$)

$$\sigma(s, m_t) = \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_{4m_t^2}^{s} \frac{d\hat{s}}{s}\, ff_{ij}\left(\frac{\hat{s}}{s}, \mu\right) \hat{\sigma}_{ij}\left(\frac{4m_t^2}{\hat{s}}, \mu\right)$$

$$\approx \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_{4m_t^2}^{s} \frac{d\hat{s}}{s}\, ff_{ij}\left(\frac{\hat{s}}{s}, \mu\right) \hat{\sigma}_{ij}\left(\frac{4m_t^2}{\hat{s}}, \mu\right)\bigg|_{\hat{s}\approx 4mt^2}$$

Approximate cross section at NNLO (quark channel)

The singular part of NNLO partonic cross section in limit $\beta \to 0$ is:

Moch, Uwer '08; Beneke, Czakon, Falgari, Mitov, Schwinn '09

$$\hat{\sigma}^{(2,0)}_{q\bar{q}} = \frac{1}{(16\pi^2)^2}\frac{\pi\beta}{9}\Big[910.22\ln^4\beta - 1315.5\ln^3\beta + 592.29\ln^2\beta + 452.52\ln\beta - \frac{1}{\beta}\Big(140.37\ln^2\beta + 18.339\ln\beta - 72.225\Big) + \hat{\sigma}^{\rm potential}_{q\bar{q}}\Big] + \cdots$$

- first piece recovered by expanding approximate NNLO formula for $z \to 1$ around $\beta \to 0$ and dropping subleading terms
- two-loop "potential" effects not recovered (would come from δ-fcn. piece)

Beneke, Czakon, Falgari, Mitov, Schwinn '09

$$\hat{\sigma}^{\rm potential}_{q\bar{q}} = \frac{3.6077}{\beta^2} + \frac{1}{\beta}(50.445\ln\beta - 68.274) + 76.033\ln\beta$$

Approximate cross section at NNLO

	Tevatron	LHC (7 TeV)
$\sigma_{\text{NLO, leading}}$	$5.34^{+0.73\,+0.28}_{-0.73\,-0.21}$	$127^{+14\,+6}_{-15\,-7}$
$\sigma_{\rm NLO}$	$5.64^{+0.73\,+0.30}_{-0.75\,-0.22}$	$126^{+19\,+7}_{-18\,-7}$
$\sigma_{\rm NLO+NNLL}$	$6.30^{+0.19\,+0.31}_{-0.19\,-0.23}$	$149^{+7\,+8}_{-7\,-8}$
$\sigma_{{\rm NNLO},\, z\to 1}$	$6.05^{+0.43\,+0.31}_{-0.50\,-0.23}$	$139^{+9\,+7}_{-9\,-7}$
$\sigma_{{\rm NNLO},\, \beta-{\rm exp.}}$	$7.37^{+0.00\,+0.39}_{-0.20\,-0.29}$	$156^{+2\,+8}_{-5\,-8}$
$\sigma_{{\rm NNLO},\, \beta-{\rm exp.+\,potential}}$	$7.30^{+0.00\,+0.39}_{-0.18\,-0.28}$	$158^{+3\,+8}_{-6\,-8}$

- $z \to 1$ and β-exp differ only by terms subleading in β
- β-exp.+potential includes additional $1/\beta^2 \ldots \ln\beta$ terms

seems that subleading terms in β are not generically small

RG equations: Hard Function

RG equation: (recall $\mathbf{H}_{IJ} \sim C_I C_J^*$)

$$\frac{d}{d\ln\mu}\mathbf{H}(M, m_t, \cos\theta, \mu) = \mathbf{\Gamma}_H(M, m_t, \cos\theta, \mu)\,\mathbf{H}(M, m_t, \cos\theta, \mu) + \mathbf{H}(M, m_t, \cos\theta, \mu)\,\mathbf{\Gamma}_H^\dagger(M, m_t, \cos\theta, \mu)$$

Solution:

$$\mathbf{H}(M, m_t, \cos\theta, \mu) = \mathbf{U}(M, m_t, \cos\theta, \mu_h, \mu)\,\mathbf{H}(M, m_t, \cos\theta, \mu_h)\,\mathbf{U}^\dagger(M, m_t, \cos\theta, \mu_h, \mu)$$

$$\mathbf{U}(M, m_t, \cos\theta, \mu_h, \mu) = \mathcal{P}\exp\int_{\mu_h}^{\mu}\frac{d\mu'}{\mu'}\,\mathbf{\Gamma}_H(M, m_t, \cos\theta, \mu'),$$

Solution to RG equation

$$\mathbf{U}(M, m_t, \cos\theta, \mu_h, \mu) = \mathcal{P} \exp \int_{\mu_h}^{\mu} \frac{d\mu'}{\mu'} \, \mathbf{\Gamma}_H(M, m_t, \cos\theta, \mu') \,,$$

$$\mathbf{\Gamma}_H(M, m_t, \cos\theta, \mu) = \Gamma_{\rm cusp}(\alpha_s) \left(\ln \frac{M^2}{\mu^2} - i\pi \right) \mathbf{1} + \boldsymbol{\gamma}^h(M, m_t, \cos\theta, \alpha_s)$$

Factor out logarithmic evolution:

$$\mathbf{U}(M, m_t, \cos\theta, \mu_h, \mu) = \exp\left[2S(\mu_h, \mu) - a_\Gamma(\mu_h, \mu) \left(\ln \frac{M^2}{\mu_h^2} - i\pi \right) \right]$$
$$\times \left\{ \mathbf{u}(M, m_t, \cos\theta, \mu_h, \mu) = \mathcal{P} \exp \int_{\alpha_s(\mu_h)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)} \, \boldsymbol{\gamma}^h(M, m_t, \cos\theta, \alpha) \right\}$$

solve for **u** using standard techniques (e.g. from B decay)

RG EVOLUTION OF HARD FUNCTION

Solution to RG equation (recall $H_{IJ} \sim C_I C_J^*$)

$$\mathbf{H}(M, m_t, \cos\theta, \mu) = \mathbf{U}(M, m_t, \cos\theta, \mu_h, \mu)\, \mathbf{H}(M, m_t, \cos\theta, \mu_h)\, \mathbf{U}^\dagger(M, m_t, \cos\theta, \mu_h, \mu)$$

$$\mathbf{U}(M, m_t, \cos\theta, \mu_h, \mu) = \mathcal{P} \exp \int_{\mu_h}^{\mu} \frac{d\mu'}{\mu'}\, \mathbf{\Gamma}_H(M, m_t, \cos\theta, \mu')\,,$$

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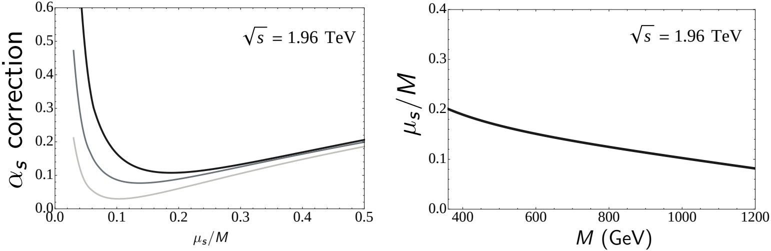
solve for **u** using standard techniques (e.g. from B decays)

THE SOFT SCALE μ_s

Choose μ_s such that α_s correction from $\tilde{\mathbf{s}}$ is smallest:



$M = 400$ GeV (dark), $M = 700$ GeV (medium), and $M = 1000$ GeV (light)