Parton Showers in SCET

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Parton showers vs. SCET

- Both parton showers and SCET claim to be correct limit of QCD in soft/collinear limit
- Both resum large logarithmic terms
- There are many obvious similarities
- Many things seem different

What is exact relationship?

Similarities

- SCET at leading order reproduces AP splitting functions
- Strongly ordered limit $\mu_1 \ll \mu_2 \ll \ldots \ll \mu_n$: interference effects in SCET cancel
  - Product of splitting functions
- Double logarithmic dependence present in both SCET and Parton Showers

Much of this discussed in earlier work with Matt

Differences

Parton shower	SCET
Has 2 scales (t_{start}, t_{end})	Knows about 3 scales (μ_H, μ_J, μ_S)
Only uses collinear limit	Knows about soft function
Simple products of AP splitting & Sudakov	Needs convolutions between functions

Point of this talk to reconcile the two approaches

Why do we care?

- Corrections to parton shower
  - In SCET NLL and power corrections tractable
  - Should give insight how to implement in parton shower
- Match parton shower with fixed order calcs
  - Short distance physics included in SCET by matching
  - Should tell how to do do same for parton shower
- Pure curiosity how both describe same limit of QCD

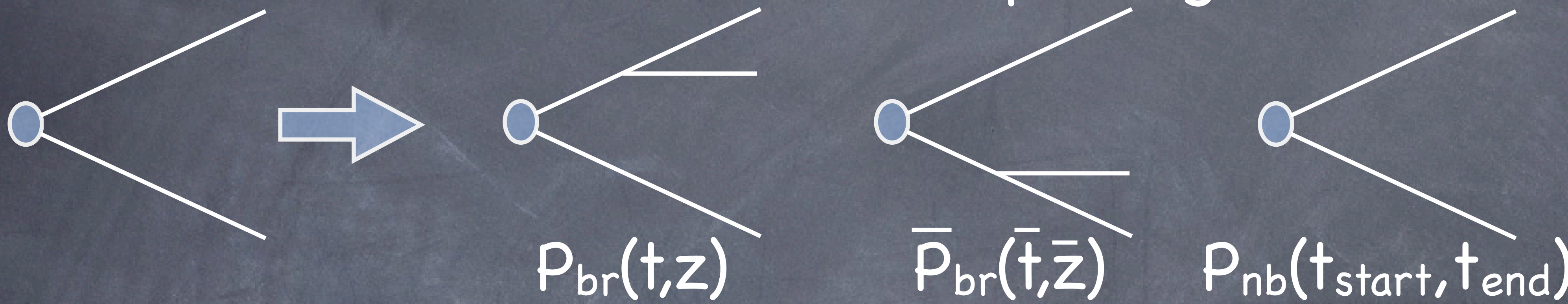
Outline

- Explain how parton shower works
- Constructing physical observables
- The SCET result and potential conflicts
  - Absence of convolutions
  - The effect of soft running in SCET
- Comparison with previous work
- Some preliminary results
- Conclusions

How a parton shower works

The parton shower

Consider at most one splitting



- Need 3 variables to describe single splitting (2+φ)
- Two non-trivial variables usually chosen as
  - t: evolution variable
  - z: splitting variable
- Need to know where shower starts and ends (t_{start}, t_{end})

How are the probabilities calculated?

The probabilities

To preserve probability, need

$$\int_{t_{end}}^{t_{start}} dt \int dz\, P_{br}(t,z) + P_{nb}(t_{start}, t_{end}) = 1$$

or

$$\int dz\, P_{br}(t,z) = d/dt\, P_{nb}(t_{start}, t)$$

To LO in PT want $P_{br}(t,z) = AP(t,z) + O(\alpha_s)$

Solution given by

$$P_{br}(t,z) = AP(t,z)\, \Delta(t_{start}, t)$$

$$P_{nb}(t_{start}, t_{end}) = \Delta(t_{start}, t_{end})$$

$$\Delta(t_{start}, t_{end}) = \exp\left\{-\int_{t_{end}}^{t_{start}} dt \int dz\, AP(t,z)\right\}$$

The probabilities

$$AP(t,z) = \frac{1}{t}\frac{1+z^2}{1-z}$$

What is the variable t?

This depends on the parton shower

Pythia6: $t = p^2$
Pythia8: $t = p_T{}^2$
Herwig: $t = E^2(1-\cos\Theta)$

What are the correct limits of integration on z?

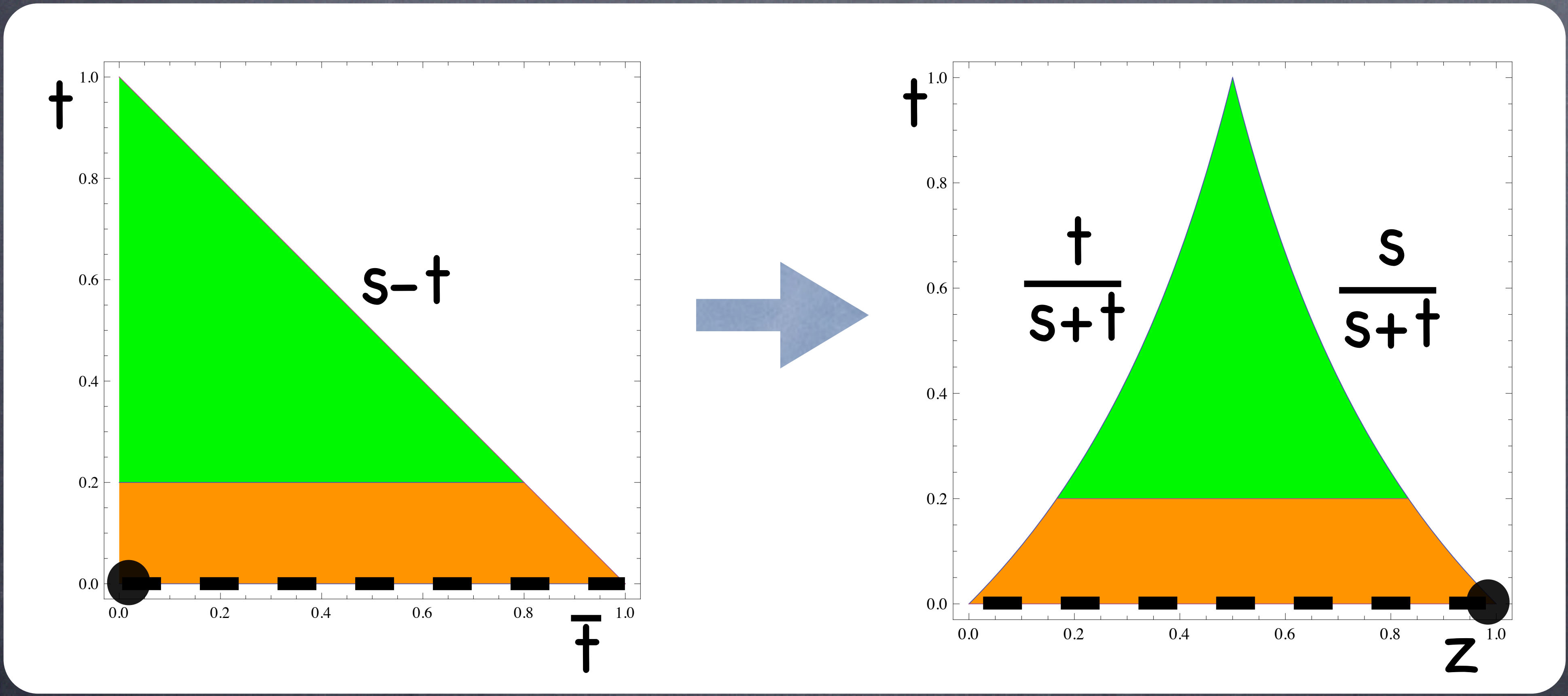
Want parton shower to cover all phase space

$z_{min} = z_{min}(t)$
$z_{max} = z_{max}(t)$

Best understood by considering an example

An example: Pythia6

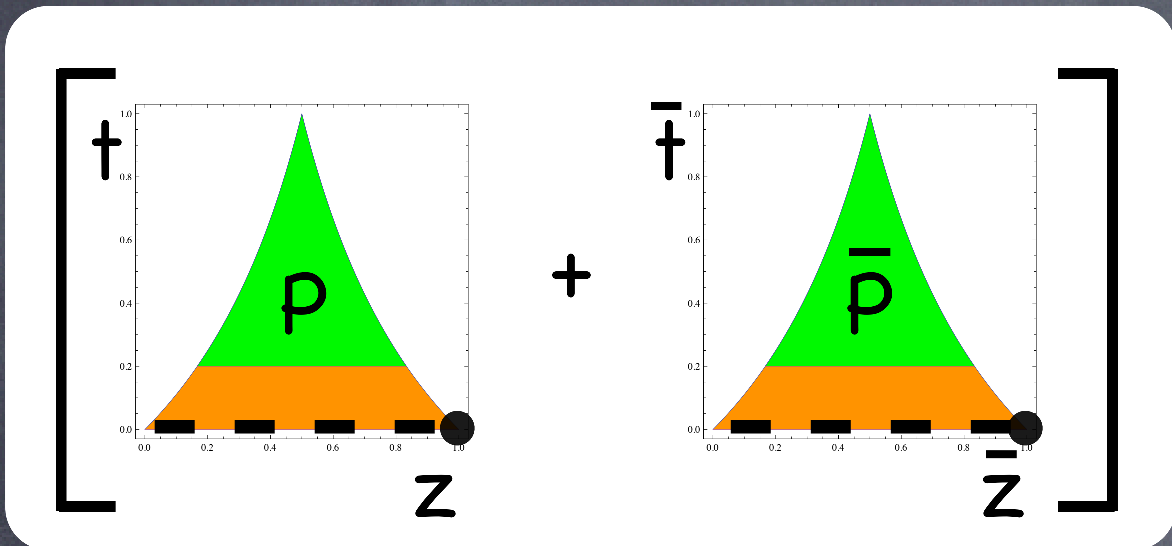
Phase space limits for first emission



AP splitting has correct singularity if t→0, but half the singularity if (t→0,z→1)

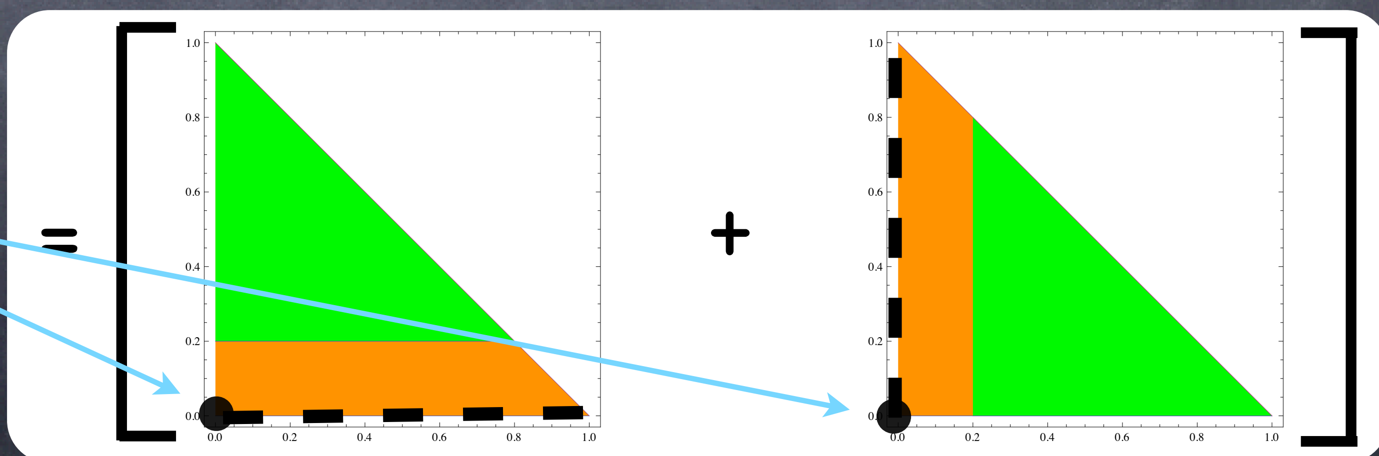
An example: Pythia6

In reality, there are two possible splittings (quark and antiquark)



Full singularities reproduced, since double singularities is half in each case

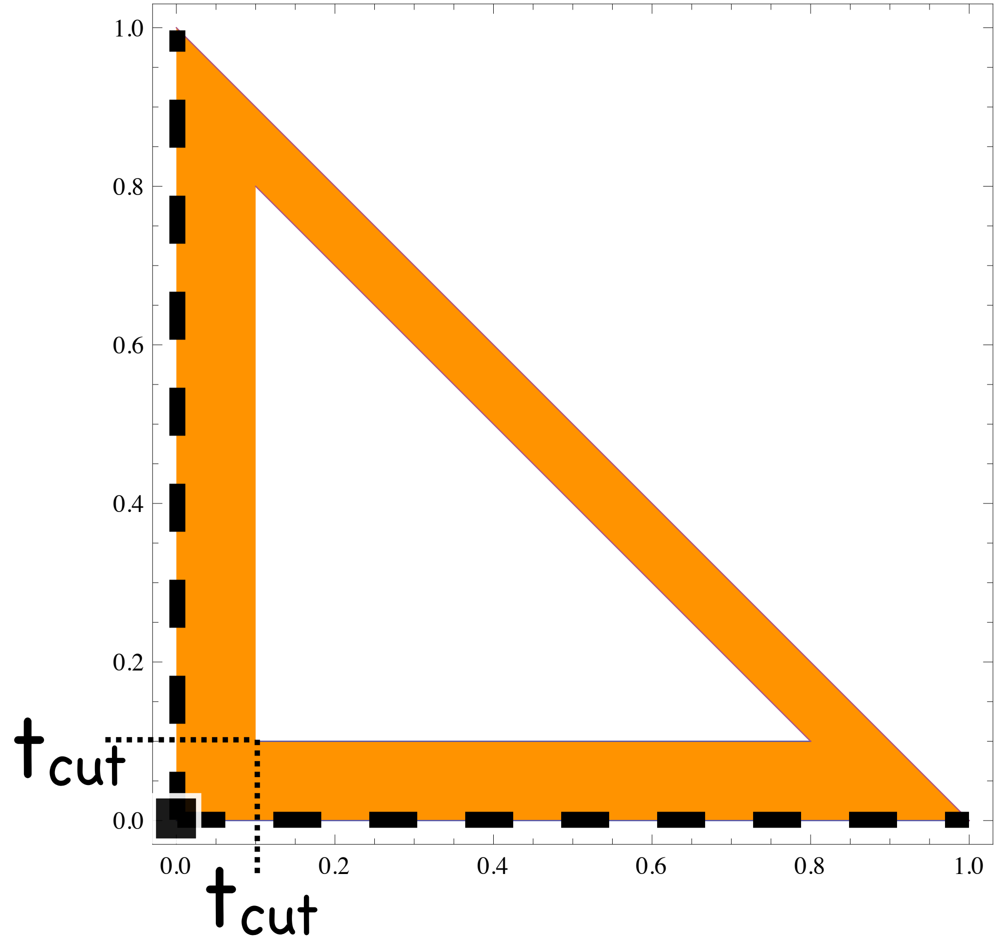
1/2



Constructing physical observables

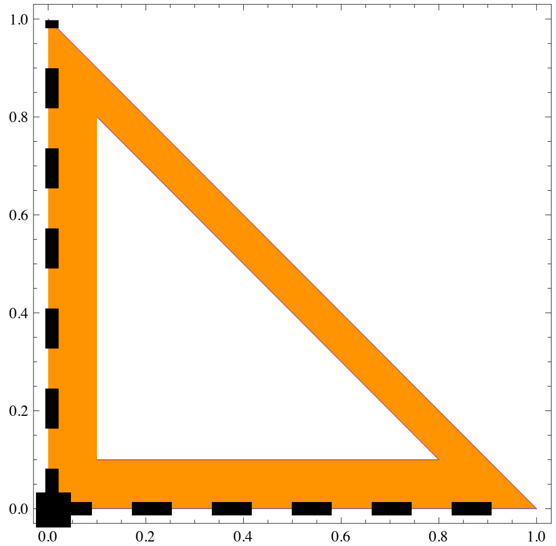
The exclusive 2-jet cross-section

1. Calculate the thrust axis of an event
2. Calculate the invariant mass in both hemispheres
3. Keep all events with m_L^2, $m_R^2 < t_{cut}$

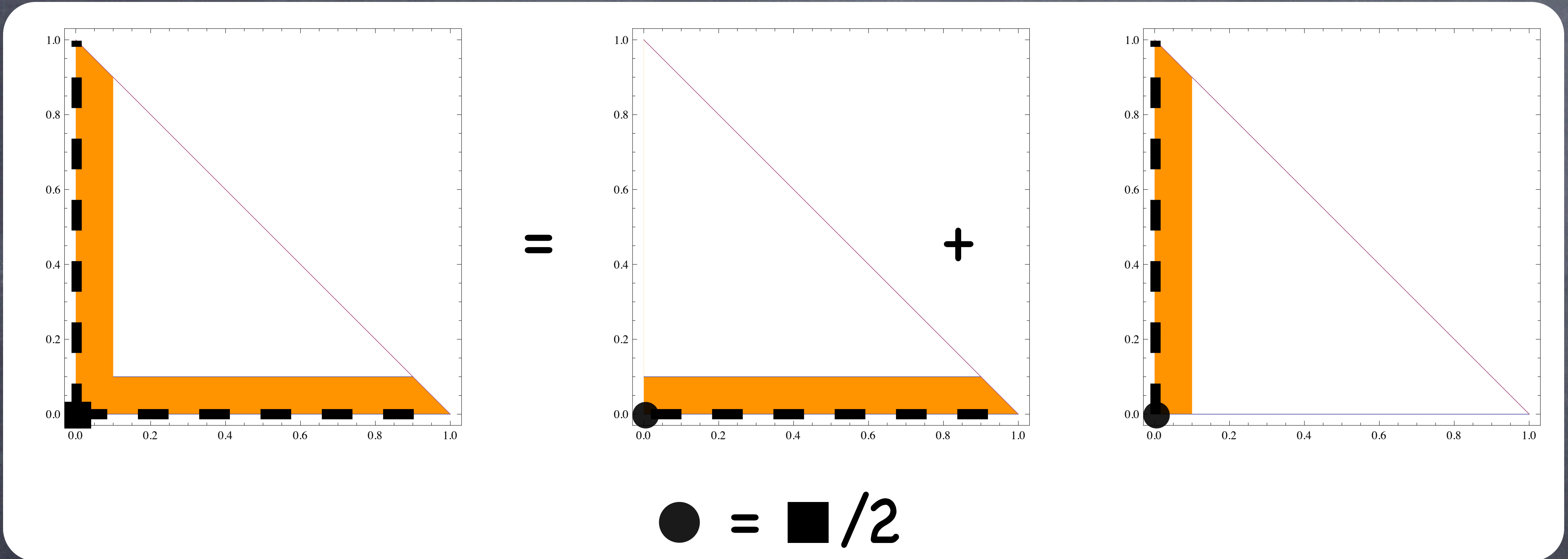


Resulting cross section $\sigma_2^{excl}(t_{cut})$

The exclusive 2-jet cross-section



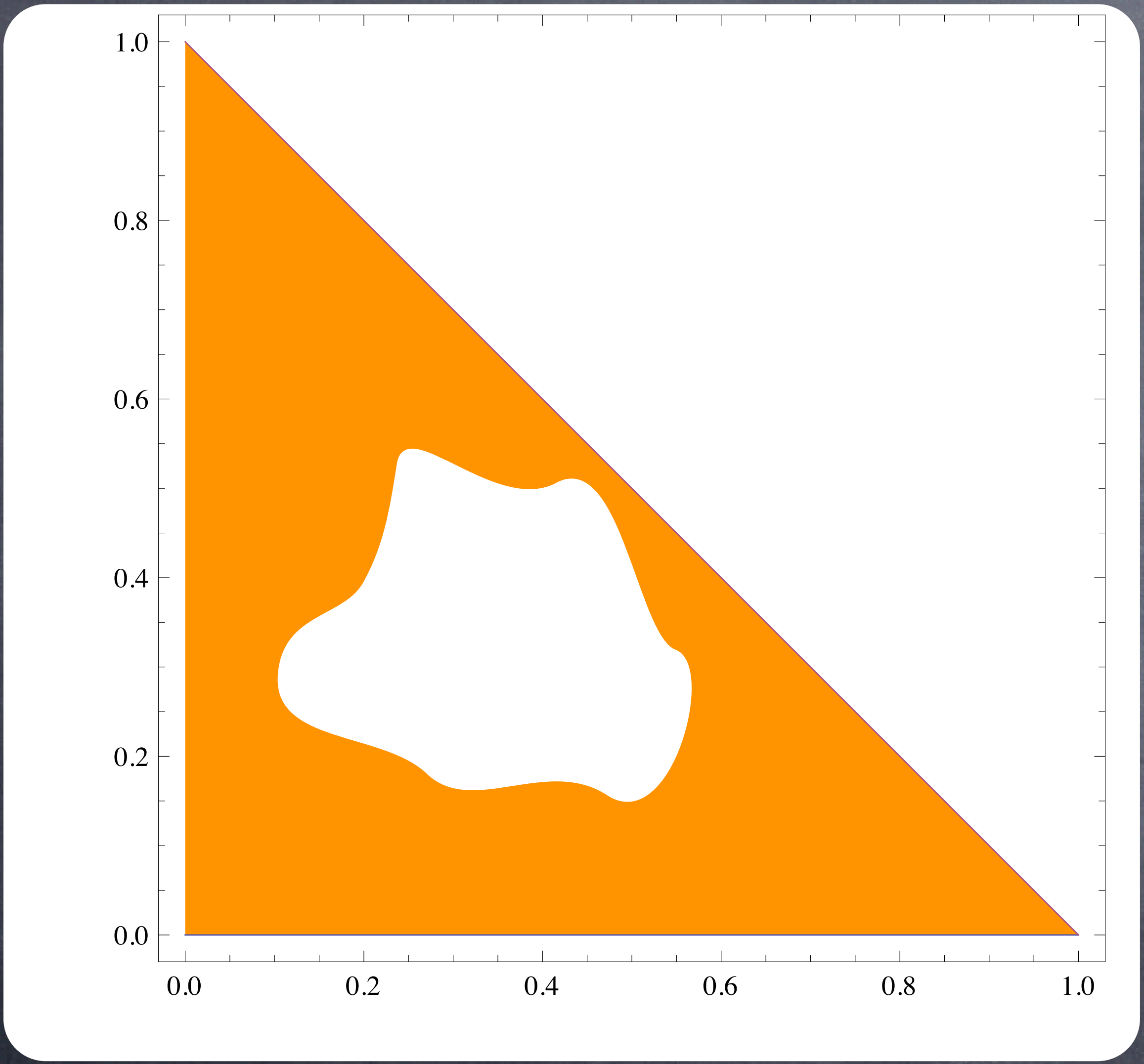
The exclusive 2-jet cross-section



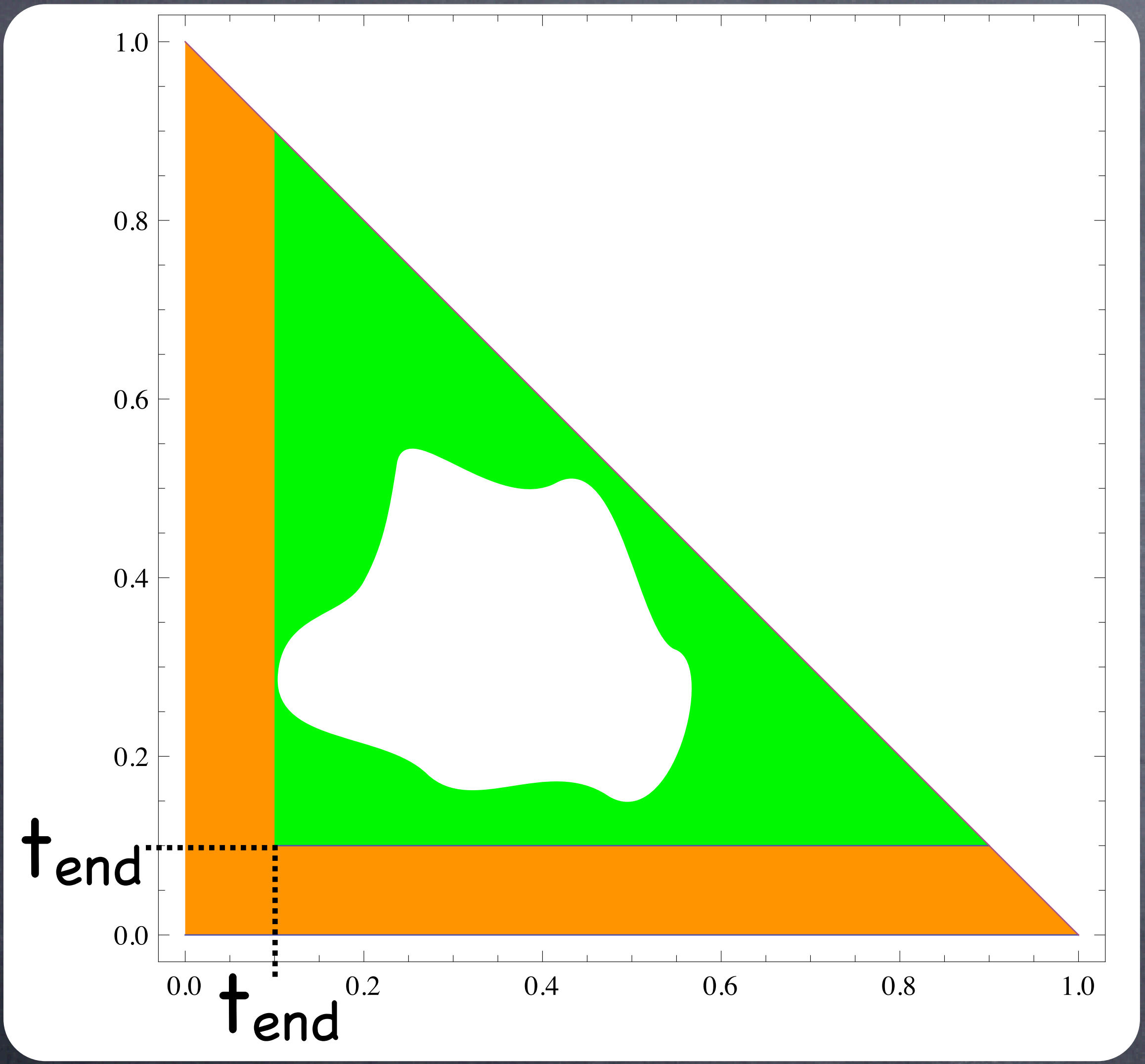
1. Running shower from $t_{start} = s$ to $t_{end} = t_{cut}$
2. Keep all unbranced events

$$\sigma_2^{excl}(t_{cut}) = B_2\, \Delta^2(s, t_{cut}) + \ldots$$

More general observables



More general observables



1. Run shower to smallest required value of t
2. Sum over all branched events outside region
3. Add to unbranched events

Any IR safe observable can be implemented

The SCET result

The issues

$$d\sigma_n(t_{\rm cut}) =$$

$$B_n\, H_n(\mu_H)\, U_H(\mu_H,\mu) \times \Big[\prod_{i=1}^{n} J_i(\mu_J) \otimes U_J(\mu_J,\mu)\Big] \otimes \big[S_n(\mu_S) \otimes U_S(\mu_S,\mu)\big]$$

Running of the hard function = Sudakov?

What happens to running of soft and jet function?

Where do the convolutions go in the parton shower?

The SCET expression

The well known factorization formula is

$$\frac{d\sigma_2^{\rm SCET}}{d\Omega}(t_{\rm cut}) = \frac{d\sigma_0}{d\Omega}\, H_2(Q,Q,\mu) \int dk_1^+ dk_2^+\, \widetilde{J}_q(t_{\rm cut} - Qk_1^+,\mu)\widetilde{J}_q(t_{\rm cut} - Qk_2^+,\mu)\, S_2(k_1^+,k_2^+,\mu)$$

$k_i^+ \sim t_{cut}/Q$

We are using the integrated jet function

$$\widetilde{J}_q(t) = \int^t dt' J_q(t',\mu)$$

The Born cross section is

$$\frac{d\sigma_0}{d\Omega} = N_c\, Q_q^2\, \frac{\alpha_{\rm em}^2}{4Q^2}\,(1+\cos^2\theta)$$

The SCET expression

The well known factorization formula is

$$\frac{d\sigma_2^{\rm SCET}}{d\Omega}(t_{\rm cut}) = \frac{d\sigma_0}{d\Omega} H_2(Q,Q,\mu) \int dk_1^+ dk_2^+ \, \widetilde{J}_q(t_{\rm cut} - Qk_1^+, \mu) \widetilde{J}_q(t_{\rm cut} - Qk_2^+, \mu) \, S_2(k_1^+, k_2^+, \mu)$$

Summing large logarithms

- Find scales μ_H, μ_J, μ_S, where no large logs in H, J, S
- Use RG to evolve each term to common scale μ
- To LL can use tree level expression for $H(\mu_H)$, $J(\mu_J)$, $S(\mu_S)$

Go through each of these steps...

The scales in SCET

The one loop expressions are

$$H(Q,Q,\mu) = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left(-\log^2\frac{Q^2}{\mu^2} + 3\log\frac{Q^2}{\mu^2} - 8 + \frac{7\pi^2}{6}\right)$$

$$\widetilde{J}_q(t,\mu) = \theta(t) + \frac{\alpha_s(\mu)C_F}{2\pi}\theta(t)\left[\frac{7}{2} - \frac{\pi^2}{2} - \frac{3}{2}\log\frac{t}{\mu^2} + \log^2\frac{t}{\mu^2}\right]$$

$$S_2(k_1^+,k_2^+,\mu) = \delta(k_1^+)\delta(k_2^+) + \frac{\alpha_s(\mu)C_F}{2\pi}\left[\frac{\pi^2}{6}\delta(k_1^+)\delta(k_2^+) -4\delta(k_1^+)\frac{1}{\mu}\left(\frac{\theta(k_2^+)\log(k_2^+/\mu)}{k_2^+/\mu}\right)_+ - 4\delta(k_2^+)\frac{1}{\mu}\left(\frac{\theta(k_1^+)\log(k_1^+/\mu)}{k_1^+/\mu}\right)_+\right]$$

No large logs at the scales

$\mu^2 = \mu_h^2 = Q^2$ | $\mu^2 = \mu_j^2 \sim t_{\rm cut}$ | $\mu^2 = \mu_s^2 \sim t_{\rm cut}^2/Q^2$

The RG equations

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}H_2(Q,Q,\mu) = \gamma_{H_2}(Q,\mu)H_2(Q,Q,\mu)$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\tilde{J}(t,\mu) = \int \mathrm{d}t'\gamma_J(t-t',\mu)\tilde{J}(t',\mu)$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}S_n(k_1^+,k_2^+,\mu) = \int \mathrm{d}k_1'^+\int \mathrm{d}k_1'^+\gamma_{S_2}(k_1^+-k_1'^+,k_2^+-k_2'^+,\mu)S_n(k_1'^+,k_2'^+,\mu)$$

Well known anomalous dimensions at LL

$$\gamma_{H_2}(Q;\mu) = \frac{\alpha_s(\mu)C_F}{2\pi}\left[-4\,\ln\frac{\mu^2}{Q^2}\right]$$

$$\gamma_J(t;\mu) = \frac{\alpha_s(\mu)C_F}{2\pi}\left[-4\,\frac{1}{\mu^2}\left(\frac{\mu^2}{t}\right)_+\right]$$

$$\gamma_{S_2}(k_1^+,k_2^+;\mu) = \frac{\alpha_s(\mu)C_F}{2\pi}\left[\frac{4}{\mu}\left(\frac{\mu}{k_1^+}\right)_+\delta(k_2^+)+\frac{4}{\mu}\left(\frac{\mu}{k_2^+}\right)_+\delta(k_1^+)\right]$$

Tree level expressions

$$H_2(Q, Q, \mu) = 1$$

$$\widetilde{J}_q(t) = \theta(t)$$

$$S_2(k_1^+, k_2^+) = \delta(k_1^+)\delta(k_2^+)$$

Makes convolutions trivial

Solutions at LL

$$H_2(Q,Q,\mu) = \exp\left[K_{H_2}(\mu_H,\mu)\right]$$

$$\tilde{J}(t,\mu) = \theta(t)\,\exp\left[K_J(\mu_J,\mu)\right]$$

$$S_2(k_1^+,k_2^+;\mu) = \delta(k_1^+)\delta(k_2^+)\,\exp\left[K_{S_2}(\mu_S,\mu)\right]$$

with functions

$$K_F(\mu_F,\mu) = \frac{4C_F}{\pi}\int_{\mu_F}^{\mu}\frac{\mathrm{d}\mu'}{\mu'}\,\alpha_s(\mu')\,\ln\frac{\mu_F}{\mu'}$$

LL in exponent

$$= -\frac{\alpha_s C_F}{2\pi}\ln^2\frac{\mu^2}{\mu_F^2} + \ldots$$

LL in x-section

Putting it together

Choose the renormalization scale $\mu = \mu_j$

$$\frac{d\sigma_2^{\rm SCET}}{d\Omega}(t_{\rm cut}) = \frac{d\sigma_0}{d\Omega}\ \exp\left[K_H(\mu_H,\mu_J) + K_S(\mu_S,\mu_J)\right]$$

Convolutions have disappeared, because tree level expression almost trivial

But still need running of soft and hard function

Correlation of scales

$$\frac{\mu_J}{\mu_H} = \frac{\mu_S}{\mu_J}$$

$$\begin{aligned} K_H(\mu_H, \mu_J) &= -\frac{\alpha_s C_F}{2\pi} \ln^2 \frac{\mu_J^2}{\mu_H^2} + \dots \\ K_S(\mu_J^2/\mu_H, \mu_J) &= -\frac{\alpha_s C_F}{2\pi} \ln^2 \frac{\mu_H^2}{\mu_J^2} + \dots \end{aligned}$$

This gives to leading log (in x-section)

$$K_H(\mu_H, \mu_J) = K_S(\mu_S, \mu_J)$$

Final result is therefore

$$\frac{\mathrm{d}\sigma_2^{\mathrm{SCET}}}{\mathrm{d}\Omega}(t_{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega} \exp\left[2K_H(\mu_H, \mu_J)\right]$$

Soft running and convolutions have disappeared!

Comparison with Parton Shower

SCET result

$$\frac{d\sigma_2^{\mathrm{SCET}}}{d\Omega}(t_{\mathrm{cut}}) = \frac{d\sigma_0}{d\Omega}\,\exp\left[2K_H(\mu_H,\mu_J)\right]$$

Parton Shower result

$$\sigma^{\mathrm{excl}}(t_{\mathrm{cut}}) = B_2\,\Delta^2(s,t_{\mathrm{cut}})$$

Agreement if

$$\Delta(s,t_{\mathrm{cut}}) = \exp[K_H(s,t_{\mathrm{cut}})]$$

How is this reconciled with previous results?

Comparison with previous work

Running of Wilson coefficient

$$\Pi_n(\mu_2,\mu_1) \equiv \frac{\mathrm{C}_n(\mu_1)}{\mathrm{C}_n(\mu_2)} = \exp\left[-\int_{\mu_1}^{\mu_2} \frac{d\mu}{\mu}\gamma_n(\mu)\right]$$

Solution well known

$$\Pi_2(Q,\mu) = \exp\left\{\frac{C_F}{\pi}\int_{\mu}^{Q} \frac{\mathrm{d}\mu'}{\mu'}\alpha_s(\mu')\left[\log\frac{-\mu'^2}{Q^2}+\frac{3}{2}\right]\right\}$$

Compare to NLL Sudakov factor

$$\Delta_q^{\mathrm{NLL}}(\tau_2,\tau_1) = \exp\left\{-\frac{C_F}{2\pi}\int_{\tau_1}^{\tau_2} \frac{d\tau'}{\tau'}\alpha_s[\sqrt{\tau'}]\int_{\frac{\sqrt{\tau'}}{Q}}^{1-\frac{\sqrt{\tau'}}{Q}} \mathrm{d}z\frac{1+z^2}{1-z}\right\}$$

Comparison with previous work

Gave us final result

$$\Pi_n^2(Q,\mu) = \Delta_q^{n_q}(Q,\mu)\, \Delta_g^{n_g}(Q,\mu)$$

In notation of this talk:

$$\Delta^2(s, t_{cut}) = \exp[K_H(s, t_{cut})]$$

This work gives

$$\Delta(s, t_{cut}) = \exp[K_H(s, t_{cut})]$$

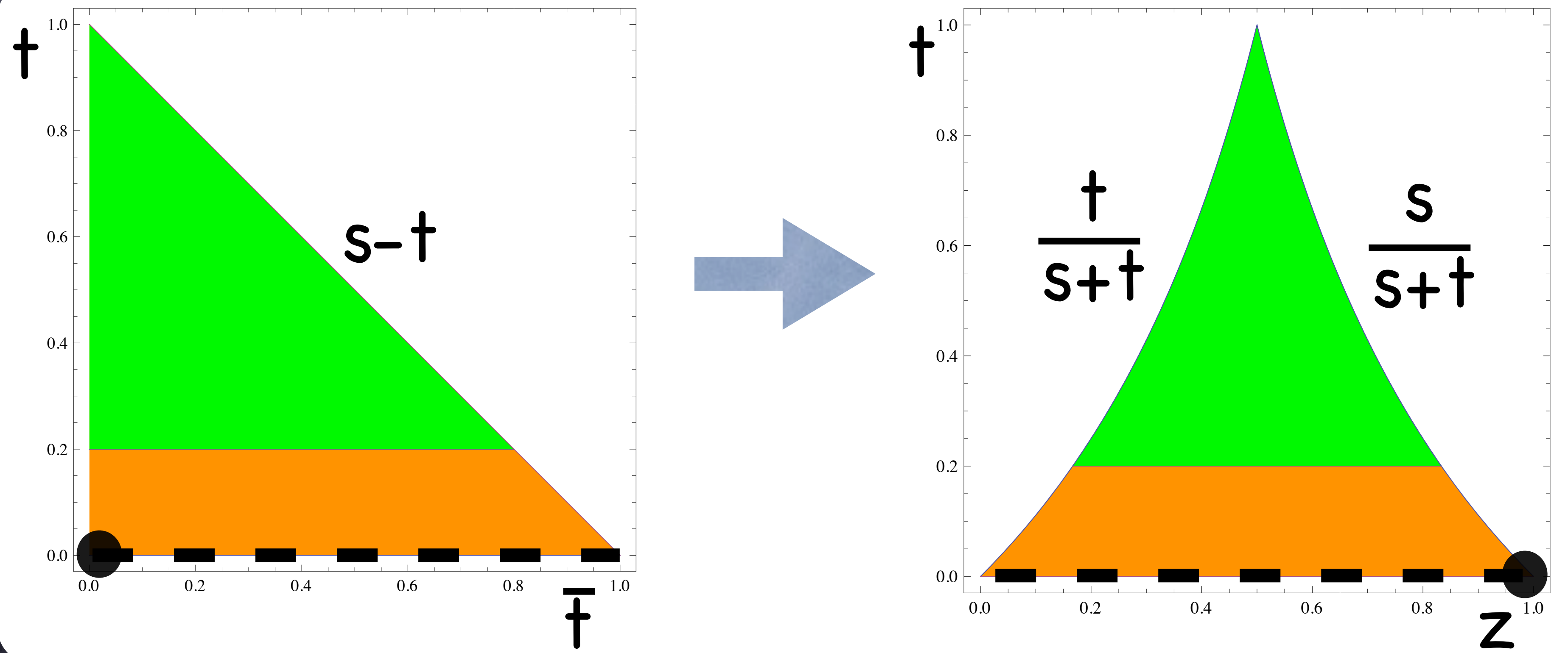
Factor of two from additional soft running

Where does the difference come from?

Comparison with previous work

Go back to definition of NLL Sudakov factor

$$\Delta_q^{\rm NLL}(\tau_2,\tau_1) = \exp\left\{ -\frac{C_F}{2\pi}\int_{\tau_1}^{\tau_2} \frac{d\tau'}{\tau'}\alpha_s[\sqrt{\tau'}]\int_{\frac{\sqrt{\tau'}}{Q}}^{1-\frac{\sqrt{\tau'}}{Q}} {\rm d}z\,\frac{1+z^2}{1-z}\right\}$$



Wednesday, April 7, 2010

Comparison with previous work

Go back to definition of NLL Sudakov factor

$$\Delta_q^{\rm NLL}(\tau_2,\tau_1) = \exp\left\{ -\frac{C_F}{2\pi}\int_{\tau_1}^{\tau_2} \frac{d\tau'}{\tau'}\alpha_s[\sqrt{\tau'}]\int_{\frac{\sqrt{\tau'}}{Q}}^{1-\frac{\sqrt{\tau'}}{Q}} {\rm d}z \frac{1+z^2}{1-z}\right\}$$

z-limits we Pythia is using:

$$\frac{t}{s+t} \le z \le \frac{s}{s+t}$$ or expanded $$\frac{t}{s} \le z \le 1 - \frac{t}{s}$$

Square root is changing double log by factor of two

Where is the root coming from?

The evolution variable of Herwig

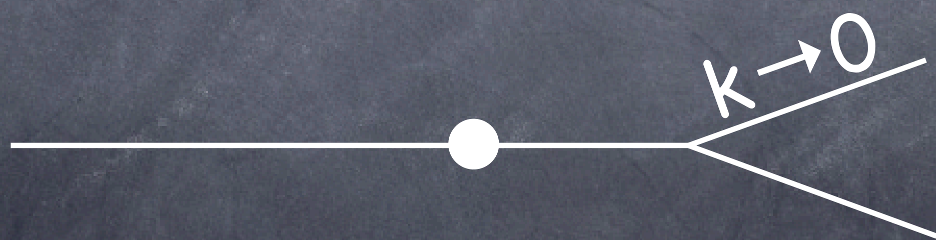
NLL Sudakov very similar to Herwig Sudakov

$$\Delta^{\rm HW}_{a\to bc}(\tilde{t}) = \exp\left\{ - \int_{4t_0}^{\tilde{t}} \frac{dt'}{t'} \int_{\sqrt{\frac{t_0}{t'}}}^{1-\sqrt{\frac{t_0}{t'}}} \frac{dz}{2\pi} \alpha_S(z^2(1-z)^2 t') \hat{P}_{ba}(z) \right\}$$

As mentioned before, angular ordering (not p_T)

$t = E^2(1-\cos\Theta)$

Angular variable can not be resolution variable



Herwig uses gluon (and quark) mass to regulate IR

The evolution variable of Herwig

At NLO, find for $\sigma_2(m_g) \equiv \sigma_{tot} - \int d\sigma_3$

$$\sigma_2^{\rm NLO}(m_g) = \sigma_0 \left\{ 1 - \frac{\alpha_s(\mu) C_F}{4\pi} \left[2 \ln^2 \frac{m_g^2}{Q^2} + 6 \ln \frac{m_g^2}{Q^2} + 9 - \frac{2\pi^2}{3} + \mathcal{O}\left(\frac{m_g^2}{Q^2} \right) \right] \right\}$$

Compare with

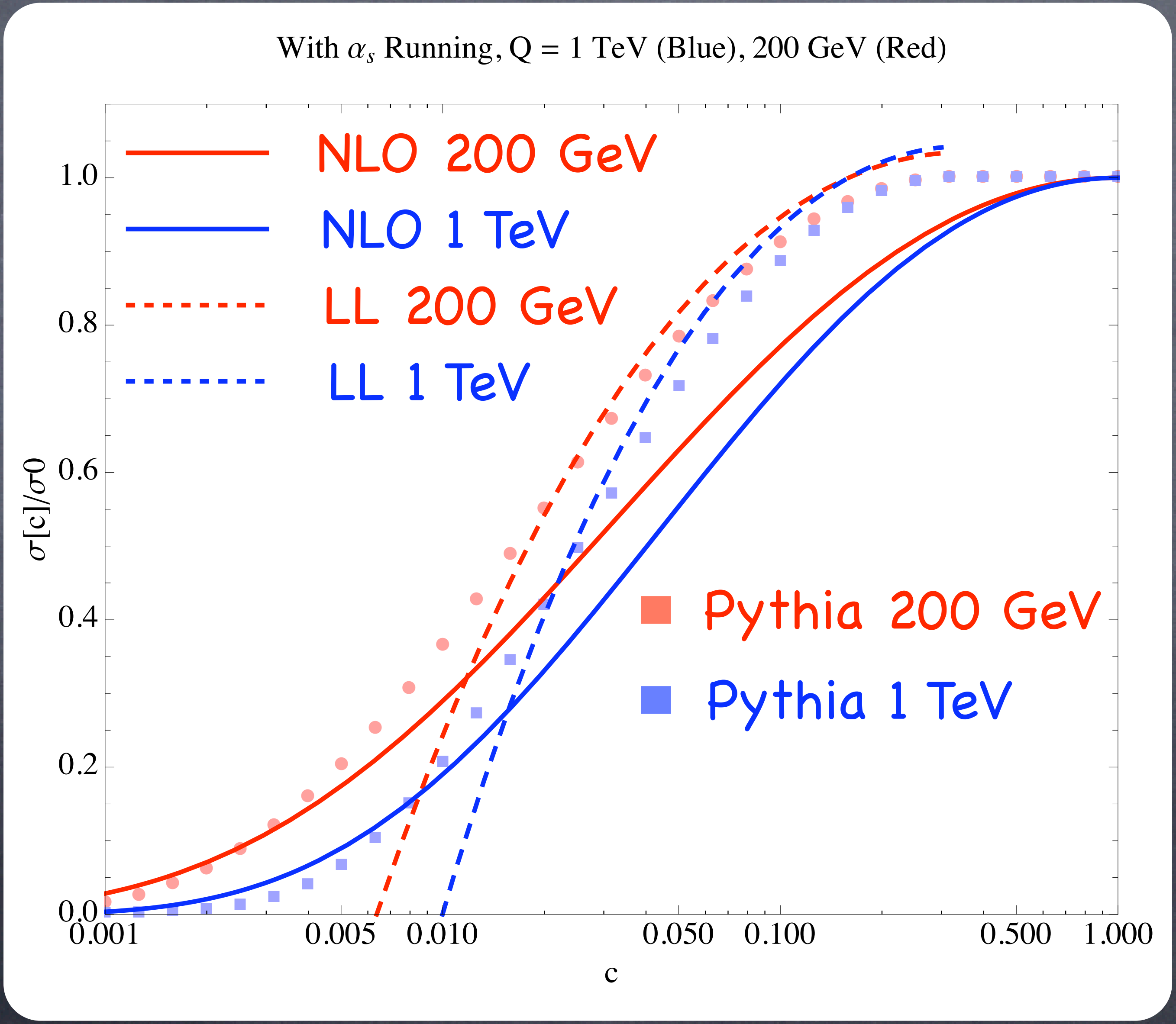
$$\sigma_2^{\rm NLO}(t_{\rm cut}) = \sigma_0 \left\{ 1 - \frac{\alpha_s(\mu) C_F}{4\pi} \left[4 \ln^2 \frac{t_{\rm cut}}{Q^2} + 6 \ln \frac{t_{\rm cut}}{Q^2} + 2 - \frac{2\pi^2}{3} + \mathcal{O}\left(\frac{t_{\rm cut}}{Q^2} \right) \right] \right\}$$

Double logs differ by factor of 2

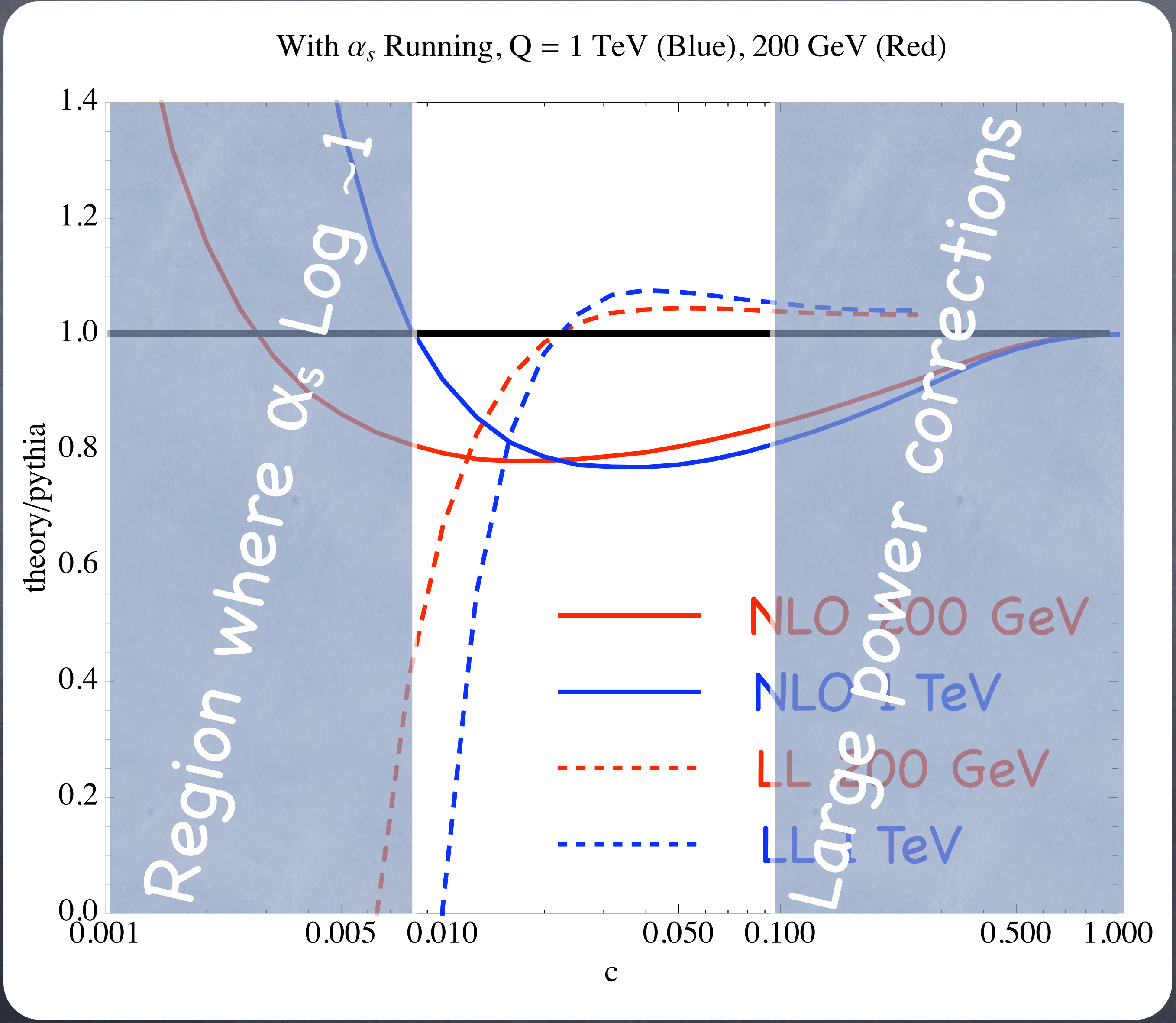
Herwig Sudakov properly reflects this

Final expression for observable will again agree

Preliminary Results



Preliminary Results



Things I did not talk about

- Kinematical logarithms
  - How to resum logs of $n_i \cdot n_j$ in soft function?
- Proof that parton shower will get LL right for any observable
- Extensions to higher jet multiplicities
- Momentum reshuffling, power corrections etc