Exclusive Jet Cross Sections with Beam Functions at the LHC

Frank Tackmann

Massachusetts Institute of Technology

SCET workshop
Ringberg Castle, April 6-9, 2010

Work with Iain Stewart, Wouter Waalewijn, Carola Berger, Claudio Marcantonini

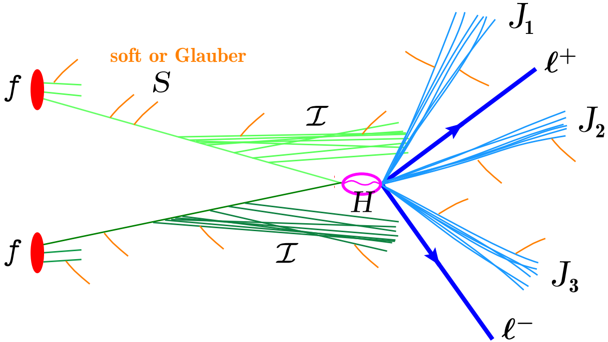
[arXiv:0910.0467, arXiv:1002.2213, (arXiv:1004:xxxx)^n]

Outline

Outline

Typical Hard Interaction at LHC

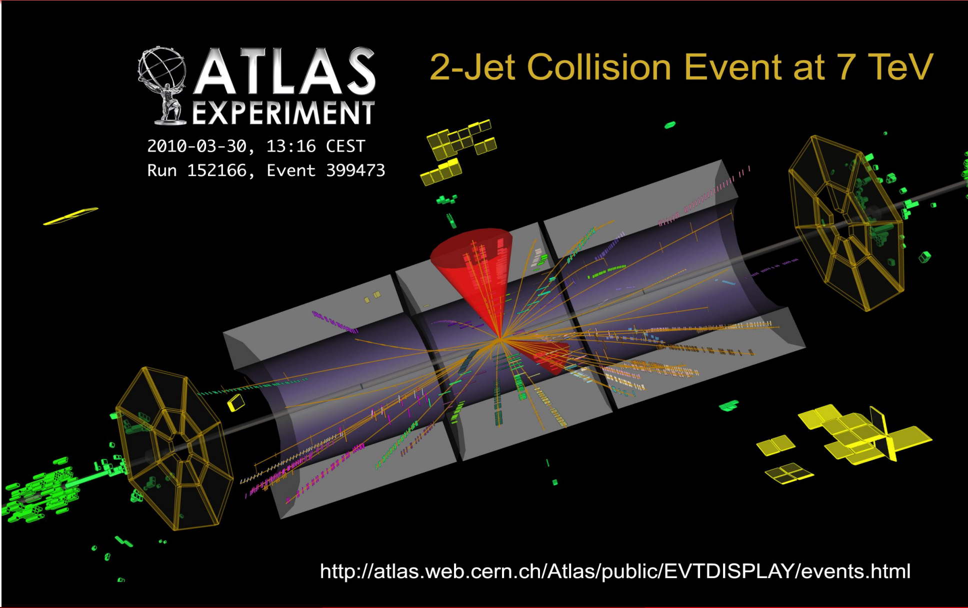
New physics hides at short distances in hard interaction



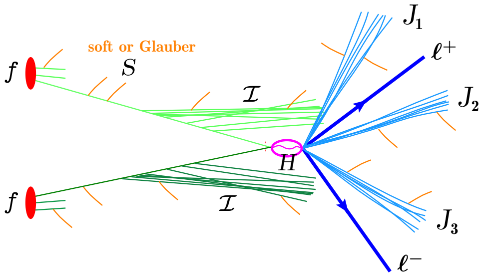
$d\sigma =$ PDFs $\otimes$ ISR $\otimes$ hard interaction $\otimes$ FSR $\otimes$ soft radiation

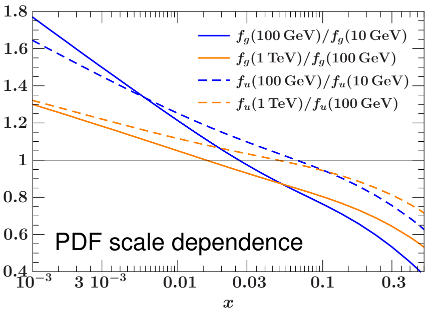
MC:	initial-state parton shower	matrix-element generator	final-state parton shower	hadronization underlying event

Actual Hard Interaction at LHC



Important to Understand Hadronic ISR

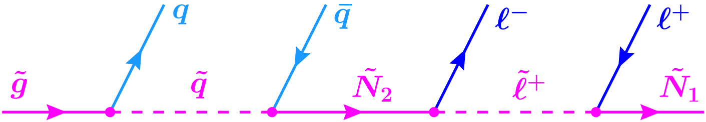




- ISR has important effects
  - Modifies parton luminosity available for hard interaction
  - Can contaminate signal jets (and leptons, photons)
- At LHC: Incoming gluons, large phase space available for ISR
- Modelled by initial-state parton shower (less understood than final-state)
- Proper physical scale to evaluate PDFs in measured cross section?

Looking for Higgs and New Physics

Hard interactions are identified by looking for signal with a characteristic number of jets plus leptons/photons



Background discrimination often requires a *veto on additional jets*

- SM processes with same signal signature and additional jets
  - $pp \to t\bar{t} \to WWb\bar{b}$ huge background for $h \to WW$
- Reconstructing new-physics masses and decay chains
  - Additional jets cause large combinatorial backgrounds
- Jets can fake signal leptons or photons (e.g. hard $\pi^0 \to \gamma\gamma$)
  - $pp \to jj$, $pp \to j\gamma$ major background to $h \to \gamma\gamma$

$\Rightarrow$ Want to measure *exclusive N-jet cross section* $pp \to XLNj$

Exclusive N-Jet Cross Section $pp \to X L N j$

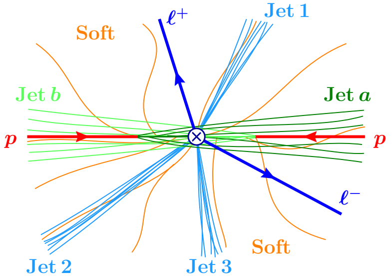
Jet veto defines what having "exactly N" jets means

- Restricts phase space of underlying inclusive N-jet cross section
- Restriction causes large phase-space double logs $\alpha_s^n \ln^m(\mu_J^2/\mu_H^2)$
  - μ_H: Hard interaction scale
  - μ_J: Jet resolution scale from distinction of N vs. $\geq N+1$ jets
  - $\mu_S \simeq \mu_J^2/\mu_H$: Induced soft scale

$\Rightarrow$ Forces kinematics into "SCET region" of N-jet phase space

Most natural cross section for SCET to compute

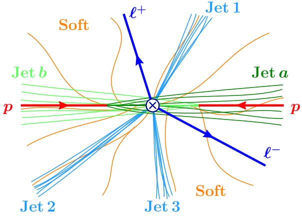
- 2 incoming collinear sectors (ISR)
- N outgoing collinear sectors (FSR)
- Soft sector (soft radiation)



How to Veto Jets

Analysis looking for signal with N jets

- Run jet algorithm to find all jets in the event
  - $<$ N jets: Cannot be signal $\longrightarrow$ throw away
  - N jets: Could be signal $\longrightarrow$ keep
  - $>$ N jets: Likely background $\longrightarrow$ throw away



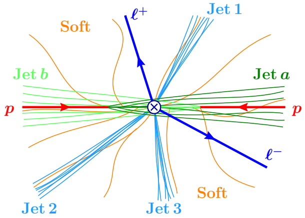
- Jet veto via jet algorithm yields very complicated phase-space restriction

$\Rightarrow$ Usually rely on parton shower Monte Carlo to sum leading logs

How to Veto Jets

Analysis looking for signal with N jets

- Run jet algorithm to find all jets in the event
  - $< N$ jets: Cannot be signal → throw away
  - N jets: Could be signal → keep
  - $> N$ jets: Likely background → throw away



- Jet veto via jet algorithm yields very complicated phase-space restriction

⇒ Usually rely on parton shower Monte Carlo to sum leading logs

Theory control of phase-space restriction in jet veto allows for

- Systematic summation of phase-space logs (beyond parton shower and leading log)
- Theory treatment of soft effects (beyond hadronization and underlying event models)

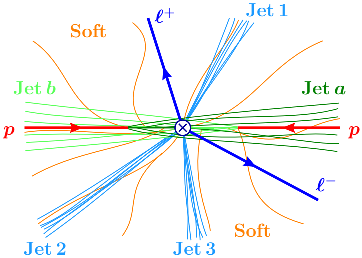
⇒ Want to veto jets by cutting on simple kinematic variable

N-Jettiness for $pp \to X L N j$

N-jettiness $$\tau_N = \frac{2}{Q^2} \sum_k \min\Bigl\{q_a \cdot p_k,\ q_b \cdot p_k,\ q_1 \cdot p_k,\ \ldots,\ q_N \cdot p_k\Bigr\}$$

- Use jet algorithm to select N signal jets
  Gives jet energies E_J and directions $\vec{n}_J$
- Define label momenta for beams and jets

$$q_a^\mu = x_a E_{\rm cm} \tfrac{1}{2} n_a^\mu \qquad n_a^\mu = (1, \vec{z})$$
$$q_b^\mu = x_b E_{\rm cm} \tfrac{1}{2} n_b^\mu \qquad n_b^\mu = (1, -\vec{z})$$
$$q_J^\mu = E_J n_J^\mu \qquad n_J^\mu = (1, \vec{n}_J)$$



- x_a and x_b defined by label (partonic) momentum conservation with q total momentum in L

$$q_a^\mu + q_b^\mu = q_1^\mu + \cdots + q_N^\mu + q^\mu \qquad Q^2 \equiv (q_a + q_b)^2 = x_a x_b E_{\rm cm}^2$$

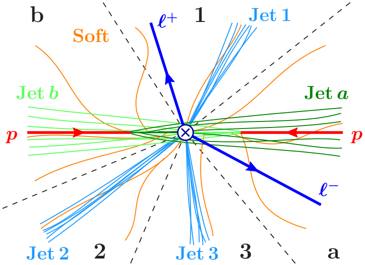
- $Y = \frac{1}{2} \ln(x_a / x_b)$ is boost of partonic center-of-mass frame

N-Jettiness for $pp \to X\,L\,N j$

N-jettiness
$$\tau_N = \frac{2}{Q^2}\sum_k \min\Big\{q_a\cdot p_k,\; q_b\cdot p_k,\; q_1\cdot p_k,\; \dots,\; q_N\cdot p_k\Big\}$$

Particles get associated with closest jet/beam

- Divides phase space into union of 2 beam regions and N jet regions
- Large contributions only from energetic particles not collinear to any beam or jet
  - $\tau_N \sim 1$: Additional hard jets
  - $\tau_N \ll 1$: N-jet SCET region



- Different jet algorithms only differ in treatment of soft particles
  - (must) give same label q_m up to p.c. $\Rightarrow \tau_N^{\rm alg.1} = \tau_N^{\rm alg.2} + \mathcal{O}(\tau_N^2)$

$\Rightarrow$ Implements jet veto via inclusive event shape, can sum logs $\alpha_s^n \ln^m \tau_N$

Factorization for N-Jettiness

$$\tau_N Q^2 = \sum_k \min_m \{2q_m \cdot p_k\}$$

Factorization for N-Jettiness

$$\tau_N Q^2 = \sum_k \min_m \{2q_m \cdot p_k\}$$

$$= \Big[\sum_{k\in \mathrm{coll}_a} + \sum_{k\in \mathrm{coll}_b} + \sum_J \sum_{k\in \mathrm{coll}_J} + \sum_{k\in \mathrm{soft}} \Big] \min_m \{2q_m \cdot p_k\}$$

- Separate into sum over 2 beam ($q_{a,b}$), N jet(q_J), soft sectors

Factorization for N-Jettiness

$$\tau_N Q^2 = \sum_k \min_m \{2q_m \cdot p_k\}$$

$$= \Bigg[\underbrace{\sum_{k \in \mathrm{coll}_a}} + \underbrace{\sum_{k \in \mathrm{coll}_b}} + \underbrace{\sum_J \sum_{k \in \mathrm{coll}_J}} + \sum_{k \in \mathrm{soft}} \Bigg] \min_m \{2q_m \cdot p_k\}$$

$$= \quad t_a \quad + \quad t_b \quad + \sum_J \quad s_J$$

- Separate into sum over 2 beam $(q_{a,b})$, N jet(q_J), soft sectors
- By p.c. collinear particles are always associated with their own sector

$$\sum_{k \in \mathrm{coll}_J} \min_m \{2q_m \cdot p_k\} = \sum_{k \in \mathrm{coll}_J} 2q_J \cdot p_k = s_J$$

$\Rightarrow$ Distribution of total collinear invariant masses t_a, t_b, s_J described by inclusive beam and jet functions $B(t)$ and $J(s)$

Factorization for N-Jettiness

$$\tau_N Q^2 = \sum_k \min_m \{2q_m \cdot p_k\}$$

$$= \Bigl[\underbrace{\sum_{k\in \mathrm{coll}_a}}_{} + \underbrace{\sum_{k\in \mathrm{coll}_b}}_{} + \underbrace{\sum_J \sum_{k\in \mathrm{coll}_J}}_{} + \underbrace{\sum_{k\in \mathrm{soft}}}_{} \Bigr] \min_m \{2q_m \cdot p_k\}$$

$$= t_a + t_b + \sum_J s_J + \tau_N^{\mathrm{soft}} Q^2$$

- Separate into sum over 2 beam $(q_{a,b})$, N jet(q_J), soft sectors
- By p.c. collinear particles are always associated with their own sector

$$\sum_{k\in \mathrm{coll}_J} \min_m \{2q_m \cdot p_k\} = \sum_{k\in \mathrm{coll}_J} 2q_J \cdot p_k = s_J$$

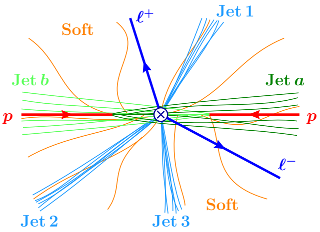
$\Rightarrow$ Distribution of total collinear invariant masses t_a, t_b, s_J described by inclusive beam and jet functions $B(t)$ and $J(s)$

- τ_N^{soft} described by τ_N-dependent soft function $S_N(\tau_N^{\mathrm{soft}}, \{q_m\})$

Factorization for N-Jet Cross Section ($\tau_N \ll 1$)

$$\frac{d\sigma^{F_N}}{d\tau_N} = \int dx_a dx_b \int d^4q\, d\Phi_L(q) \int d\Phi_N(\{q_J\})(2\pi)^4 \delta^4\Big(q_a + q_b - \sum_J q_J - q\Big)$$
$$\times F_N(\{q_m\}, L) \sum_{ij,\kappa} \mathrm{tr}\, \widehat{H}_{ij\to\kappa}(\{q_m\}, L, \mu)$$
$$\times \int dt_a\, B_i(t_a, x_a, \mu) \int dt_b\, B_j(t_b, x_b, \mu) \prod_J \int ds_J\, J_{\kappa_J}(s_J, \mu)$$
$$\times \widehat{S}_N^{ij\to\kappa}\Big(\tau_N - \frac{t_a + t_b + \sum_J s_J}{Q^2}, \{q_m\}, \mu\Big)$$

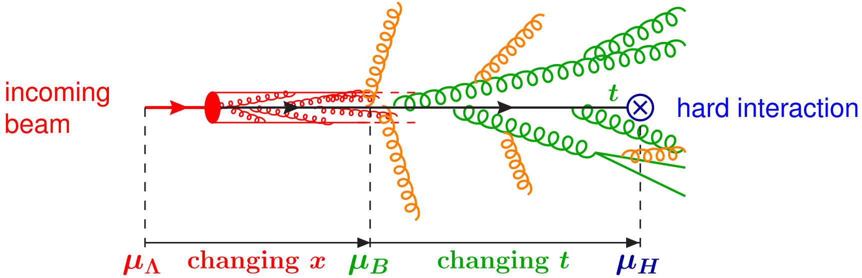
- $\widehat{H}_{ij\to\kappa}(\{q_m\}, L)$ contains hard scattering $i(q_a)j(q_b) \to L(q)\kappa_1(q_1)\cdots\kappa_N(q_N)$
- Measurement function $F_N(\{q_m\}, L)$ encodes signal requirements on L and N jets
- B_i, B_j describe ISR, J_{κ_J} describe FSR
- $\widehat{S}_N^{ij\to\kappa}$ describes soft radiation



Outline

Physical Picture of Initial State

Exclusive jet measurements probe PDFs at some intermediate scale μ_B



$\mu < \mu_B$: On-shell partons “inside” incoming proton

- ISR captured by PDF evolution redistributing x

$\mu > \mu_B$: Off-shell parton ($-t < 0$) inside incoming jet

- Colliding parton emits collinear and soft ISR outside proton
- Beam-function evolution redistributes t at fixed x

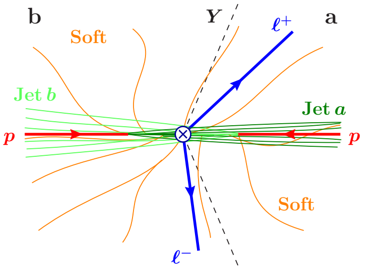
Beam Thrust for "Isolated" Drell-Yan ($N = 0$)

Simplest case to study ISR: $pp \to X\ell^+\ell^-\ 0j$

$Q = q^2$ (or q^+q^-) $\quad Y \equiv \ell^+\ell^-$ rapidity

$$q_a = x_a E_{\rm cm}\frac{n_a}{2} = e^{+Y} Q\,\frac{1}{2}(1, \vec{z})$$

$$q_b = x_b E_{\rm cm}\frac{n_b}{2} = e^{-Y} Q\,\frac{1}{2}(1, -\vec{z})$$



Beam thrust: ISR analog of thrust

$$\tau_B = \frac{2}{Q^2}\sum_k \min\{q_a\cdot p_k,\ q_b\cdot p_k\} = \sum_k \frac{|\vec{p}_{kT}|}{Q}\min\{e^{Y-\eta_k},\ e^{-Y+\eta_k}\}$$

Cut on $\tau_B \le e^{-2y_B^{\rm cut}} \ll 1$: 0-jet SCET region of Drell-Yan

- Vetos radiation with energy $\sim Q$ at rapidities $|Y - \eta_k| \lesssim y_B^{\rm cut} \sim 1$
- Allows energetic radiation for $|Y - \eta_k| \gtrsim y_B^{\rm cut} \sim 1$

Hemisphere Plus Momenta

$$\tau_B = \sum_k \frac{|\vec{p}_{kT}|}{Q} \min\{e^{Y-\eta_k}, e^{-Y+\eta_k}\}$$

$$= \frac{e^Y B_a^+(Y) + e^{-Y} B_b^+(Y)}{Q}$$

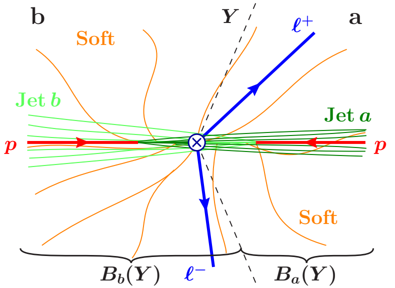
$$B_a^+(Y) = \sum_{\eta_k > Y} E_k (1 + \tanh\eta_k) e^{-2\eta_k}$$

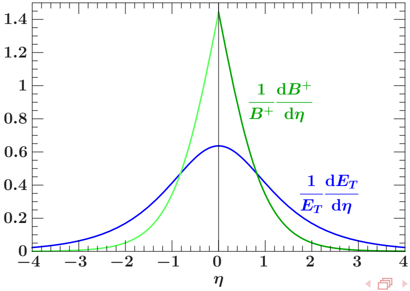
$$B_b^+(Y) = \sum_{\eta_k < Y} E_k (1 - \tanh\eta_k) e^{+2\eta_k}$$

- Accounts for boost Y of partonic cm
- Detector rapidity limit is no problem

$$\eta_{\rm det}^{\rm LHC} = 5 : 14\,\mathrm{TeV} e^{-10} = 0.6\,\mathrm{GeV}$$

$$\eta_{\rm det}^{\rm Tev} = 4 : \quad 2\,\mathrm{TeV} e^{-8} = 0.7\,\mathrm{GeV}$$





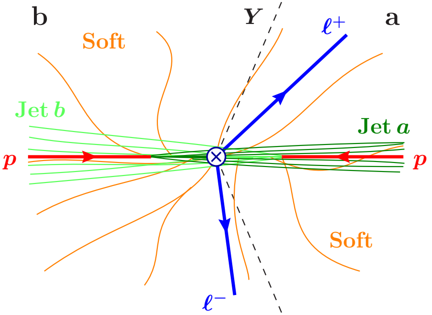
Beam Thrust Factorization Theorem

Factorization theorem for $\tau_B \ll 1$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y\,\mathrm{d}\tau_B} = \frac{8\pi\alpha_{\mathrm{em}}^2}{3N_c E_{\mathrm{cm}}^2 Q} \sum_{ij=\{q\bar{q},\bar{q}q\}} H_{ij}(Q^2,\mu)$$

$$\times \int \mathrm{d}t_a B_i(t_a, x_a, \mu) \int \mathrm{d}t_b B_j(t_b, x_b, \mu)\, S_B^{q\bar{q}}\Big(\tau_B - \frac{t_a + t_b}{Q^2}, \mu\Big)$$

$$\times \Big[1 + \mathcal{O}\Big(\frac{\Lambda_{\mathrm{QCD}}}{Q}, \sqrt{\tau_B}\Big)\Big]$$

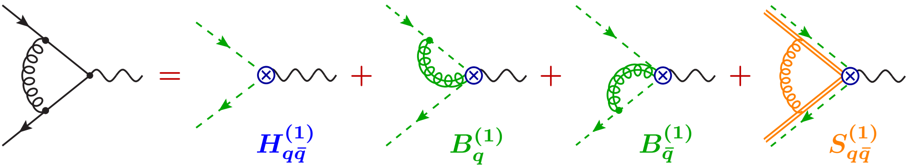


Relevant physical scales

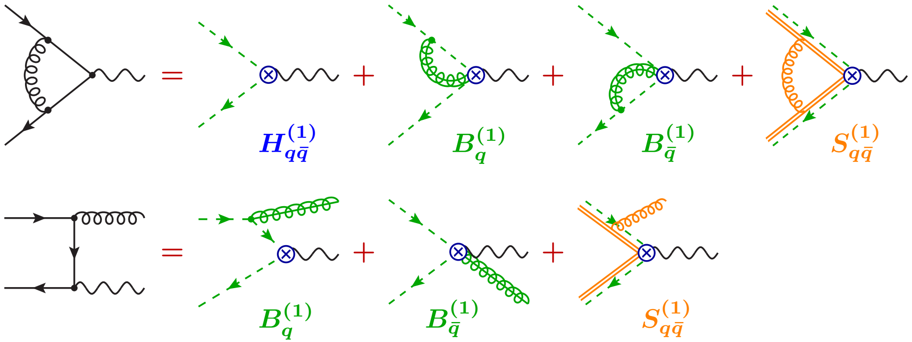
- Hard scale: $\mu_H \simeq Q$
- Beam scale: $\mu_B \simeq \sqrt{t} \simeq \sqrt{\tau_B} Q$
- Soft scale: $\mu_S \simeq \tau_B Q$

$\Rightarrow$ RGE running sums logs of τ_B

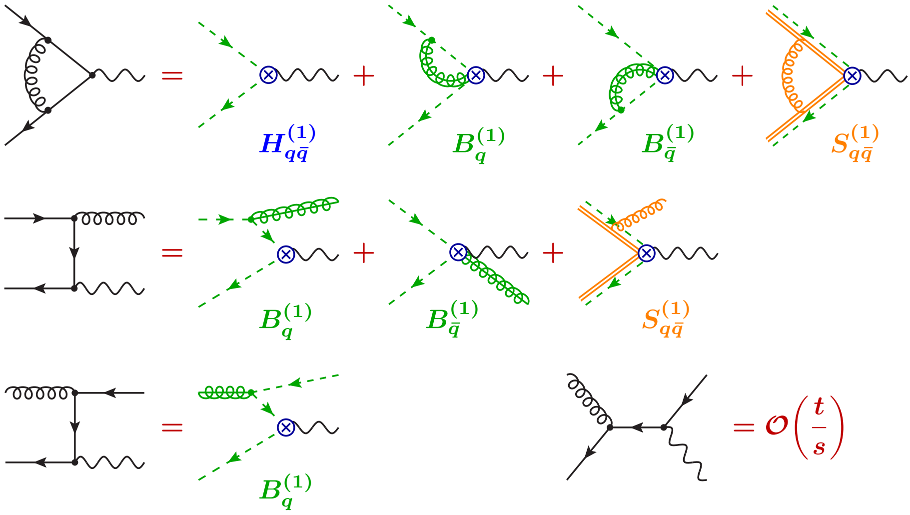
Correspondence with Fixed-Order Calculation



Correspondence with Fixed-Order Calculation



Correspondence with Fixed-Order Calculation

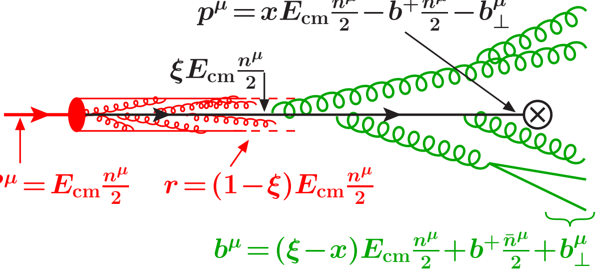


$\Rightarrow$ $B(t)$ resums t-channel singularities (vs. $J(s)$ resums s-channel sing.)

Outline

Beam Function Overview

$$B_i(t,x,\mu_B) = \sum_j \int_x^1 \frac{d\xi}{\xi}\, \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mu_B\Big) f_j(\xi,\mu_B) \Big[1 + \mathcal{O}\Big(\frac{\Lambda^2_{\rm QCD}}{t}\Big)\Big]$$



$-t = xE_{\rm cm}(-b^+) \leq 0$: (transverse) virtuality of annihilated parton

LO: $B_i(t,x,\mu_B) = \delta(t)\, f_i(x,\mu_B) + \cdots$ (already fixes PDF scale)

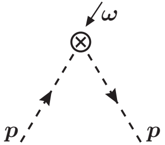
NLO: $\mu_B \simeq \sqrt{t}$, $t \neq 0$, $x \neq \xi$, $i \neq j$

SCET Operator Definition of PDF

SCET operator definition for arbitrary ξ

$$\mathcal{Q}_q^{\rm bare}(\omega) = \theta(\omega)\, \bar\chi_n(0)\frac{\bar{n}\!\!\!/}{2}\big[\delta(\omega - \overline{\mathcal{P}}_n)\chi_n(0)\big]$$

$$f_q(\xi = \omega/P^-, \mu) = \big\langle p_n(P^-)\big|\mathcal{Q}_q(\omega,\mu)\big|p_n(P^-)\big\rangle$$



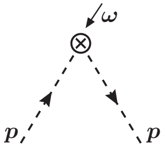
- Fields require *no zero-bin subtractions* (soft region cancels in PDF)
- RPI-III invariance: f only depends on $\xi = \omega/P^-$

SCET Operator Definition of PDF

SCET operator definition for arbitrary ξ

$$\mathcal{Q}_q^{\rm bare}(\omega) = \theta(\omega)\, \bar\chi_n(0) \frac{\bar{n}\!\!\!/}{2} \big[\delta(\omega - \overline{\mathcal{P}}_n)\chi_n(0)\big]$$

$$f_q(\xi = \omega/P^-, \mu) = \big\langle p_n(P^-) \big| \mathcal{Q}_q(\omega, \mu) \big| p_n(P^-) \big\rangle$$



- Fields require *no zero-bin subtractions* (soft region cancels in PDF)
- RPI-III invariance: f only depends on $\xi = \omega/P^-$

Standard QCD definition

$$f_q(\omega'/P^-, \mu) = \theta(\omega') \int \frac{\mathrm{d}y^+}{4\pi}\, e^{-\mathrm{i}\omega' y^+/2} \times \Big\langle p_n(P^-) \Big| \Big[\bar\psi\Big(y^+\frac{\bar n}{2}\Big) W\Big(y^+\frac{\bar n}{2}, 0\Big) \frac{\bar{n}\!\!\!/}{2} \psi(0)\Big]_\mu \Big| p_n(P^-) \Big\rangle$$

- Position-space version of $W_n \delta(\omega - \overline{\mathcal{P}}_n) W_n^\dagger$

$\Rightarrow$ SCET PDF is *exactly equivalent* to QCD PDF

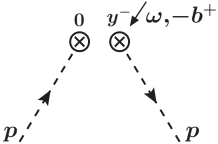
Operator Definition of Beam Function

Bare beam-function operator

$$\widetilde{\mathcal{O}}_q^{\mathrm{bare}}(y^-,\omega) = e^{-\mathrm{i}\hat{p}^+ y^-/2}\,\bar{\chi}_n\Big(y^-\frac{n}{2}\Big)\frac{\bar{n}\!\!\!/}{2}\big[\delta(\omega-\overline{\mathcal{P}}_n)\chi_n(0)\big]$$

$$\mathcal{O}_q^{\mathrm{bare}}(t,\omega) = \frac{1}{2\pi}\int\frac{\mathrm{d}y^-}{2|\omega|}\,e^{\mathrm{i}ty^-/2\omega}\,\widetilde{\mathcal{O}}_q^{\mathrm{bare}}(y^-,\omega)$$

$$= \bar{\chi}_n(0)\,\delta(t-\omega\hat{p}^+)\,\frac{\bar{n}\!\!\!/}{2}\big[\delta(\omega-\overline{\mathcal{P}}_n)\chi_n(0)\big]$$



- Fields separated in y^- coordinate, defined *with zero-bin subtractions*
- $\hat{p}^+$ residual momentum operator

Beam function defined by proton matrix element of renormalized operator

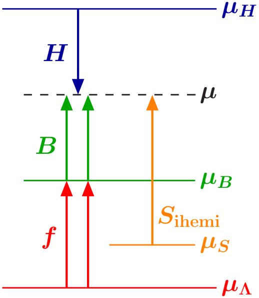
$$B_q(t, x=\omega/P^-,\mu) = \big\langle p_n(P^-)\big|\theta(\omega)\mathcal{O}_q(t,\omega,\mu)\big|p_n(P^-)\big\rangle$$

- RPI-III invariance: B only depends on t and $x=\omega/P^-$

Beam Function Renormalization

RGE for PDF (DGLAP) mixes ξ and i, sums single logs

$$\mu \frac{d}{d\mu} f_i(\xi, \mu) = \sum_j \int \frac{d\xi'}{\xi'} \gamma^f_{ij}\Big(\frac{\xi}{\xi'}, \mu\Big)\, f_j(\xi', \mu)$$



Beam function RGE sums double logs of t

$$\mu \frac{d}{d\mu} B_i(t, \omega, \mu) = \int dt' \gamma^i_B(t - t', \mu)\, B_i(t', \omega, \mu)$$

$$\gamma^i_B(t, \mu) = -2\Gamma^i_{\rm cusp}(\alpha_s)\, \frac{1}{\mu^2} \Big[\frac{\theta(t)}{t/\mu^2}\Big]_+ + \gamma^i_B(\alpha_s)\, \delta(t)$$

All-order structure follows from consistency and Wilson-line renormalization

- No mixing between operators with different ω or i (in contrast to PDF)
- $\gamma^i_B(t) = \gamma^i_J(t)$ (jet function can be related to the same operator)

OPE and Matching onto PDFs

Perform OPE about $y^- \to 0$ limit

$$\widetilde{\mathcal{O}}_i(y^-, \omega, \mu) = \widetilde{J}_i\Big(\frac{y^-}{2\omega}, \mu\Big) 1 + \sum_j \int \frac{d\omega'}{\omega'} \widetilde{\mathcal{I}}_{ij}\Big(\frac{y^-}{2\omega}, \frac{\omega}{\omega'}, \mu\Big) \mathcal{Q}_j(\omega', \mu) + \mathcal{O}\Big(\frac{y^-}{\omega}\Big)$$

$$\mathcal{O}_i(t, \omega, \mu) = \widehat{J}_i(-t, \mu) 1 + \sum_j \int \frac{d\omega'}{\omega'} \mathcal{I}_{ij}\Big(t, \frac{\omega}{\omega'}, \mu\Big) \mathcal{Q}_j(\omega', \mu) + \mathcal{O}\Big(\frac{y^-}{\omega}\Big)$$

- Structure fixed by SCET symmetries (RPI-III)
- $\widehat{J}$ and $\widetilde{J}$ related to jet function (drops out in matrix element)

OPE and Matching onto PDFs

Perform OPE about $y^- \to 0$ limit

$$\widetilde{\mathcal{O}}_i(y^-,\omega,\mu) = \widetilde{J}_i\Big(\frac{y^-}{2\omega},\mu\Big)\mathbb{1} + \sum_j \int \frac{d\omega'}{\omega'}\,\widetilde{\mathcal{I}}_{ij}\Big(\frac{y^-}{2\omega},\frac{\omega}{\omega'},\mu\Big)\mathcal{Q}_j(\omega',\mu) + \mathcal{O}\Big(\frac{y^-}{\omega}\Big)$$

$$\mathcal{O}_i(t,\omega,\mu) = \widehat{J}_i(-t,\mu)\mathbb{1} + \sum_j \int \frac{d\omega'}{\omega'}\,\mathcal{I}_{ij}\Big(t,\frac{\omega}{\omega'},\mu\Big)\mathcal{Q}_j(\omega',\mu) + \mathcal{O}\Big(\frac{y^-}{\omega}\Big)$$

- Structure fixed by SCET symmetries (RPI-III)
- $\widehat{J}$ and $\widetilde{J}$ related to jet function (drops out in matrix element)

Taking proton matrix element yields

$$B_i(t,x,\mu) = \sum_j \int_x^1 \frac{d\xi}{\xi}\,\mathcal{I}_{ij}\Big(t,\frac{x}{\xi},\mu\Big) f_j(\xi,\mu)\Big[1 + \mathcal{O}\Big(\frac{\Lambda^2_{\rm QCD}}{t}\Big)\Big]$$

- First obtained for $\mathcal{I}_{gg}$ by Fleming, Leibovich, Mehen (2006) from moment-space OPE

One-Loop Matching Calculation

Compute $\mathcal{I}_{ij}(t, z)$ perturbatively by taking partonic external states

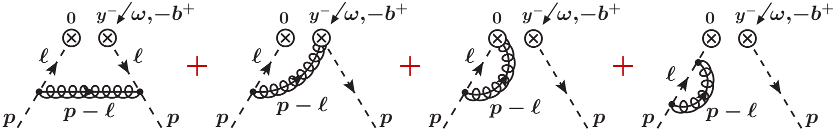
$$\langle \mathcal{O}_i(t, \omega, \mu) \rangle = \sum_j \int \frac{\mathrm{d}\omega'}{\omega'} \mathcal{I}_{ij}\Big(t, \frac{\omega}{\omega'}, \mu\Big) \langle \mathcal{Q}_j(\omega', \mu) \rangle$$

One-Loop Matching Calculation

Compute $\mathcal{I}_{ij}(t, z)$ perturbatively by taking partonic external states

$$\langle \mathcal{O}_i(t, \omega, \mu) \rangle = \sum_j \int \frac{d\omega'}{\omega'} \mathcal{I}_{ij}\Big(t, \frac{\omega}{\omega'}, \mu\Big) \langle \mathcal{Q}_j(\omega', \mu) \rangle$$

$\mathcal{I}_{qq}^{(1)} =$

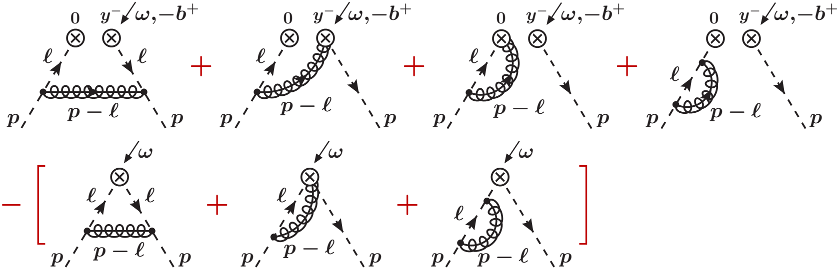


One-Loop Matching Calculation

Compute $\mathcal{I}_{ij}(t, z)$ perturbatively by taking partonic external states

$$\langle \mathcal{O}_i(t, \omega, \mu) \rangle = \sum_j \int \frac{\mathrm{d}\omega'}{\omega'} \mathcal{I}_{ij}\Big(t, \frac{\omega}{\omega'}, \mu\Big) \langle \mathcal{Q}_j(\omega', \mu) \rangle$$

$\mathcal{I}_{qq}^{(1)} =$

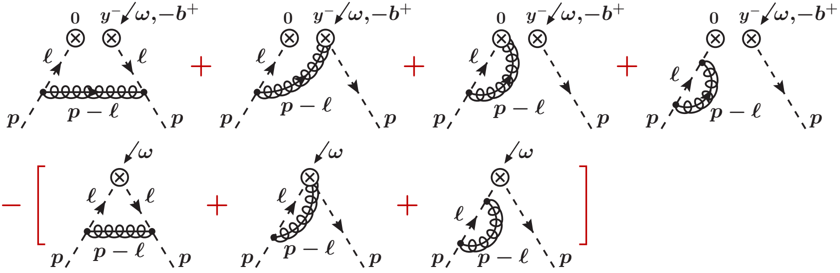


One-Loop Matching Calculation

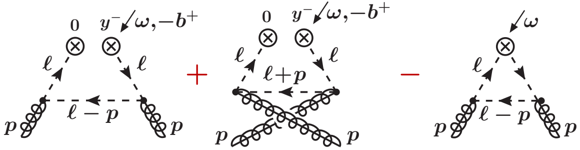
Compute $\mathcal{I}_{ij}(t, z)$ perturbatively by taking partonic external states

$$\langle \mathcal{O}_i(t, \omega, \mu) \rangle = \sum_j \int \frac{d\omega'}{\omega'} \mathcal{I}_{ij}\Big(t, \frac{\omega}{\omega'}, \mu\Big) \langle \mathcal{Q}_j(\omega', \mu) \rangle$$

$\mathcal{I}_{qq}^{(1)} =$



$\mathcal{I}_{qg}^{(1)} =$



NLO Result

$$B_i(t, x, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mu\Big) f_j(\xi, \mu)$$

$$\mathcal{I}_{qq}(t, z, \mu) = \delta(t)\,\delta(1-z) + \frac{\alpha_s(\mu) C_F}{2\pi}\,\theta(z)$$
$$\times \Big\{ \frac{2}{\mu^2}\mathcal{L}_1\Big(\frac{t}{\mu^2}\Big)\delta(1-z) + \frac{1}{\mu^2}\mathcal{L}_0\Big(\frac{t}{\mu^2}\Big)\Big[P_{qq}(z) - \frac{3}{2}\,\delta(1-z)\Big]$$
$$+ \delta(t)\Big[\mathcal{L}_1(1-z)(1+z^2) - \frac{\pi^2}{6}\,\delta(1-z) + \theta(1-z)\Big(1-z - \frac{1+z^2}{1-z}\ln z\Big)\Big]\Big\}$$

$$\mathcal{I}_{qg}(t, z, \mu) = \frac{\alpha_s(\mu)}{4\pi}\theta(z)\Big\{\frac{1}{\mu^2}\mathcal{L}_0\Big(\frac{t}{\mu^2}\Big) P_{qg}(z) + \delta(t)\Big[P_{qg}(z)\Big(\ln\frac{1-z}{z} - 1\Big) + \theta(1-z)\Big]\Big\}$$

- $P_{qq}(z)$, $P_{qg}(z)$ AP splitting functions, $\mathcal{L}_n(y) = [\theta(y)(\ln y)^n/y]_+$
- Logs are minimized for $\mu^2 \simeq t$
- Nontrivial check: μ dependence of $\mathcal{I}_{ij}(t, z, \mu)$ indeed converts PDF running into beam-function running

Relation to Hadronic Threshold Limit

Hadronic threshold limit $Q^2 \to E_{\mathrm{cm}}^2$

- Indirectly forces *total energy* outside signal jets to be small
  - Vetoes additional jets and collinear ISR, only allows soft ISR
  - Defines even more exclusive N-jet cross section
- Can resum phase-space double logs $\ln(1 - Q^2/E_{\mathrm{cm}}^2)$

Relation to Hadronic Threshold Limit

Hadronic threshold limit $Q^2 \to E_{\rm cm}^2$

- Indirectly forces *total energy* outside signal jets to be small
  - Vetoes additional jets and collinear ISR, only allows soft ISR
  - Defines even more exclusive N-jet cross section
- Can resum phase-space double logs $\ln(1 - Q^2/E_{\rm cm}^2)$
- Corresponds to taking $x \to 1$ in $B_i(t, x, \mu)$

$$\mathcal{I}_{qq}^{\rm thresh}(t, z, \mu) = \delta(t)\delta(1-z) + \frac{\alpha_s(\mu)C_F}{2\pi}\,\theta(z)\Big\{\frac{2}{\mu^2}\mathcal{L}_1\Big(\frac{t}{\mu^2}\Big)\delta(1-z) + \frac{2}{\mu^2}\mathcal{L}_0\Big(\frac{t}{\mu^2}\Big)\mathcal{L}_0(1-z) + \delta(t)\Big[2\mathcal{L}_1(1-z) - \frac{\pi^2}{6}\delta(1-z)\Big]\Big\}$$

$$\mathcal{I}_{qg}^{\rm thresh}(t, z, \mu) = 0$$

Our full result contains fixed-order threshold terms and in addition

- Resums t-channel singularities
- Includes parton mixing contributions

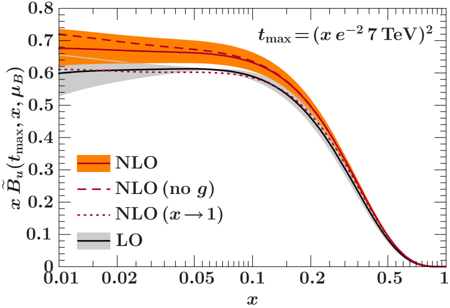
Beam Function at NLO

Consider integrated beam function

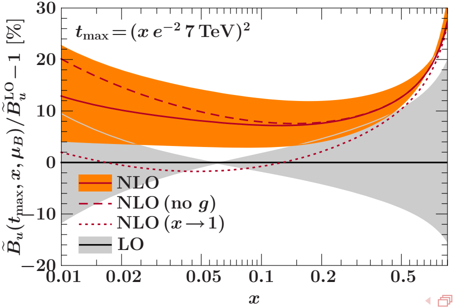
$$\widetilde{B}_i(t_{\max}, x, \mu_B) = \int \mathrm{d}t\, B_i(t, x, \mu_B)\, \theta(t_{\max} - t)$$

- Take $\sqrt{t_{\max}} = e^{-y^{\rm cut}} Q = e^{-2}\, x 7\,\mathrm{TeV}$
- Vary $\sqrt{t_{\max}}/2 \le \mu_B \le 2\sqrt{t_{\max}}$

$x\widetilde{B}_u(t_{\max}, x, \mu_B)$



$\widetilde{B}_u(t_{\max}, x, \mu_B)/f_u(x, \mu_B) - 1$



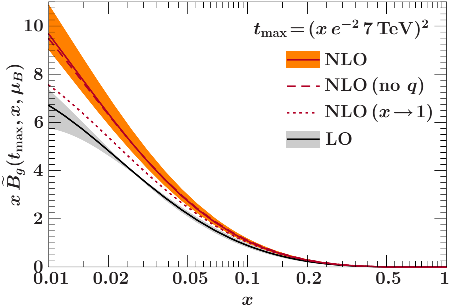
Beam Function at NLO

Consider integrated beam function

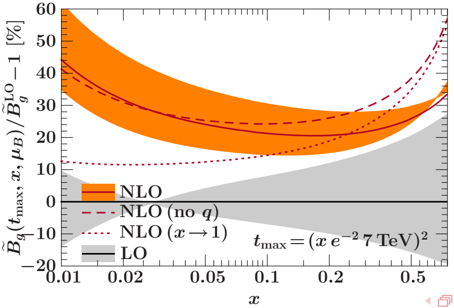
$$\widetilde{B}_i(t_{\max}, x, \mu_B) = \int \mathrm{d}t\, B_i(t, x, \mu_B)\, \theta(t_{\max} - t)$$

- Take $\sqrt{t_{\max}} = e^{-y^{\mathrm{cut}}} Q = e^{-2}\, x 7\,\mathrm{TeV}$
- Vary $\sqrt{t_{\max}}/2 \le \mu_B \le 2\sqrt{t_{\max}}$

$x\widetilde{B}_g(t_{\max}, x, \mu_B)$



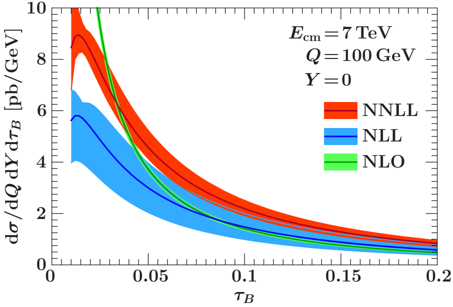
$\widetilde{B}_g(t_{\max}, x, \mu_B)/f_g(x, \mu_B) - 1$



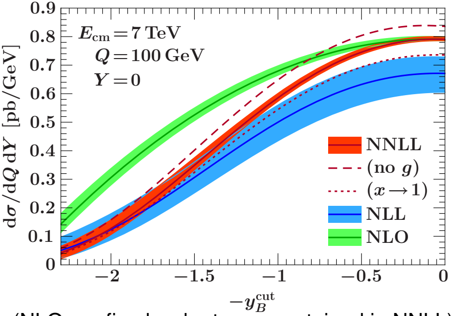
Outline

Drell-Yan Beam Thrust Cross Section

differential in τ_B



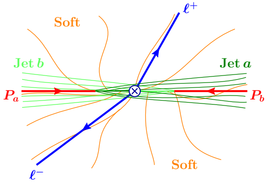
integrated over $\tau_B \leq e^{-2y_B^{cut}}$



[MSTW2008 NLO PDFs]

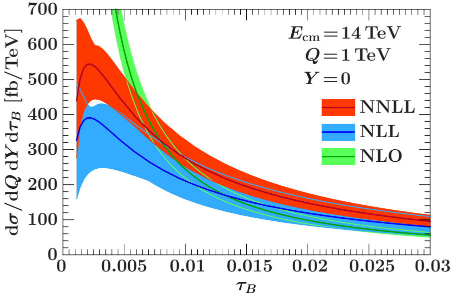
(NLO are fixed-order terms contained in NNLL)

- Most of cross section comes from small τ_B
- Summation of $\ln \tau_B$ is crucial
- Perturbative uncertainties are envelope of separate μ_H, μ_B, μ_S variation
- S_B perturbative in tail, nonperturbative in peak

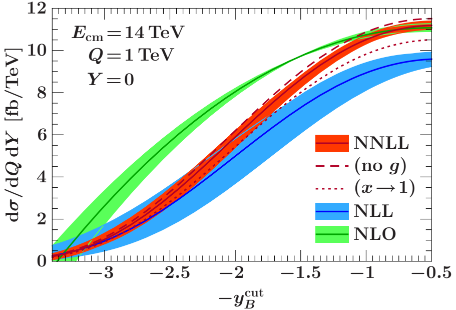


Drell-Yan Beam Thrust Cross Section

differential in τ_B



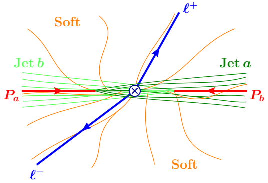
integrated over $\tau_B \leq e^{-2y_B^{cut}}$



[MSTW2008 NLO PDFs]

(NLO are fixed-order terms contained in NNLL)

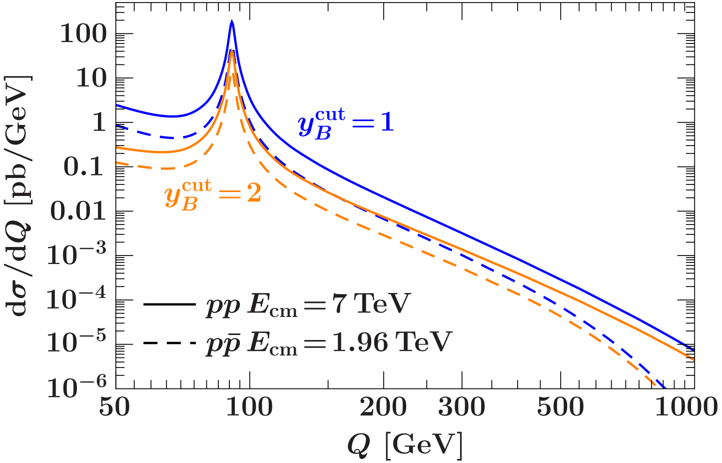
- Most of cross section comes from small τ_B
- Summation of $\ln \tau_B$ is crucial
- Perturbative uncertainties are envelope of separate μ_H, μ_B, μ_S variation
- S_B perturbative in tail, nonperturbative in peak



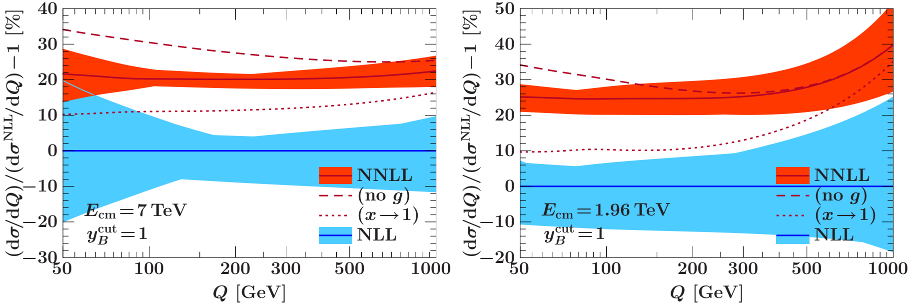
Drell-Yan Cross Section With Jet Veto at NNLL

Cross section integrated over Y with inclusive jet veto $\tau_B \leq e^{-2y_B^{\mathrm{cut}}}$

[MSTW2008 NLO PDFs]



Theory Uncertainties



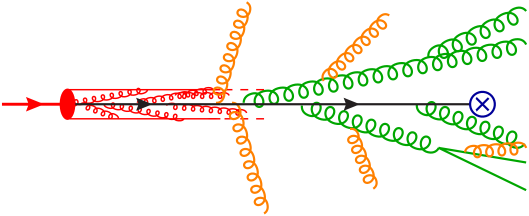
Fixed-order gives sizable correction from NLL to NNLL

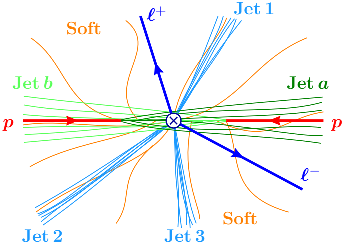
- Gluon contribution of f_g to B_q can be important, reduces cross section
- Threshold terms give poor approximation

Uncertainties

- $5\% - 10\%$ at NNLL ($10\% - 20\%$ for $y_B^{cut} = 2$) perturbative uncertainties
- PDF uncertainties are $\sim 5\%$

Summary





Experiments need to veto jets, measure exclusive jet cross sections

- Phase-space cuts in jet vetoes cause large logs

$\Rightarrow$ Need to go beyond parton shower and LL

N-jettiness: Inclusive event shape to veto jets

- Can systematically sum phase-space logarithms
- Beam Thrust: Factorization and NNLL resummation

Beam functions describe ISR in excl. jet cross section

- Renormalization and NLO matching for quark and gluon beam functions

Outline

Backup Slides

Generalizations of τ_N

$$\tau_N^d = \sum_k \min\{d_a(p_k), d_b(p_k), d_1(p_k), \ldots, d_N(p_k)\}$$

Can use any soft-collinear distance measure $d_m(p_k)$

- Previously invariant mass: $d_m(p_k) = 2q_m \cdot p_k / Q^2$
- $d_m(p_k) \to d_m(p_k)^\alpha$: Same regions with different relative weight between center and periphery
- Boost invariant geometric measure independent of $|\vec{q}_{JT}|$

$$d_{a,b}(p_k) = \frac{|\vec{p}_{kT}|}{Q} e^{\pm(Y-\eta_k)} \qquad d_J(p_k) = \frac{|\vec{p}_{kT}|}{Q} 2(\cosh\Delta\eta_{Jk} - \cos\Delta\phi_{Jk})$$

$\Rightarrow$ Use as N-jet algorithm by minimizing τ_N with respect to Y and $\vec{n}_J$ (N-jet analog of thrust axis in $e^+e^- \to$ jets)

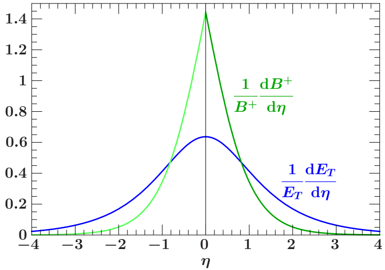
Compare B^+ to Transverse Energy and Momentum

$$B_a^+ = \sum_{\eta_k>0} (E_k - p_k^z) = \sum_{\eta_k>0} E_k (1 + \tanh\eta_k) e^{-2\eta_k}$$

$$E_T = \sum_k |\vec{p}_{k\,T}| = \sum_k E_k (1 + \tanh\eta_k) e^{-\eta_k}$$

$$\vec{p}_T = \sum_k \vec{p}_{k\,T}$$

	B^+	E_T	$\vec{p}_T$
Linear in momenta	✓	×	✓
Constrains central radiation	✓	(✓)	×
Insensitive to very forward (detector limit)	✓	(✓)	✓
Boost invariant along z	×	✓	✓



⇒ B^+ is natural scalar quantity complementary to vector $\vec{p}_T$
(Boost invariance is fixed by τ_B)

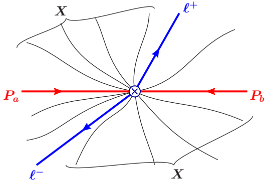
Beam Thrust Cross Section in Perturbation Theory

Integrate over $\tau_B \leq \tau_B^{\mathrm{cut}} = e^{-2y_B^{\mathrm{cut}}}$ with $L = \ln \tau_B^{\mathrm{cut}} = -2y_B^{\mathrm{cut}}$

$$\begin{aligned}\sigma(\tau_B^{\mathrm{cut}}) = \sigma_0\big[1 &+ \alpha_s(c_{12}L^2 + c_{11}L + c_{10}) \\ &+ \alpha_s^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) \\ &+ \alpha_s^3(c_{36}L^6 + c_{35}L^5 + c_{34}L^4 + c_{33}L^3 + \cdots) \\ &+ \cdots\big]\end{aligned}$$

$\tau_B^{\mathrm{cut}} \sim 1$ or $y_B^{\mathrm{cut}} \sim 0$: No veto on central jets

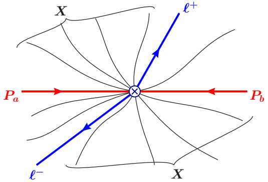
- Total (inclusive) cross section, logs don't matter
- Can calculate to fied order in α_s: LO, NLO



Beam Thrust Cross Section in Perturbation Theory

Integrate over $\tau_B \leq \tau_B^{\rm cut} = e^{-2y_B^{\rm cut}}$ with $L = \ln \tau_B^{\rm cut} = -2y_B^{\rm cut}$

$$\begin{aligned}\sigma(\tau_B^{\rm cut}) = \sigma_0\big[1 &+ \alpha_s(c_{12}L^2 + c_{11}L + c_{10}) \\ &+ \alpha_s^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) \\ &+ \alpha_s^3(c_{36}L^6 + c_{35}L^5 + c_{34}L^4 + c_{33}L^3 + \cdots) \\ &+ \cdots\big]\end{aligned}$$



$\tau_B^{\rm cut} \sim 1$ or $y_B^{\rm cut} \sim 0$: No veto on central jets

- Total (inclusive) cross section, logs don't matter
- Can calculate to fied order in α_s: LO, NLO

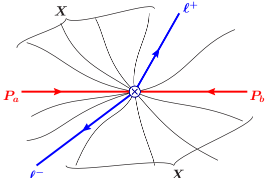
Beam Thrust Cross Section in Perturbation Theory

Integrate over $\tau_B \leq \tau_B^{\rm cut} = e^{-2y_B^{\rm cut}}$ with $L = \ln \tau_B^{\rm cut} = -2y_B^{\rm cut}$

$$\sigma(\tau_B^{\rm cut}) = \sigma_0 \Big[1 + \alpha_s(c_{12}L^2 + c_{11}L + c_{10}) + \alpha_s^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) + \alpha_s^3(c_{36}L^6 + c_{35}L^5 + c_{34}L^4 + c_{33}L^3 + \cdots) + \cdots\Big]$$

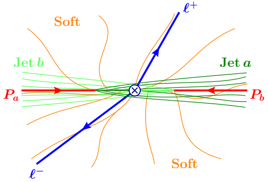
$\tau_B^{\rm cut} \sim 1$ or $y_B^{\rm cut} \sim 0$: No veto on central jets

- Total (inclusive) cross section, logs don't matter
- Can calculate to fied order in α_s: LO, NLO



$\tau_B^{\rm cut} \ll 1$ or $y_B^{\rm cut} \gtrsim 1$: Veto central jets

- Large logs spoil convergence, must sum them
- LL (parton shower), NLL, NNLL



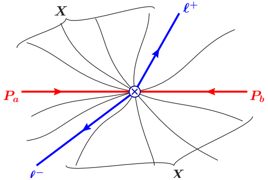
Beam Thrust Cross Section in Perturbation Theory

Integrate over $\tau_B \leq \tau_B^{\rm cut} = e^{-2y_B^{\rm cut}}$ with $L = \ln \tau_B^{\rm cut} = -2y_B^{\rm cut}$

$$\begin{aligned}\sigma(\tau_B^{\rm cut}) = \sigma_0\Big[1 &+ \alpha_s(c_{12}L^2 + c_{11}L + c_{10}) \\ &+ \alpha_s^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) \\ &+ \alpha_s^3(c_{36}L^6 + c_{35}L^5 + c_{34}L^4 + c_{33}L^3 + \cdots) \\ &+ \cdots\Big]\end{aligned}$$

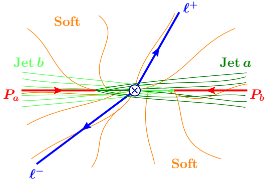
$\tau_B^{\rm cut} \sim 1$ or $y_B^{\rm cut} \sim 0$: No veto on central jets

- Total (inclusive) cross section, logs don't matter
- Can calculate to fied order in α_s: LO, NLO



$\tau_B^{\rm cut} \ll 1$ or $y_B^{\rm cut} \gtrsim 1$: Veto central jets

- Large logs spoil convergence, must sum them
- LL (parton shower), NLL, NNLL



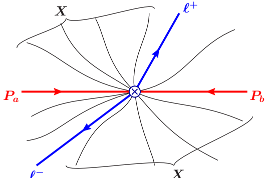
Beam Thrust Cross Section in Perturbation Theory

Integrate over $\tau_B \leq \tau_B^{\mathrm{cut}} = e^{-2y_B^{\mathrm{cut}}}$ with $L = \ln \tau_B^{\mathrm{cut}} = -2y_B^{\mathrm{cut}}$

$$
\begin{aligned}
\sigma(\tau_B^{\mathrm{cut}}) = \sigma_0 \Big[1 &+ \alpha_s (c_{12} L^2 + c_{11} L + c_{10}) \\
&+ \alpha_s^2 (c_{24} L^4 + c_{23} L^3 + c_{22} L^2 + c_{21} L + c_{20}) \\
&+ \alpha_s^3 (c_{36} L^6 + c_{35} L^5 + c_{34} L^4 + c_{33} L^3 + \cdots) \\
&+ \cdots \Big]
\end{aligned}
$$

$\tau_B^{\mathrm{cut}} \sim 1$ or $y_B^{\mathrm{cut}} \sim 0$: No veto on central jets

- Total (inclusive) cross section, logs don't matter
- Can calculate to fied order in α_s: LO, NLO



$\tau_B^{\mathrm{cut}} \ll 1$ or $y_B^{\mathrm{cut}} \gtrsim 1$: Veto central jets

- Large logs spoil convergence, must sum them
- LL (parton shower), NLL, NNLL