Glauber Gluons & SCET

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earlier stages:
Christopher Lee, Zoltan Ligeti

Introduction

Bowdin, Brodsky, Lepage, `81
Collins, Soper, Sterman, `82
Bowdin, `85

Where the "Glauber" Issue Arises?

- Factorization of the Drell-Yan process
- Loop diagrams contain a Glauber region which gives a leading order IR divergent contribution (on top of Soft and Collinear regions)
- "Glaubers" break the traditional factorization of the exclusive Drell-Yan cross-section
- In the inclusive cross-section this contribution cancels: $G+G^{*}=0$ and factorization is restored

Do we need Glauber modes in the Effective Theory?

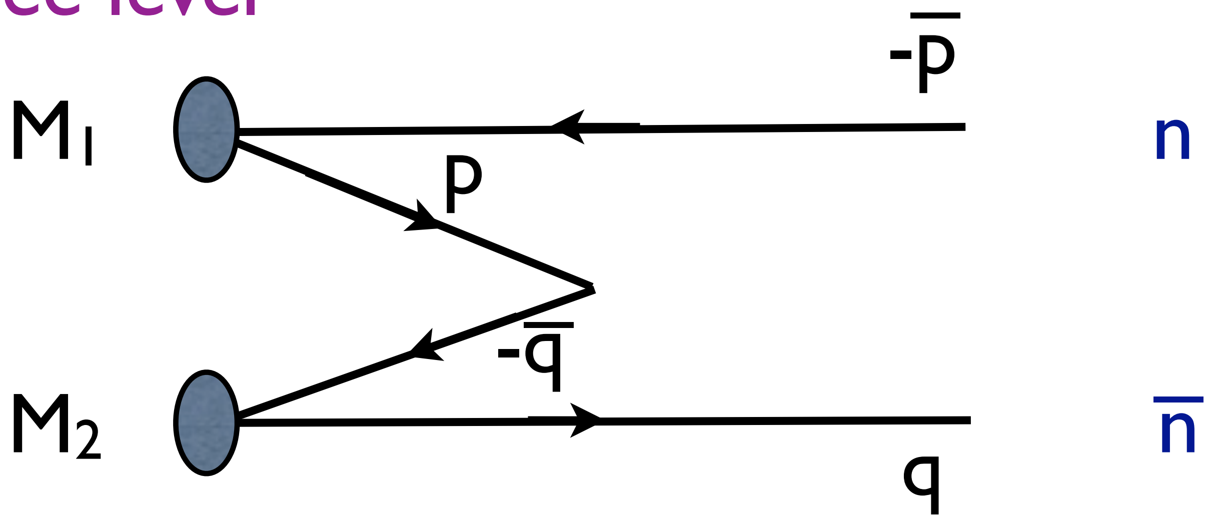
Introduction

Why is the presence of Glauber modes important?

- Glauber interactions happen between initial state spectator partons and they break the simple factorization in the exclusive cross-section
- Factorization is the key ingredient to make predictions for high energy QCD cross-sections
- Factorization of any process in hadron-hadron collisions needs analysis of Glauber modes
- Conseptual issue: do we have all the necessary low energy modes included into SCET?
- "Glaubers" play an important role for jet propagating in dense QCD media (Idilbi, Majumder `08)

Introduction

Drell-Yan: tree level



$\bar{p}, p \propto (1, \lambda^2, \lambda)$

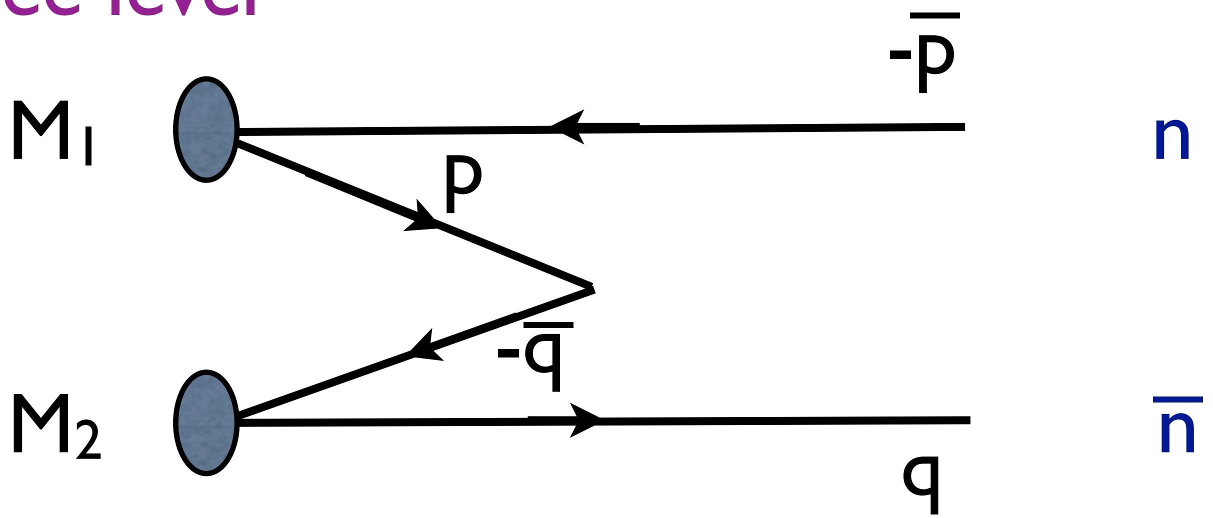
$\bar{q}, q \propto (\lambda^2, 1, \lambda)$

$P_{M1} = \bar{p} + p$

$P_{M2} = \bar{q} + q$

Introduction

Drell-Yan: tree level



$\bar{p}, p \propto (1,\lambda^2,\lambda)$ $\qquad P_{M1}=\bar{p}+p$

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Glauber gluon: $l(\lambda^2, \lambda^2,\lambda)$

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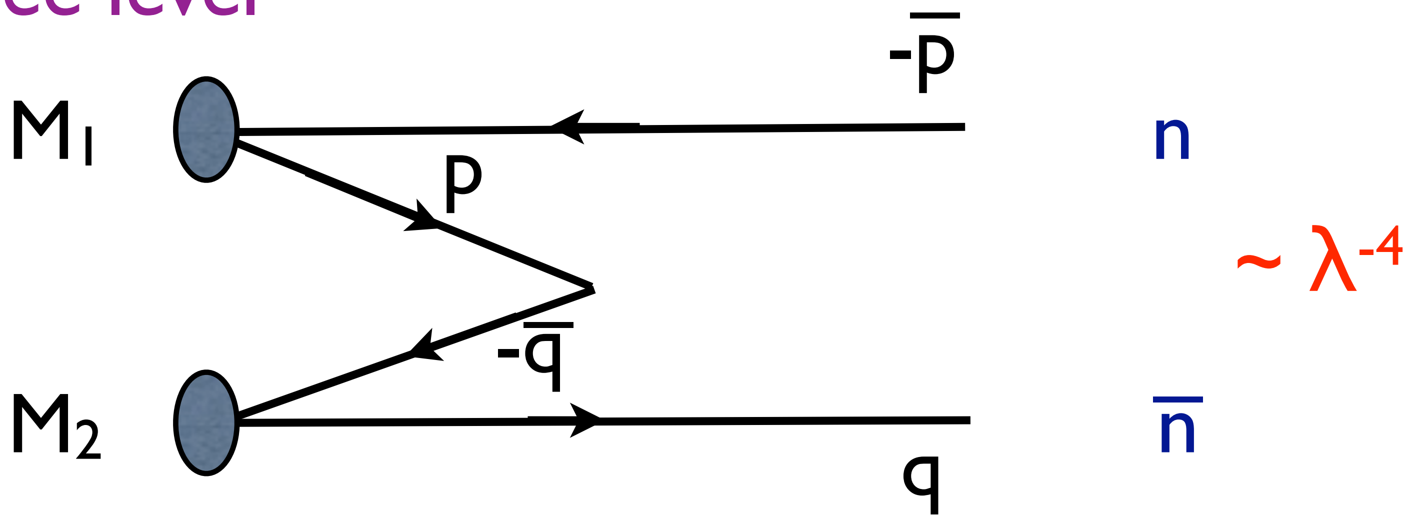

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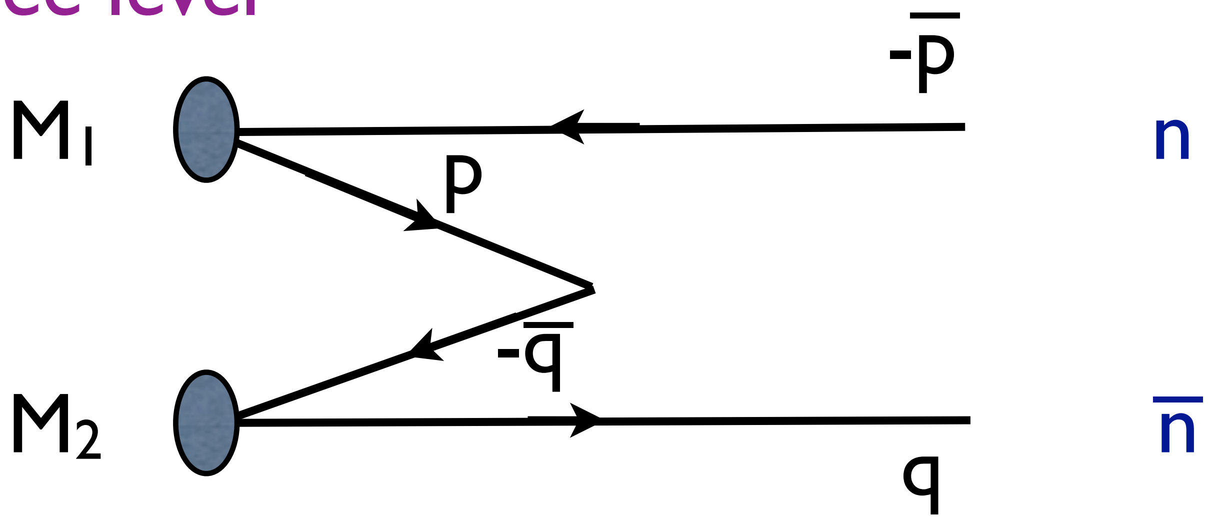


$\sim \lambda^{-4}$

$\bar{p}, p \propto (1,\lambda^2,\lambda\ )$

$\bar{q}, q \propto (\lambda^2,1,\lambda\ )$

$P_{M1}=\bar{p}+p$

$P_{M2}=\bar{q}+q$

Glauber gluon: $l(\lambda^2, \lambda^2,\lambda\ )$

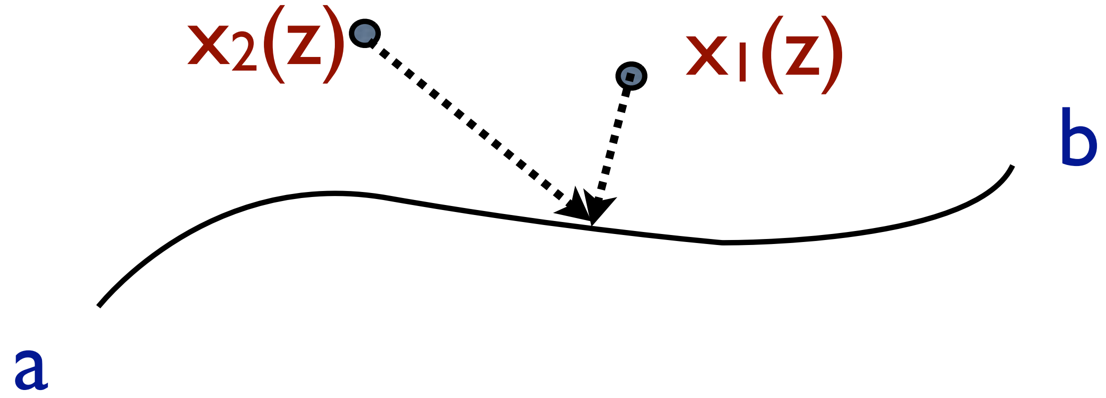
Off-shellness as an infrared regulator

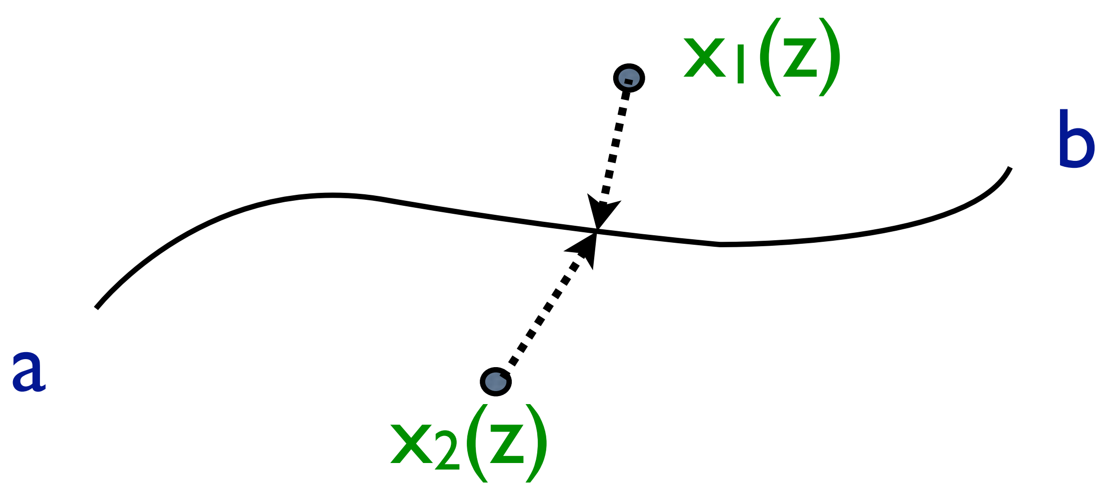
Introduction

Pinch analysis of loop integrals

$$I(z) = \int_C \mathrm{d}x\, f(x,z)$$

$$I(z \to z_0) = ?$$





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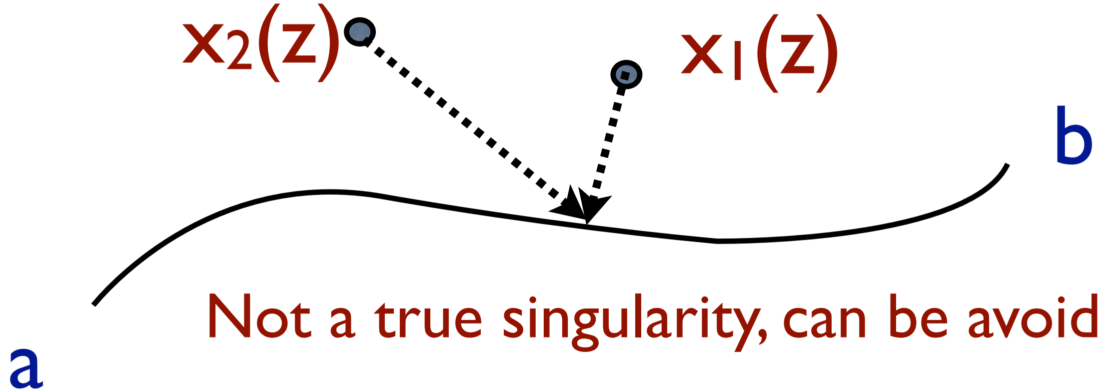
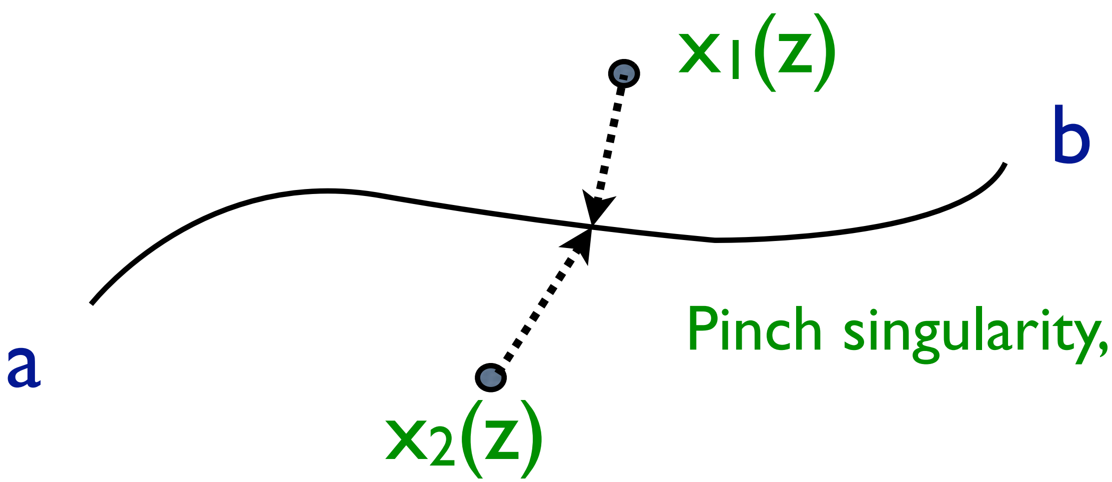


Not a true singularity, can be avoided by deforming the contour

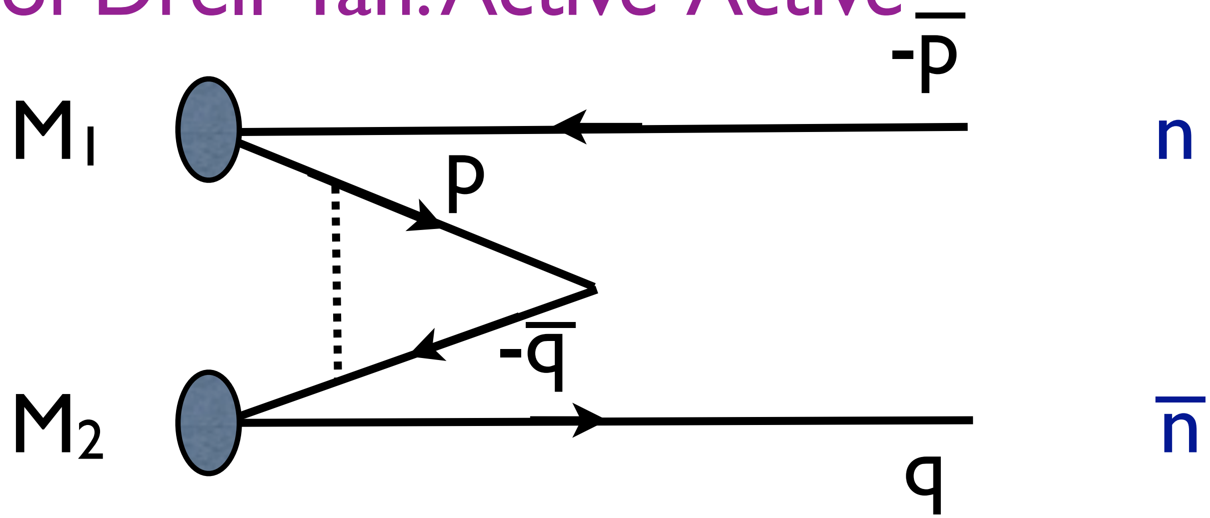


Pinch singularity, leads to a true pole

Introduction

Collins, Soper, Sterman, `82

Factorization of Drell-Yan: Active-Active

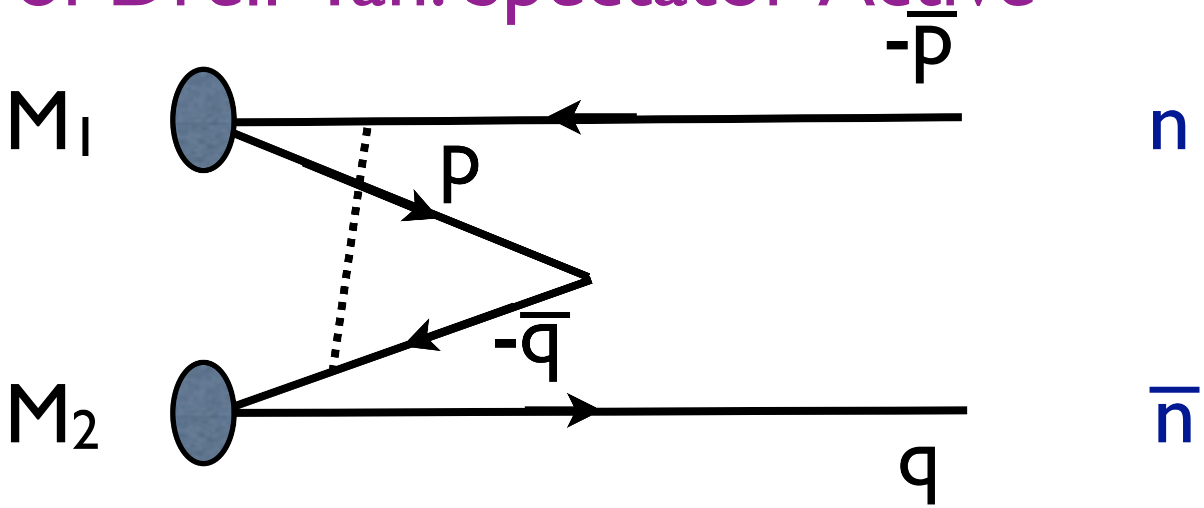


- The pinched singularities appear only in the collinear n, collinear $\bar{n}$ and soft regions
- Glauber region is not pinched, thus no infrared divergence comes from solely Glauber region

Introduction

Collins, Soper, Sterman, `82

Factorization of Drell-Yan: Spectator-Active



- The leading pinched singularities appear only in the collinear n and soft regions
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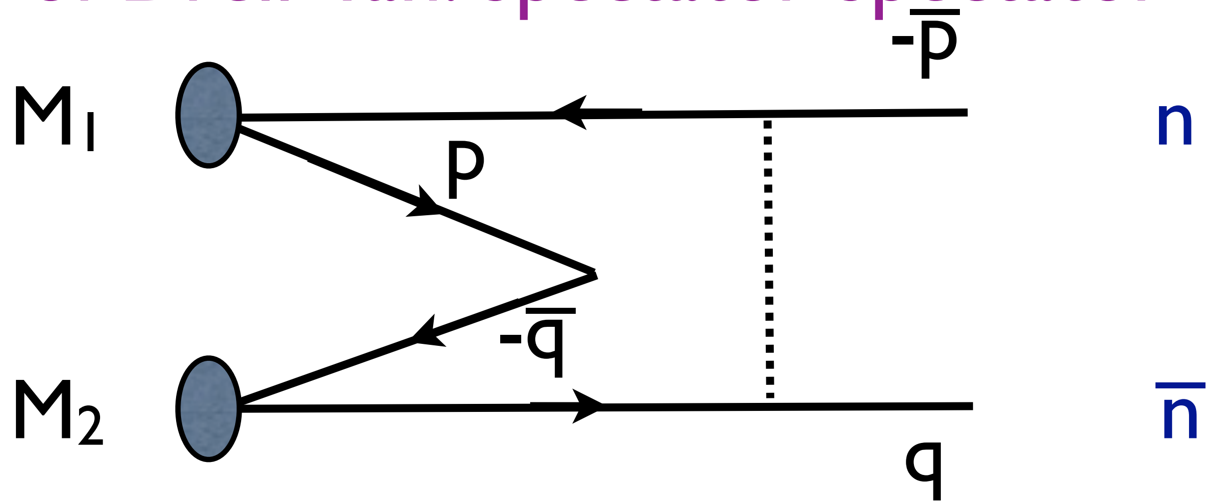
Introduction

Collins, Soper, Sterman, `82

Factorization of Drell-Yan: Spectator-Spectator

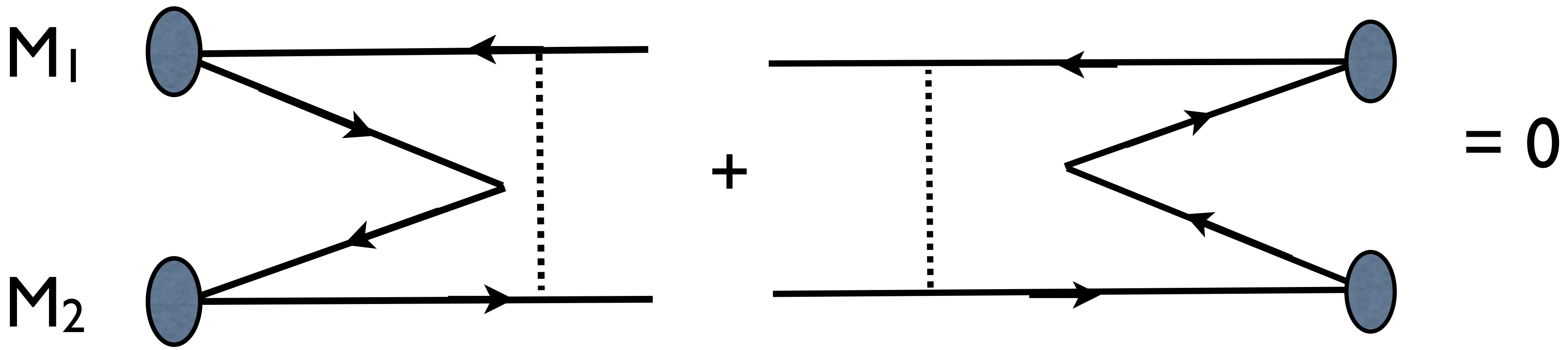


- The pinched singularities appear in the Soft and Glauber regions
- This mode break the simple factorization of Drell-Yan exclusive cross-section

Introduction

Collins, Soper, Sterman, `82

Factorization and cancellation of Glaubers



- Glauber contribution is shown to be purely imaginary
- Thus it cancels in the inclusive cross-section
- These one-loop results are generalized to all orders

Our goal: should we include Glauber modes into Effective Theory?

A Matching calculation for Drell-Yan

Idea of the calculation

- We want to set up a matching calculation which involves Drell-Yan one loop diagrams
- Should be a matching between QCD and EFT_1, and EFT_2, where

$$EFT_1 = SCET \quad \text{(collinear, ultrasoft)}$$

$$EFT_2 = SCET + \text{Glaubers}$$

- By comparing the two matching calculations we should be able to find out which effective theory consistently describes the Drell-Yan amplitude

Operator O_2

Definition

Operator O_2 arises from matching the QCD current onto a 2-jet Effective Theory:

$$\mathcal{J} = \bar{q}\,\Gamma q$$

$$\mathcal{J} = C_2 O_2$$

where O_2 is defined as:

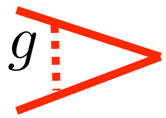
$$O_2 = \chi_n \Gamma \chi_{\bar{n}}$$

C_2 is Wilson coefficient which is well known to higher orders

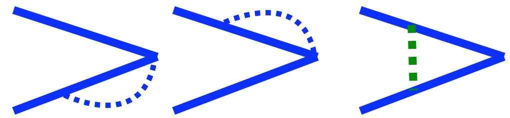
Operator O_2

The Idea

Simplest final states to calculate C_2 are $\langle 0|$ and $|q\bar{q}\rangle$:



$$\langle 0|J|q\bar{q}\rangle = C_2 \langle 0|O_2|q\bar{q}\rangle$$

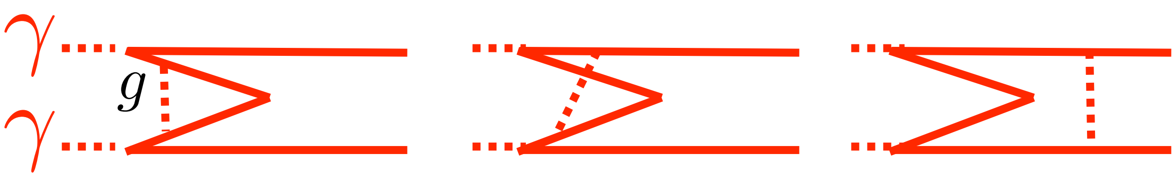


For our purpose we will chose instead states: $\langle\gamma\gamma|$ and $|q\bar{q}\rangle$

$$\langle\gamma\gamma|J|q\bar{q}\rangle = C_2 \langle\gamma\gamma|O_2|q\bar{q}\rangle$$

EFT$_1$ $\quad C_2=?$

EFT$_2$ $\quad C_2=?$



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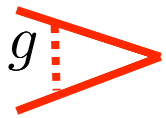

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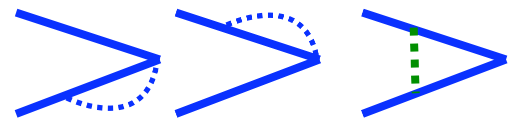

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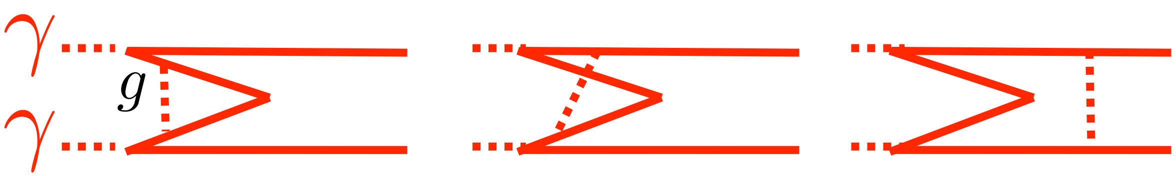
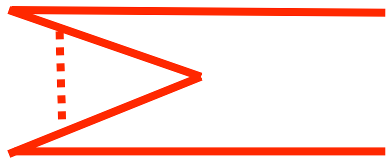


We know the answer for C_2

Active-Active topology

QCD=I_3= $\int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2(l+p)^2(l-\bar{q})^2}$



n-collinear $(1,\lambda^2,\lambda)$:

$$C_n = \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2(l+p)^2[-\bar{q}^- l^+]} \propto \lambda^{-4}$$

$\bar{n}$-collinear $(\lambda^2,1,\lambda)$:

$$C_{\bar{n}} = \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2[p^+ l^-](l-\bar{q})^2} \propto \lambda^{-4}$$

EFT_1

Soft $(\lambda^2,\lambda^2,\lambda^2)$:

$$S = \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2[p^+ l^- + p^2][-\bar{q}^- l^+ + \bar{q}^2]} \propto \lambda^{-4}$$

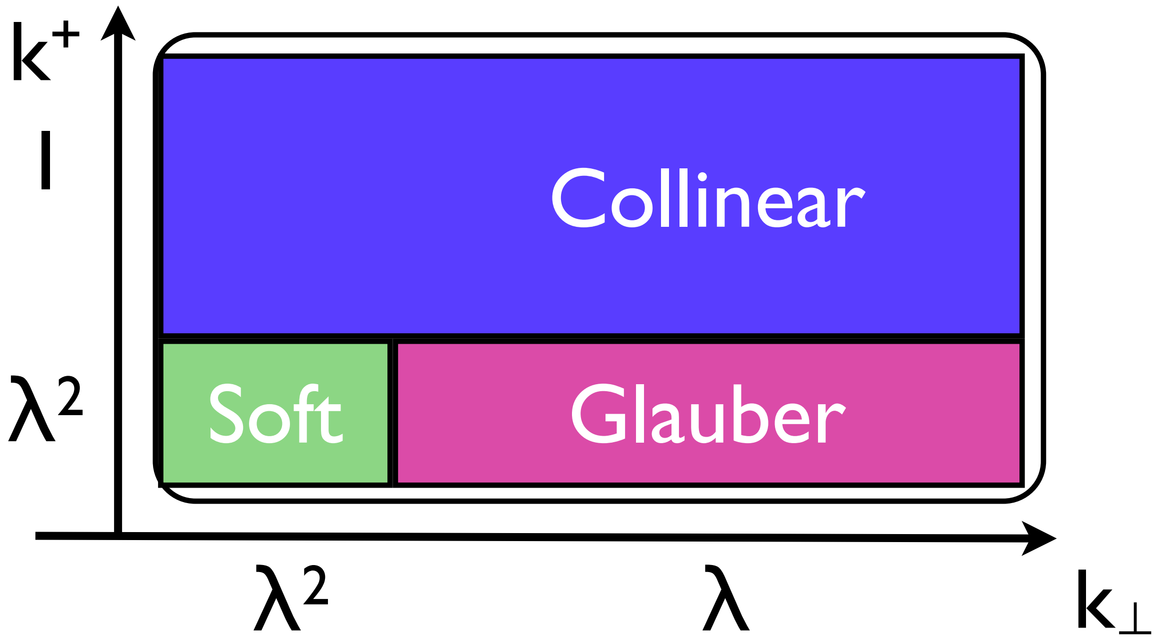
EFT_2

Glauber $(\lambda^2,\lambda^2,\lambda)$:

$$G_n = \int \frac{d^D l}{(2\pi)^D} \frac{1}{[-l_\perp^2][p^+(l^- + p^-) - (l_\perp + p_\perp)^2][-\bar{q}^-(l^+ - \bar{q}^+) - (l_\perp - \bar{q}_\perp)^2]} \propto \lambda^{-4}$$

Zero-bin subtractions in EFT_2

Manohar, Stewart (`06)



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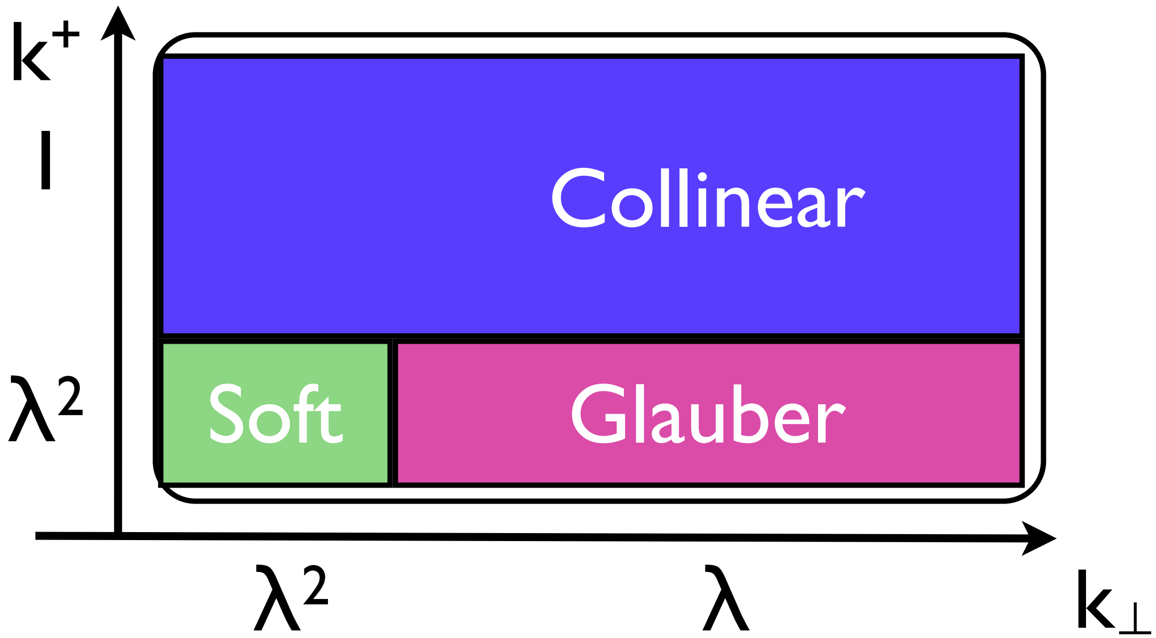

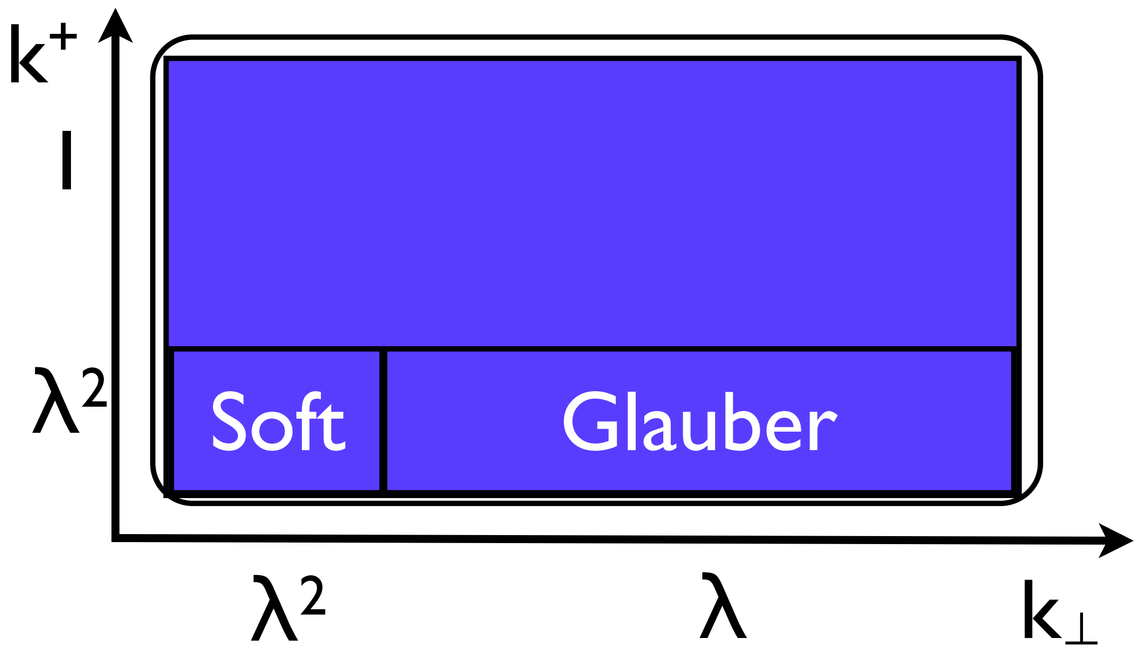

Zero-bin subtractions in EFT_2

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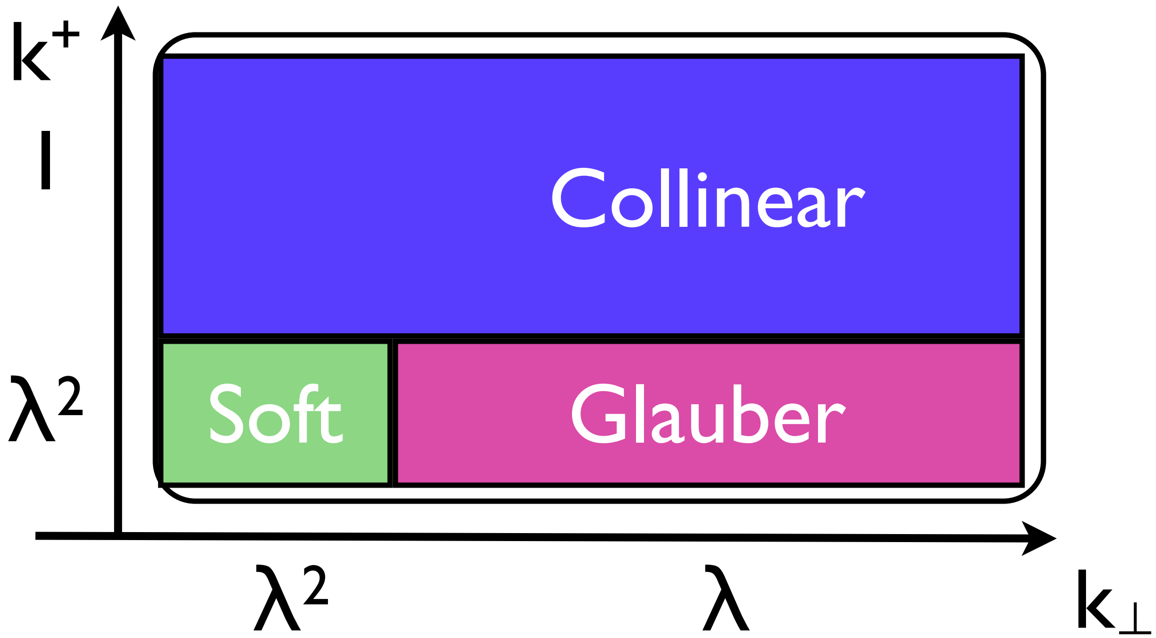
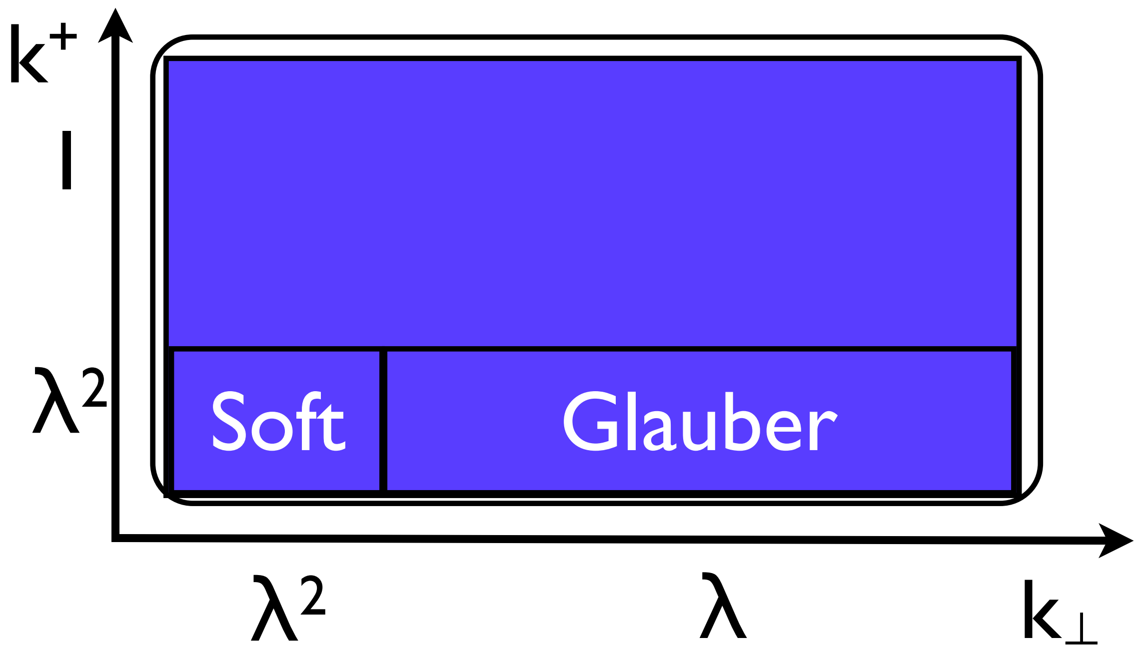
$C_n = C - (C_g - C_{gs} + C_s)$





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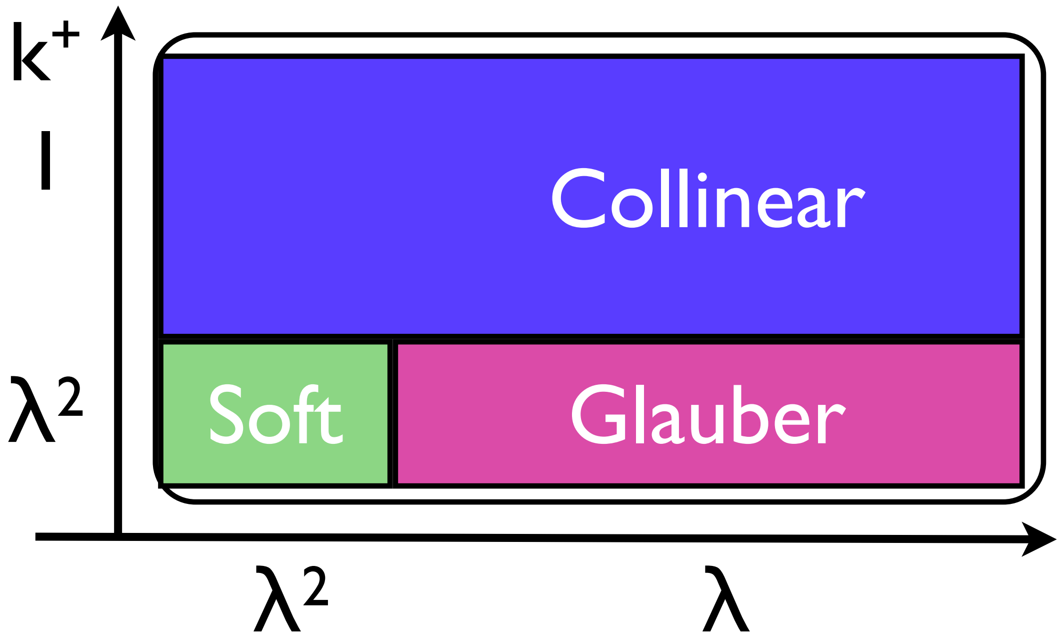
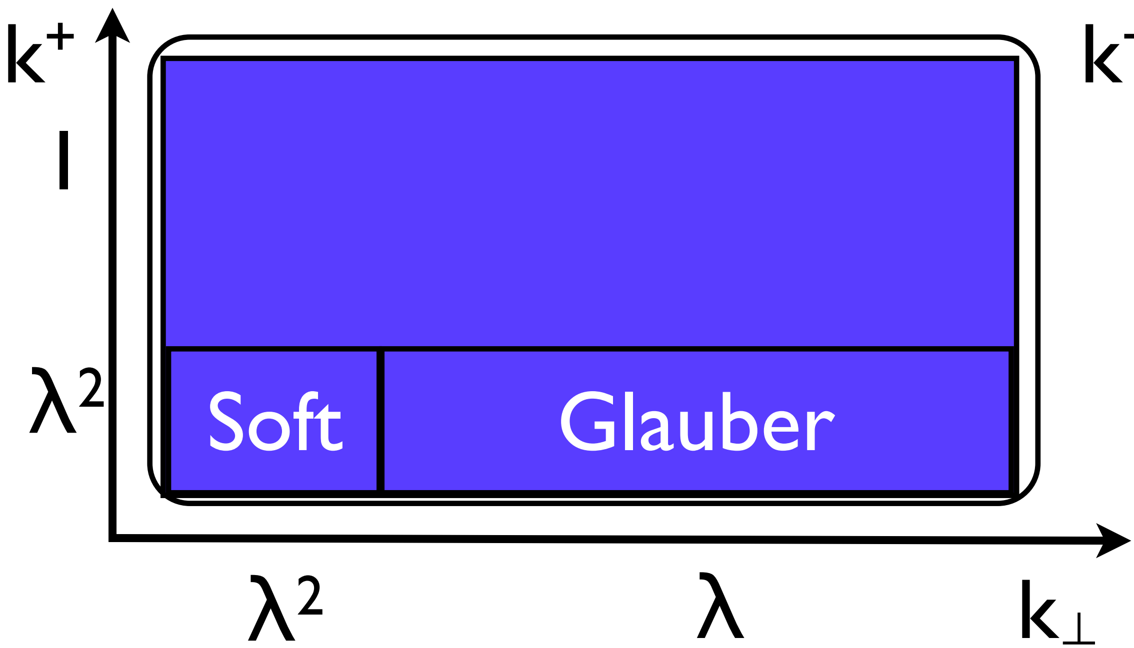
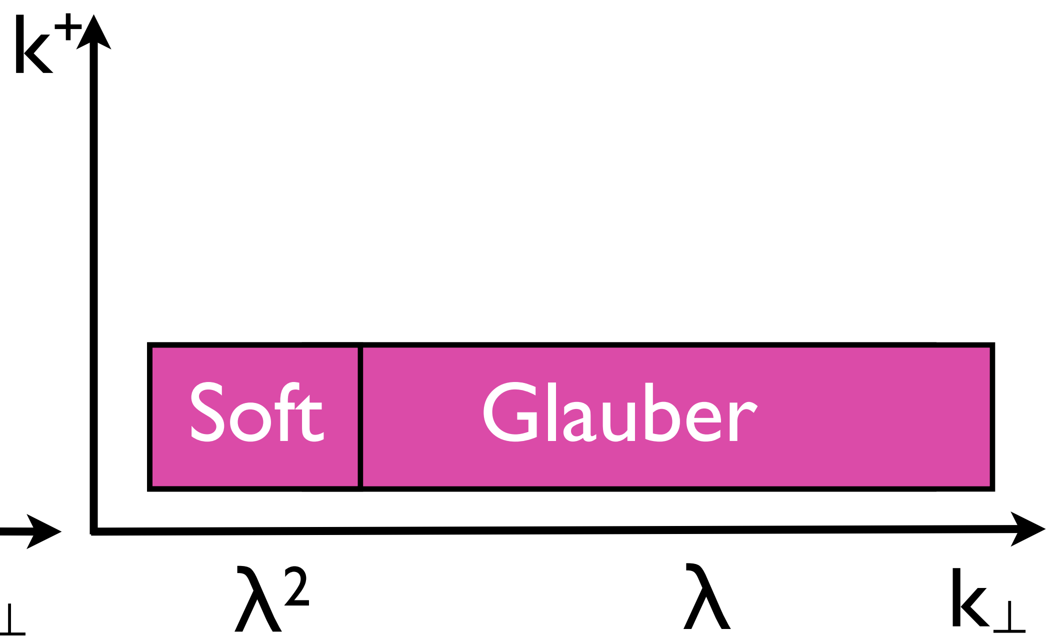




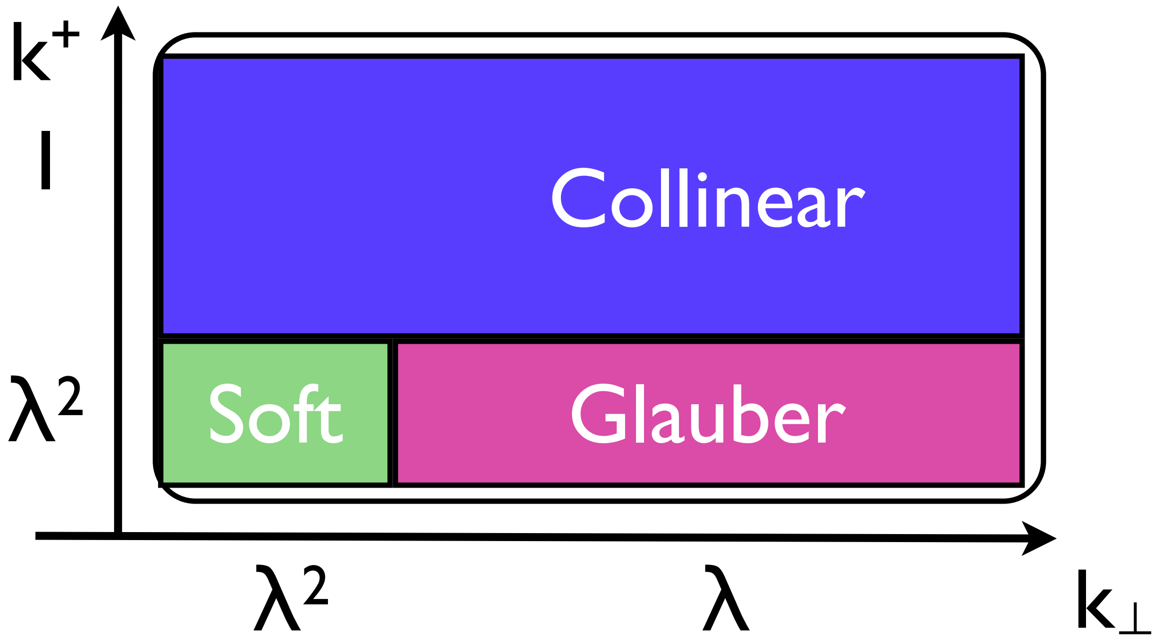


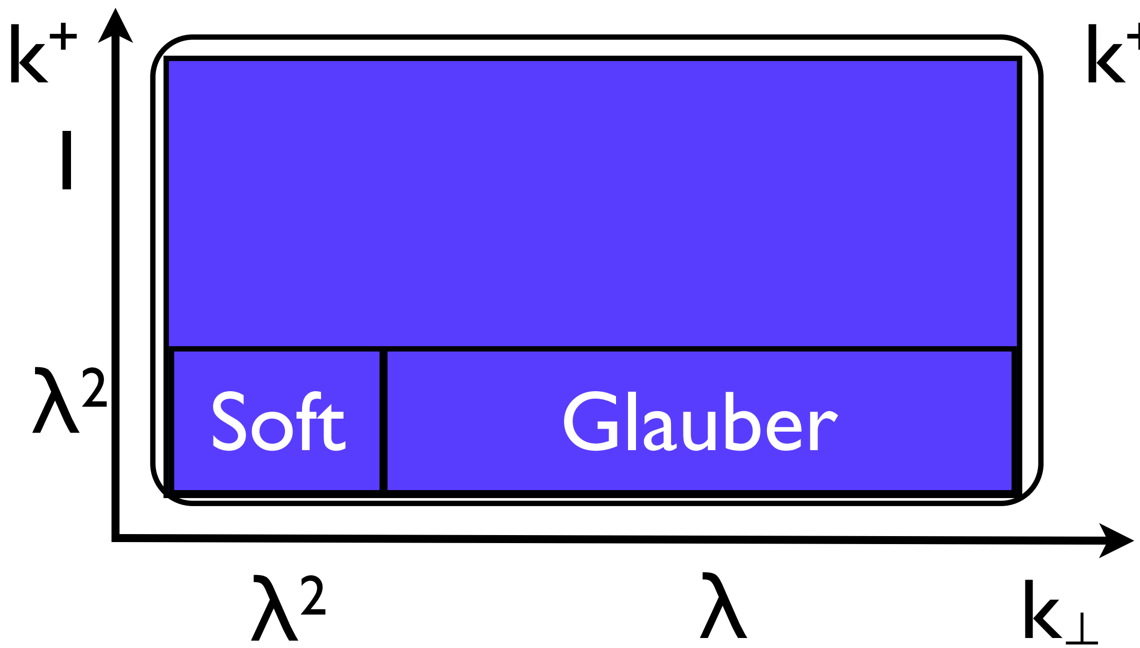
Zero-bin subtractions in EFT$_2$

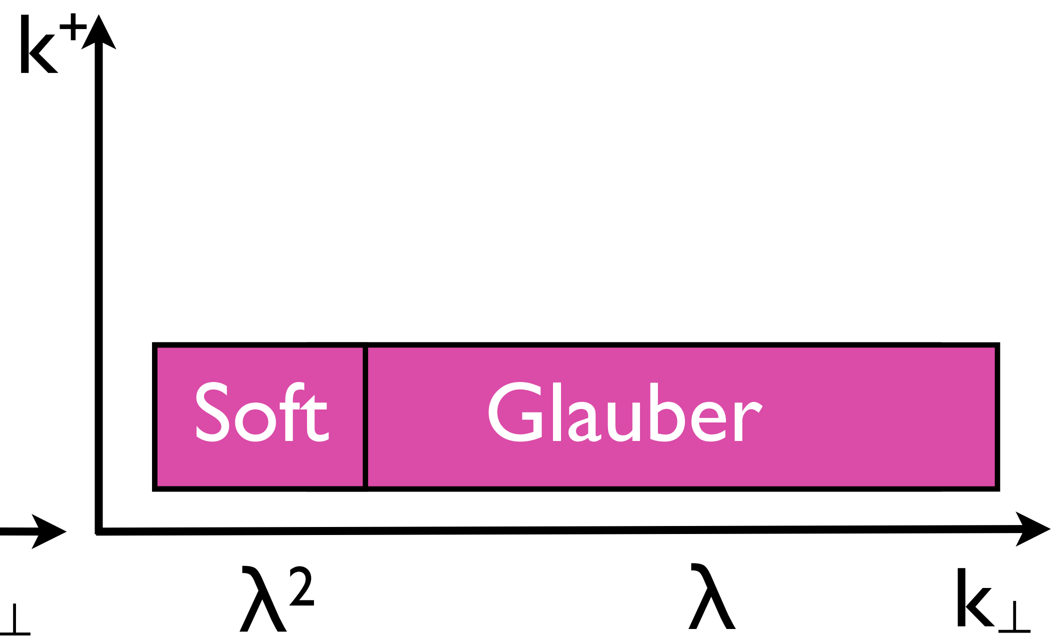
Manohar, Stewart (`06)

$C_n = C - (C_g - C_{gs} + C_s)$

$G_n = G - G_s$

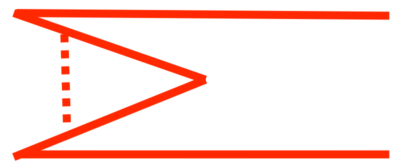






Active-Active topology

Contribution to the matching



In the First effective theory all modes including overlaps equal:

$$EFT_1 = C_n - C_s + C_{\bar{n}} - C_s + S$$

In the Second effective theory all modes including overlaps equal:

$$EFT_2 = C_n - (C_{ng} - C_{ngs} + C_s) + C_{\bar{n}} - (C_{\bar{n}g} - C_{\bar{n}gs} + C_s) + G_n - G_s + S \;=$$

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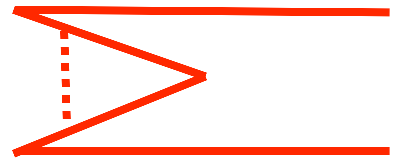

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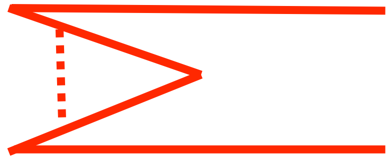

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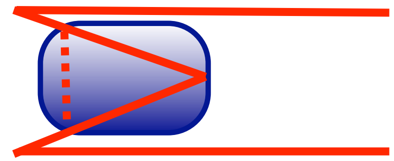

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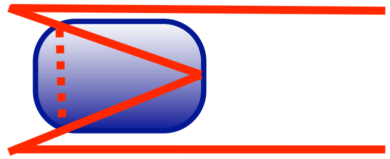

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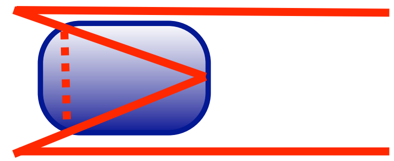

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$$G_s = G_n$$

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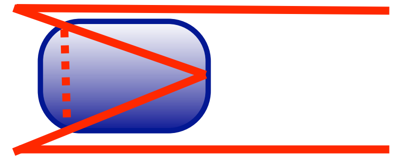

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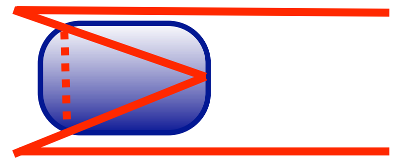
$$C_s = S \qquad\qquad = C_n + C_{\bar{n}} - S$$

$$C_{ng} = C_{ngs} = G_n$$

$$G_s = G_n$$

Active-Active topology

Contribution to the matching



In the First effective theory all modes including overlaps equal:

$$EFT_{I} = C_{n} - C_{s} + C_{\bar{n}} - C_{s} + S = C_{n} + C_{\bar{n}} - S$$

$$C_{s} = S \qquad I_{3} - EFT_{I} = C_{2}$$

In the Second effective theory all modes including overlaps equal:

$$EFT_{2} = C_{n} - (C_{ng} - C_{ngs} + C_{s}) + C_{\bar{n}} - (C_{\bar{n}g} - C_{\bar{n}gs} + C_{s}) + G_{n} - G_{s} + S = $$

$$= C_{n} + C_{\bar{n}} - S$$

$$C_{s} = S$$

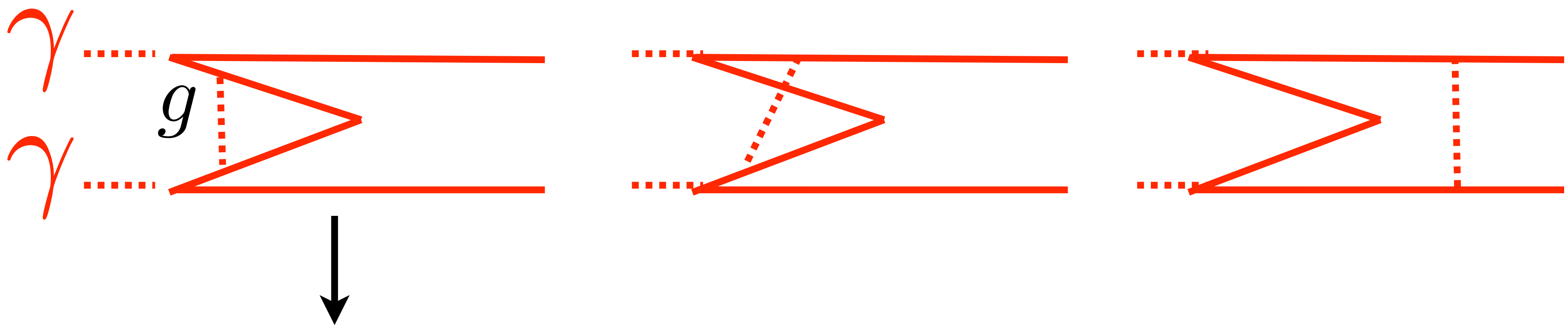
$$C_{ng} = C_{ngs} = G_{n}$$

$$G_{s} = G_{n}$$

So, for active-active graph we find:

$$EFT_{I} \equiv EFT_{2}$$

Status of the calculation



gives C_2

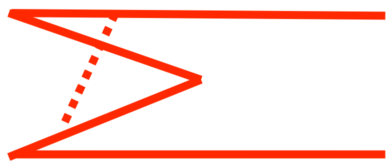
$\langle\gamma\gamma|J|q\bar{q}\rangle = C_2\langle\gamma\gamma|O_2|q\bar{q}\rangle$

EFT$_1$ $C_2 = C_2 + ? + ?$

EFT$_2$ $C_2 = C_2 + ? + ?$

Spectator-Active topology

QCD=I_4= $$\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[l^2][(l-\bar{p})^2][(l+p)^2][(l-\bar{q})^2]}$$



n-collinear $(1,\lambda^2,\lambda)$:

$$C_n = \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[l^2][(l-\bar{p})^2][(l+p)^2][-l^+\bar{q}^-]} \propto \lambda^{-4}$$

$\bar{n}$-collinear $(\lambda^2,1,\lambda)$:

$$C_{\bar{n}} = \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[l^2][-\bar{p}^+ l^-][p^+ l^- + p^2][(l-\bar{q})^2]} \propto \lambda^{-2}$$

Soft $(\lambda^2,\lambda^2, \lambda^2)$:

$$S = \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[l^2][-\bar{p}^+ l^- + \bar{p}^2][p^+ l^- + p^2][-\bar{q}^- l^+ + \bar{q}^2]} \propto \lambda^{-4}$$

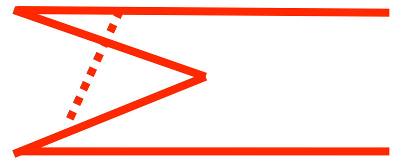
Glauber $(\lambda^2,\lambda^2,\lambda)$:

$$G_n = \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[-l_\perp^2][-\bar{p}^+(l^- - \bar{p}^-) - (l_\perp - \bar{p}_\perp)^2][p^+(l^- + p^-) - (l_\perp + p_\perp)^2][-\bar{q}^-(l^+ - \bar{q}^+) - (l_\perp - \bar{q}_\perp)^2]} \propto \lambda^{-4}$$

EFT_1 EFT_2

Spectator-Active topology

Contribution to the matching



In the First effective theory all modes including overlaps equal:

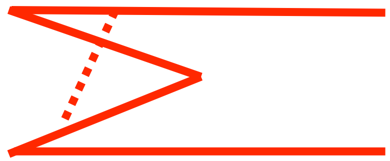
$$EFT_1 = C_n - C_s + S$$

In the Second effective theory all modes including overlaps equal:

$$EFT_2 = C_n - (C_{ng} - C_{ngs} + C_s) + G_n - G_s + S$$

Spectator-Active topology

Contribution to the matching



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relationship between zero-bins: $C_s = S$

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Spectator-Active topology

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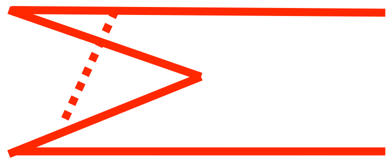

In the First effective theory all modes including overlaps equal:

$$EFT_1 = C_n - C_s + S \; = C_n$$

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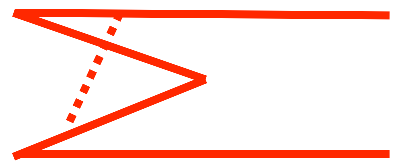
Spectator-Active topology

Contribution to the matching



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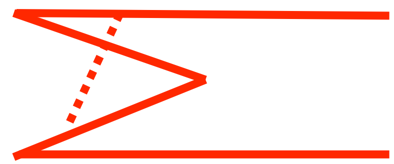
$C_{ng} = G_n$

$C_{ngs} = G_{ns}$

Spectator-Active topology

Contribution to the matching



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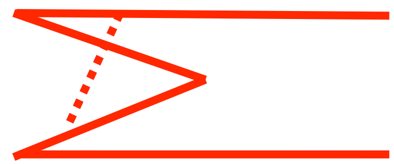
Spectator-Active topology

Contribution to the matching



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$C_s = S$

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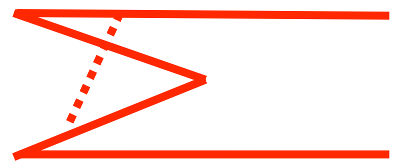
So, for spectator-active graph we find: $EFT_1 \equiv EFT_2$

Spectator-Active topology

Contribution to the matching



In the First effective theory all modes including overlaps equal:

$$EFT_{I} = C_{n} - C_{s} + S = C_{n}$$

relationship between zero-bins: $C_s = S$

In the Second effective theory all modes including overlaps equal:

$$EFT_{2} = C_{n} - (C_{ng} - C_{ngs} + C_{s}) + G_{n} - G_{s} + S = C_{n}$$

relationship between zero-bins:

$C_s = S$

$C_{ng} = G_n$

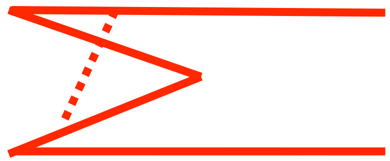
$C_{ngs} = G_{ns}$

So, for spectator-active graph we find: $EFT_{I} \equiv EFT_{2}$

$\Delta C_2 = (I_4 - C_n)/\text{Tree} = ?$

Spectator-Active topology

Contribution to the matching

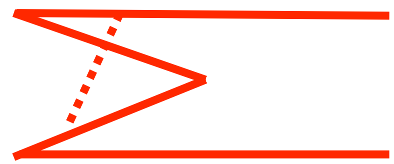


$$I_4 = \int \frac{d^D l}{(2\pi)^D} \frac{1}{[l^2][(l-\bar{p})^2][(l+p)^2][(l-\bar{q})^2]} =$$

$$= \frac{i}{16\pi^2} \cdot \frac{1}{\bar{q}^-} \cdot \frac{1}{\bar{p}^2 p^+ + p^2 \bar{p}^+} \left[\frac{\pi^2}{3} - 2\,\mathrm{Li}_2\left(\frac{1}{2 - \frac{p^2 p^+}{\bar{p}^2 \bar{p}^+}} \right) + \left(\ln\left(\frac{\bar{p}^2 p^+}{p^2 \bar{p}^+} \right) - i\,\pi \right) \ln\left(\frac{2\frac{\bar{p}^2 p^+}{p^2 \bar{p}^+} + 1}{2\frac{\bar{p}^2 p^+}{p^2 \bar{p}^+} - 1} \right) \right]$$

Spectator-Active topology

Contribution to the matching



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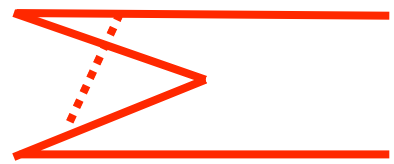
Comparing reduced to 2 Feynman parameter integrals there is an exact relation between the I_4 and C_n:

$$I_4 \equiv I_4\left[p^2, (\bar{p}-\bar{q})^2, \bar{p}^2, (p+\bar{p})^2, (p+\bar{q})^2, \bar{q}^2\right]$$

$$C_n \equiv I_4\left[p^2, \bar{q}^2 - \bar{p}^+\bar{q}^-, \bar{p}^2, (p+\bar{p})^2, p^+\bar{q}^- + \bar{q}^2, \bar{q}^2\right] \equiv I_4\left(1 + \mathcal{O}(\lambda^2)\right)$$

Spectator-Active topology

Contribution to the matching



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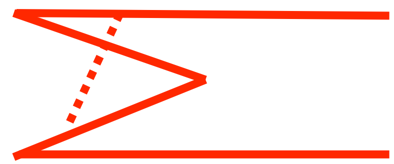
$\Delta C_2 = (I_4 - C_n)/\mathrm{Tree} = 0$

Spectator-Active topology

Contribution to the matching



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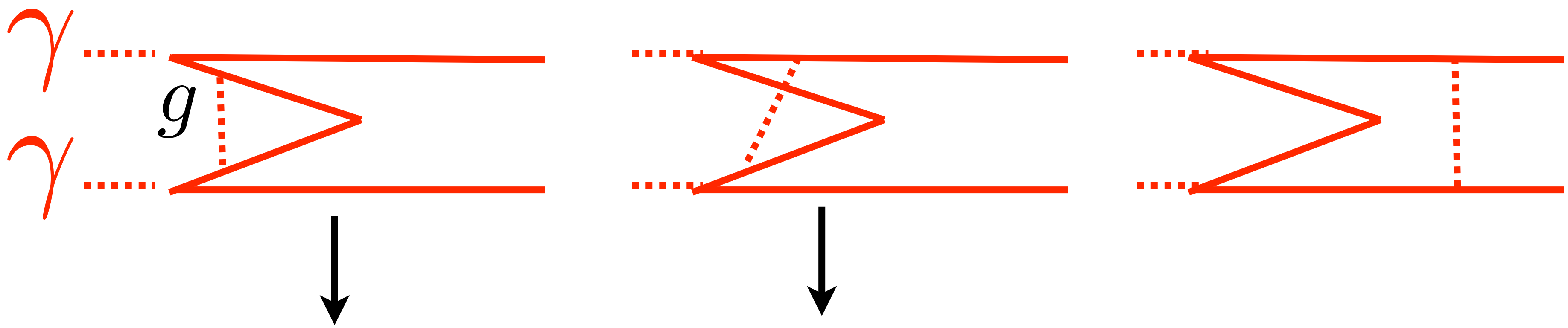
$\Delta C_2 = (I_4 - C_n)/\mathrm{Tree} = 0$

In both effective theories contribution to the matching coefficient is zero

Status of the calculation



gives C_2
(both theories)

gives 0
(both theories)

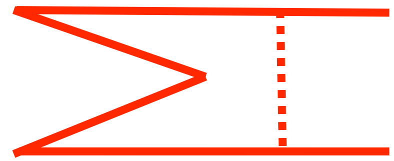
$\langle\gamma\gamma|J|q\bar{q}\rangle = C_2\langle\gamma\gamma|O_2|q\bar{q}\rangle$

EFT$_1$ $C_2 = C_2 + 0 + ?$

EFT$_2$ $C_2 = C_2 + 0 + ?$

Spectator-Spectator topology

QCD= I_5 = $\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{l^2 (l-\bar{p})^2 (l+p)^2 (l-\bar{q})^2 (l+q)^2}$



n-collinear $(1,\lambda^2,\lambda)$: $\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{l^2 (l-\bar{p})^2 (l+p)^2 [-l^+\bar{q}^-][l^+ q^-]} \propto \lambda^{-2}$

$\bar{n}$-collinear $(\lambda^2,1,\lambda)$: $\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{l^2 [-l^-\bar{p}^+][l^- p^+] (l-\bar{q})^2 (l+q)^2} \propto \lambda^{-2}$

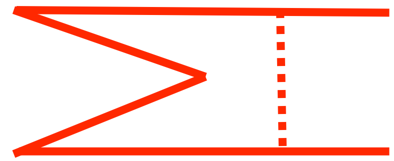
Soft $(\lambda^2,\lambda^2,\lambda^2)$: $\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{l^2 [-l^-\bar{p}^+ + \bar{p}^2][l^- p^+ + p^2][-l^+\bar{q}^- + \bar{q}^2][l^+ q^- + q^2]} \propto \lambda^{-4}$

Glauber $(\lambda^2,\lambda^2,\lambda)$:

$$\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[-l_\perp^2][-\bar{p}^+(l^- - \bar{p}^-) - (l_\perp - \bar{p}_\perp)^2][p^+(l^- + p^-) - (l_\perp + p_\perp)^2][-\bar{q}^-(l^+ - \bar{q}^+) - (l_\perp - \bar{q}_\perp)^2]} \frac{1}{[q^-(l^+ + q^+) - (l_\perp + q_\perp)^2]} \propto \lambda^{-4}$$

Spectator-Spectator topology

Contribution to the matching



In the First effective theory we have only Soft mode present:

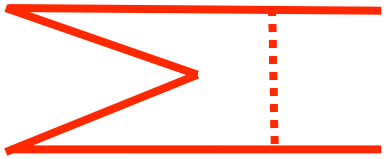
$$EFT_1 = S = (1/\varepsilon_{UV} + 1/\varepsilon_{IR} + \text{finite})$$

In the Second effective theory all modes including overlaps equal:

$$EFT_2 = G_n - G_s + S \quad (G_s = 0 \text{ and } G_n \neq 0)$$

Spectator-Spectator topology

Contribution to the matching



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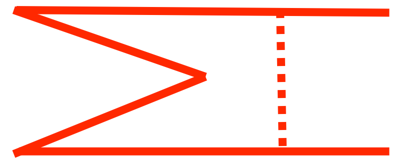
- So, for spectator-active graph EFT_1 and EFT_2 are Not equivalent

Spectator-Spectator topology

Contribution to the matching



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- So, for spectator-active graph EFT_1 and EFT_2 are Not equivalent
- In the matching I_5-EFT_1 there is an extra UV divergence which will change the anomalous dimension of C_2

Spectator-Spectator topology

Matching contribution in EFT_2

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)$$

Spectator-Spectator topology

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$$\frac{i}{16\pi^2}\frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)$$

$$S = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

Spectator-Spectator topology

Matching contribution in EFT_2

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$$\frac{i}{16\pi^2}\frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)$$

$$S = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon} + \ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right) + i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right) + 3\pi^2\right]$$

$$G_n = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon} - 2\pi^2 + 2\pi\, i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right) +\right.$$

$$\left. + \frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

Spectator-Spectator topology

Matching contribution in EFT_2

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$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)$$

$$S = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

$$G_n = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon}-2\pi^2+2\pi\, i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right)+\right.$$

$$\left.+\frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $G_s = 0$

Spectator-Spectator topology

Matching contribution in EFT_2 UV divergence

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right) + \mathcal{O}\left(\epsilon,\frac{1}{\lambda^2}\right)$$

$$S = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

$$G_n = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon}-2\pi^2+2\pi\,i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right)+\right.$$

$$\left.+\ \frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $G_s = 0$

Spectator-Spectator topology

Matching contribution in EFT_2 IR divergence

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right) + \mathcal{O}\left(\epsilon,\ \frac{1}{\lambda^2}\right)$$

$$S = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

$$G_n = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon}-2\pi^2+2\pi\, i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right)+\right.$$

$$\left. + \frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $G_s=0$

Spectator-Spectator topology

Matching contribution in EFT_2 IR divergence

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right) + i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right) + \pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right) + \mathcal{O}\left(\epsilon, \frac{1}{\lambda^2}\right)$$

$$S = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon} + \ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right) + i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right) + 3\pi^2\right]$$

$$G_n = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon} - 2\pi^2 + 2\pi\, i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right) + \right.$$

$$\left. + \frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $G_s=0$

Spectator-Spectator topology

Matching contribution in EFT_2

IR divergence

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right) + \mathcal{O}\left(\epsilon,\frac{1}{\lambda^2}\right)$$

$$S = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

$$G_n = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon}-2\pi^2+2\pi\, i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right) +\right.$$

$$\left. + \frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $G_s=0$

Spectator-Spectator topology

Matching contribution in EFT_2

Finite term

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right) + \mathcal{O}\left(\epsilon, \frac{1}{\lambda^2}\right)$$

$$S = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

$$G_n = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon} - 2\pi^2 + 2\pi\, i\, \ln\left(\frac{\mu^2}{Q^2}\right)\right) +\right.$$

$$\left. + \frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $G_s = 0$

Spectator-Spectator topology

Matching contribution in EFT_2 — Finite term

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right) + i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right) + \pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right) + \mathcal{O}\left(\epsilon, \frac{1}{\lambda^2}\right)$$

$$S = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon} + \ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right) + i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right) + 3\pi^2\right]$$

$$G_n = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon} - 2\pi^2 + 2\pi\, i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right) +\right.$$

$$\left. + \frac{2\pi i\, Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

$$\Delta C_2 = (I_5 - S - G + G_s)/\text{Tree} = 0$$

Spectator-Spectator topology

$$\mathrm{EFT}_1 \neq \mathrm{EFT}_2$$

EFT_1

EFT_2

Spectator-Spectator topology

$$\mathrm{EFT}_1 \neq \mathrm{EFT}_2$$

EFT_1

$$\Delta C_2 = (I_5 - S)/\mathrm{Tree} = 1/\varepsilon_{UV} + 1/\varepsilon_{IR}$$

EFT_2

Spectator-Spectator topology

$$\mathrm{EFT}_1 \neq \mathrm{EFT}_2$$

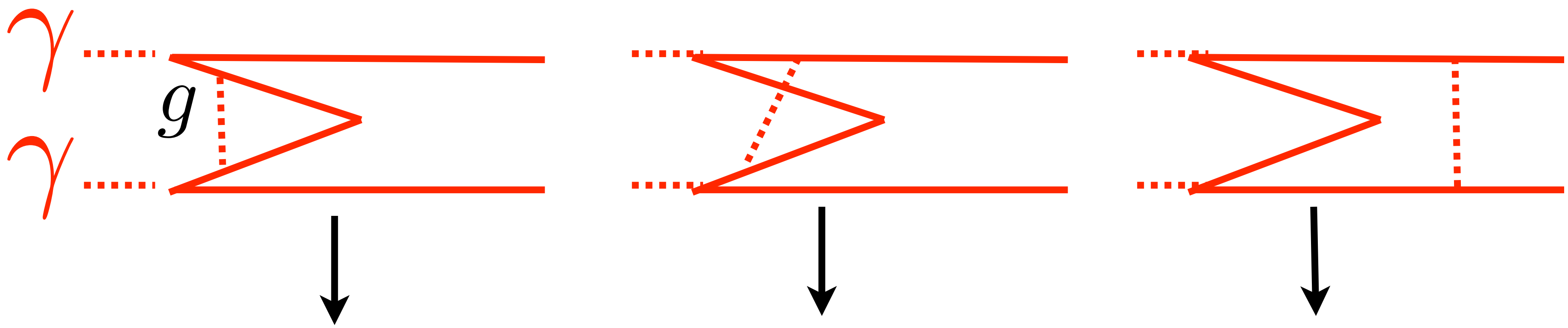
EFT_1

$$\Delta C_2 = (I_5 - S)/\mathrm{Tree} = 1/\varepsilon_{UV} + 1/\varepsilon_{IR}$$

EFT_2

$$\Delta C_2 = (I_5 - S - G + G_s)/\mathrm{Tree} = 0$$

Status of the calculation



gives C_2
(both theories)

gives 0
(both theories)

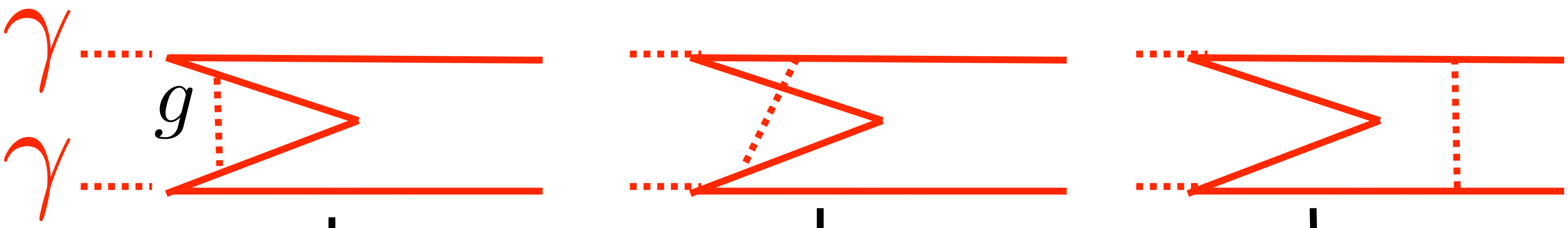
gives 0 in EFT_2
but not in EFT_1

$$\langle\gamma\gamma|J|q\bar{q}\rangle = C_2\langle\gamma\gamma|O_2|q\bar{q}\rangle$$

EFT_1 $C_2 = C_2 + 0 + 1/\varepsilon_{UV} + 1/\varepsilon_{IR}$

EFT_2 $C_2 = C_2 + 0 + 0$

Status of the calculation



gives C_2 (both theories)

gives 0 (both theories)

gives 0 in EFT_2 but not in EFT_1

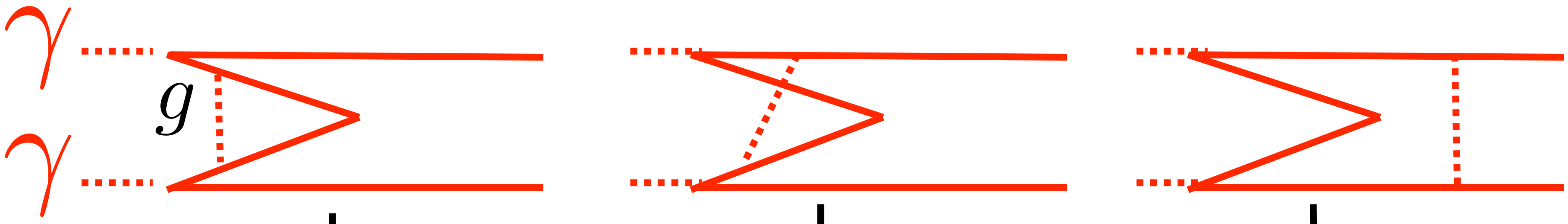
$\langle\gamma\gamma|J|q\bar{q}\rangle = C_2\langle\gamma\gamma|O_2|q\bar{q}\rangle$

EFT_1 $\quad C_2 = C_2 + 0 + 1/\varepsilon_{UV} + 1/\varepsilon_{IR}$

EFT_2 $\quad C_2 = C_2 + 0 + 0$

EFT_2 is an Effective Theory with Glauber modes, and it is the Right one!

Status of the calculation



gives C_2	gives 0	gives 0 in EFT_2
(both theories)	(both theories)	but not in EFT_1

$\langle\gamma\gamma|J|q\bar{q}\rangle = C_2 \langle\gamma\gamma|O_2|q\bar{q}\rangle$

EFT_1 $C_2 = C_2 + 0 + 1/\varepsilon_{UV} + 1/\varepsilon_{IR}$ ×

EFT_2 $C_2 = C_2 + 0 + 0$ √

EFT_2 is an Effective Theory with Glauber modes,
and it is the Right one!

Matching calculation

Summary of the matching calculation

Matching calculation

Summary of the matching calculation

- In active-active and spectator-active topologies putting the Glauber mode or not into SCET doesn't make any difference
- In spectator-spectator topology, the presence of Glauber mode makes a non-trivial contribution to the Drell-Yan amplitude and only including Glaubers into effective theory we get the right answer for the matching coefficient C_2
- Our results are in no conflict with Collins, Soper, Sterman's "pinch" analysis of Drell-Yan loop integrals
- Taking into account the zero-bins, or the overlaps between the different modes was crucial for our analysis

Conclusions

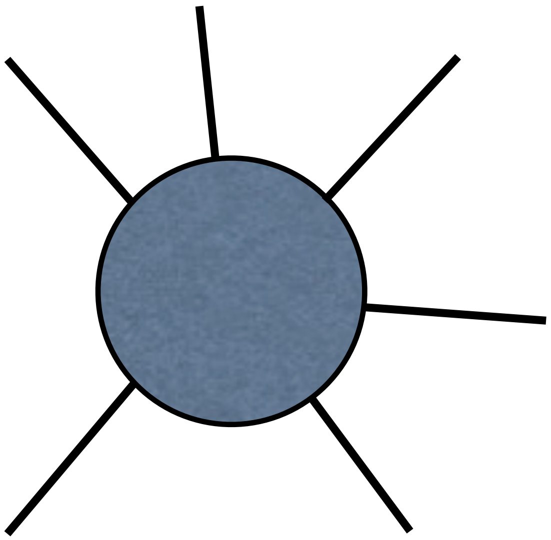
- We completed a one-loop matching calculation for the operator O_2 with special partonic final states
- For consistency of Effective Theory to get the right matching coefficient, SCET has to be expanded by Glauber modes
- Understanding the cancellation of Glauber gluons in the Drell-Yan cross-section using Effective Theory hasn't been achieved(yet)

backup

Coleman Norton Theorem

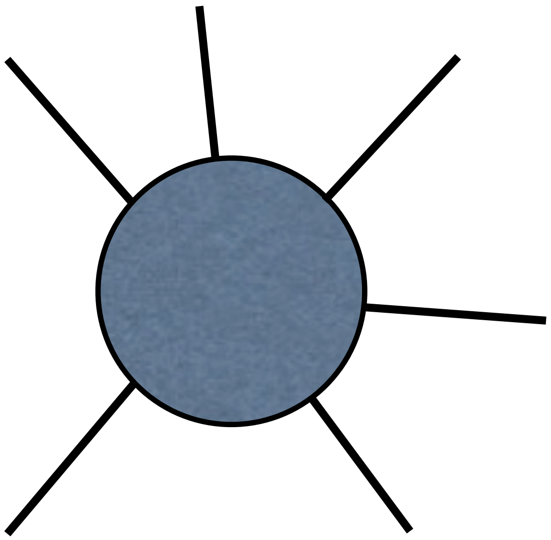
- Because of the scaling of the Glauber mode it is always off-shell
- According to Coleman-Norton theorem the infrared poles can come only from on-shell propagating modes
- As we review below the apparent contradiction is resolved by a trivial observation that the amount of off-shellness of the Glauber mode is infinitesimally small

Coleman Norton Theorem



$$\int \prod_i d\alpha_i \; \delta\Big(\sum_i \alpha_i - 1\Big)\Big(\sum_j \alpha_j (q_j^2 - m_j^2)\Big)^{-1}$$

Coleman Norton Theorem

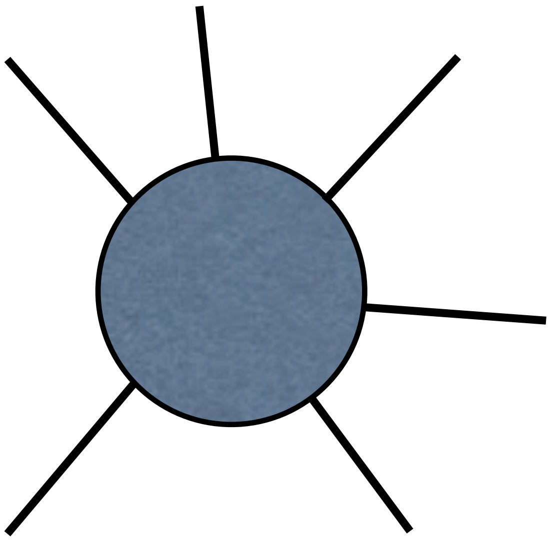


$$\int \prod_i d\alpha_i \, \delta(\sum_i \alpha_i - 1) \, (\sum_j \alpha_j (q_j^2 - m_j^2))^{-1}$$

Infrared singularities come from conditions known as Landau Equations:

$q_j^2 - m_j^2 = 0$ or $\alpha_j = 0$ and $\sum_j \alpha_j q_j = 0$ $\quad q_j^* = q_j \quad \alpha_j \geq 0$

Coleman Norton Theorem



$$\int \prod_i d\alpha_i \, \delta(\sum_i \alpha_i - 1) \, (\sum_j \alpha_j (q_j^2 - m_j^2) \,)^{-1}$$

Infrared singularities come from conditions known as Landau Equations:

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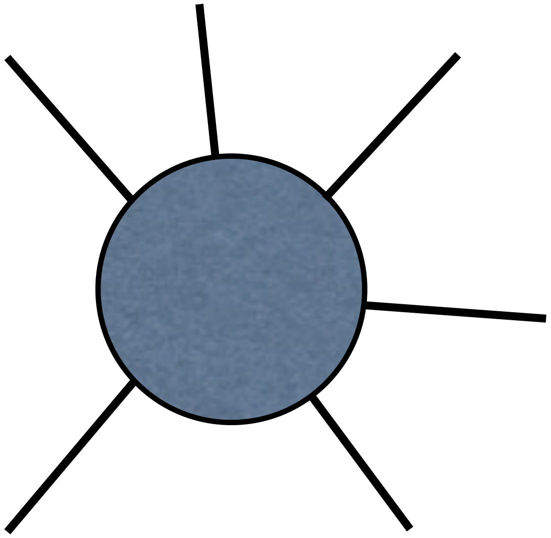
Coleman Norton `65

$\Delta X_j = \alpha_j q_j$

Coleman Norton Theorem



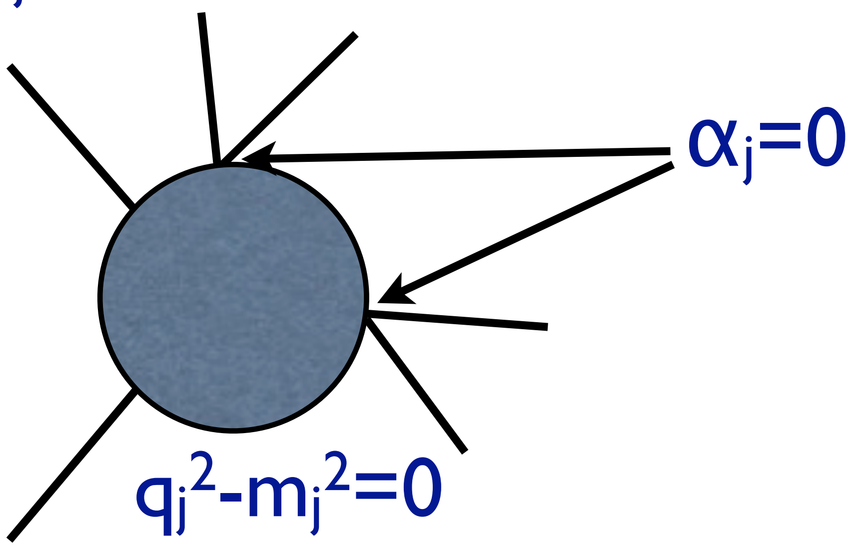
$$\int \prod_i d\alpha_i \; \delta(\sum_i \alpha_i - 1) \, (\sum_j \alpha_j (q_j^2 - m_j^2) \,)^{-1}$$

Infrared singularities come from conditions known as Landau Equations:

$q_j^2 - m_j^2 = 0$ or $\alpha_j = 0$ and $\sum_j \alpha_j q_j = 0$ $\quad q_j^* = q_j$ $\quad \alpha_j \geq 0$

Coleman Norton `65

$\Delta X_j = \alpha_j q_j$



Example: Triangle Graph

$$I_3 = \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{l^2 (l+p)^2 (l-\bar{q})^2}$$

$$\alpha_1 l^\mu + \alpha_2 (l+p)^\mu + \alpha_3 (l-\bar{q})^\mu = 0$$

$$\alpha_1 l^2 = 0 \qquad \alpha_2 (l+p)^2 = 0 \qquad \alpha_3 (l-\bar{q})^2 = 0$$

Example: Triangle Graph

$$I_3 = \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2 (l+p)^2 (l-\bar{q})^2}$$

$$\alpha_1 l^\mu + \alpha_2 (l+p)^\mu + \alpha_3 (l-\bar{q})^\mu = 0$$

$$\alpha_1 l^2 = 0 \qquad \alpha_2 (l+p)^2 = 0 \qquad \alpha_3 (l-\bar{q})^2 = 0$$

Solution to Landau's Equations:

Collinear: $l = \xi p \quad \alpha_3 = 0$

Soft: $l^\mu = 0 \quad \alpha_2 = \alpha_3 = 0$

Glauber: $l^\mu = 0 \quad \alpha_2 = \alpha_3 = 0$

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Landau Equations do not distinguish between Glauber and Soft regions. More analysis needed

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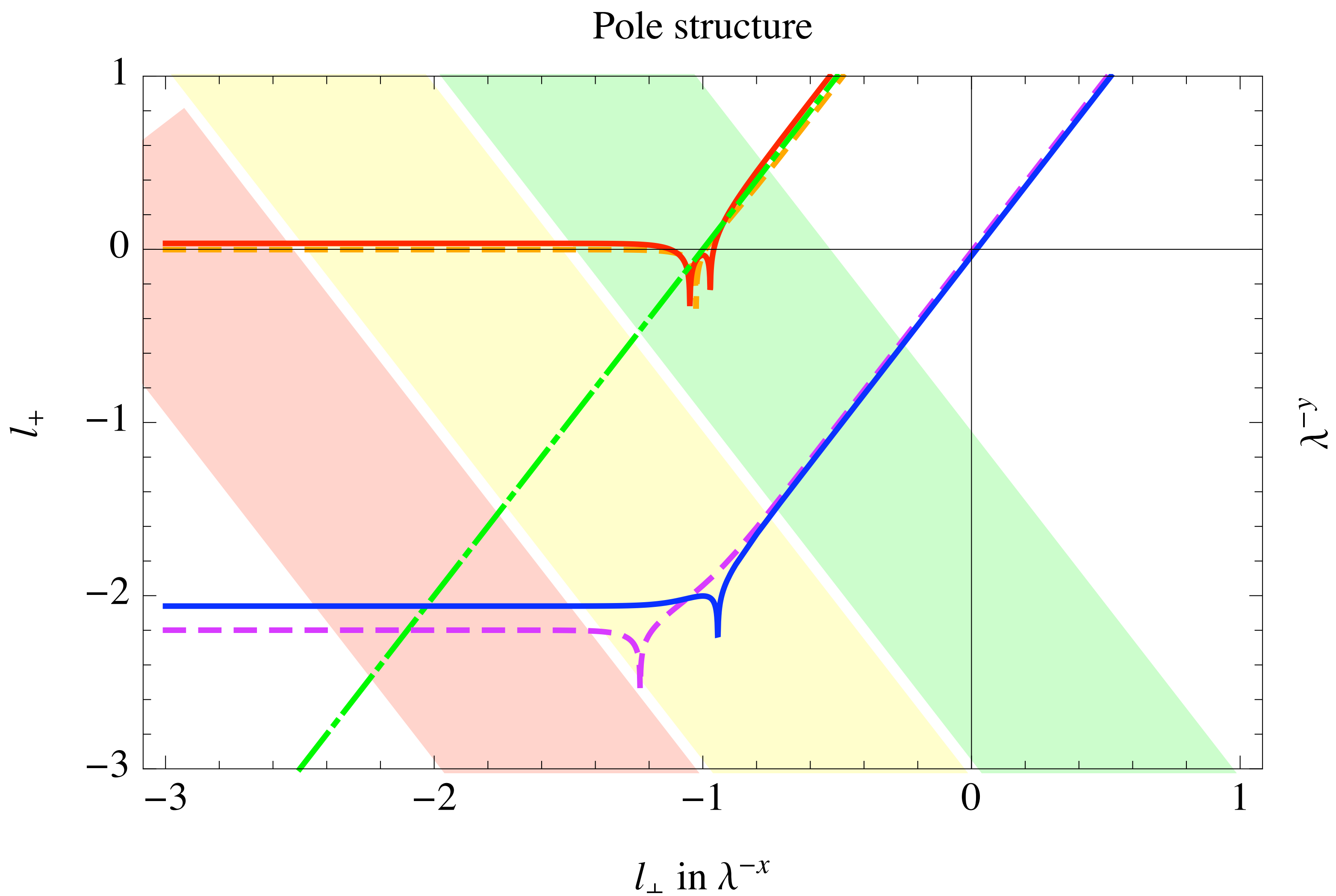
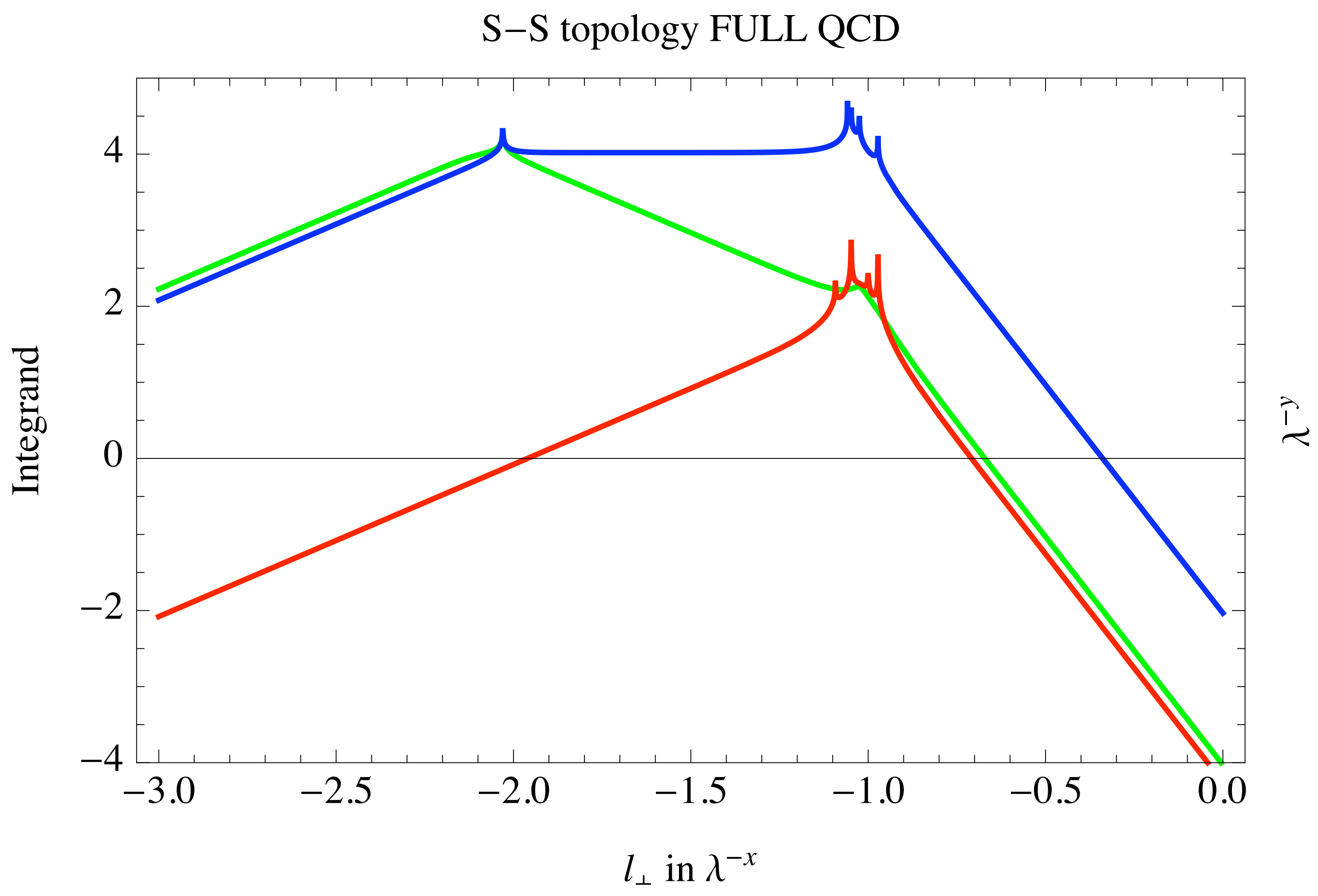
Pinches



Pinches



S−S topology FULL QCD
Integrand
4
2
0
−2
−4
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
$l_\perp$ in λ^{-x}
λ^{-y}