Factorization and Resummation of Higgs Boson Differential Distributions

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Outline

- Introductory Remarks
- Collins-Soper-Sterman Approach
- Effective field theory Approach.

-Factorization and resummation formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

• Conclusions

Higgs Boson

- The Higgs boson is the last missing piece of the SM.
- Search strategy complicated by decay properties:



- Typically there are three search regions:
 - (*i*) 90GeV < M_H < 130 GeV,
 - (*ii*) 130GeV < M_H < $2 \cdot M_{Z^0}$,
 - (iii) $2 \cdot M_{Z^0} < M_H < 800$ GeV.

• Search strategies vary in different mass regions.



• We restrict the transverse momentum of the Higgs:

 $m_h \gg p_T \gg \Lambda_{QCD}$

 Such pT restrictions can play an important role in Higgs searches.

Higgs Search at the LHC

• For the Higgs mass range: $130~{
m GeV} < m_h < 180~{
m GeV}$ • Higgs search channel:

$$gg \rightarrow h \rightarrow W^+W^- \rightarrow \ell^+ \nu \ell^- \bar{\nu}$$

- Large backgrounds from: $pp \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow \ell^+\nu\ell^-\bar{v} + jets$ Jet Veto enhances signal to background ratio
- Background elimination requires jet vetoes:

veto events with jets of $p_T > 20 \text{ GeV}$

LHC 14 TeV		Accepted event fraction		
reaction $pp \to X$	$\sigma \times BR^2$ [pb]	cut 1-3	cut 4- 6	cut 7
$pp \rightarrow H \rightarrow W^+W^- (m_H = 170 \text{ GeV})$	1.24	0.21	0.18	0.080
$pp \to W^+W^-$	7.4	0.14	0.055	0.039
$pp \to t\bar{t} \ (m_t = 175 \text{ GeV})$	62.0	0.17	0.070	0.001
$pp \to Wtb \ (m_t = 175 \text{ GeV})$	≈ 6	0.17	0.092	0.013

(Dittmar, Dreiner)

Low pT Region

 ${\mbox{ \bullet The schematic perturbative series for the pT distribution for }pp \longrightarrow h + X$



• Resummation of large logarithms required.

 Resummation has been studied in great detail in the Collins-Soper-Sterman formalism.

(Davies, Stirling; Arnold, Kauffman; Berger, Qiu; Ellis, Veseli, Ross, Webber; Ladinsky, Yuan; Fai, Zhang; Catani, Emilio, Trentadue; Hinchliffe, Novae; Florian, Grazzini,)

Collins-Soper-Sterman Formalism

$$A(P_A) + B(P_B) \rightarrow C(Q) + X$$
, $C = \gamma^*, W^{\pm}, Z, h$

• The transverse momentum distribution in the CSS formalism is schematically given by:





- Important in region of small Q_T .
- Treated with resummation.

- Obtained from fixed order calculation.
- Less Singular terms.
- Important in region of large Q_{T} .

 The CSS factorization and resummation formula takes the form:

$$\frac{d^{2}\sigma}{dp_{T} dY} = \sigma_{0} \int \frac{d^{2}b_{\perp}}{(2\pi)^{2}} e^{-i\vec{p}_{T}\cdot\vec{b}_{\perp}} \sum_{a,b} \left[C_{a} \otimes f_{a/P} \right] (x_{A}, b_{0}/b_{\perp}) \left[C_{b} \otimes f_{b/P} \right] (x_{B}, b_{0}/b_{\perp}) \\ \times \exp\left\{ \int_{b_{0}^{2}/b_{\perp}^{2}}^{\hat{Q}^{2}} \frac{d\mu^{2}}{\mu^{2}} \left[\ln \frac{\hat{Q}^{2}}{\mu^{2}} A(\alpha_{s}(\mu^{2})) + B(\alpha_{s}(\mu^{2})) \right] \right\}.$$

Coefficients with well defined perturbative expansions



- The integration over the impact parameter introduces a Landau pole.
- Landau pole present even for perturbative pT values.

 Treatment of Landau pole is prescription dependent. (Collins, Soper, Sterma; Kulesza, Laenen, Vogelsang; Qiu, Zhang,...) EFT Approach

EFT framework

• The low transverse momentum distribution is affected by physics at the scales:

$$m_h \gg p_T \gg \Lambda_{QCD}$$

• Hierarchy of scales suggests EFT approach with well defined power counting.

• The most singular pT emissions recoiling against the Higgs are soft and collinear emissions whose dynamics may be addressed in Soft-Collinear Effective Theory (SCET).

 Resummation has also been previously studied in SCET. (Idilbi, Ji, Juan; Gao, Li, Liu)

EFT framework

 $QCD(n_f = 6) \rightarrow QCD(n_f = 5) \rightarrow SCET_{p_T} \rightarrow SCET_{\Lambda_{QCD}}$



EFT framework

 $QCD(n_f = 6) \rightarrow QCD(n_f = 5) \rightarrow SCET_{p_T} \rightarrow SCET_{\Lambda_{QCD}}$



SCET Factorization Formula

• Factorization formula derived in SCET in schematic form:



- Large logarithms are summed via RG equations in EFTs.
- Formulation is free of Landau poles.

Integrating out the top



Leading term in the Higgs effective interaction with Gluons:

$$\mathcal{L}_{m_t} = C_{GGh} \frac{h}{v} G^a_{\mu\nu} G^{\mu\nu}_a \ , \qquad C_{GGh} = \frac{\alpha_s}{12\pi} \left\{ 1 + \frac{11}{4} \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right\}$$

$$Two \ \text{loop result for}$$

$$Two \ \text{loop result for}$$

$$Wilson \ \text{coefficient.}$$

(Chetyrkin, Kniehl, Kuhn, Schroder, Steinhauser, Sturm)

SCET

• Effective theory with soft and collinear degrees of freedom:

$$p_n \sim m_h(\eta^2, 1, \eta), \ p_{\bar{n}} \sim m_h(1, \eta^2, \eta), \ p_s \sim m_h(\eta, \eta, \eta),$$

$$\eta \sim \frac{p_T}{m_h}$$

• These modes describe the soft and collinear pT emissions recoiling against the Higgs.



Matching onto SCET



• Effective SCET operator:

 $\mathcal{O}(\omega_1, \omega_2) = g_{\mu\nu} h T \{ \operatorname{Tr} \left[S_n (g B_{n\perp}^{\mu})_{\omega_1} S_n^{\dagger} S_{\bar{n}} (g B_{\bar{n}\perp}^{\nu})_{\omega_2} S_{\bar{n}}^{\dagger} \right] \}$

Schematic form of SCET cross-section:





Factorization in SCET

We are here

 $\frac{d^2\sigma}{dp_T^2 dY} \sim \int PS ||\widehat{C} \otimes \langle \mathcal{O} \rangle|^2$

Factorize cross-section using soft-collinear decoupling in SCET

 $\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$ Hard matching coefficient squared Decoupled collinear and soft functions





 iBFs and soft functions field theoretically defined as the fourier transform of:

$$J_{n}^{\alpha\beta}(\omega_{1},x^{-},x_{\perp},\mu) = \sum_{\text{initial pols.}} \langle p_{1} | \left[gB_{1n\perp\beta}^{A}(x^{-},x_{\perp})\delta(\bar{\mathcal{P}}-\omega_{1})gB_{1n\perp\alpha}^{A}(0) \right] | p_{1} \rangle$$

$$J_{\bar{n}}^{\alpha\beta}(\omega_{1},y^{+},y_{\perp},\mu) = \sum_{\text{initial pols.}} \langle p_{2} | \left[gB_{1n\perp\beta}^{A}(y^{+},y_{\perp})\delta(\bar{\mathcal{P}}-\omega_{2})gB_{1n\perp\alpha}^{A}(0) \right] | p_{2} \rangle$$

$$S(z,\mu) = \langle 0 | \bar{T} \left[\text{Tr} \left(S_{\bar{n}}T^{D}S_{\bar{n}}^{\dagger}S_{n}T^{C}S_{n}^{\dagger} \right) (z) \right] T \left[\text{Tr} \left(S_{n}T^{C}S_{n}^{\dagger}S_{\bar{n}}T^{D}S_{\bar{n}}^{\dagger} \right) (0) \right] | 0 \rangle$$

iBFs



• iBFs are similar to the **Beam Functions**.

(Stewart, Tackmann, Waalewijin; Fleming, Leibovich, Mehen)

$$J_{n}^{\alpha\beta}(\omega_{1}, x^{-}, x_{\perp}, \mu) = \sum_{\text{initial pols.}} \langle p_{1} | [gB_{1n\perp\beta}^{A}(x^{-}, x_{\perp})\delta(\bar{\mathcal{P}} - \omega_{1})gB_{1n\perp\alpha}^{A}(0)] | p_{1} \rangle$$
Transverse index structure Transverse spatial separation

• iBFs are in general gauge dependent. However, the product of iBFs and the soft function is still gauge invariant.

 $B_n \otimes B_{\bar{n}} \otimes S$

Equivalence of Zero-Bin & Soft Subtractions

• Zero-bin iBF reproduces soft graphs. This is the equivalence of zero-bin and soft subtractions in SCET. (Lee, Sterman; Idilbi, Mehen; Chiu, Fuhrer, Kelly, Hoang, Manohar;...)

$$\frac{d^2\sigma}{dp_T^2dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

Zero-bin Subtraction in order to avoid double counting the soft region.

Equivalent to soft graphs

Equivalence of Zero-Bin & Soft Subtractions

$$B_{n,\bar{n}}^{\alpha\beta}(\omega, k^{\pm}, b_{\perp}, \mu) = \tilde{B}_{n,\bar{n}}^{\alpha\beta}(\omega, k^{\pm}, b_{\perp}, \mu) - B_{\{n0,\bar{n}0\}}^{\alpha\beta}(\omega, k^{\pm}, b_{\perp}, \mu)$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$
Purely Collinear iBF "Naive" iBF Zero-bin iBF

• Factorization can be reformulated with naive iBFs and an inverse Soft Function (iSF):

$$\frac{d^2\sigma}{dp_T^2dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

$$\swarrow \int \int f$$
Naive iBFs iSF

 This structure with an iSF is crucial for reproducing the known QCD cross-section.

Factorization in SCET We are here $\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$ iBFs are proton matrix elements and sensitive to the non-perturbative scale

iBF

PDF

• The iBFs are matched onto PDFs to separate the perturbative and non-perturbative scales:



iBFs to PDFs



• iBF is matched onto the PDF with matching coefficient defined as:

$$\tilde{B}_{n}^{\alpha\beta}(z,t_{n}^{+},b_{\perp},\mu) = -\frac{1}{z} \sum_{i=g,q,\bar{q}} \int_{z}^{1} \frac{dz'}{z'} \mathcal{I}_{n;g,i}^{\alpha\beta}(\frac{z}{z'},t_{n}^{+},b_{\perp},\mu) \frac{f_{i/P}(z',\mu)}{f_{i/P}(z',\mu)}$$

• The PDF is known to be scaleless and defined as:

Scaleless
$$\longrightarrow f_{g/P}(z,\mu) = \frac{-z\bar{n}\cdot p_1}{2} \sum_{\text{spins}} \langle p_1 | \left[\text{Tr}\{B^{\mu}_{\perp}(0)\delta(\bar{\mathcal{P}}-z\,\bar{n}\cdot p_1)B_{\perp\mu}(0)\} \right] | p_1 \rangle$$

• The matching coefficient is given by:

$$\mathcal{I}_{n;g,i}^{\beta\alpha}(\frac{z}{z'}, t_n^+, b_\perp, \mu) = -z \left[\tilde{B}_n^{\alpha\beta}(\frac{z}{z'}, z't_n^+, b_\perp, \mu) \right]_{\text{finite part in dim-reg}}$$

Factorization in SCET



• After matching the iBFs to the PDFs we get:

 $(\text{QCD} (n_f = 5))$

 SCET_{p_T}

iSF

iBF

PDF

iBF

PDF

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes [\mathcal{I}_{n,i} \otimes f_i] \otimes [\mathcal{I}_{\bar{n},j} \otimes f_j] \otimes S^{-1}$$

 Group the perturbative pT scale functions into transverse momentum dependent function(TMF):



Factorization Formula

• Factorization formula in full detail:

$$\frac{d^{2}\sigma}{dp_{T}^{2} dY} = \frac{\pi^{2}}{4(N_{c}^{2}-1)^{2}Q^{2}} \int_{0}^{1} \frac{dx_{1}}{x_{1}} \int_{0}^{1} \frac{dx_{2}}{x_{2}} \int_{x_{1}}^{1} \frac{dx'_{1}}{x'_{1}} \int_{x_{2}}^{1} \frac{dx'_{2}}{x'_{2}}$$

$$\times \frac{H(x_{1}, x_{2}, \mu_{Q}; \mu_{T})}{4} \mathcal{G}^{ij}(x_{1}, x'_{1}, x_{2}, x'_{2}, p_{T}, Y, \mu_{T})} \frac{f_{i/P}(x'_{1}, \mu_{T})f_{j/P}(x'_{2}, \mu_{T})}{4}$$
Hard function. Transverse momentum function(TMF).

• The transverse momentum function is a convolution of the iBF matching coefficients and the soft function:

$$\begin{aligned} \mathcal{G}^{ij}(x_1, x_1', x_2, x_2', p_T, Y, \mu_T) &= \int dt_n^+ \int dt_{\bar{n}}^- \int \frac{d^2 b_\perp}{(2\pi)^2} J_0(|\vec{b}_\perp| p_T) \\ &\times \mathcal{I}_{n;g,i}^{\beta\alpha}(\frac{x_1}{x_1'}, t_n^+, b_\perp, \mu_T) \mathcal{I}_{\bar{n};g,j}^{\beta\alpha}(\frac{x_2}{x_2'}, t_{\bar{n}}^-, b_\perp, \mu_T) \\ &\times \mathcal{S}^{-1}(x_1 Q - e^Y \sqrt{p_T^2 + m_h^2} - \frac{t_{\bar{n}}^-}{Q}, x_2 Q - e^{-Y} \sqrt{p_T^2 + m_h^2} - \frac{t_n^+}{Q}, b_\perp, \mu_T) \end{aligned}$$

Fixed order and Matching Analysis

One loop Matching onto SCET



• Wilson Coefficient obtained from finite part in dimensional regularization of the QCD result for gg->h. At one loop we have: $C(\bar{n} \cdot \hat{p}_1 n \cdot \hat{p}_2, \mu) = \frac{c \, \bar{n} \cdot \hat{p}_1 n \cdot \hat{p}_2}{v} \left\{ 1 + \frac{\alpha_s}{4\pi} C_A \left[\frac{11}{2} + \frac{\pi^2}{6} - \ln^2 \left(-\frac{\bar{n} \cdot \hat{p}_1 n \cdot \hat{p}_2}{\mu^2} \right) \right] \right\}$

(Ahrens, Becher, Neubert, Yang; Harlander)

iBFs



Soft function



Soft function definition:

 $S(z) = \langle 0 | \operatorname{Tr} \left(\bar{T} \{ S_{\bar{n}} T^D S_{\bar{n}}^{\dagger} S_n T^C S_n^{\dagger} \} \right) (z) \operatorname{Tr} \left(T \{ S_n T^C S_n^{\dagger} S_{\bar{n}} T^D S_{\bar{n}}^{\dagger} \} \right) (0) | 0 \rangle$



Running

Running

• Factorization formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

Schematic picture of running:



Running

• Factorization formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

• Schematic picture of running:



Limit of very small pT

• We derived a factorization formula in the limit:

 $m_h \gg p_T \gg \Lambda_{QCD}$

• For smaller values of pT, one can introduce a non-perturbative model for the transverse momentum function:





 Derived factorization formula for the Higgs transverse momentum distribution in an EFT approach:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- Resummation via RG equations in EFTs.
- Formulation is free of Landau poles.
- Limit of very small pT can be accommodated with a model for the transverse momentum dependent function (TMF).
- Formalism applies to the pT distribution of any other color neutral particles