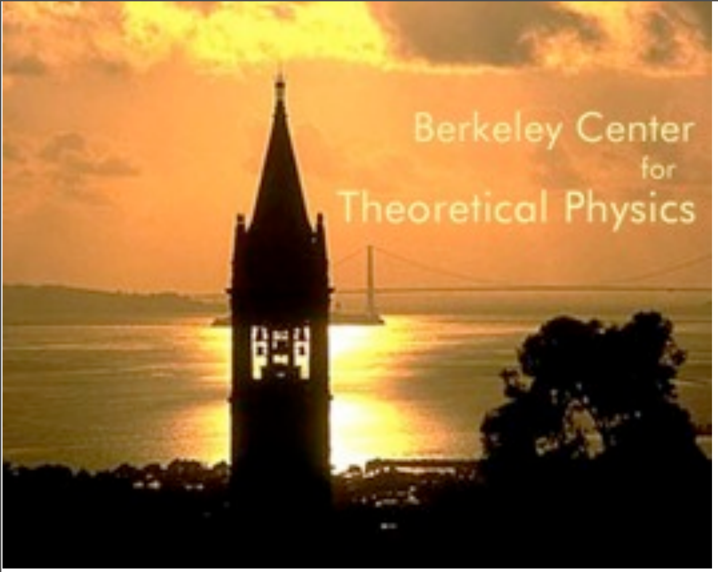


Jet Shapes in SCET

in collaboration with:

Steve Ellis

Chris Lee

Chris Vermilion

Jonathan Walsh

arXiv:0912.0262

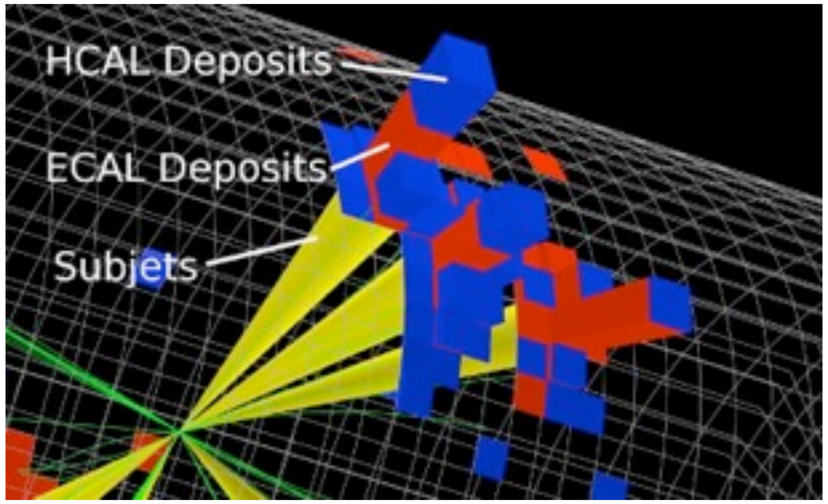
arXiv:1001.0014

Andrew Hornig
Berkeley Center for Theoretical Physics
& Lawrence Berkeley National Laboratory

SCET 2010 Workshop
April 8, 2010

Intro

- much recent work on jet substructure to find boosted tops, new physics, etc. (see Steve Ellis' talk)



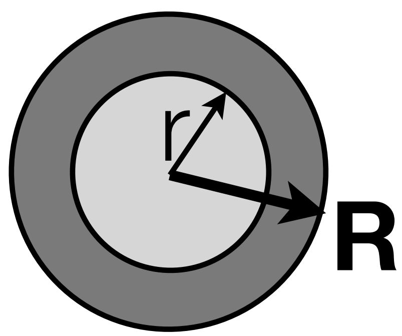
- often times it relies on Monte Carlo (e.g., 1→2 splittings in Pythia)
- question: can we use SCET to predict and systematically improve our understanding of QCD jets???
- we begin by trying to determine to what extent the ***shapes*** of quark and gluon jets are different...

“The” (Original) Jet Shape

- frac. of p_T inside subcone of radius r (gives energy profile of jet):

Ellis, Kunzst, Soper

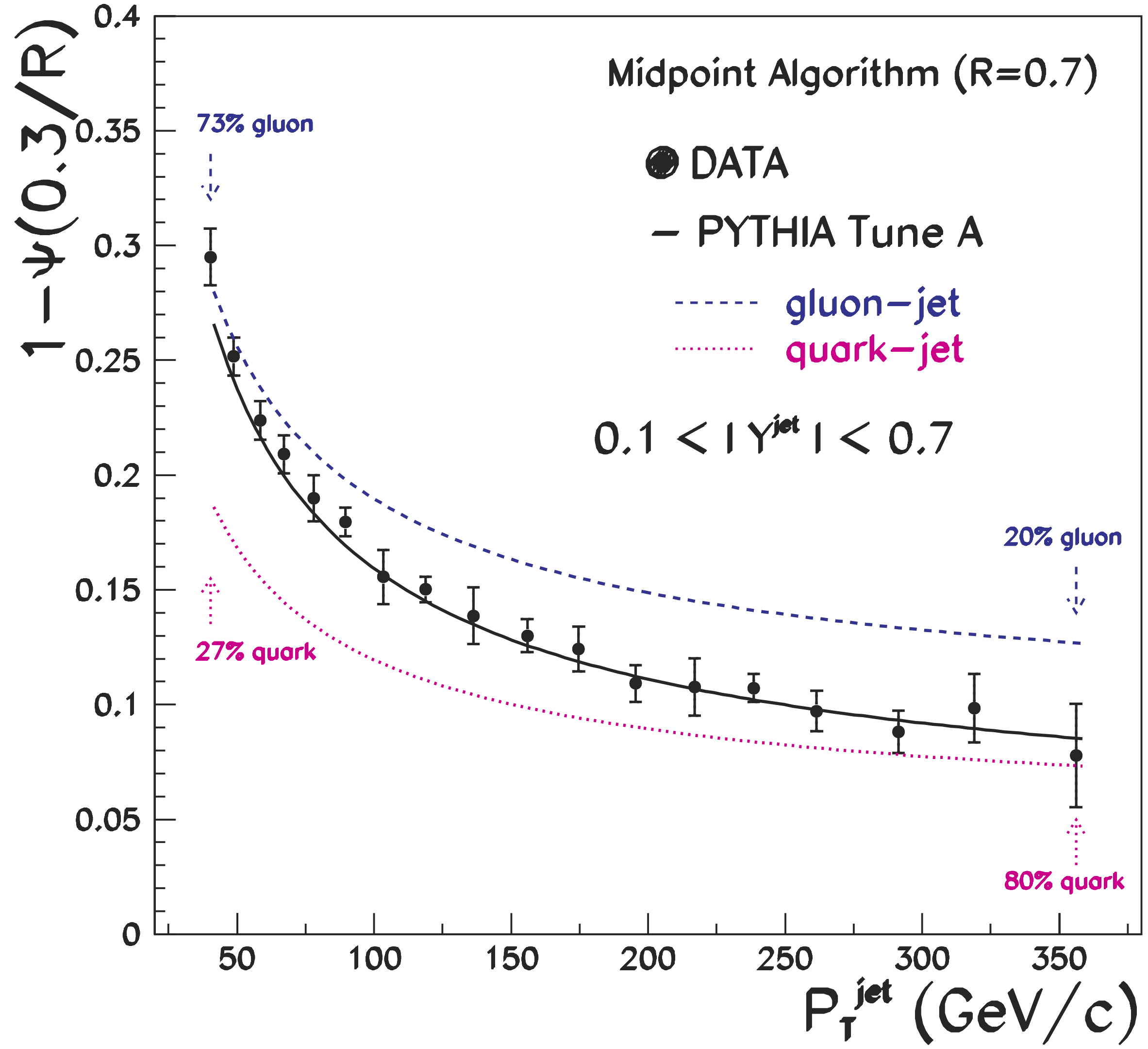
$$\psi(r/R):$$



q

g

CDF:



Angularities as jet shapes

Berger, Kucs, Sterman (2003)

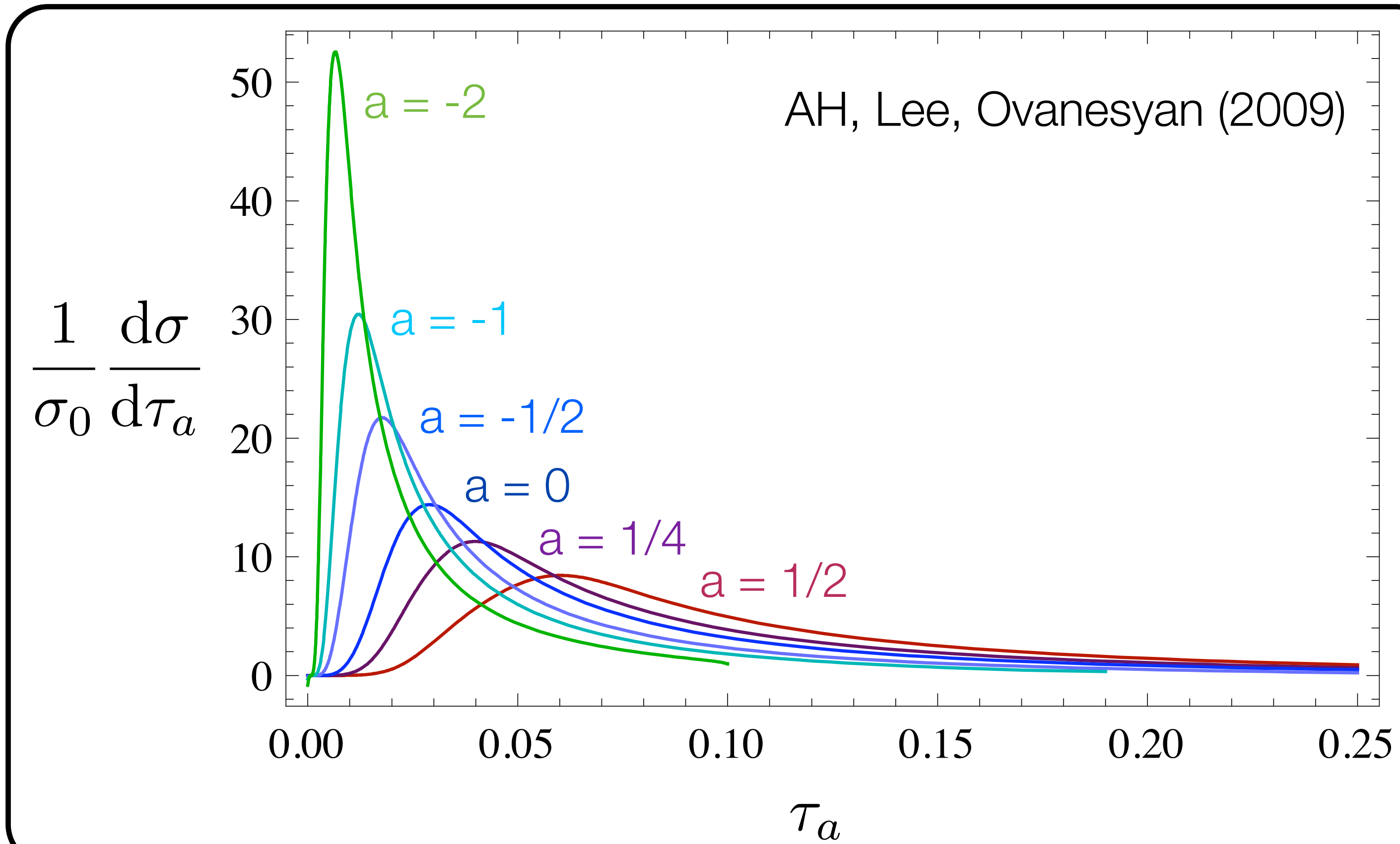
$$\tau_a = \frac{1}{Q} \sum_{i \in \text{jet}} E_i (\sin\theta_i)^a (1 - |\cos\theta_i|)^{1-a} = \frac{1}{Q} \sum_{i \in \text{jet}} |\mathbf{p}_i^T| e^{-|\eta_i|(1-a)}$$

↗ sum only over jet

a = 0 thrust
a = 1 broadening

infrared safety: $-\infty < a < 2$
factorizability: $-\infty < a < 1$

- Knowing distribution for multiple “a” also gives profile:



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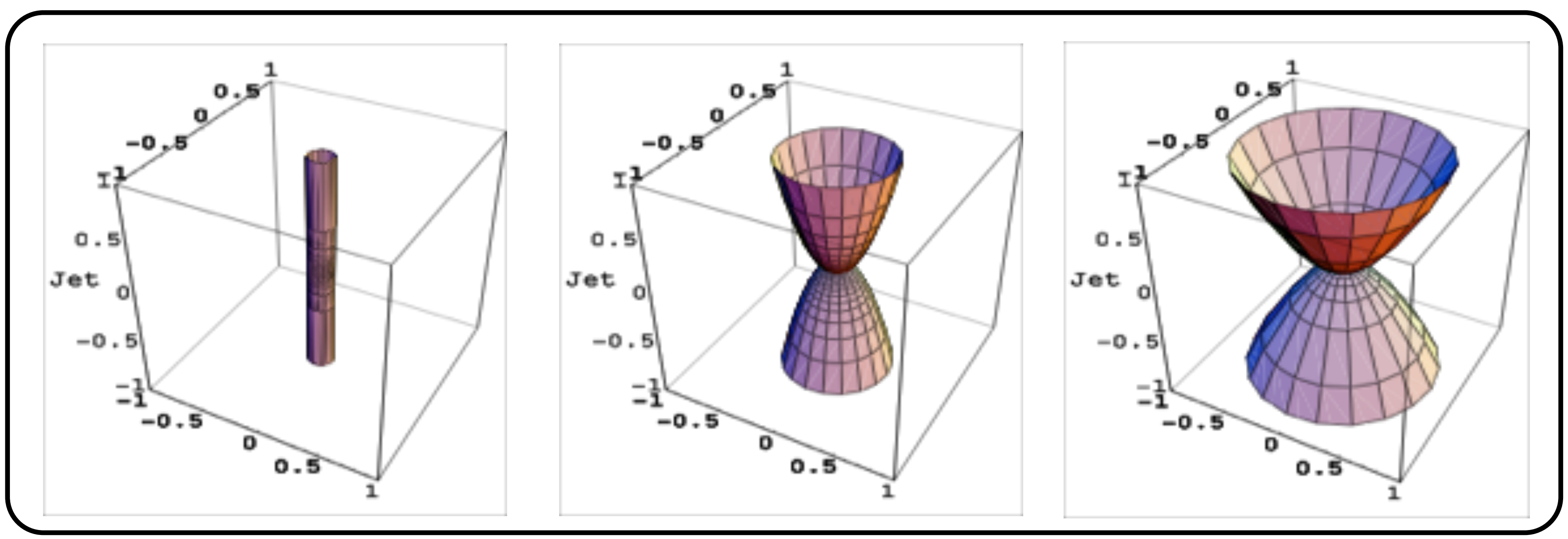
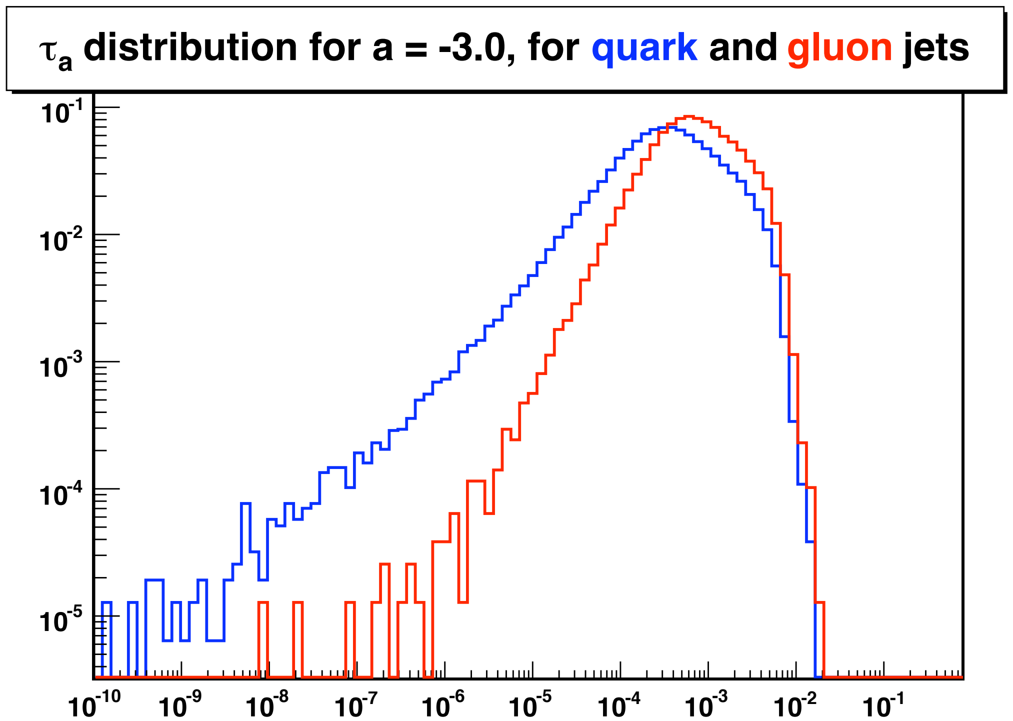
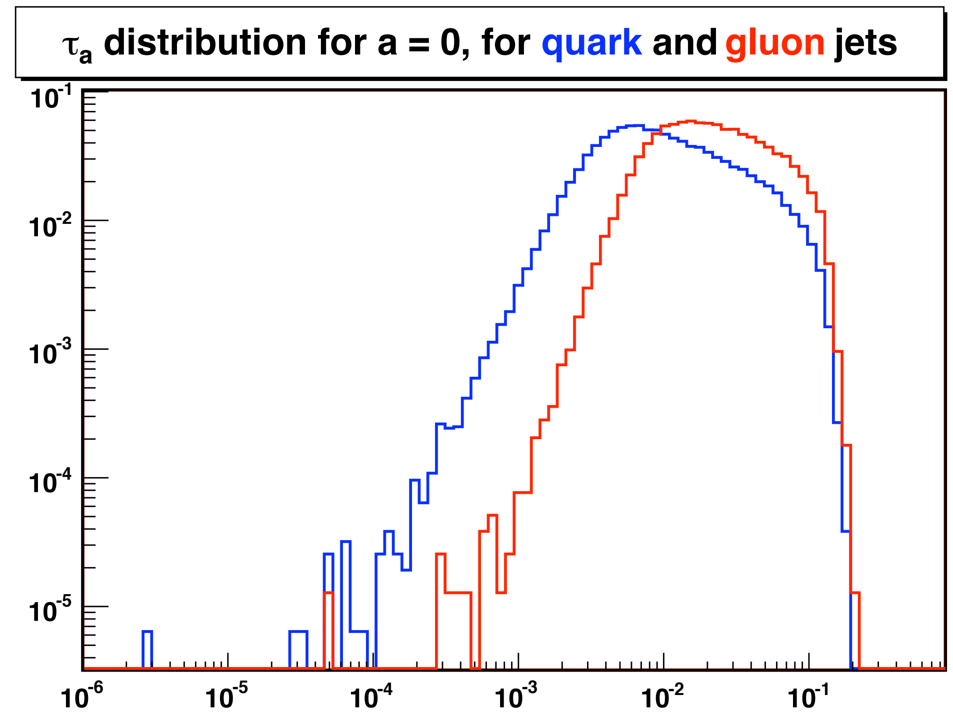


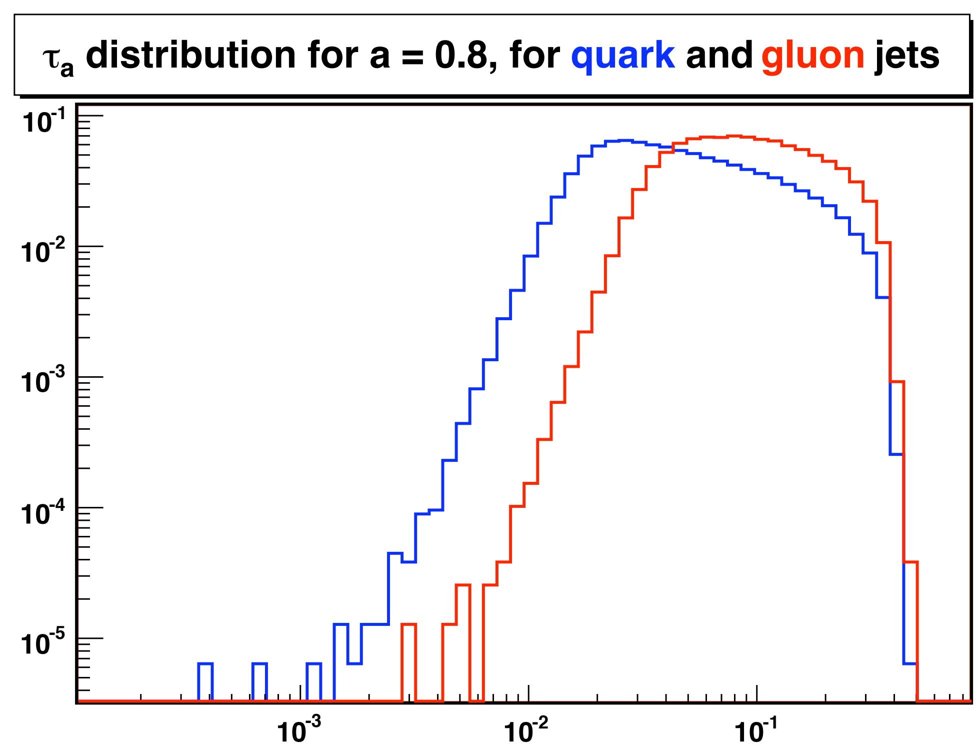
a = 1 a = 0 a = -1

Using angularities to distinguish quark and gluon jets

PRELIMINARY







Studied quark v. gluon jets in e+e- to 3 well-separated jets in PYTHIA

cuts exist keeping ~2% of gluon jets and ~20% of quark jets,

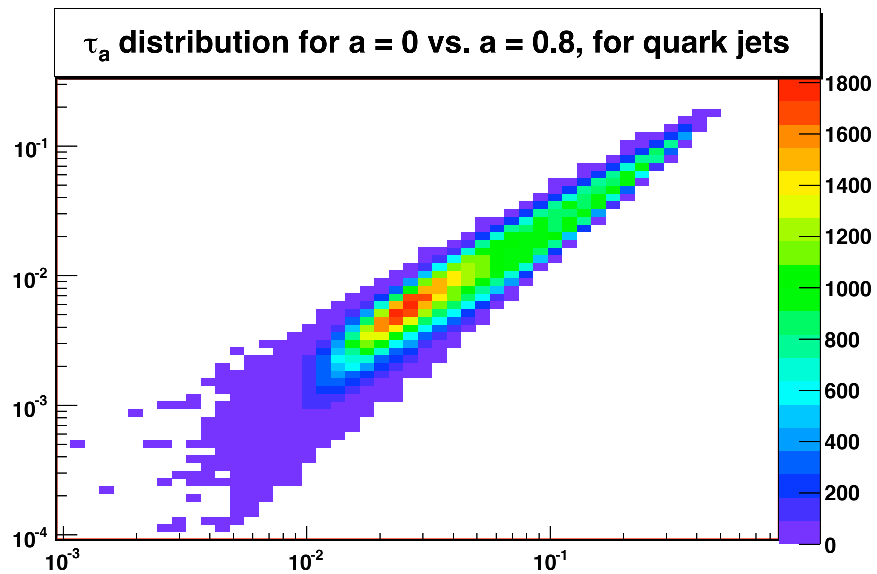
or ~15% of gluons and ~8% quarks.

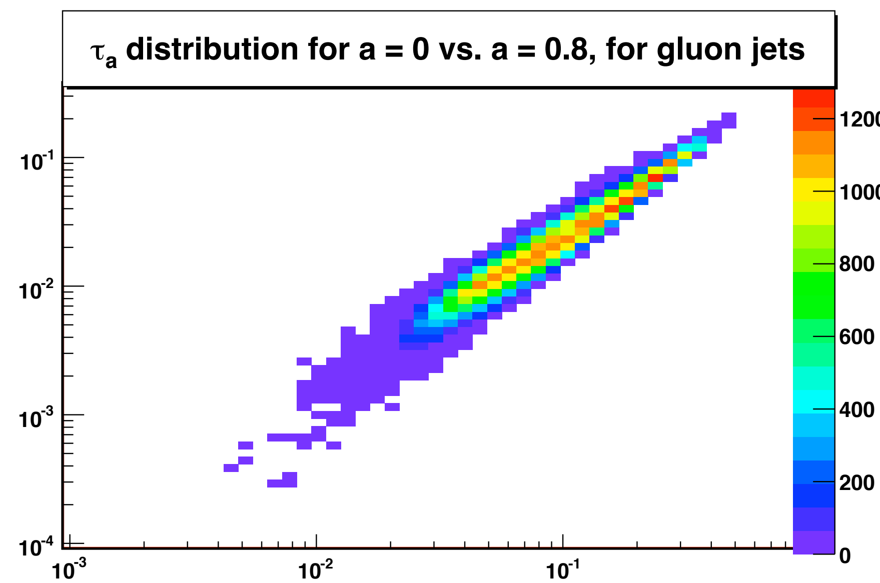
Greater discriminating power in correlated distributions for multiple values of a?...

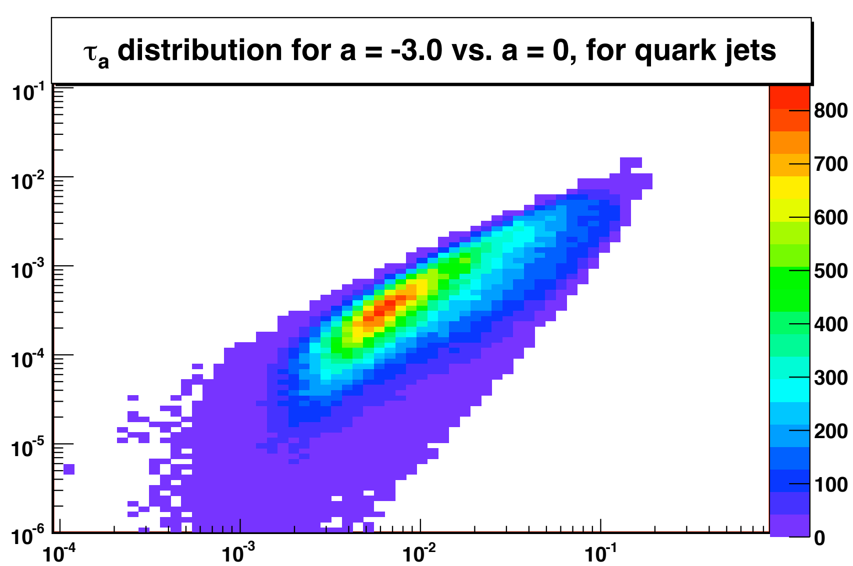
Using angularities to distinguish quark and gluon jets

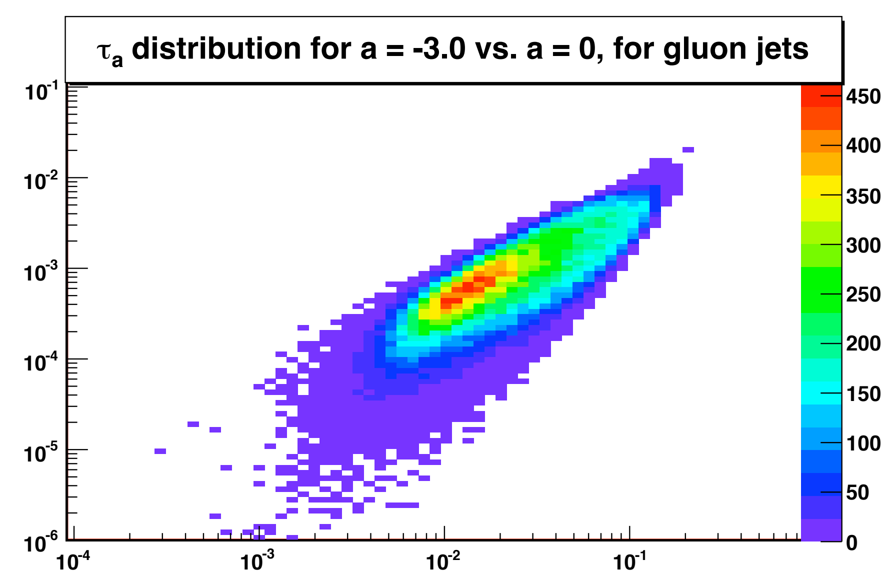
PRELIMINARY

- 2d-cuts (or multivariate analysis) may have greater distinguishing power than 1d-cuts *(work in progress):*



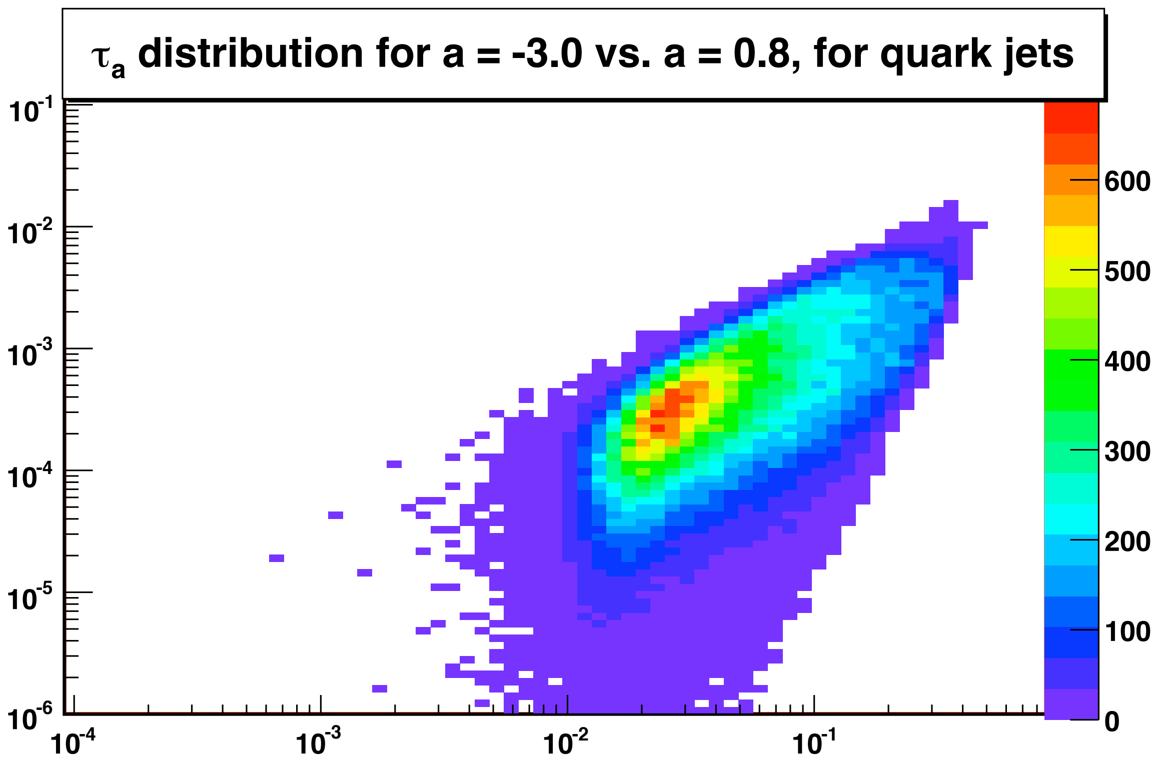


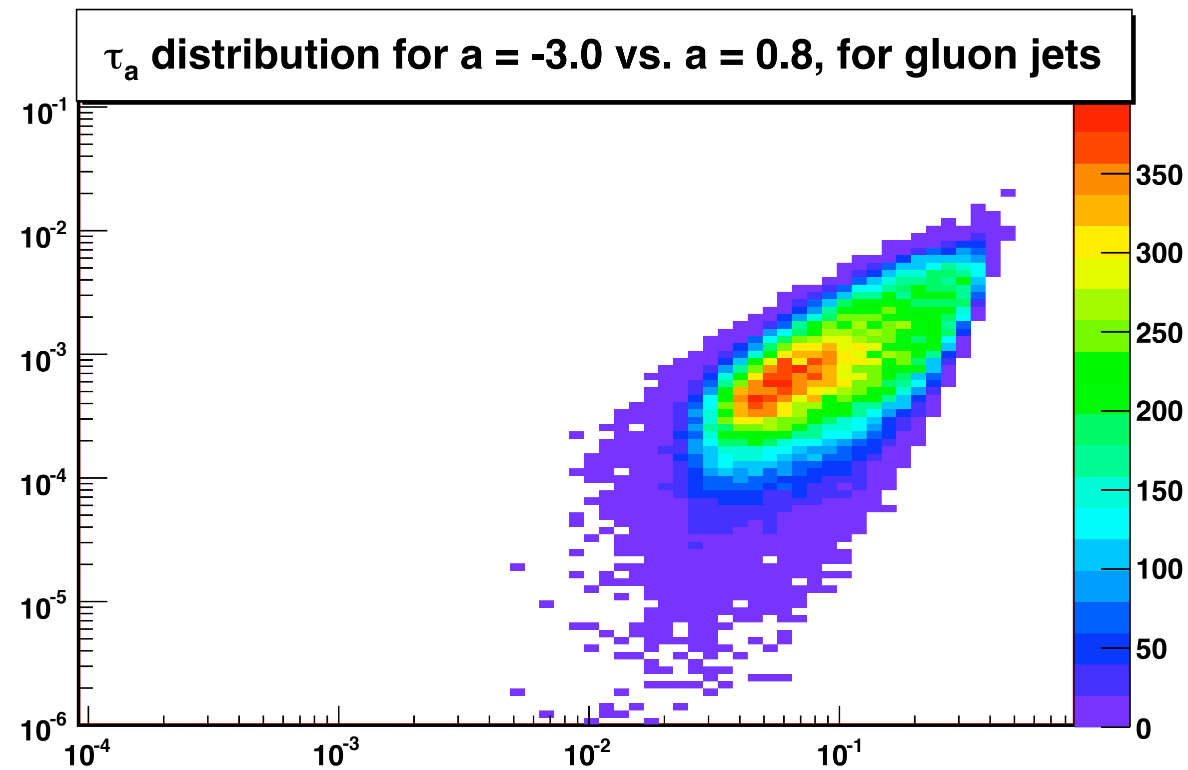




Using angularities to distinguish quark and gluon jets

PRELIMINARY





- our starting focus: likelihood fnc. from analytical, singly-differential distributions from SCET

Jet Algorithms

- N jets ⇒ need jet algorithms in factorization (or global "N-jet shapes")
- Examples:
  - k_T
  - Cambridge-Aachen (CA)
  - anti-k_T
  - SISCone
  - Snowmass
  - Sterman-Weinberg (SW)
  - JADE
  - ...
- not a zoology of all algorithms (see talk by Saba for more...)
- its up to the algorithm to act at higher orders as it should

Jet Algorithms

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  - ...

Our Focus:

“k_T-type” (for k_T, Cambridge-Aachen, anti-k_T)

“cone-type” (for SISCone, Snowmass)

- not a zoology of all algorithms (see talk by Saba for more...)
- its up to the algorithm to act at higher orders as it should

k_T-type Algorithms

- sequential recombination:
  - for all "particles", make list of d_i for each particle and d_{ij} for each pair of particles

$$d_{ij} = \min\{d_i, d_j\}\frac{\Delta R_{ij}}{R}$$

d_i energy metric (E^α or p_T^α for e^+e^- or pp, $\alpha = \pm 1, 0$ for k_T, anti-k_T, or CA)

ΔR_{ij} angular metric (θ_{ij} or $\sqrt{\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2}$)

R number (typically, 0.7 or 1)

  - if d_{ij} is smallest, merge i & j, call result a "particle"; if d_i is smallest, remove from list and call i a jet

k_T-type Algorithms

- for 2 particle (coll & soft or coll & coll), metric cancels:
  - to merge, need $d_{ij} < d_i$ and d_j

$$\Rightarrow \; d_{ij} = \min\{d_i, d_j\} \frac{\Delta R_{ij}}{R} < \min\{d_i, d_j\}$$

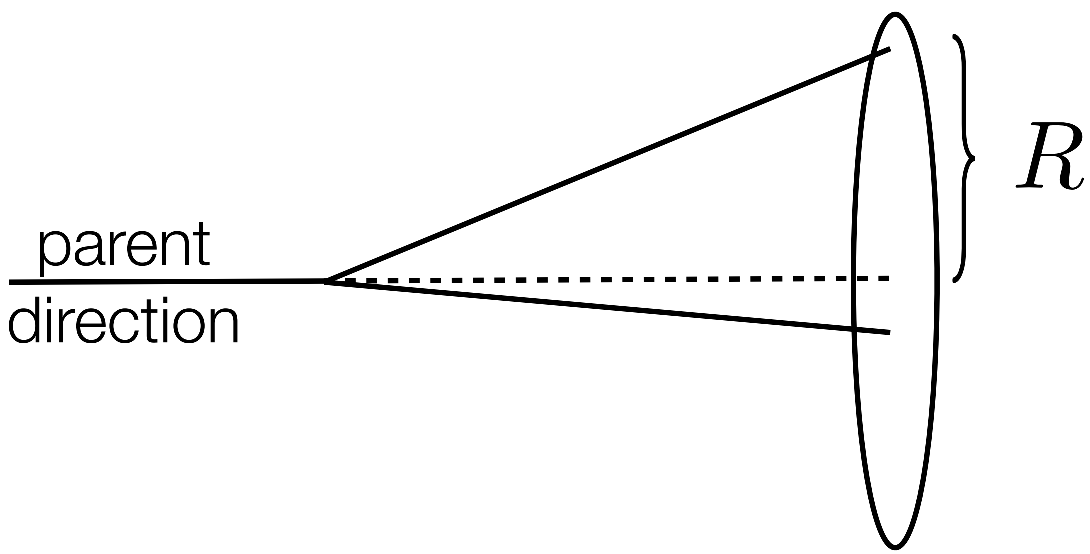
$$\Rightarrow \; \Delta R_{ij} < R$$

note: we focus on this "inclusive" type of recombination (also are "exclusive": d_{ij} is compared to some fixed number - see Saba's talk)

- metric matters for order in multi-particle state: anti-k_T groups hardest first, k_T groups softest

Cone-type algorithms

- modern example: SISCone
- find "stable" cones: parent direction = center of cone



$\Rightarrow$ need only to impose individual restrictions $\Delta R_{i,n} < R$

- nicer? so far, but split/merge issue for overlapping stable jets

Goal: Factorize & Resum N-jet distribution (M "measured" and N-M "unmeasured")

- "unmeasured jet": jet whose direction and energy (label momenta) are measured, but otherwise unprobed

- "measured jet": (singly) differential in angularity of jet (+ labels)

- reasons for having both:
  - unmeasured jets related to total cross-section (see Saba's talk)
  - unmeasured jets mimic beams w/ no measurement
  - study what's needed in general for consistency of factorization

Overview of Factorization

- **recall**: 2 hemisphere-jet factorization:

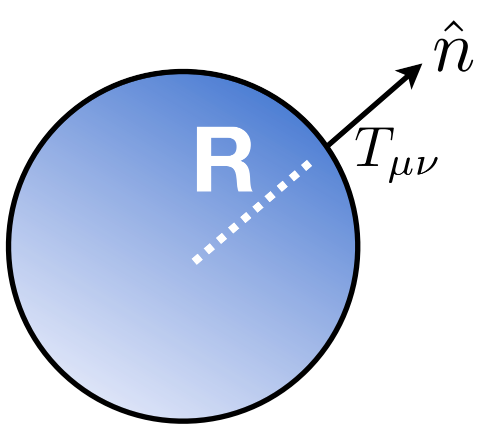
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q,\mu)\int d\tau_n d\tau_{\bar{n}} \underbrace{J_n(\tau_n;\mu) J_{\bar{n}}(\tau_{\bar{n}};\mu)}_{\text{inclusive jet functions}} \underbrace{S(\tau - \tau_n - \tau_{\bar{n}};\mu)}_{\text{hemisphere soft function}}$$

  - thrust:

$$J_n(\tau) = \frac{1}{N_C}\text{Disc}\left[\int \mathrm{d}^4x\, e^{il\cdot x}\,\text{Tr}\,\langle 0|\text{T}\,\chi_{n,Q}(x)\,\bar{\chi}_{n,Q}(0)|0\rangle\right]\Bigg|_{l^+ = Q\tau}$$

  - in general (e.g., angularity) need $\delta(\tau - \hat{\tau})$ insertion w/

$$\hat{\tau} = \tau(\hat{\mathcal{E}}) = \tau(\hat{\mathcal{E}}_s + \sum_i \hat{\mathcal{E}}_{n_i}) = \tau(\hat{\mathcal{E}}_s) + \sum_i \tau(\hat{\mathcal{E}}_{n_i})$$



$$\mathcal{E}(\hat{n}) = \lim_{R\to\infty}\int_0^\infty dt\,\hat{n}_i T_{0i}(t, R\hat{n})$$

$$\mathcal{E}(\hat{n})|N\rangle = \sum_{i\in N} E_i \delta^2(\hat{n} - \hat{n}_i)|N\rangle$$

Korchemsky, Oderda, Sterman (1997); cf. Sveshnikov, Tkachov (1996)

Overview of Factorization

- **now**: also need that only E in jet contributes:
  - jet algorithm → jets and out-of-jets/soft

$$\mathcal{J}(\hat{\mathcal{E}}) = \{\mathcal{J}_{n_1}(\mathcal{E}), \ldots, \mathcal{J}_{n_N}(\mathcal{E}), \mathcal{J}_s(\mathcal{E})\} \simeq \{\mathcal{J}_{n_1}(\mathcal{E}_{n_1}), \ldots, \mathcal{J}_{n_N}(\mathcal{E}_{n_N}), \mathcal{J}_s(\mathcal{E}_s)\}$$

  - tells which pts in phase-space belong to jet i:

$$\hat{\theta}_{n_i} = \theta(\mathcal{J}(\hat{\mathcal{E}}_{n_i}))$$

$$\left.\begin{aligned} \hat{\tau}_{s_i} &= \hat{\theta}_{n_i} \hat{\tau}_s \\ \hat{\tau}_{\text{out}} &= (1 - \sum_i \hat{\theta}_i) \hat{\tau}_s \end{aligned}\right\}$$

same as before, but nonzero τ_{out}

$$\hat{\tau}_{n_i} \longrightarrow \hat{\theta}_{n_i} \hat{\tau}_{n_i}$$

no longer "inclusive jet fncs"

Overview of Factorization

- additional multijet assumptions (⇔ power corrections):

  1. jet algorithms respect factorization (soft doesn't know about collinear splittings)

  2. jets are well-collimated and well-separated: not (N-1)-jet

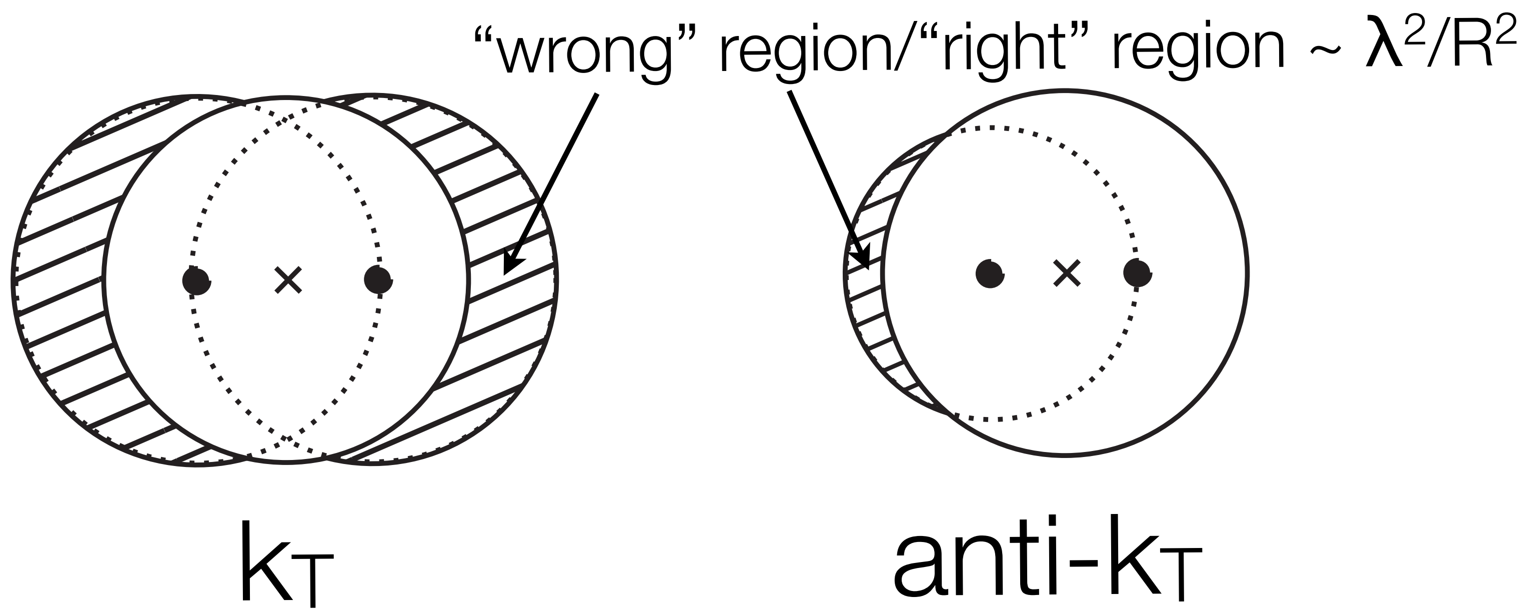
  3. energy outside jets is cut off by Λ : not (N+1)-jet

⇒ not just a single, global parameter $\tau_{\mathrm{event}} \ll 1$

⇒ many scales $\tau_{\mathrm{jet}}^1, \tau_{\mathrm{jet}}^2, \ldots, \Lambda \ll 1$ and $R \ll n_i \cdot n_j$ (more later....)

Power Corrections from Algorithm

- need soft fnc. depend only on "n" of jets (not coll. splitting details)
- k_T-type algorithms: all orders of soft emission for 2 collinear splittings (similar story at all orders in collinear splittings):



- cone-type better (soft only need to know about "n"), but again split merge issue for borderline cases....

⇒ take R ~ 1 for observables that are sensitive to soft momenta (also, calorimeter cell itself has R ~ .1 @ LHC)

Power Corrections from Jet Separation

- we will find that consistency (μ-independence) to $\mathcal{O}(1/t^2)$ where

$$t = \frac{\tan\frac{\psi}{2}}{\tan\frac{R}{2}}$$

ψ angle between jets

R angular size of jet

- suggests that this is the meaning of “well-separated” (but, no $\mathcal{L}$)
- note: t → ∞ for back-to-back jets
- $1/t^2$ can be small with R ~ 1:
  - e.g., for 3 jet, mercedes-benz events with R = .7, $1/t^2 = .044$
- @ LHC, this is improved for non-central jets (R → R/cosh η)

Power Corrections from Jet Separation

- consistency for arbitrary t if all jets are measured (unmeasured jets need large t since there is no other handle like τ)

- however, finite parts of the form

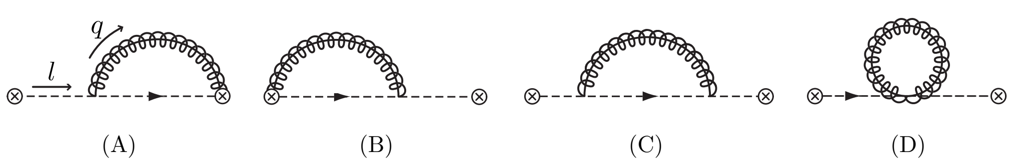
$$f(t)\log(\Lambda/Q) \qquad\qquad f(t)\sim 1/t^2$$

- again, suggestive that the “true” expansion is in $1/t^2$

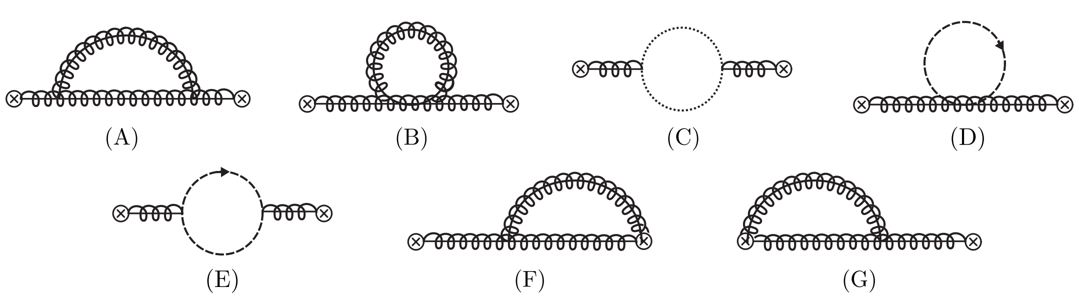
New Calculations

- graphs with jet algorithm in N-jet calc:

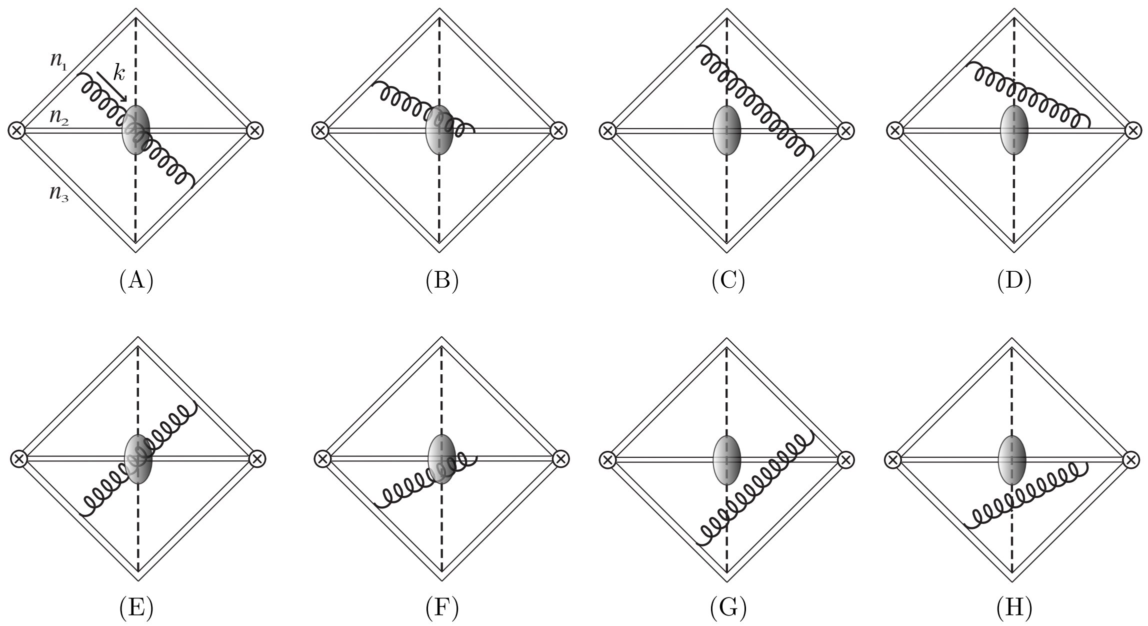
quark jet:



gluon jet:



soft:



Jet Function (& Zero-Bin)

- out-of-jet contributions: suppressed by Λ/Q

- algorithm introduces new scales ⇒ nonzero zero-bin!

  (μ-independence/consistency of anom. dim. requires this)

- should not take scaling limits of theta functions; can take any limit on full (naive - zero-bin) limit of our results (for $R \gg \tau$ to get incl. jet function)

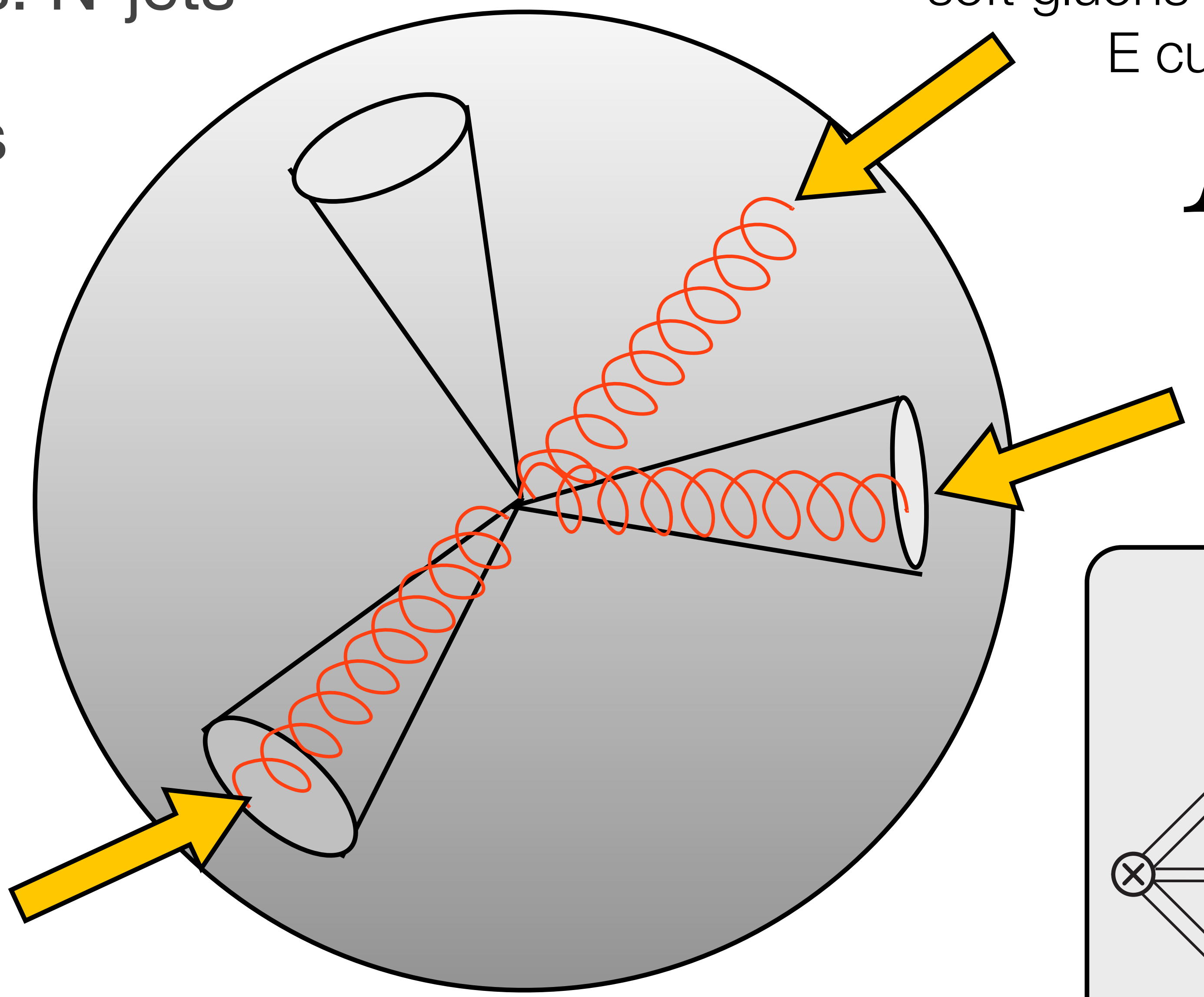
- see Teppo's talk for more discussion/details

Soft Function

- calculations: N-jets
- plots: 3-jets

soft gluons outside all jets:
E cut off by

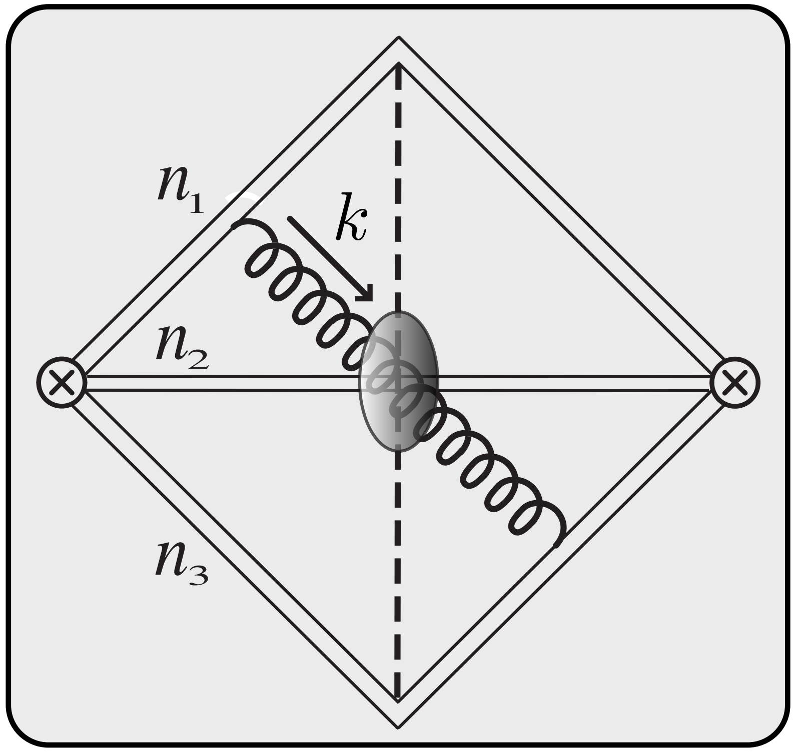
Λ



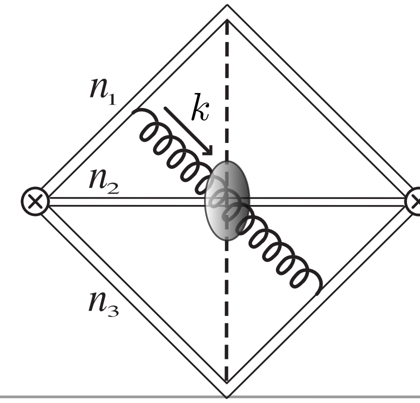
soft gluons in
unmeasured jets:
any E ⇒ scaleless

soft gluons in
measured jet #i
contribute

τ_s^i



Soft Function

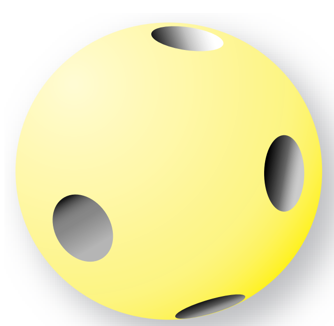
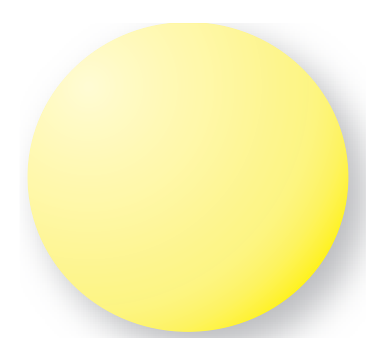
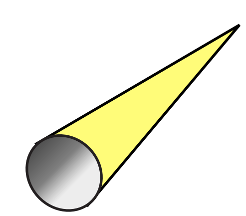


$$S_{(1)}(\tau_a^1,\tau_a^2,\ldots,\tau_a^M) = \sum_{i\neq j}\Bigg[\underbrace{\sum_{k\in\text{meas}} S_{ij}^{\text{meas}}(\tau_a^k)\prod_{l\neq k}^{M}\delta(\tau_a^l)}_{\text{measured}}\Bigg] + \sum_{i\neq j}\Bigg[\Bigg(\underbrace{S_{ij}^{\text{incl}} - \sum_{k\in\text{meas}}\bar{S}_{ij}^k}_{\text{out of meas., } E<\Lambda} + \underbrace{\sum_{k\notin\text{meas}} S_{ij}^k}_{\text{in unmeas., } E>\Lambda}\Bigg)\prod_{l}^{M}\delta(\tau_a^l)\Bigg]$$

$$= \sum_{k\in\text{meas}} S_{(1)}^{\text{meas}}(\tau_a^k)\prod_{l\neq k}^{M}\delta(\tau_a^l) + S_{(1)}^{\text{unmeas}}\prod_{l}^{M}\delta(\tau_a^l)$$

universal "swiss cheese"

- Using $\bar{S}_{ij}^k = -S_{ij}^k$ (scaleless), $S_{(1)}^{\text{unmeas}} \equiv \sum_{i\neq j}\left(S_{ij}^{\text{incl}} + \sum_{k=1}^{N} S_{ij}^k\right)$

 $=$  $-\sum_{i}^{N}$ 

Results for Anomalous Dimensions to $\mathcal{O}(1/t^2)$

$$\mu \frac{d}{d\mu} H(Q;\mu) = \gamma_H(\mu) H(Q;\mu) \qquad \mu \frac{d}{d\mu} F(\tau_a;\mu) = \int d\tau_a' \gamma_F(\tau_a - \tau_a';\mu) F(\tau_a';\mu)$$

hard	$\gamma_H = -\sum_{i=1}^{N}\left(\Gamma \ln \frac{\mu^2}{\bar{\omega}_H^2}\mathbf{T}_i^2 + \gamma_i\right) - \Gamma \sum_{i \neq j}^{N} \mathbf{T}_i \cdot \mathbf{T}_j \ln \frac{n_i \cdot n_j}{2}$
unmeasured jet	$\gamma_{J_i}^{\text{unmeas}} = \Gamma \mathbf{T}_i^2 \ln \frac{\mu^2}{\omega_i^2 \tan^2 \frac{R}{2}} + \gamma_i$
universal soft (“swiss cheese”)	$\gamma_S^{\text{unmeas}} = \Gamma \sum_{i=1}^{N} \mathbf{T}_i^2 \ln \tan^2 \frac{R}{2} + \Gamma \sum_{i\neq j}^{N} \mathbf{T}_i \cdot \mathbf{T}_j \ln \frac{n_i \cdot n_j}{2}$
measured jet	$\gamma_{J_i}^{\text{meas}}(\tau_a^i) = \mathbf{T}_i^2 \left[\Gamma \frac{2-a}{1-a} \ln \frac{\mu^2}{\omega_i^2} + \gamma_i\right] \delta(\tau_a^i) - 2\Gamma \mathbf{T}_i^2 \frac{1}{1-a}\left[\frac{\Theta(\tau_a^i)}{\tau_a^i}\right]_+$
measured soft	$\gamma_S^{\text{meas}}(\tau_a^i;\mu) = \sum_{i=1}^{M}\left\{-\Gamma \mathbf{T}_i^2 \frac{1}{1-a} \ln\left(\frac{\mu^2 \tan^{2(1-a)} \frac{R}{2}}{\omega_i^2}\right)\delta(\tau_a^i) + 2\Gamma \mathbf{T}_i^2 \frac{1}{1-a}\left[\frac{\Theta(\tau_a^i)}{\tau_a^i}\right]_+\right\}$

$$\bar{\omega}_H = \prod_{i=1}^{N} \omega_i^{\mathbf{T}_i^2/\mathbf{T}^2} \qquad \Gamma = \frac{\alpha_s}{\pi}\,, \quad \gamma_q = \frac{3\alpha_s}{2\pi}\,, \quad \gamma_g = \frac{\alpha_s}{\pi}\frac{11 C_A - 2N_F}{6}$$

Results for Anomalous Dimensions to $\mathcal{O}(1/t^2)$

Requirement for consistency:

$$0 = \Big(\gamma_H(\mu) + \gamma_S^{\text{unmeas}}(\mu) + \sum_{i \notin \text{meas}} \gamma_{J_i}(\mu)\Big)\delta(\tau_a^i) + \sum_{i \in \text{meas}} \Big(\gamma_{J_i}(\tau_a^i;\mu)) + \gamma_S^{\text{meas}}(\tau_a^i;\mu)\Big)$$

hard	$\gamma_H = -\sum_{i=1}^{N}\left(\Gamma \ln \frac{\mu^2}{\bar{\omega}_H^2}\mathbf{T}_i^2 + \gamma_i\right) - \Gamma \sum_{i \neq j}^{N} \mathbf{T}_i \cdot \mathbf{T}_j \ln \frac{n_i \cdot n_j}{2}$
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$$\bar{\omega}_H = \prod_{i=1}^{N} \omega_i^{\mathbf{T}_i^2/\mathbf{T}^2} \qquad \Gamma = \frac{\alpha_s}{\pi}\,, \quad \gamma_q = \frac{3\alpha_s}{2\pi}\,, \quad \gamma_g = \frac{\alpha_s}{\pi}\frac{11C_A - 2N_F}{6}$$

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Requirement for consistency:

$$0=\Big(\gamma_H(\mu)+\gamma_S^{\rm unmeas}(\mu)+\sum_{i\notin\rm meas}\gamma_{J_i}(\mu)\Big)\delta(\tau_a^i)+\sum_{i\in\rm meas}\Big(\gamma_{J_i}(\tau_a^i;\mu)\big)+\gamma_S^{\rm meas}(\tau_a^i;\mu)\Big)$$

hard	$\gamma_H=-\sum_{i=1}^{N}\left(\Gamma\ln\frac{\mu^2}{\bar\omega_H^2}\mathbf{T}_i^2+\gamma_i\right)-\Gamma\sum_{i\neq j}^{N}\mathbf{T}_i\cdot\mathbf{T}_j\ln\frac{n_i\cdot n_j}{2}$
unmeasured jet	$\gamma_{J_i}^{\rm unmeas}=\Gamma\mathbf{T}_i^2\ln\frac{\mu^2}{\omega_i^2\tan^2\frac{R}{2}}+\gamma_i$
universal soft ("swiss cheese")	$\gamma_S^{\rm unmeas}=\Gamma\sum_{i=1}^{N}\mathbf{T}_i^2\ln\tan^2\frac{R}{2}+\Gamma\sum_{i\neq j}^{N}\mathbf{T}_i\cdot\mathbf{T}_j\ln\frac{n_i\cdot n_j}{2}$
measured jet	$\gamma_{J_i}^{\rm meas}(\tau_a^i)=\mathbf{T}_i^2\left[\Gamma\frac{2-a}{1-a}\ln\frac{\mu^2}{\omega_i^2}+\gamma_i\right]\delta(\tau_a^i)-2\Gamma\mathbf{T}_i^2\frac{1}{1-a}\left[\frac{\Theta(\tau_a^i)}{\tau_a^i}\right]_+$
measured soft	$\gamma_S^{\rm meas}(\tau_a^i;\mu)=\sum_{i=1}^{M}\left\{-\Gamma\mathbf{T}_i^2\frac{1}{1-a}\ln\left(\frac{\mu^2\tan^{2(1-a)}\frac{R}{2}}{\omega_i^2}\right)\delta(\tau_a^i)+2\Gamma\mathbf{T}_i^2\frac{1}{1-a}\left[\frac{\Theta(\tau_a^i)}{\tau_a^i}\right]_+\right\}$

$$\bar\omega_H=\prod_{i=1}^{N}\omega_i^{\mathbf{T}_i^2/\mathbf{T}^2}\qquad\Gamma=\frac{\alpha_s}{\pi},\quad\gamma_q=\frac{3\alpha_s}{2\pi},\quad\gamma_g=\frac{\alpha_s}{\pi}\frac{11C_A-2N_F}{6}$$

Results for Anomalous Dimensions to $\mathcal{O}(1/t^2)$

Requirement for consistency:

$$0 = \Big(\gamma_H(\mu) + \gamma_S^{\rm unmeas}(\mu) + \sum_{i\notin {\rm meas}} \gamma_{J_i}(\mu)\Big)\delta(\tau_a^i) + \sum_{i\in{\rm meas}} \Big(\gamma_{J_i}(\tau_a^i;\mu)) + \gamma_S^{\rm meas}(\tau_a^i;\mu)\Big)$$

hard	$\gamma_H = -\sum_{i=1}^N \left(\Gamma \ln \frac{\mu^2}{\bar\omega_H^2}\mathbf{T}_i^2 + \gamma_i\right) - \Gamma \sum_{i\neq j}^N \mathbf{T}_i\cdot\mathbf{T}_j \ln\frac{n_i\cdot n_j}{2}$
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measured soft	$\gamma_S^{\rm meas}(\tau_a^i;\mu) = \sum_{i=1}^M\left\{-\Gamma\mathbf{T}_i^2\frac{1}{1-a}\ln\left(\frac{\mu^2\tan^{2(1-a)}\frac{R}{2}}{\omega_i^2}\right)\delta(\tau_a^i) + 2\Gamma\mathbf{T}_i^2\frac{1}{1-a}\left[\frac{\Theta(\tau_a^i)}{\tau_a^i}\right]_+\right\}$

$$\bar\omega_H = \prod_{i=1}^N \omega_i^{\mathbf{T}_i^2/\mathbf{T}^2} \qquad \Gamma = \frac{\alpha_s}{\pi}\,, \quad \gamma_q = \frac{3\alpha_s}{2\pi}\,, \quad \gamma_g = \frac{\alpha_s}{\pi}\frac{11C_A - 2N_F}{6}$$

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measured soft	$\gamma_S^{\rm meas}(\tau_a^i;\mu) = \sum_{i=1}^M \Big\{ -\Gamma \mathbf{T}_i^2 \frac{1}{1-a} \ln \Big(\frac{\mu^2 \tan^{2(1-a)} \frac{R}{2}}{\omega_i^2}\Big) \delta(\tau_a^i) + 2\Gamma \mathbf{T}_i^2 \frac{1}{1-a} \Big[\frac{\Theta(\tau_a^i)}{\tau_a^i}\Big]_+ \Big\}$

$$\bar\omega_H = \prod_{i=1}^N \omega_i^{\mathbf{T}_i^2/\mathbf{T}^2} \qquad \Gamma = \frac{\alpha_s}{\pi}, \quad \gamma_q = \frac{3\alpha_s}{2\pi}, \quad \gamma_g = \frac{\alpha_s}{\pi} \frac{11 C_A - 2 N_F}{6}$$

Results for Anomalous Dimensions to $\mathcal{O}(1/t^2)$

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hard	$\gamma_H = -\sum_{i=1}^{N}\Big(\Gamma \ln\frac{\mu^2}{\bar\omega_H^2}\mathbf{T}_i^2 + \gamma_i\Big) - \Gamma\sum_{i\neq j}^{N}\mathbf{T}_i\cdot\mathbf{T}_j \ln\frac{n_i\cdot n_j}{2}$
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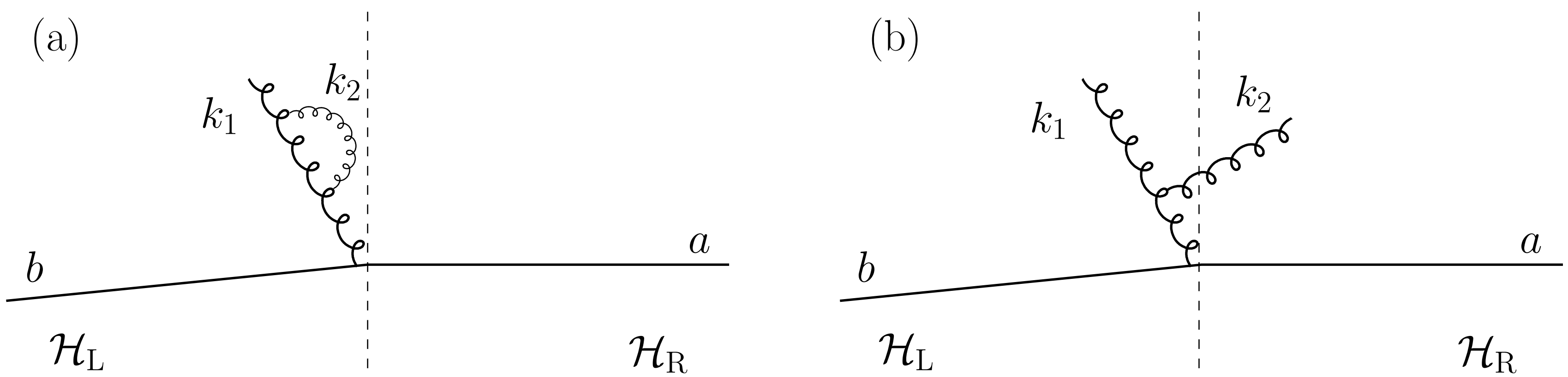
$$\bar\omega_H = \prod_{i=1}^{N}\omega_i^{\mathbf{T}_i^2/\mathbf{T}^2} \qquad \Gamma = \frac{\alpha_s}{\pi}\,, \quad \gamma_q = \frac{3\alpha_s}{2\pi}\,, \quad \gamma_g = \frac{\alpha_s}{\pi}\frac{11C_A - 2N_F}{6}$$

Non-Global Logs

- incomplete cancellation due to radiation in restricted region:
- classic example - R/L jet mass:

Dasgupta, Salam



- another classic example: out-of-jet radiation w/ cutoff ("η-gaps")...
- however, can write $\Sigma_{2\text{ng}}(Q, V, E_{out}) = \Sigma(Q, V) \cdot \Sigma_{\text{out}}(VQ, E_{\text{out}})$

Yu, Dokshitzer, Marchesini

→ no non-global logs for $\omega_i \tau_i \sim \Lambda$

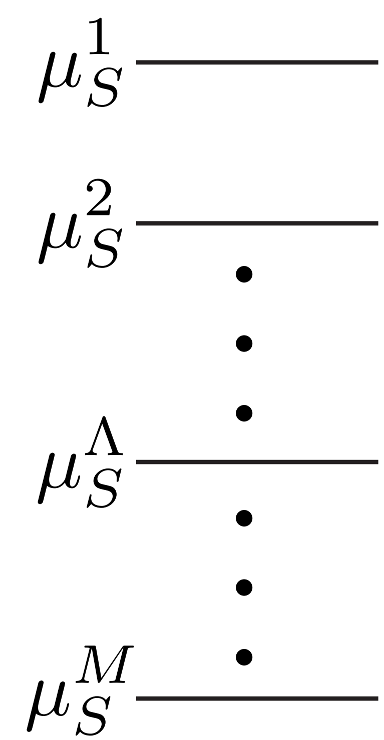
$$\begin{cases} E_{\text{out}} \leftrightarrow \Lambda \\ V \leftrightarrow \tau_{\text{jet}} \end{cases}$$

Refactorization

- the limit $\omega_1\tau_1 \sim \omega_2\tau_2 \sim \cdots \sim \Lambda$ is very restrictive
- consider other extreme $\omega_1\tau_1 \ll \omega_2\tau_2 \ll \cdots \ll \Lambda \ll \cdots \ll \omega_M\tau_M$
- 1st write $S(\tau_a^1,\tau_a^2,\ldots,\tau_a^M;\mu) = \langle 0|\mathcal{O}_S^\dagger \Theta(\Lambda-\hat\Lambda)\prod_{i=1}^{M}\delta(\tau_a^i-\hat\tau_a^i)\mathcal{O}_S|0\rangle$
  where $\mathcal{O}_S = Y_1\ldots Y_M Y_{M+1}\ldots Y_N$
- below μ_M, set $\mu_M \to \infty$ ⇒ matching coeff. is $S^{\rm meas}(\tau_a^M,\mu)$
- likewise, below Λ write $\theta(\Lambda-\hat\Lambda) = \theta(\Lambda)+\cdots$
- this gives $S(\tau_a^1,\ldots,\tau_a^M;\mu) = S^{\rm unmeas}(\mu)\prod_{i=1}^{M} S^{\rm meas}(\tau_a^i;\mu)\,\langle 0|\,\mathcal{O}_S^\dagger\mathcal{O}_S\,|0\rangle$

$$S^{\rm unmeas}(\mu) = U_S^{\rm unmeas}(\mu,\mu_\Lambda)S^{\rm unmeas}(\mu_\Lambda)$$

$$S^{\rm meas}(\tau_a^i,\mu) = \int d\tau' U_S^i(\tau_a^i-\tau';\mu,\mu_S^i)S^{\rm meas}(\tau',\mu_S^i)$$

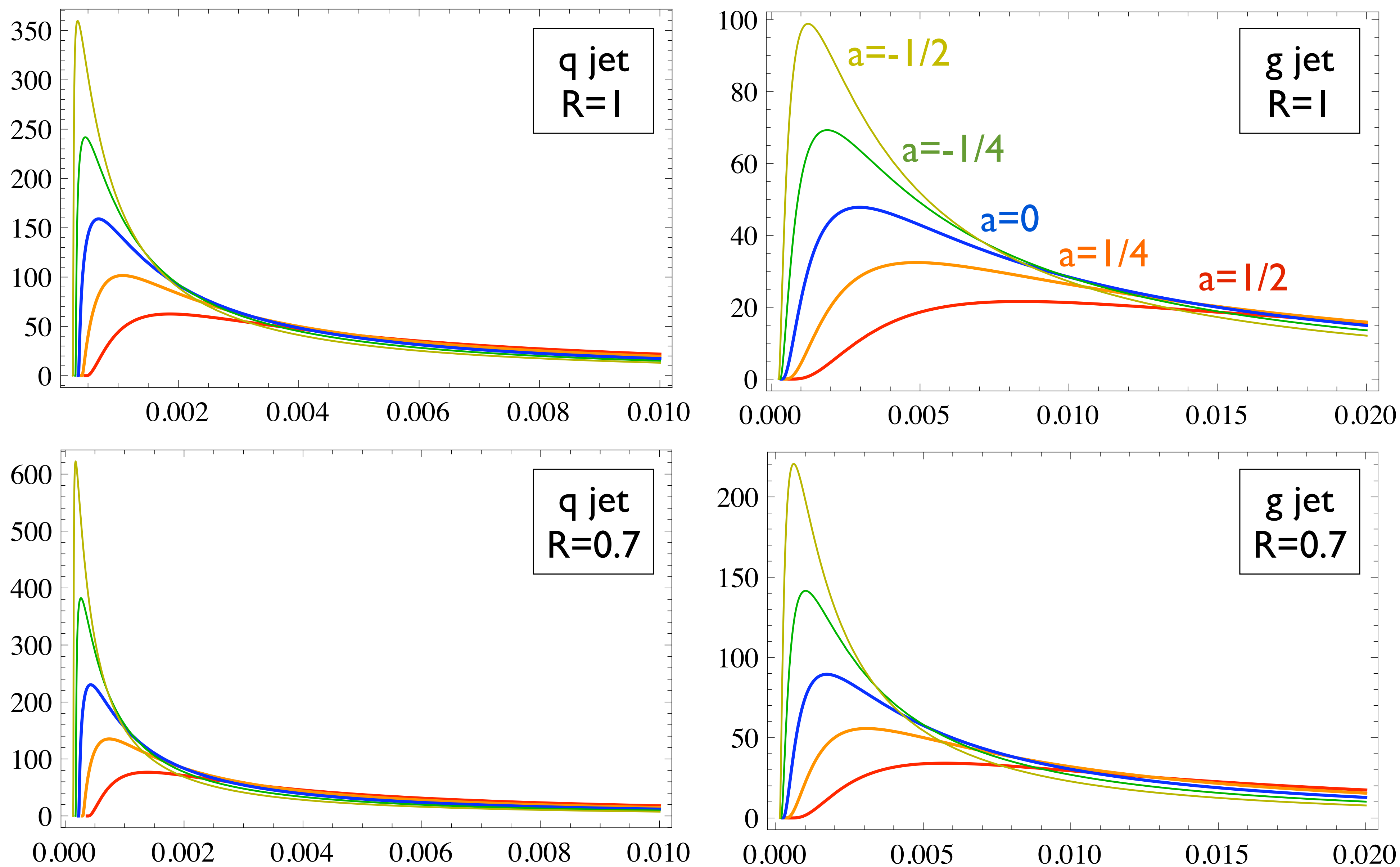


- use this result to interpolate between extremes

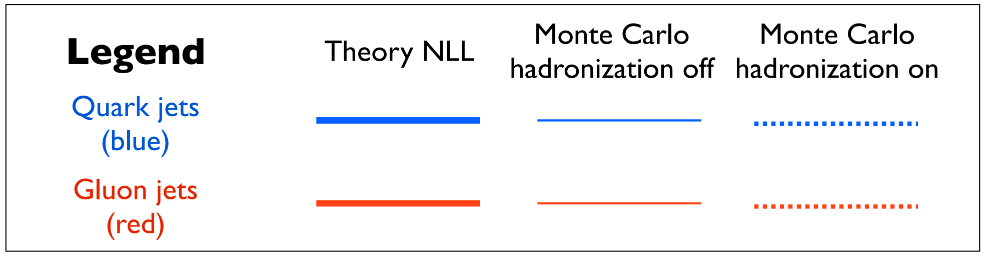
Plots of Results

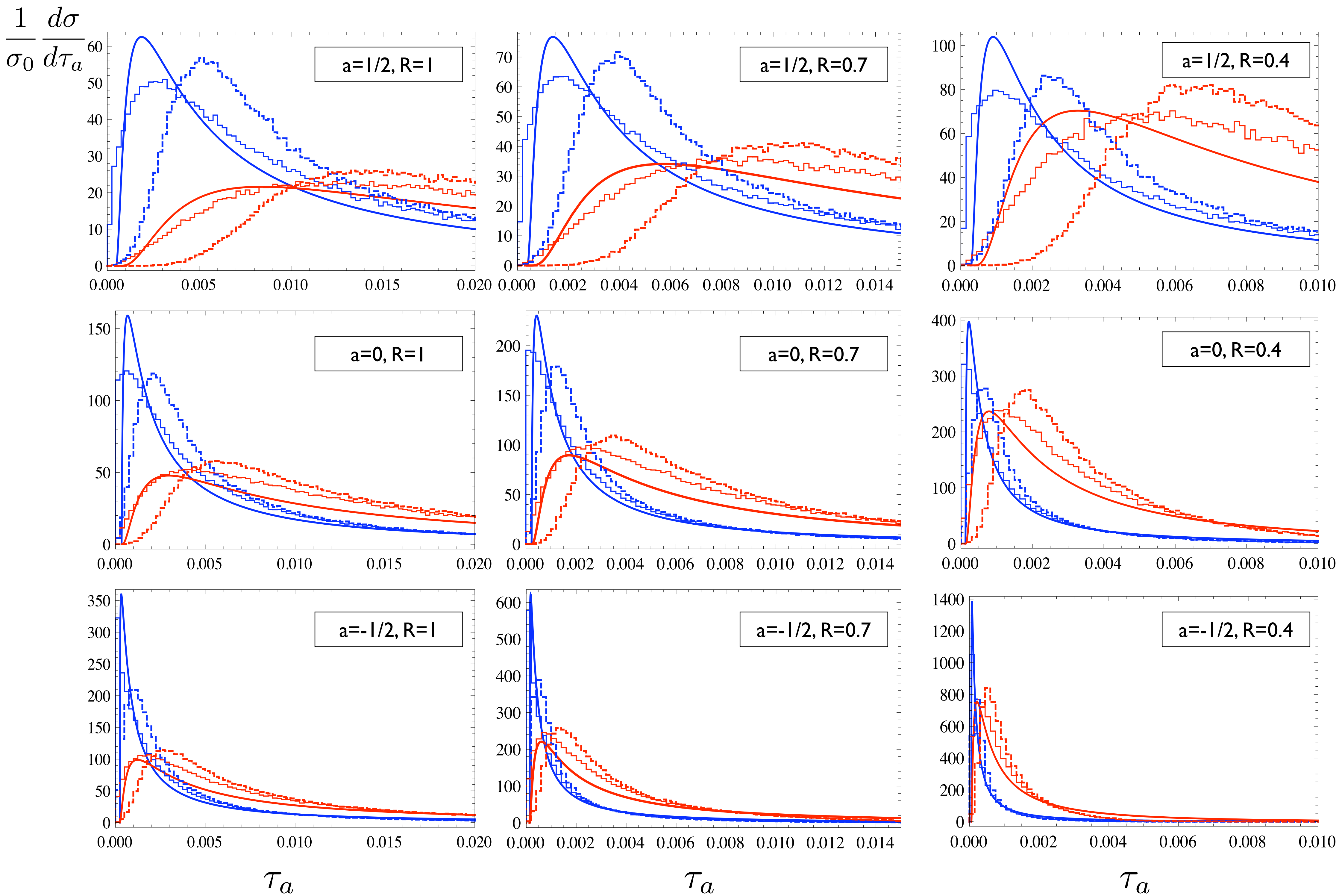
after "refactorization":

$$\frac{1}{\sigma^{(0)}}\frac{d\sigma}{d\tau_a}$$



Comparison to Pythia





More Jets

- our calculations are valid when there are more than 3 jets (e.g., did not assume jets were in a plane)
- written in terms of color-correlation operators $\mathbf{T}_i \cdot \mathbf{T}_j$
- lead to mixing for n > 3 jets (n > 1 @ LHC)
- however, mixing matrices computed for all n=5 (e.g., 2 → 3)
- e.g., # of indep. operators for gg → ggg is 44 (giving a 44x44 matrix)

Sjodahl, ...

Conclusions

- consistency and no large logs for M measured, N-M unmeasured (with power corrections as $1/t^2$) as long as:

  I) in region $\omega_1\tau_1 \sim \omega_2\tau_2 \sim \cdots \sim \Lambda$

  II) in region $\omega_1\tau_1 \ll \omega_2\tau_2 \ll \cdots \ll \Lambda \ll \cdots \ll \omega_M\tau_M$

  } interpolate between

- universal "swiss cheese" soft function (fill w/ anything)

- qualitative agreement w/ pythia across R, "a", jet algorithm, etc.

- raises many interesting questions & still much to do....

Outlook

- application to likelihood fnc. of q vs. g
  - hadronization uncertainty (hurts pure q)
  - large angle emission uncertainty (hurts pure g)
- calculation extensions:
  - doubly-differential $\frac{d\sigma}{d\tau_a d\tau_b}$
  - 2-loop (algorithms different?, anom. dim. dependence on R?)
  - pp (use boost-inv measure; can lift some results: cf. Nick's talk)
- open questions: non-global logs in SCET, refactorization, ...

Backup

Anomalous Dimensions

gluon inside jet: $\Theta\left(\tan^2\frac{R}{2} - \frac{q^+}{q^-}\right)$

gluon outside jets: $\Theta\left(\frac{q^+}{q^-} - \tan^2\frac{R}{2}\right)\Theta(\Lambda - q^0)$

	with no assigned scalings	$R \sim \lambda^0$
measured jet (naive)	$\left(\Gamma \mathbf{T}_i^2 \ln\frac{\mu^2}{\omega_i^2\tan^2\frac{R}{2}} + \gamma_i\right)\delta(\tau_a)$	$\left(\frac{\Gamma(2-a)}{1-a}\mathbf{T}_i^2\ln\frac{\mu^2}{\omega_i^2} + \gamma_i\right)\delta(\tau_a) - \frac{2\Gamma\mathbf{T}_i^2}{1-a}\left[\frac{\theta(\tau_a)}{\tau_a}\right]_+$
measured jet (0-bin)	$-\frac{\Gamma\mathbf{T}_i^2}{1-a}\left\{\ln\frac{\mu^2\tan^{2(1-a)}\frac{R}{2}}{\omega_i^2}\delta(\tau_a) + 2\left[\frac{\theta(\tau_a)}{\tau_a}\right]_+\right\}$	0 (scaleless)
unmeasured jet	$\Gamma\mathbf{T}_i^2\ln\frac{\mu^2}{\omega_i^2\tan^2\frac{R}{2}} + \gamma_i$	0 (scaleless)
soft in measured jet	$-\frac{\Gamma}{1-a}\mathbf{T}_i^2\left\{\ln\left(\frac{\mu^2\tan^{2(1-a)}\frac{R}{2}}{\omega_i^2}\right)\delta(\tau_a^i) - 2\left[\frac{\Theta(\tau_a^i)}{\tau_a^i}\right]_+\right\}$	$-\frac{\Gamma}{1-a}\mathbf{T}_i^2\left\{\ln\left(\frac{\mu^2\tan^{2(1-a)}\frac{R}{2}}{\omega_i^2}\right)\delta(\tau_a^i) - 2\left[\frac{\Theta(\tau_a^i)}{\tau_a^i}\right]_+\right\}$
soft "swiss cheese"	$\Gamma\sum_{i=1}^N\mathbf{T}_i^2\ln\tan^2\frac{R}{2} + \Gamma\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\ln\frac{n_i\cdot n_j}{2}$	$\Gamma\sum_{i=1}^N\mathbf{T}_i^2\ln\tan^2\frac{R}{2} + \Gamma\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\ln\frac{n_i\cdot n_j}{2}$

can take $\Lambda \sim \omega\lambda^2$ or assign no scaling

unmeasured jet R-dependence left uncancelled

Anomalous Dimensions

gluon inside jet: $\Theta\left(\tan^2\frac{R}{2}-\frac{q^+}{q^-}\right)$ gluon outside jets: $\Theta\left(\frac{q^+}{q^-}-\tan^2\frac{R}{2}\right)\Theta(\Lambda-q^0)$

	with no assigned scalings	$R\sim\lambda^1$
measured jet (naive)	$\left(\Gamma\mathbf{T}_i^2\ln\frac{\mu^2}{\omega_i^2\tan^2\frac{R}{2}}+\gamma_i\right)\delta(\tau_a)$	$\left(\Gamma\mathbf{T}_i^2\ln\frac{\mu^2}{\omega_i^2\tan^2\frac{R}{2}}+\gamma_i\right)\delta(\tau_a)$
measured jet (0-bin)	$-\frac{\Gamma\mathbf{T}_i^2}{1-a}\left\{\ln\frac{\mu^2\tan^{2(1-a)}\frac{R}{2}}{\omega_i^2}\delta(\tau_a)+2\left[\frac{\theta(\tau_a)}{\tau_a}\right]_+\right\}$	0 (zero cone size)
unmeasured jet	$\Gamma\mathbf{T}_i^2\ln\frac{\mu^2}{\omega_i^2\tan^2\frac{R}{2}}+\gamma_i$	$\Gamma\mathbf{T}_i^2\ln\frac{\mu^2}{\omega_i^2\tan^2\frac{R}{2}}+\gamma_i$
soft in measured jet	$-\frac{\Gamma}{1-a}\mathbf{T}_i^2\left\{\ln\left(\frac{\mu^2\tan^{2(1-a)}\frac{R}{2}}{\omega_i^2}\right)\delta(\tau_a^i)-2\left[\frac{\Theta(\tau_a^i)}{\tau_a^i}\right]_+\right\}$	0 (zero cone size)
soft "swiss cheese"	$\Gamma\sum_{i=1}^{N}\mathbf{T}_i^2\ln\tan^2\frac{R}{2}+\Gamma\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\ln\frac{n_i\cdot n_j}{2}$	$\Gamma\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\ln\frac{n_i\cdot n_j}{2}$

can take $\Lambda\sim\omega\lambda^2$ or assign no scaling

no soft or 0-bin contributions; R-dep. uncancelled