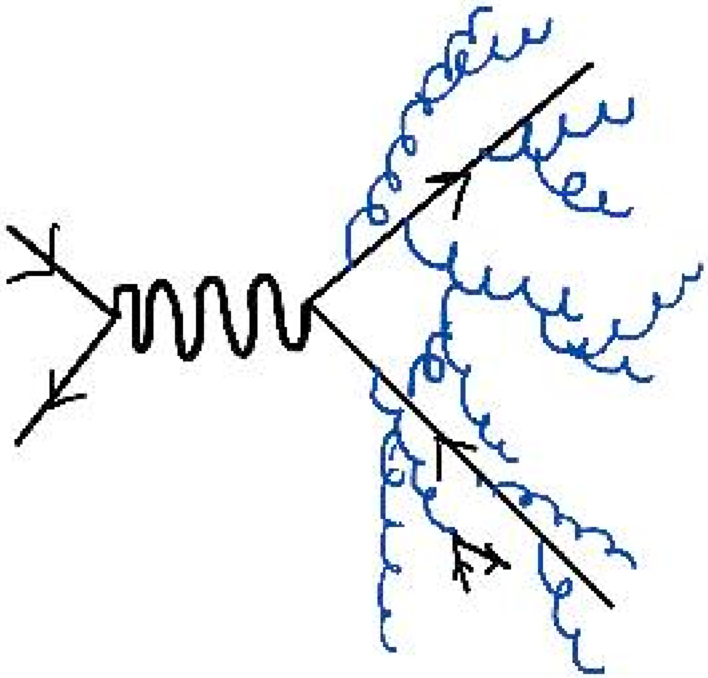
Phase Space and Jet Definitions in SCET

Saba Zuberi
University of Toronto

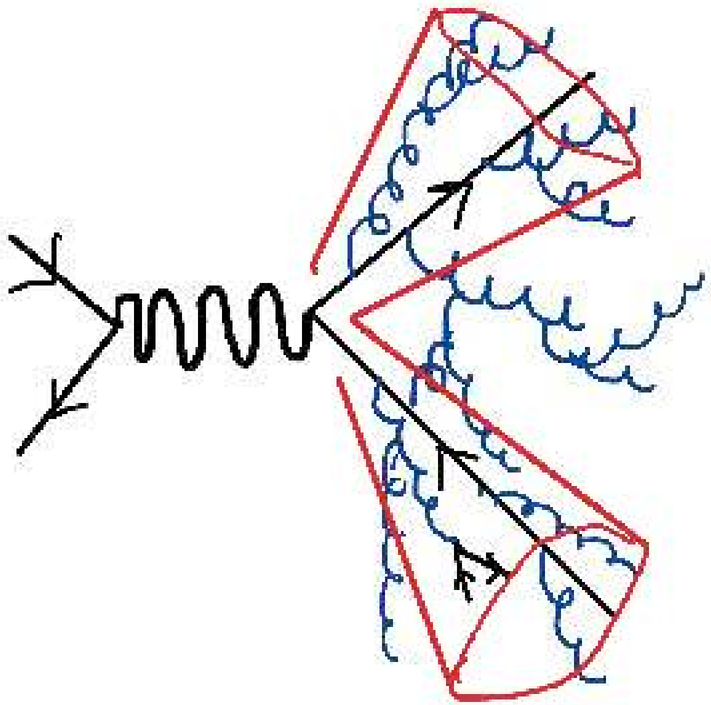
W. Cheung, M. Luke and SZ Phys. Rev. D80:114021,2009

$$e^+e^- \to jets$$



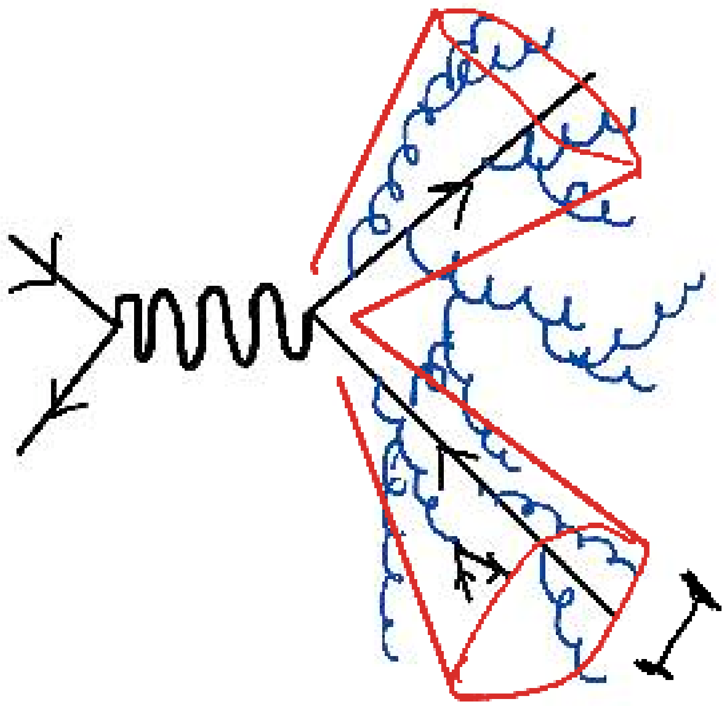
y_{cut}

$$e^+e^- \to jets$$



$$f_2 \equiv \frac{\sigma^{2\,jet}}{\sigma^0} \sim 1 + \alpha_s \ln^2 y_{cut} + \alpha_s^2 \ln^4 y_{cut}$$

$$e^+e^- \to jets$$



$$f_2 \equiv \frac{\sigma^{2\,jet}}{\sigma^0} \sim 1 + \alpha_s \ln^2 y_{cut} + \alpha_s^2 \ln^4 y_{cut}$$

$$H \cdot J \otimes S$$

Consider the dijet rate $\sigma\left(e^+e^- \to 2jets\right)$

Question: Can we resum the large logs arising from jet algorithm phase space cuts?

In order to begin to address this question we calculate the NLO result for several 'traditional' jet algorithms in SCET.

Consider the dijet rate $\sigma\left(e^{+}e^{-} \rightarrow 2jets\right)$

Question: Can we resum the large logs arising from jet algorithm phase space cuts?

In order to begin to address this question we calculate the NLO result for several 'traditional' jet algorithms in SCET.

OUTLINE

➡ Jet algorithms - cone and clustering

➡ How to treat phase space constraints?

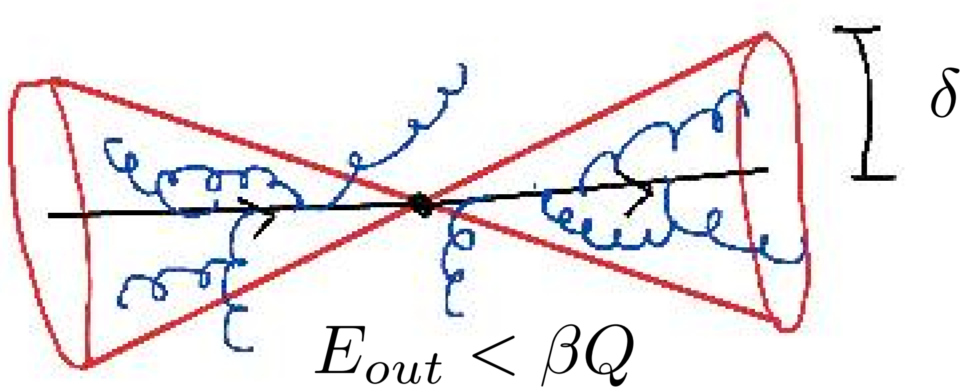
- Show approach with **consistent** power counting of PS constraints. Demonstrates relationship between cutoffs in EFT and phase space limits & how to treat jet algorithm parameters

- Consistency with non-trivial zero-bin subtraction - logs of homogeneous scale in collinear sector, simplifies jet algorithm combinatorics

- Connection between IR safety of soft and collinear contributions & UV regulator.

The Algorithms

- **Cone algorithm:** Sterman-Weinberg (1977)



$$f_2^{\rm SW} = 1 + \frac{\alpha_s C_F}{\pi}\left(-4\ln 2\beta \ln\delta - 3\ln\delta - \frac{\pi^2}{3} + \frac{5}{2}\right)$$

(Bauer, Lee, Manohar, Wise, 2004)

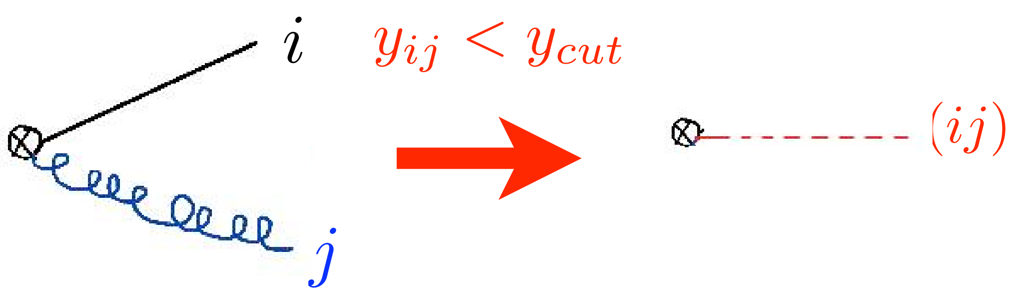
(Trott, 2006)

(Joutennus, 2010)

The Algorithms

- **Clustering algorithm**: JADE, k_T

Define discriminant y_{ij} and cut parameter y_{cut} (and recombination scheme)
Combine particles with smallest $y_{ij} < y_{cut}$ to form pseudoparticle
Repeat until all $y_{ij} > y_{cut}$



JADE: $y_{ij} = \dfrac{M_{ij}^2}{Q^2}$

k_T: $y_{ij} = \dfrac{M_{ij}^2}{Q^2} \min\left(\dfrac{\mathrm{E_i}}{\mathrm{E_j}}, \dfrac{\mathrm{E_j}}{\mathrm{E_i}}\right)$

$$f_2^{\rm JADE} = 1 + \frac{\alpha_s C_F}{2\pi}\left(-2\ln^2 j - 3\ln j + \frac{\pi^2}{3} - 1\right)$$

$$f_2{}^{k_\perp} = 1 + \frac{\alpha_s C_F}{2\pi}\left(-\ln^2 y_c - 3\ln y_c - 6\ln 2 + \frac{\pi^2}{6} - 1\right)$$

- exclusive algorithms for e+ e-

Question: Can we resume the large logs arising from jet algorithm phase space cuts?

Status SCET:

- Fully differential jet cross sections independent of jet observables *(Bauer, Hornig, Tackmann, 2009)*
- Sufficiently inclusive
  - DIS *(Manohar, 2004)*
  - invariant mass distributions for top jets with hemisphere jet definition *(Fleming, Hoang, Mantry, Stewart, 2007)*
  - angularity distributions *(Hornig, Lee, Ovanesyan, 2009)*
  - jet shape distributions with non-hemisphere jet algorithms *(Ellis, Hornig, Lee, Vermillion, Walsh, 2010)*

Question: Can we resume the large logs arising from jet algorithm phase space cuts?

Status SCET:

- Fully differential jet cross sections independent of jet observables (Bauer, Hornig, Tackmann, 2009)
- Sufficiently inclusive
  - DIS (Manohar, 2004)
  - invariant mass distributions for top jets with hemisphere jet definition (Fleming, Hoang, Mantry, Stewart, 2007)
  - angularity distributions (Hornig, Lee, Ovanesyan, 2009)
  - jet shape distributions with non-hemisphere jet algorithms (Ellis, Hornig, Lee, Vermillion, Walsh, 2010)

Status QCD (SW, JADE, k_T):

- JADE: Does not exponentiate, not known how to resum *(Brown, Stirling, 1990)*
- k_T: 2 jet fraction exponentiates *(Brown, Stirling, 1992)*

  Resummation of NLL *(Catani, Dokshitzer, Olsson, Turnock, Webber, 1991)*
- Sterman-Weinberg : Resum logs of energy and angle *(Mukhi, Sterman, 1982)*

Ingredients:

- Independent emission approximation to obtain matrix element square
- Phase space factorization in soft limit

$$\Theta\left(1,\cdots,n;y_{cut}\right)\simeq\prod_{i=1}^{n}\Theta\left(i;y_{cut}\right)$$

Question: Can we resume the large logs arising from jet algorithm phase space cuts?

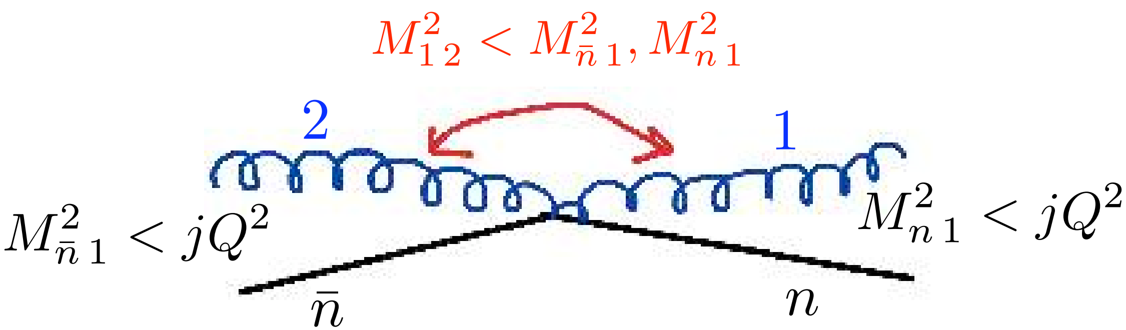
Status SCET:

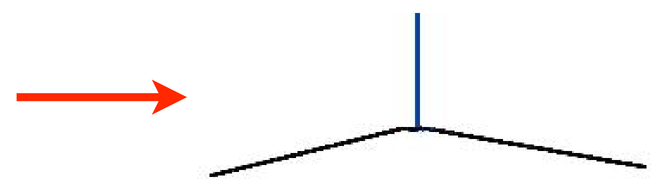
- Fully differential jet cross sections

Status QCD (SW, JADE, k_T):

- JADE: Does not exponentiate, not

The Trouble with JADE: No phase space factorization





$\mathcal{O}(\alpha_s^2 \ln^4 j)$

- angularity distributions (Hornig, Lee, Ovanesyan, 2009)
- jet shape distributions with non-hemisphere jet algorithms (Ellis, Hornig, Lee, Vermillion, Walsh, 2010)

Ingredients:

- Independent emission approximation to obtain matrix element square
- Phase space factorization for observable in soft limit

$$\Theta\left(1, \cdot, n; y_{cut}\right) \simeq \prod_{i=1}^{n} \Theta\left(i; y_{cut}\right)$$

Question: Can we resume the large logs arising from jet algorithm phase space cuts?

Status SCET:

- Fully differential jet cross sections independent of jet observables (Bauer, Hornig, Tackmann, 2009)
- Sufficiently inclusive
  - DIS (Manohar, 2004)
  - invariant mass distributions for top jets with hemisphere jet definition (Fleming, Hoang, Mantry, Stewart, 2007)
  - angularity distributions (Hornig, Lee, Ovanesyan, 2009)
  - jet shape distributions with non-hemisphere jet algorithms (Ellis, Hornig, Lee, Vermillion, Walsh, 2010)

Status QCD (SW, JADE, k_T):

- JADE: Does not exponentiate, not known how to resum *(Brown, Stirling, 1990)*
- k_T: 2 jet fraction exponentiates *(Brown, Stirling, 1992)*

  Resummation of NLL *(Catani, Dokshitzer, Olsson, Turnock, Webber, 1991)*
- Sterman-Weinberg : Resum logs of energy and angle *(Mukhi, Sterman, 1982)*

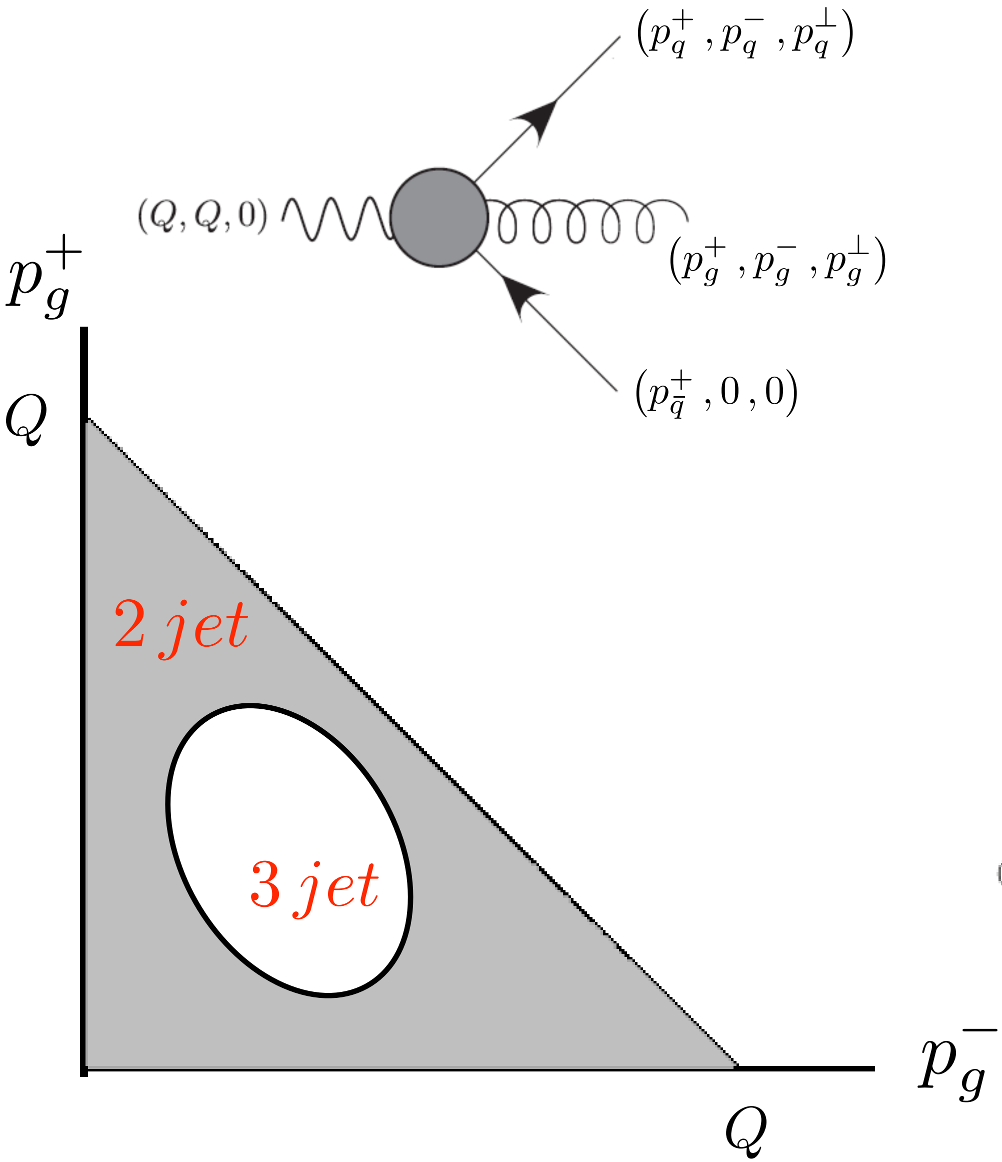
More recently:

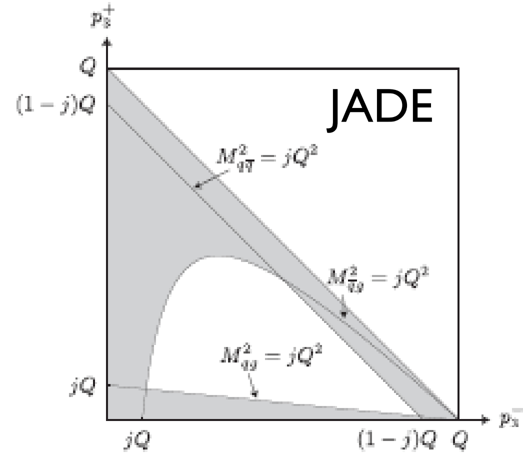
- Non-global observables - only sensitive to emissions in a restricted angular region of PS *(Dasgupta, Salam, 2001)*

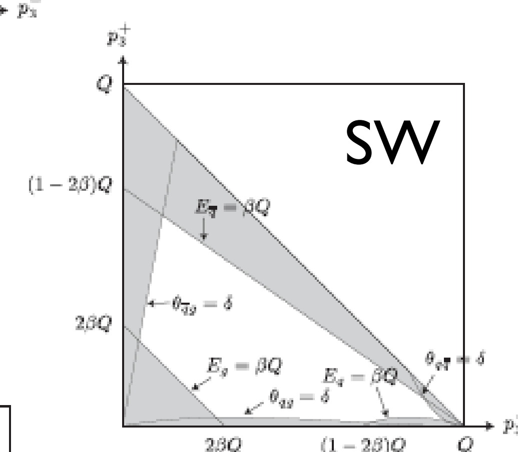
  $\alpha_s^n \ln^n y_{cut}$ not resummed.

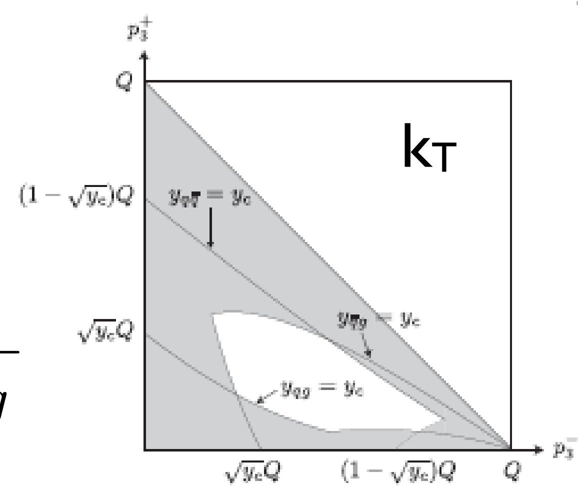
Phase Space

- At $\mathcal{O}(\alpha_s)$ the jet algorithm separates the 2 and 3 jet region







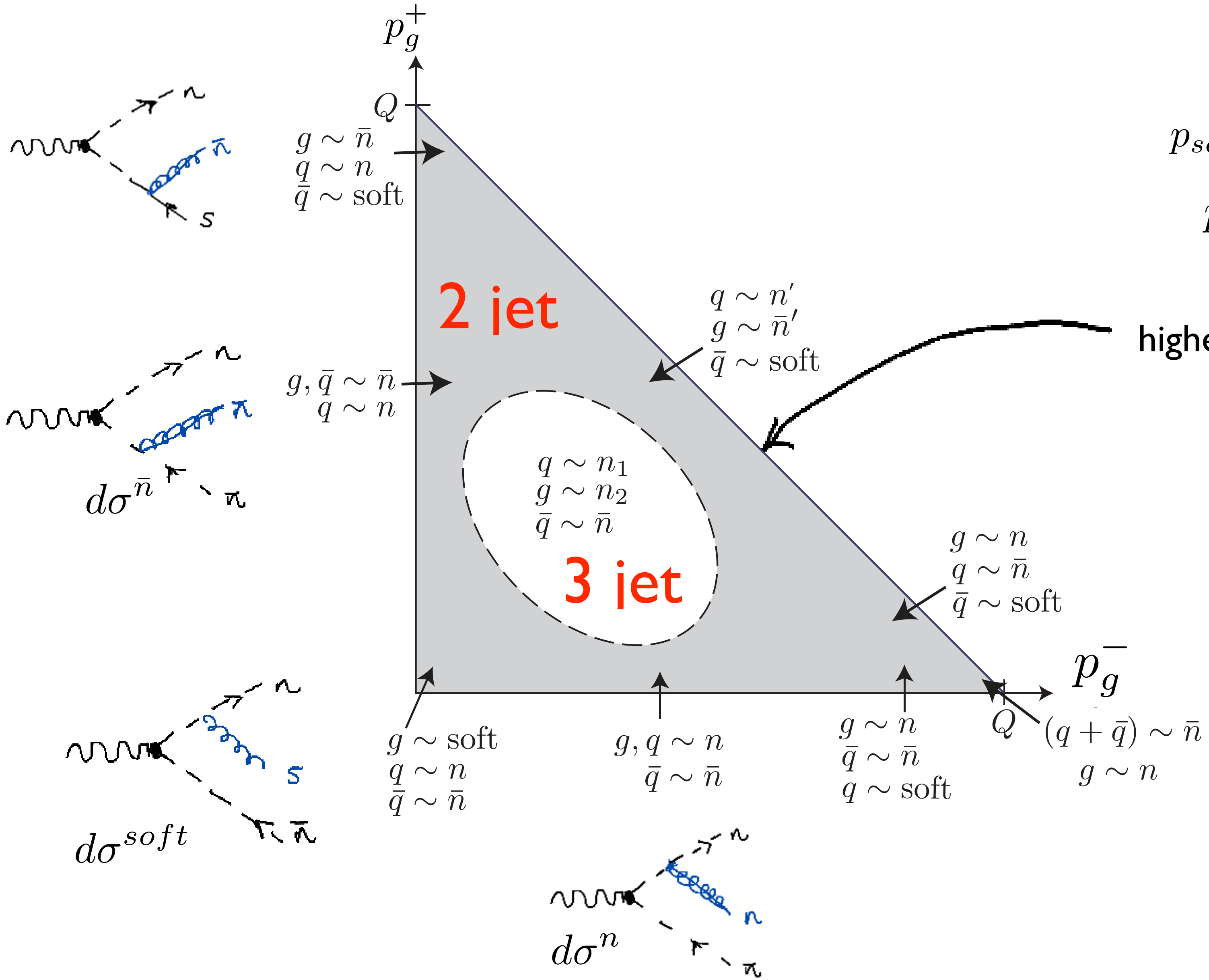


Phase Space in SCET

$$p = (p^+, p^-, p^\perp)$$

$$p_{soft} \sim Q\,(\lambda^2, \lambda^2, \lambda^2)$$

$$p_c^n \sim Q\,(\lambda^2, 1, \lambda)$$

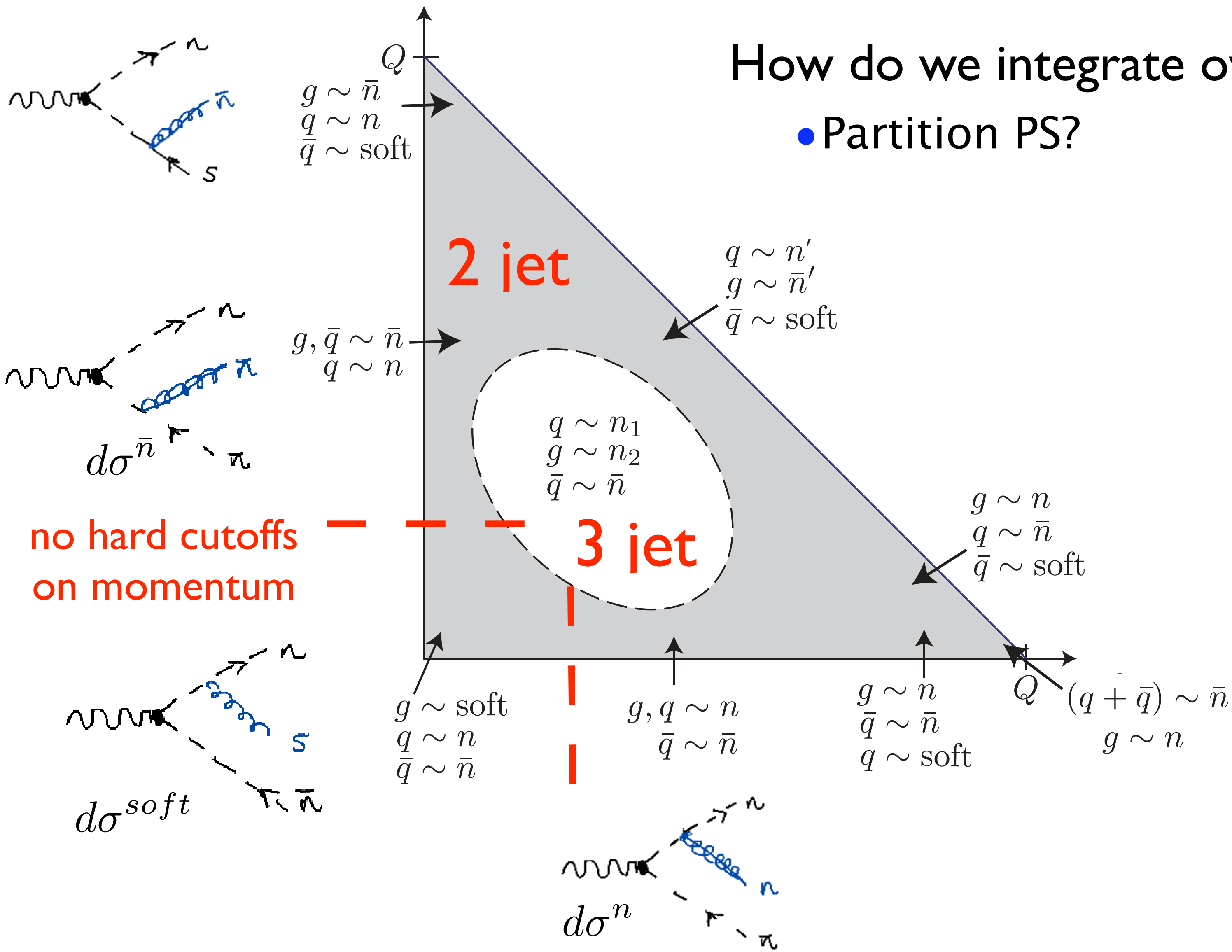


higher order in λ

Phase Space in SCET

How do we integrate over PS in SCET:

- Partition PS?

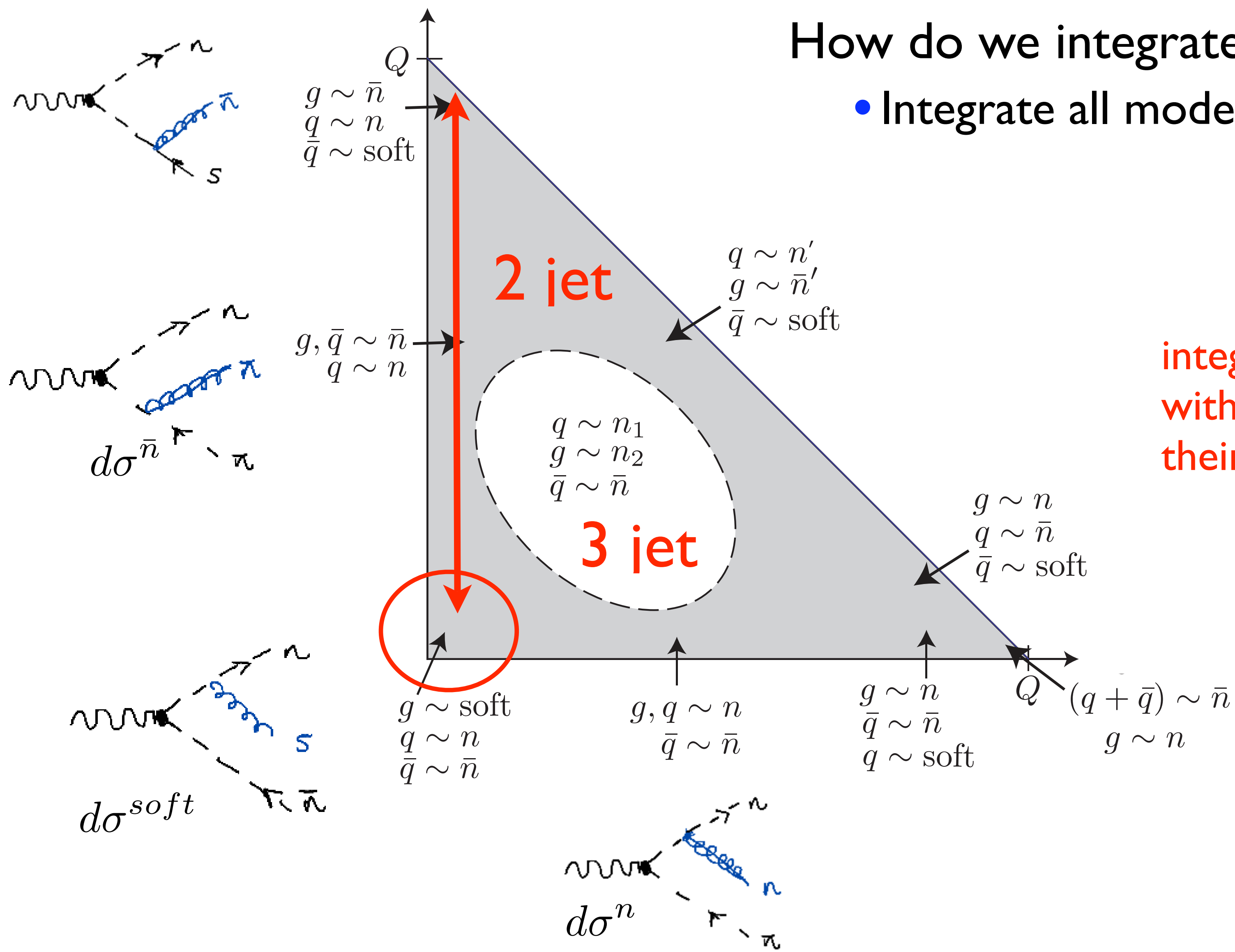


Phase Space in SCET

How do we integrate over PS in SCET:

- Integrate all modes over all PS?

(Manohar, Stewart, 2007)



integrates over modes with momentum above their cutoff

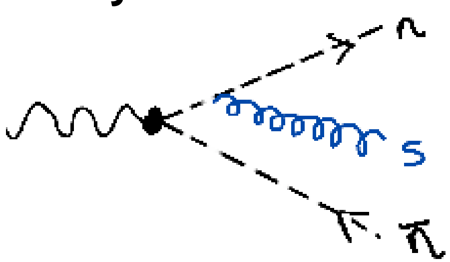
$$p_g^{soft} \to Q$$

How do we integrate over PS in SCET:

- Instead a PS integral which extends above the cutoff of the relevant mode is replaced by a UV divergence.

- Occurs naturally in SCET - **consistently multipole expand PS constraints**

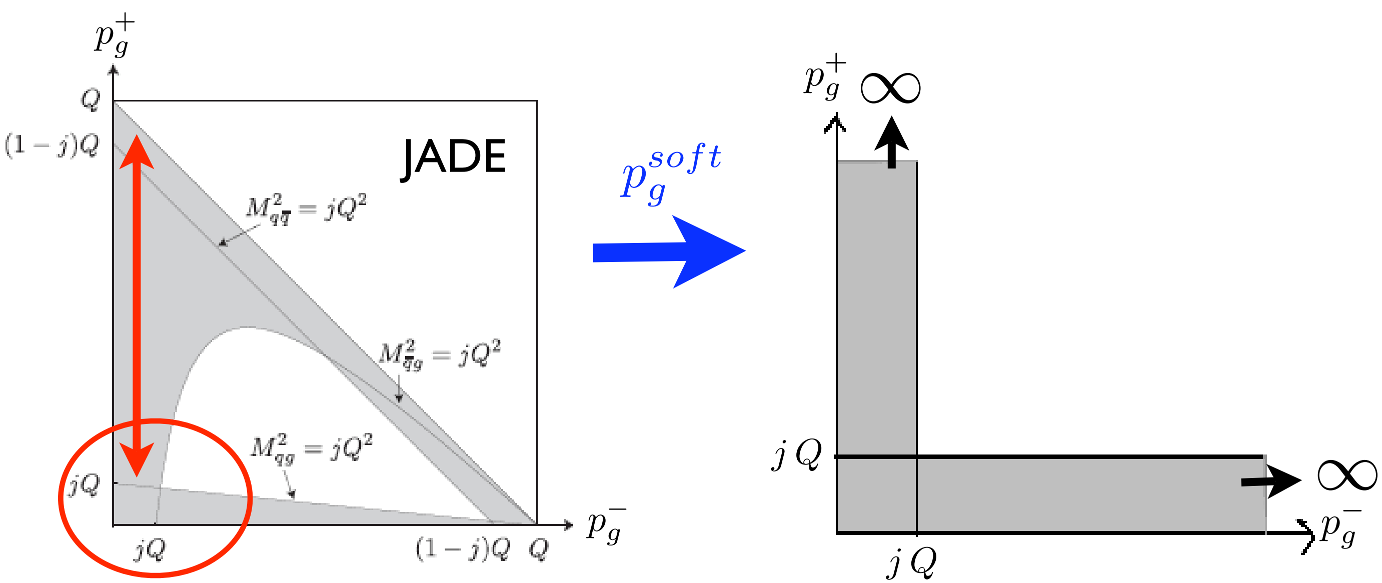
- Ex. JADE



$p_q = Q\,(\lambda^2,\,1,\,\lambda)$

$p_g = Q\,(\lambda^2,\,\lambda^2,\,\lambda^2)$

$$y_{q\,g} = \frac{M^2_{q\,g}}{Q^2} = \frac{k_g^+}{Q} + \mathcal{O}(\lambda^3) < j$$

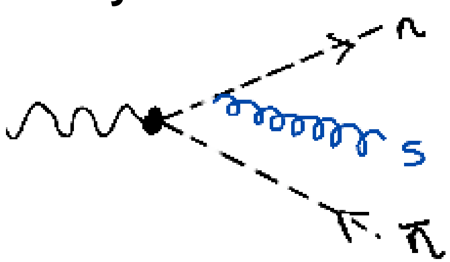


How do we integrate over PS in SCET:

- Instead a PS integral which extends above the cutoff of the relevant mode is replaced by a UV divergence.

- Occurs naturally in SCET - **consistently multipole expand PS constraints**

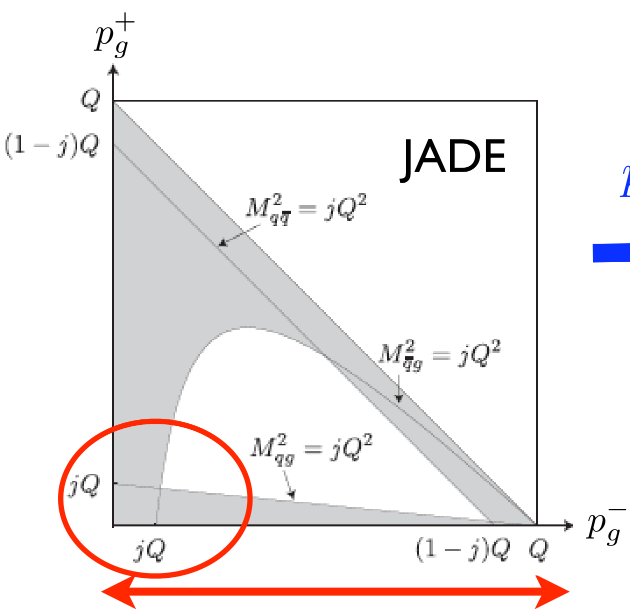
- Ex. JADE

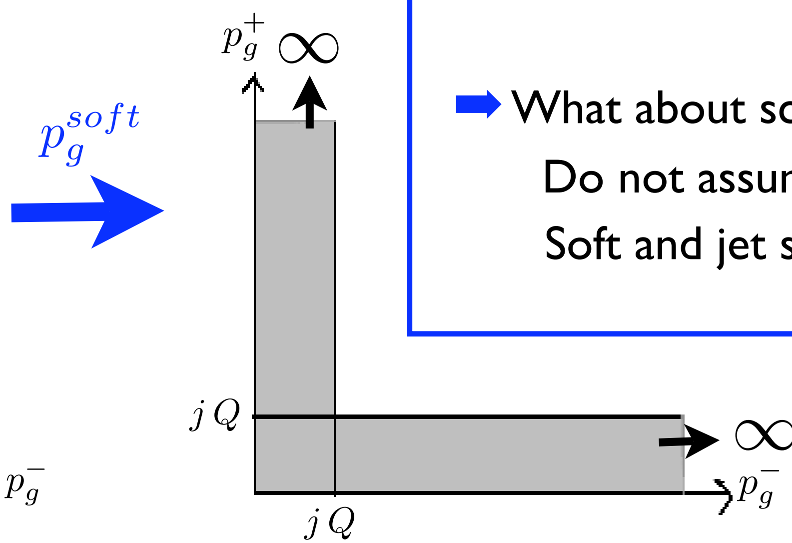


$p_q = Q\,(\lambda^2,\, 1,\, \lambda)$

$p_g = Q\,(\lambda^2,\, \lambda^2,\, \lambda^2)$

$$y_{q\,g} = \frac{M^2_{q\,g}}{Q^2} = \frac{k_g^+}{Q} + \mathcal{O}(\lambda^3) < j$$





➡ Expect no structure in PS above cut off

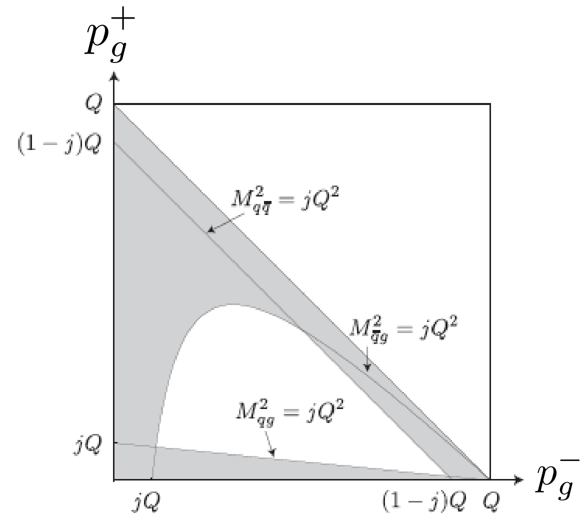
➡ PS of each mode different to full QCD

➡ What about scaling of jet algorithm parameters?

Do not assume scaling a priori, only $y_{cut} \ll 1$

Soft and jet scale are determined by dynamics

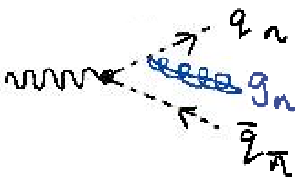
- JADE



$$k_g^+ < j\,(Q - p_g^-)$$
$$p_g^- < j\,Q$$
$$p_g^- > (1-j)\,Q$$

$$k_g^- < j\,Q$$
$$k_g^+ < j\,Q$$

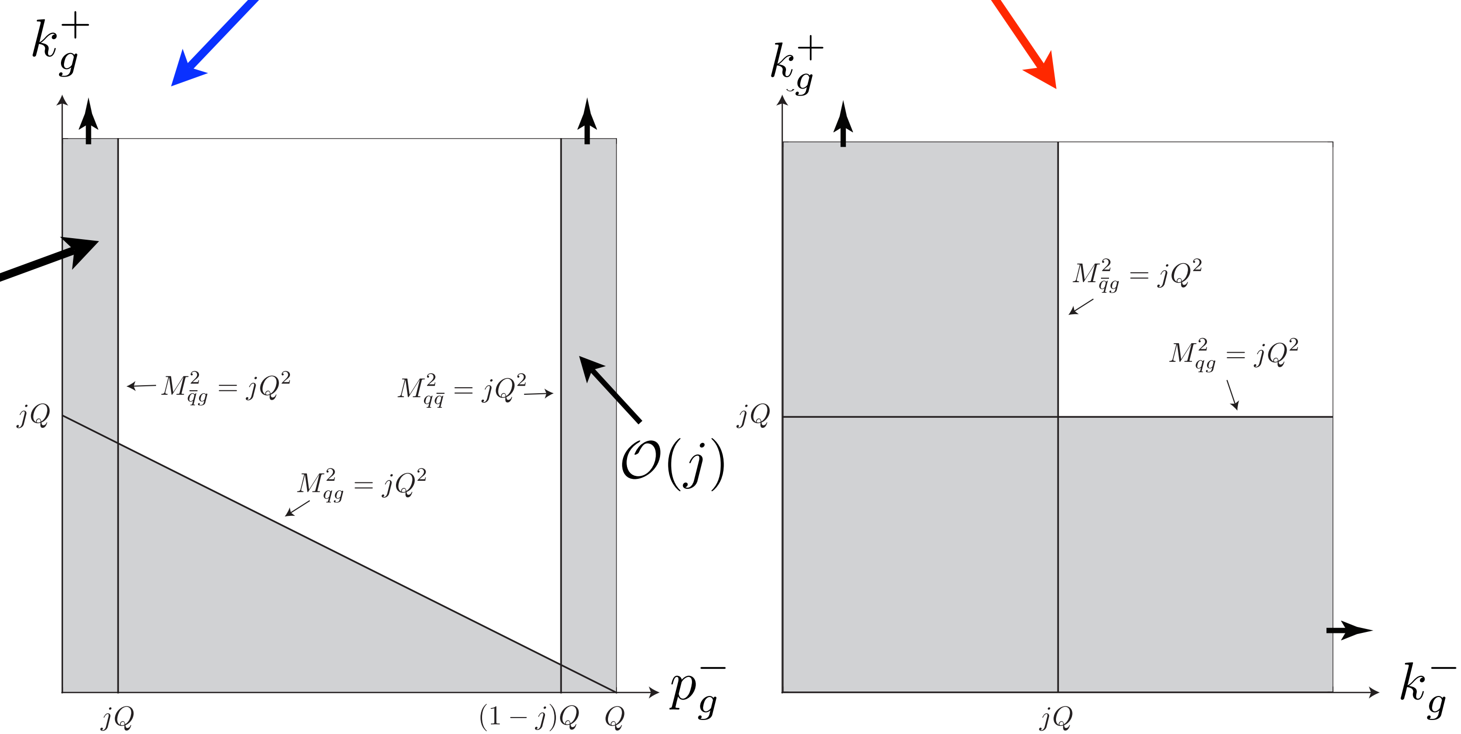
Zero-bin is the same as soft



n collinear

soft

n collinear gluon and $\bar{n}$ collinear anti-quark jet exactly cancelled by zero bin*



(a) (b)

*thanks to S. Freedman

- JADE dijet rate to $\mathcal{O}(\alpha_s)$

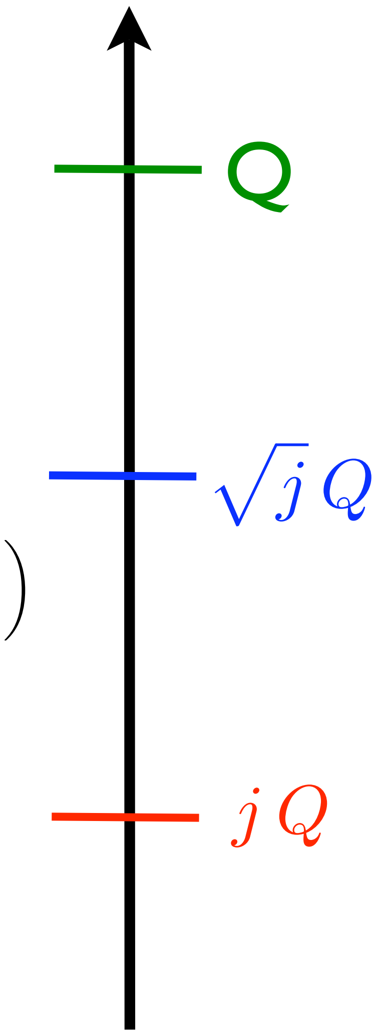
using dim-reg $d = 4 - \epsilon$

$$O_2 = \bar{\xi}_n W_n \gamma^\mu W^\dagger_{\bar{n}} \xi_{\bar{n}}$$

(Manohar, 2003)

$$Z_2 = 1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{1}{\epsilon}\ln\frac{\mu^2}{-Q^2}\right)$$

$$C_2 = 1 + \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\ln^2\frac{\mu^2}{-Q^2} - \frac{3}{2}\ln\frac{\mu^2}{-Q^2} - 4 + \frac{\pi^2}{12}\right)$$



naive collinear

$$\frac{1}{\sigma_0}\tilde{\sigma}^n_{\mathrm{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln j + \frac{3}{2}\ln\frac{\mu^2}{jQ^2} + 2\ln\frac{\mu^2}{Q^2}\ln j - 3\ln^2 j - \frac{\pi^2}{3} + \frac{7}{2}\right)$$

zero bin

$$\frac{1}{\sigma_0}\sigma^{n0}_{\mathrm{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2}{j^2Q^2} - \ln^2\frac{\mu^2}{j^2Q^2} + \frac{\pi^2}{6}\right)$$

$$\frac{1}{\sigma_0}\sigma^{n}_{\mathrm{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln\frac{\mu^2}{jQ^2} + \frac{3}{2}\ln\frac{\mu^2}{jQ^2} + \ln^2\frac{\mu^2}{jQ^2} - \frac{\pi^2}{2} + \frac{7}{2}\right)$$

$$\frac{1}{\sigma_0}\sigma^{soft}_{\mathrm{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2}{j^2Q^2} - \ln^2\frac{\mu^2}{j^2Q^2} + \frac{\pi^2}{6}\right)$$

virtual vertex and wave function are scaleless and vanish

- JADE dijet rate to $\mathcal{O}(\alpha_s)$

$$f_2^{\mathrm{JADE}} = \frac{|C_2|^2}{|Z_2|^2}\left(1 + \frac{1}{\sigma_0}\left(\sigma^n_{\mathrm{JADE}} + \sigma^{\bar{n}}_{\mathrm{JADE}} + \sigma^{\mathrm{soft}}_{\mathrm{JADE}}\right)\right)$$
$$= 1 + \frac{\alpha_s C_F}{2\pi}\left(-2\ln^2 j - 3\ln j + \frac{\pi^2}{3} - 1\right)$$

➡ Zero-bin necessary to get logs of a single scale in collinear sector

➡ It is useful to look at divergences with off-shellness for q and $\bar{q}$

Real Emission $$\frac{1}{\sigma_0}\sigma^n_{\mathrm{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon}\ln\frac{p_1^2}{jQ^2} - \ln^2\frac{p_1^2}{Q^2} + 2\ln\frac{\mu^2}{Q^2}\ln\frac{p_1^2}{Q^2} + \frac{3}{2}\ln\frac{p_1^2}{Q^2}\right) + \ldots$$

$$\frac{1}{\sigma_0}\sigma^{\mathrm{soft}}_{\mathrm{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon}\left(\ln\frac{p_1^2}{jQ^2} + \ln\frac{p_2^2}{jQ^2}\right) + \left(\ln\frac{p_1^2}{Q^2} + \ln\frac{p_2^2}{Q^2}\right)^2 - 2\left(\ln\frac{p_1^2}{Q^2} + \ln\frac{p_2^2}{Q^2}\right)\ln\frac{\mu^2}{Q^2}\right) + \cdots$$

$$\frac{1}{\sigma_0}\sigma^R_{\mathrm{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(2\ln\frac{p_1^2}{Q^2}\ln\frac{p_2^2}{Q^2} + \frac{3}{2}\ln\frac{p_1^2}{Q^2} + \frac{3}{2}\ln\frac{p_2^2}{Q^2}\right) + \ldots$$

UV divergences from phase space cancel between the soft and collinear real emission

- JADE: Buyer Beware!

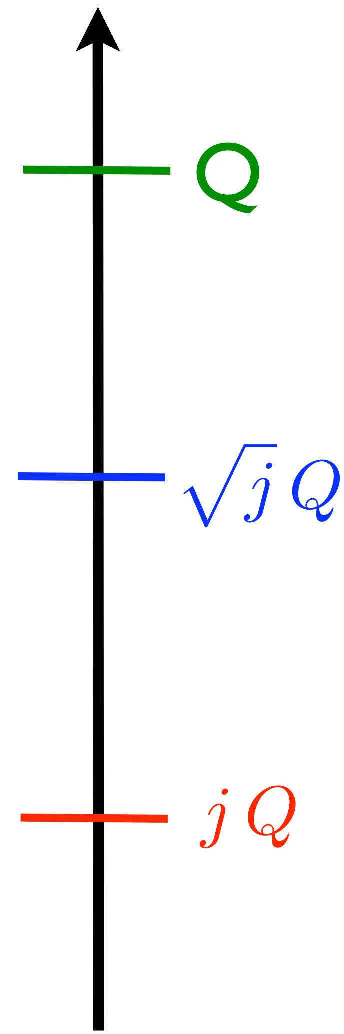
(Manohar, 2003) $O_2 = \bar{\xi}_n W_n \gamma^\mu W_{\bar{n}}^\dagger \xi_{\bar{n}}$ $\qquad Z_2 = 1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{1}{\epsilon}\ln\frac{\mu^2}{-Q^2}\right)$

$$C_2 = 1 + \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\ln^2\frac{\mu^2}{-Q^2} - \frac{3}{2}\ln\frac{\mu^2}{-Q^2} - 4 + \frac{\pi^2}{12}\right)$$

$$\frac{1}{\sigma_0}\sigma^n_{\text{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln\frac{\mu^2}{jQ^2} + \frac{3}{2}\ln\frac{\mu^2}{jQ^2} + \ln^2\frac{\mu^2}{jQ^2} - \frac{\pi^2}{2} + \frac{7}{2}\right)$$

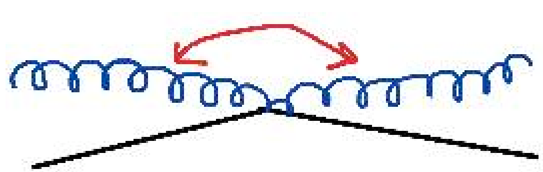
$$\frac{1}{\sigma_0}\sigma^{soft}_{\text{JADE}} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2}{j^2Q^2} - \ln^2\frac{\mu^2}{j^2Q^2} + \frac{\pi^2}{6}\right)$$

$$\alpha_s : \; Z_s\, Z_J\, Z_{\bar{J}} = 2Z_2$$



Can not resum leading logs using RG for hard, jet and soft contributions.
We know JADE has problematic correlations in soft gluon emission.

➡ Appropriate soft theory? Additional scales? Break down of factorization?

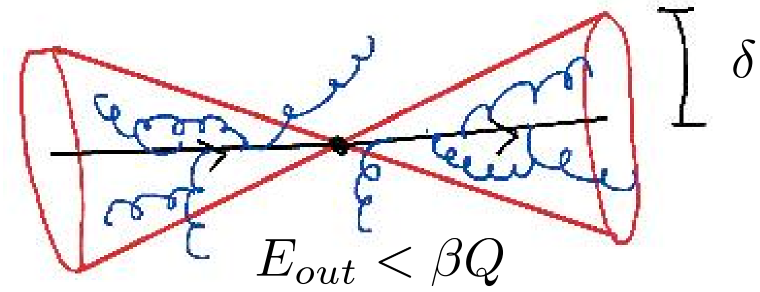


- Sterman-Weinberg

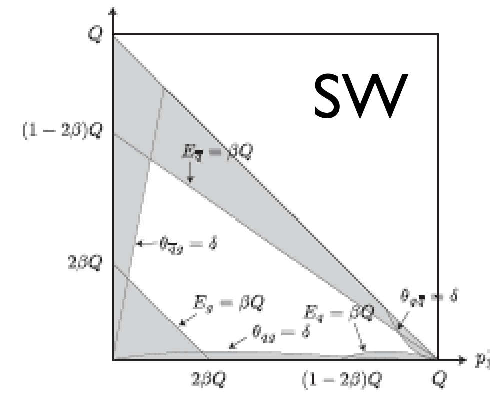
Previous work: unrestricted soft

Bauer, Lee, Manohar, Wise, 2004 : $\beta \sim \delta \sim \lambda$

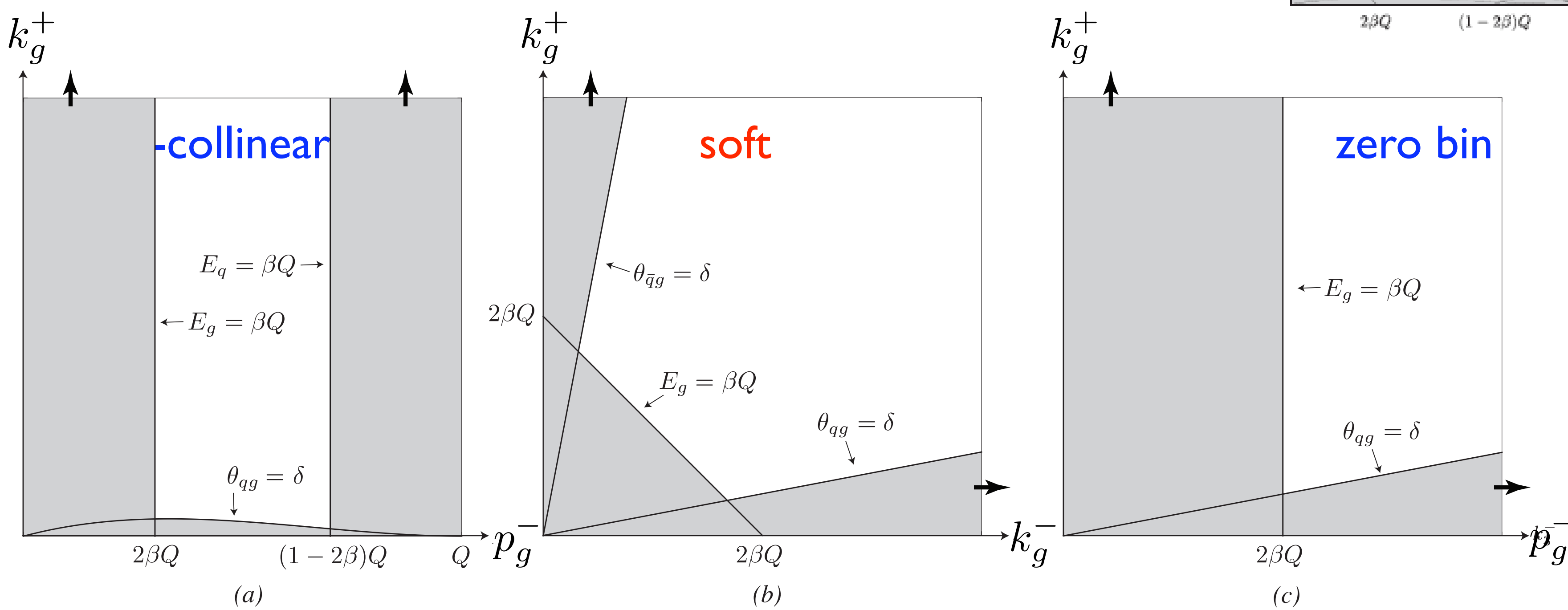
Trott, 2006 : $\beta \sim \delta^2 \sim \lambda^2 \quad p_{soft} \sim \Lambda_{QCD}$



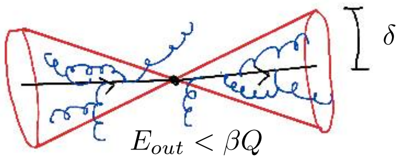
We do not assume scaling of parameters, only $\beta, \delta \ll 1$



Consistently multipole expand PS constraints



- Sterman Weinberg dijet rate to $\mathcal{O}(\alpha_s)$



naive collinear

$$\frac{1}{\sigma_0}\tilde{\sigma}^n_{\rm SW} = \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon}\left(\frac{3}{2} + 2\ln 2\beta\right) + 3\ln\frac{\mu}{\delta Q} + 2\ln 2\beta \ln\frac{\mu^2}{2\beta\delta^2 Q^2} + \frac{13}{2} - \frac{2\pi^2}{3}\right)$$

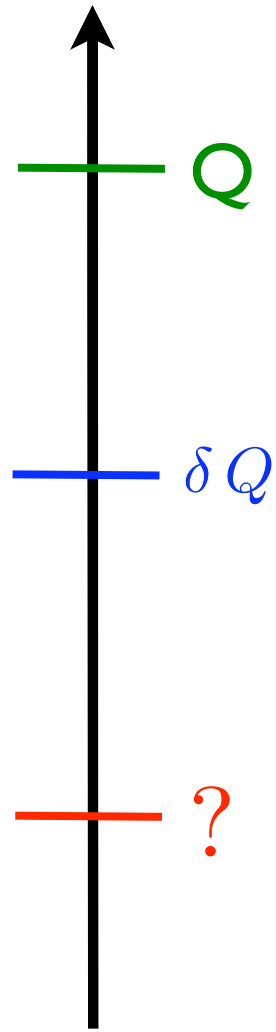
zero bin

$$\frac{1}{\sigma_0}\sigma^{n0}_{\rm SW} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu}{2\beta\delta Q} - 2\ln^2\frac{\mu}{2\beta\delta Q} + \frac{\pi^2}{12}\right)$$

$$\frac{1}{\sigma_0}\sigma^n_{\rm SW} = \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{2}{\epsilon}\ln\frac{\mu}{\delta Q} + 3\ln\frac{\mu}{\delta Q} + 2\ln^2\frac{\mu}{\delta Q} - \frac{3\pi^2}{4} + \frac{13}{2}\right)$$

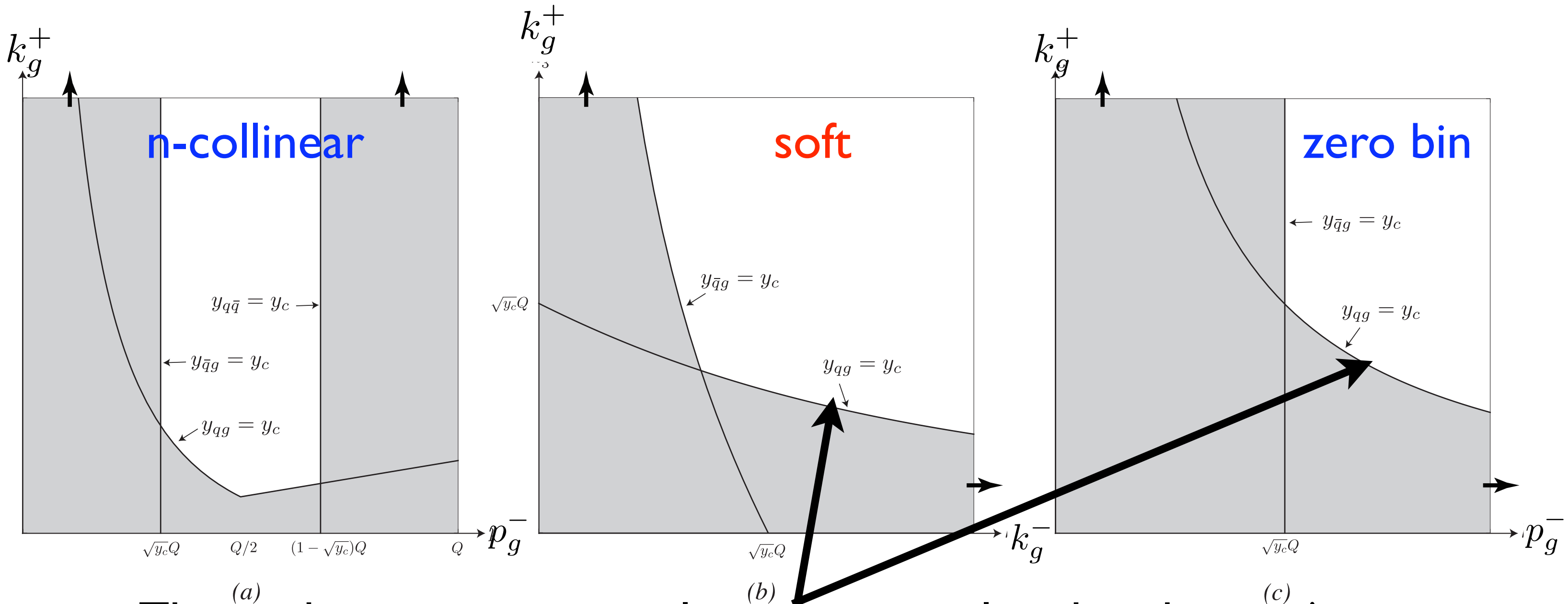
$$\frac{1}{\sigma_0}\sigma^{\rm soft}_{\rm SW} = \frac{\alpha_s C_F}{2\pi}\left(\frac{4}{\epsilon}\ln\delta - 4\ln^2\delta + 8\ln\delta\ln\frac{\mu}{2\beta Q} - \frac{\pi^2}{3}\right)$$

$$f_2^{\rm SW} = 1 + \frac{\alpha_s C_F}{\pi}\left(-4\ln 2\beta\ln\delta - 3\ln\delta - \frac{\pi^2}{3} + \frac{5}{2}\right)$$



virtual vertex and wave function are scaleless and vanish

- Finally consider k_T : Soft and collinear sectors are not separately IR safe in dim-reg!



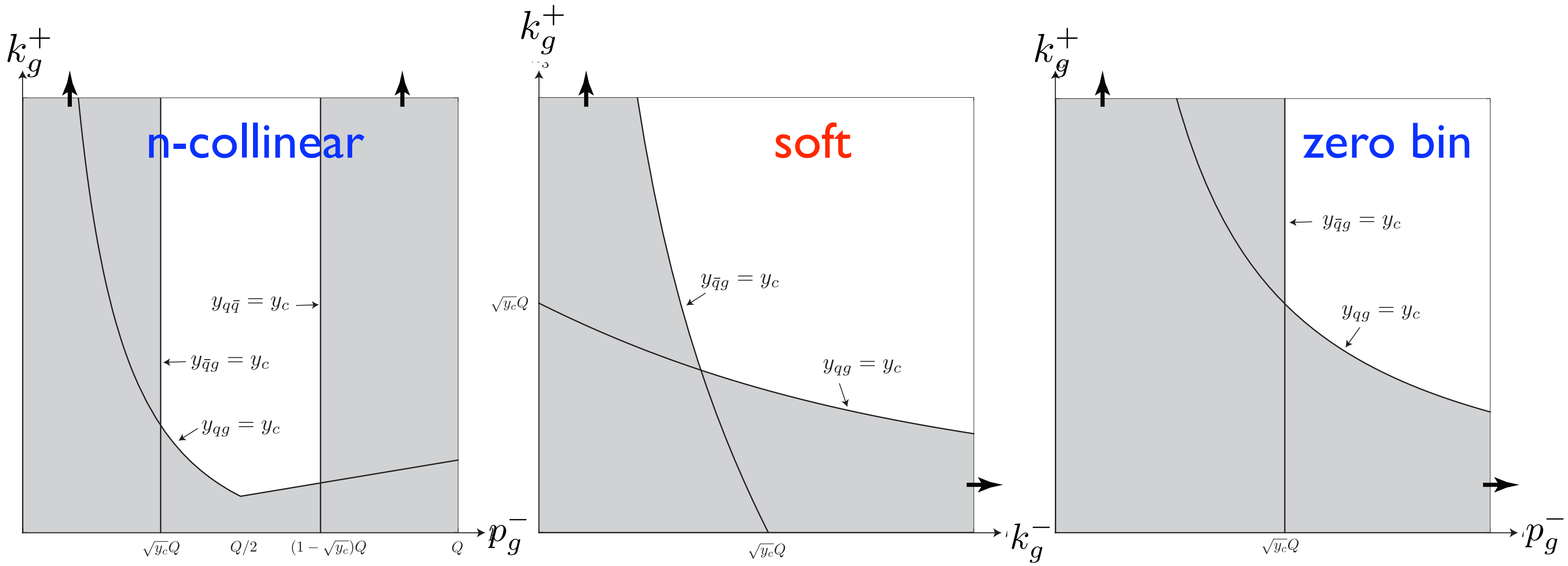
These phase space integrals are not regulated in dim-reg!

But the asymptotic behaviour of soft and zero-bin is the same → combine.

$$\frac{1}{\sigma_0}\sigma_{k_\perp} = \frac{1}{\sigma_0}\left(\tilde{\sigma}^{\bar{n}}_{k_\perp} + \tilde{\sigma}^{n}_{k_\perp} + \underbrace{\sigma^{s}_{k_\perp} - \sigma^{n0}_{k_\perp} - \sigma^{\bar{n}0}_{k_\perp}}_{\text{regulated}}\right)$$

$$f_2^{k_\perp} = 1 + \frac{\alpha_s C_F}{2\pi}\left(-\ln^2 y_c - 3\ln y_c - 6\ln 2 + \frac{\pi^2}{6} - 1\right)$$

- Finally consider k_T : Soft and collinear sectors are not separately IR safe in dim-reg!



(a) (b) (c)

Use off-shellness to regulate IR:

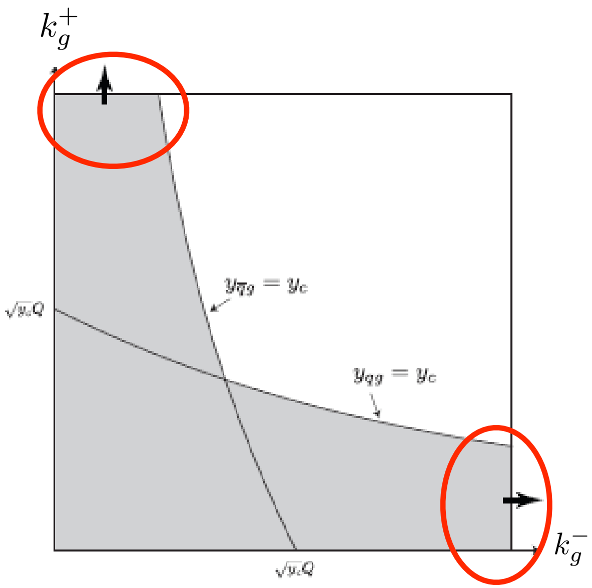
$$\frac{1}{\sigma_0}\left(\sigma^s_{k_\perp}+\sigma^s_V\right)=\frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2}-\frac{2}{\epsilon}\ln\frac{\mu^2 Q^2}{p_1^2 p_2^2}+2\ln\frac{p_1^2 p_2^2}{Q^4}\ln\frac{\mu^2}{y_c Q^2}\right)+\dots$$

UV finite, IR divergent

Total soft contribution not IR finite (same for collinear)

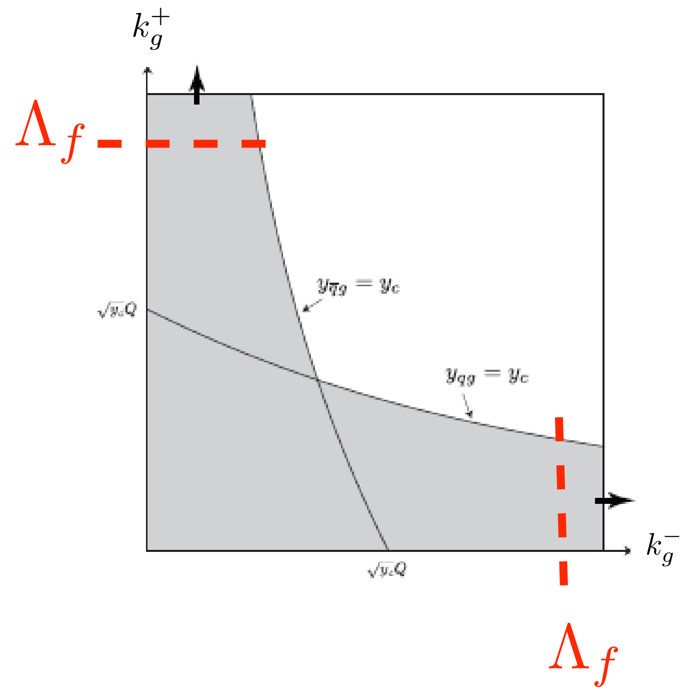
In this regularization scheme soft and collinear are not separately IR safe: Does factorization fail?

- Finally consider k_T : Does factorization fail?



UV divergent regions of PS cancel between soft and collinear. Unphysical.

Introduce cut off on $k^{\pm}$



IR divergent but sensitive to the UV

$$\frac{1}{\sigma_0}\left(\sigma^s_{k_\perp} + \sigma^s_V\right) = -\frac{\alpha_s C_F}{2\pi}\left(\ln^2 \frac{y_c Q^2}{\Lambda_f^2} + \frac{\pi^2}{6}\right)$$

Total soft contribution
IR finite

Factorization of rate in SCET in to separately IR safe soft and collinear contributions is UV regulator dependent.

IR regulator dependence (Hornig, Lee, Ovanesyan, 2009) (Chiu, Fuhrer, Kelley, Manohar, 2009)

k_T may factorize.

In conclusion:

➡ Shown a consistent power counting of PS integrals in SCET

- UV divergences cancel between soft and collinear sectors
- Zerobin - non-trivial, necessary to control logs in collinear sector, simplifies jet algorithm combinatorics

➡ Factorization of soft and collinear in to separately IR safe contributions is UV regulator dependent.

➡ Question: Can we resum all large logs arising from jet algorithm phase space cuts? Not yet.

Collinear sector under control, but can not resum all logs in soft sector: presence of additional scales? failure of factorization? non-global logs?

Backup Slides

JADE with off-shellness

Real Emission $$\frac{1}{\sigma_0}\tilde{\sigma}^n_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\left(\ln\frac{p_1^2}{jQ^2}-\ln\frac{\mu^2}{j^2Q^2}\right)-\ln^2\frac{p_1^2}{Q^2}+2\ln\frac{\mu^2}{Q^2}\ln\frac{p_1^2}{Q^2}+\frac{3}{2}\ln\frac{p_1^2}{Q^2}\right)+\ldots$$

$$\frac{1}{\sigma_0}\sigma^{n0}_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2}-\frac{2}{\epsilon}\ln\frac{\mu^2}{j^2Q^2}\right)+\ldots$$

$$\frac{1}{\sigma_0}\sigma^{n}_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(\frac{2}{\epsilon}\ln\frac{p_1^2}{jQ^2}-\ln^2\frac{p_1^2}{Q^2}+2\ln\frac{\mu^2}{Q^2}\ln\frac{p_1^2}{Q^2}+\frac{3}{2}\ln\frac{p_1^2}{Q^2}\right)+\ldots$$

$$\frac{1}{\sigma_0}\sigma^{\rm soft}_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon}\left(\ln\frac{p_1^2}{jQ^2}+\ln\frac{p_2^2}{jQ^2}\right)+\left(\ln\frac{p_1^2}{Q^2}+\ln\frac{p_2^2}{Q^2}\right)^2-2\left(\ln\frac{p_1^2}{Q^2}+\ln\frac{p_2^2}{Q^2}\right)\ln\frac{\mu^2}{Q^2}\right)+\cdots$$

$$\frac{1}{\sigma_0}\sigma^{R}_{\rm JADE} = \frac{\alpha_s C_F}{2\pi}\left(2\ln\frac{p_1^2}{Q^2}\ln\frac{p_2^2}{Q^2}+\frac{3}{2}\ln\frac{p_1^2}{Q^2}+\frac{3}{2}\ln\frac{p_2^2}{Q^2}\right)+\ldots$$

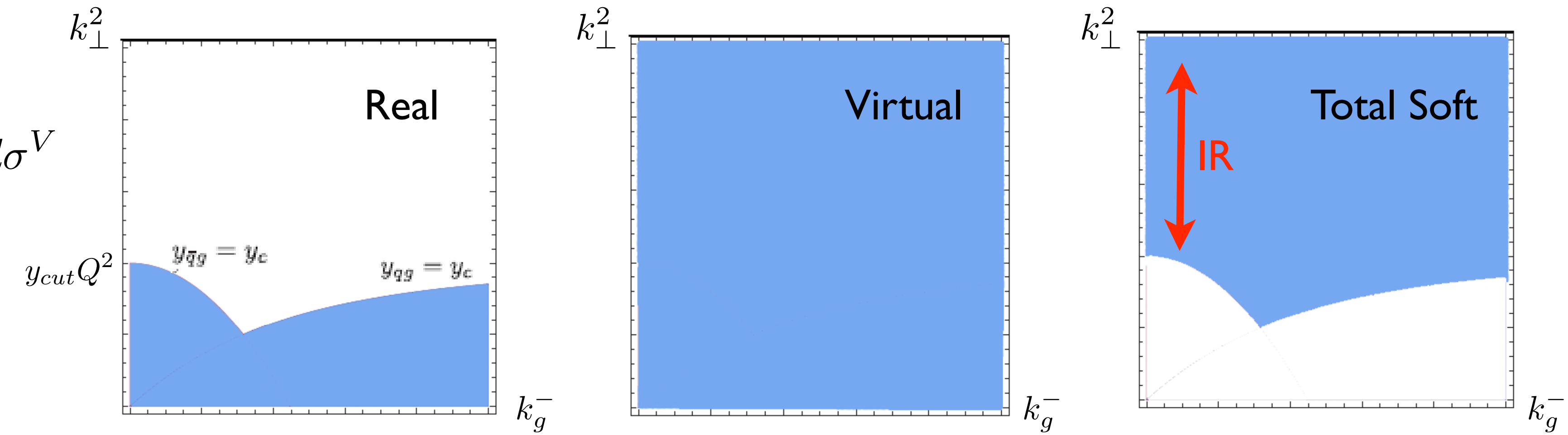
$$\frac{1}{\sigma_0}\sigma^{s}_{V} = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2}-\frac{2}{\epsilon}\ln\left(-\frac{\mu^2Q^2}{p_1^2p_2^2}\right)-\ln^2\left(-\frac{\mu^2Q^2}{p_1^2p_2^2}\right)\right)\ldots$$

$$\frac{1}{\sigma_0}\left(\sigma^{s}_{\rm JADE}+\sigma^s_V\right) = \frac{\alpha_s C_F}{2\pi}\left(-\frac{2}{\epsilon^2}-\frac{4}{\epsilon}\ln\frac{\mu}{jQ}\right)+\ldots$$

- **Finally consider k_T : Does factorization fail?**

Using the method introduced in (Hornig, Lee, Ovanesyan, 2009) we would conclude that the k_T soft contribution is IR divergent, as we found using dim-reg to regulate UV

$$d\sigma^R = -d\sigma^V$$



However with a cut-off on k+ k-, the PS becomes: