N3LL Analysis of Thrust Distributions: Determination of $\alpha_s(M_Z)$

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Outline

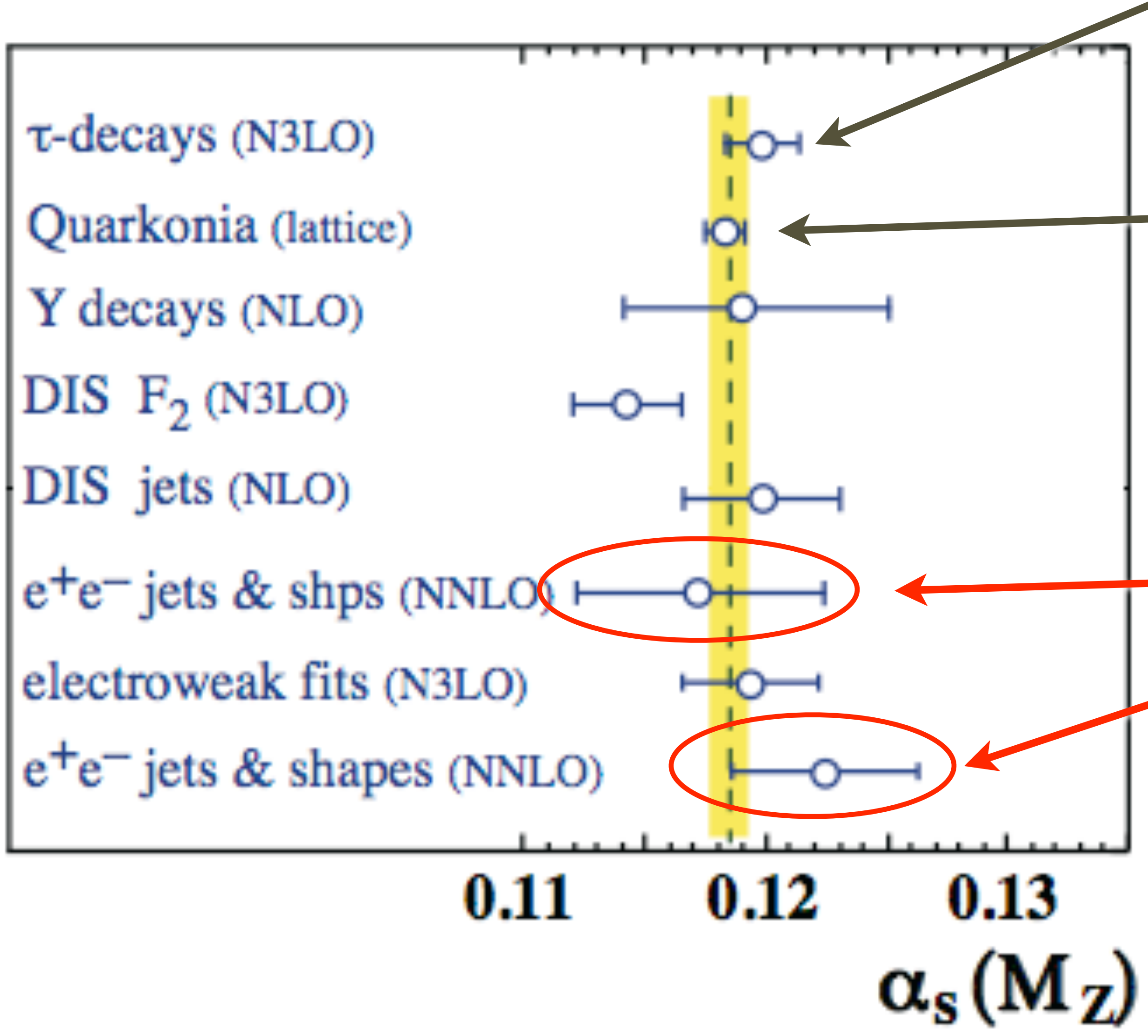
- Motivations
  - World Averages
  - Recent Thrust Analyses
- Theoretical Developments
  - Factorization Theorem
  - Running of the scales and Profile Functions
  - Non-perturbative corrections: Soft Function and Renormalon
- Global Thrust Fit
- Cross Checks and Comparisons

Latest World Average

S. Bethke, arXiv:0908.1135

$$\alpha_s(m_Z) = 0.1184 \pm 0.0007$$

errors inflated to account for variation in literature



fit to Υ-splittings, Wilson loops

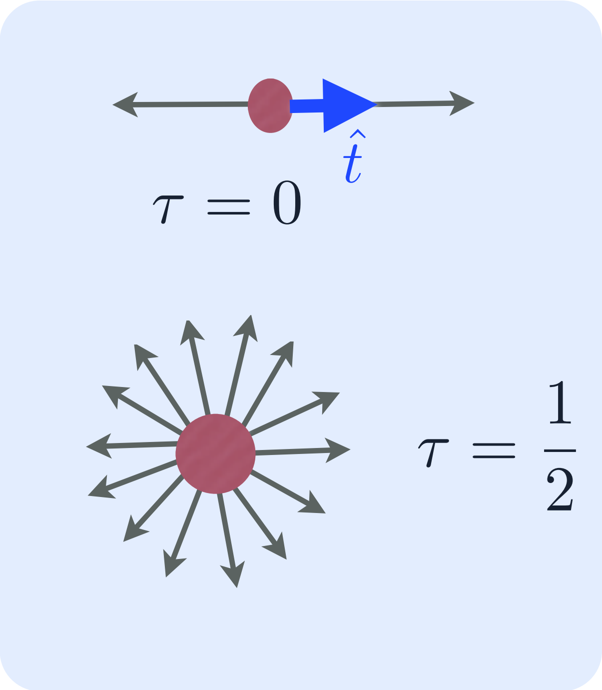
$$\alpha_s(m_Z) = 0.1183 \pm 0.0008$$

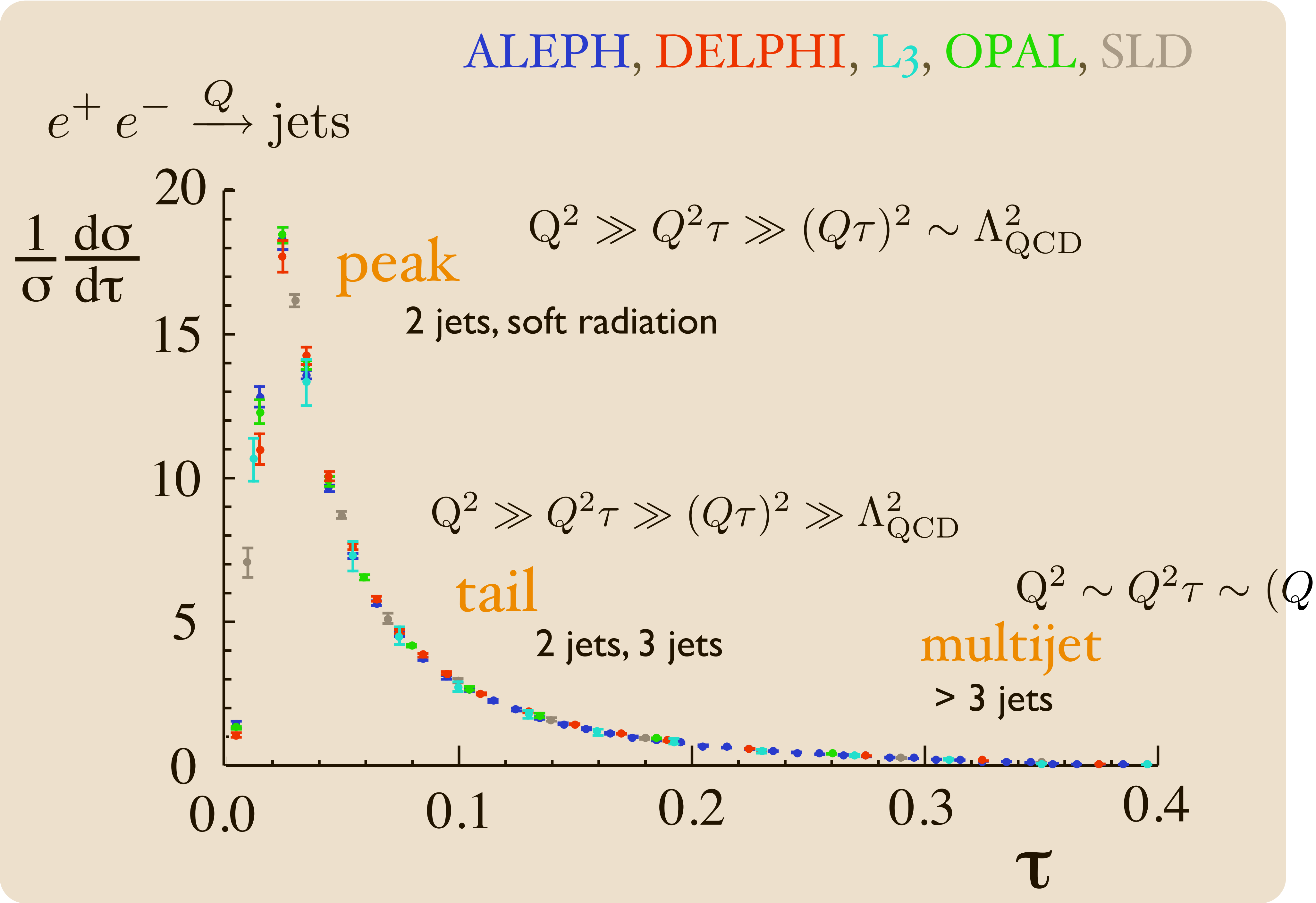
HPQCD 0807.1687

event shape results at $\mathcal{O}(\alpha_s^3)$

Event Shape Variables: Thrust

$$T = \max_{\hat{t}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{Q} \qquad \tau = 1 - T$$





peak

$Q^2 \gg Q^2\tau \gg (Q\tau)^2 \sim \Lambda^2_{\mathrm{QCD}}$

2 jets, soft radiation

$Q^2 \gg Q^2\tau \gg (Q\tau)^2 \gg \Lambda^2_{\mathrm{QCD}}$

tail

2 jets, 3 jets

$Q^2 \sim Q^2\tau \sim (Q\tau)^2 \gg \Lambda^2_{\mathrm{QCD}}$

multijet

> 3 jets

$Q = 91.2\,\mathrm{GeV}$

Recent $\alpha_s(M_Z)$ analyses: Tail Fits

	sum logs	power corrections	data	$\alpha_s(M_Z)$
Dissertori et al. 0712.0327	no	Monte Carlo	ALEPH	0.1240 ± 0.0034
Dissertori et al. 0906.3436	NLL	Monte Carlo	ALEPH	0.1224 ± 0.0039
Becher, Schwartz 0803.0342	N^3LL	Monte Carlo	ALEPH, OPAL	0.1172 ± 0.0021
Davison, Webber 0809.3326	NLL	effective coupling model	Most of data	0.1164 ± 0.0028

Improvements over earlier works

AFHMS

- Factorization theorem
  - peak
  - tail
  - multijet
- power corrections: $\overline{\Omega}_1 \equiv \frac{1}{2N_c}\langle 0\big|\mathrm{tr}\ \overline{Y}_{\bar n}(0)Y_n(0)\, i\partial_\tau\, Y_n^\dagger(0)\overline{Y}^\dagger_{\bar n}(0)\big|0\rangle$ $(\overline{MS})$

  $$i\partial_\tau \equiv \theta(i\bar n\cdot\partial - in\cdot\partial)\, in\cdot\partial + \theta(in\cdot\partial - i\bar n\cdot\partial)\, i\bar n\cdot\partial$$

  renormalon subtraction
- Finite bottom quark mass corrections included in factorization theorem
- QED Sudakov and final state radiation at NNLL
- Use of the full 3-loop hard function
- Axial anomaly corrections at $\mathcal{O}(\alpha_s^2)$ from top quarks

Factorization Theorem for Thrust

$$e^+ e^- \xrightarrow{Q} \text{jets}$$

singular partonic cross section

nonsingular partonic cross section

b-mass correction

AFHMS

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\Delta \mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) \times \left[1 + \mathcal{O}\left(\alpha_s \frac{\Lambda_{\mathrm{QCD}}}{Q} \right) \right]$$

nonperturbative: large angle soft radiation

gap parameter $\bar{\Delta}$

moment parameters Ω_i

more on those later

Schematically,

$$\frac{1}{\hat{\sigma}} \frac{d\hat{\sigma}_s}{d\tau} = \sum_{n,m} \alpha_s^n \frac{\ln^m \tau}{\tau}$$

$$\frac{1}{\hat{\sigma}} \frac{d\hat{\sigma}_{\mathrm{ns}}}{d\tau} = \sum_{n,m} \alpha_s^n \ln^m \tau + \sum_{n,m} \alpha_s^n f_m(\tau)$$

Valid for all τ values !

Singular partonic cross section

$$\frac{\mathrm{d}\hat{\sigma}_{\rm s}^{\rm QCD}}{\mathrm{d}\tau} = \sum_I \sigma_0^I \, H_Q^I(Q,\mu_H)\, U_Q(Q,\mu_H,\mu_S) \times$$

$$\times \int \mathrm{d}s\, \mathrm{d}s'\, J_\tau(s',\mu_J)\, U_J^\tau(s-s',\mu_S,\mu_J)\, e^{-2\frac{\delta(R,\mu_S)}{Q}\frac{\partial}{\partial\tau}}\; S_\tau^{\rm part}\Big(Q\,\tau - \frac{s}{Q},\mu_S\Big)$$

- σ_0^I partonic e^+e^- cross section for quark pair production. need to distinguish 4 types of components $I = \{uv, dv, va, da\}$ for up, down, vector and axial parts.

- H_Q^I hard function: encodes short distance virtual corrections NEW

  include non-log term h_3 at $\mathcal{O}(\alpha_s^3)$ extracted from Baikov et al. 0902.3519

- U_Q, U_J^τ "hard" and "jet" renormalization group evolution Becher, Schwartz

  anomalous dim. known up to 3 loops.

  To resum at N^3LL, need $\Gamma_{\rm cusp}^{(3)}$ ($\mathcal{O}(\alpha_s^4)$)

Order Counting

$$\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \dots$$

LL NLL NNLL N^{3}LL

		cusp	non-cusp	matching	alphas
standard counting	LL	1	–	tree	1
	NLL	2	1	tree	2
	NNLL	3	2	1	3
	N^{3}LL	4^{pade}	3	2	4
primed counting	LL$'$	1	–	tree	1
	NLL$'$	2	1	1	2
	NNLL$'$	3	2	2	3
	N^{3}LL$'$	4^{pade}	3	3	4

when fixed order results are important primed counting is better

Singular partonic cross section

- J_τ **thrust jet function** $\quad J_\tau(s,\mu) = \frac{1}{\mu^2}\sum_{n=-1}^{\infty} J_n[\alpha_s(\mu)]\mathcal{L}_n(s/\mu^2)$

$$J_{-1}(\alpha_s) = \quad 1. - 0.608949\alpha_s - 2.26795\alpha_s^2 + \left(2.21087 + \frac{j_3}{\pi^3}\right)\alpha_s^3$$

$$\mathcal{L}_{-1}(x) = \delta(x)$$

$$\mathcal{L}_n(x) = \left[\frac{\theta(x)\log^n x}{x}\right]_+$$

unknown non-log coefficient j_3 at $\mathcal{O}(\alpha_s^3)$

- S_τ^{part} **partonic thrust soft function** $\quad S_\tau^{\mathrm{part}}(k,\mu) = \frac{1}{\mu}\sum_{n=-1}^{\infty} S_n[\alpha_s(\mu)]\mathcal{L}_n(k/\mu)$

$$\begin{aligned}
S_{-1}(\alpha_s) &= 1 + 0.349066\alpha_s + (1.26859 + 0.0126651\,s_2)\alpha_s^2 \\
&\quad + \left(1.54284 + 0.00442097\,s_2 + \frac{s_3}{\pi^3}\right)\alpha_s^3\,, \\
S_0(\alpha_s) &= 2.07321\alpha_s^2 + (4.8002 - 0.0309077\,s_2)\alpha_s^3\,, \\
S_1(\alpha_s) &= -1.69765\,\alpha_s - 6.26659\,\alpha_s^2 - (16.4676 + 0.021501\,s_2)\,\alpha_s^3
\end{aligned}$$

$s_2 = -40.1 \pm 3.2$ Becher, Schwartz

$s_2 = -39.1 \pm 2.5$ Hoang, Kluth

non-log coefficient s_2 at $\mathcal{O}(\alpha_s^2)$

unknown non-log coefficient s_3 at $\mathcal{O}(\alpha_s^3)$

Singular partonic cross section

$$\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}^{\mathrm{QCD}}}{\mathrm{d}\tau} = \sum_I \sigma_0^I \, H_Q^I(Q,\mu_H)\, U_Q(Q,\mu_H,\mu_S) \; \times$$
$$\times \int \mathrm{d}s\, \mathrm{d}s'\, J_\tau(s',\mu_J)\, U_J^\tau(s-s',\mu_S,\mu_J)\, e^{-2\frac{\delta(R,\mu_S)}{Q}\frac{\partial}{\partial\tau}}\; S_\tau^{\mathrm{part}}\Big(Q\,\tau - \frac{s}{Q},\mu_S\Big)$$

- $\exp\Big(-2\,\frac{\delta(R,\mu_S)}{Q}\frac{\partial}{\partial\tau}\Big)$ implements renormalon subtraction (more later)

- scales:
  - μ_H hard scale
  - μ_J jet scale
  - μ_S soft scale

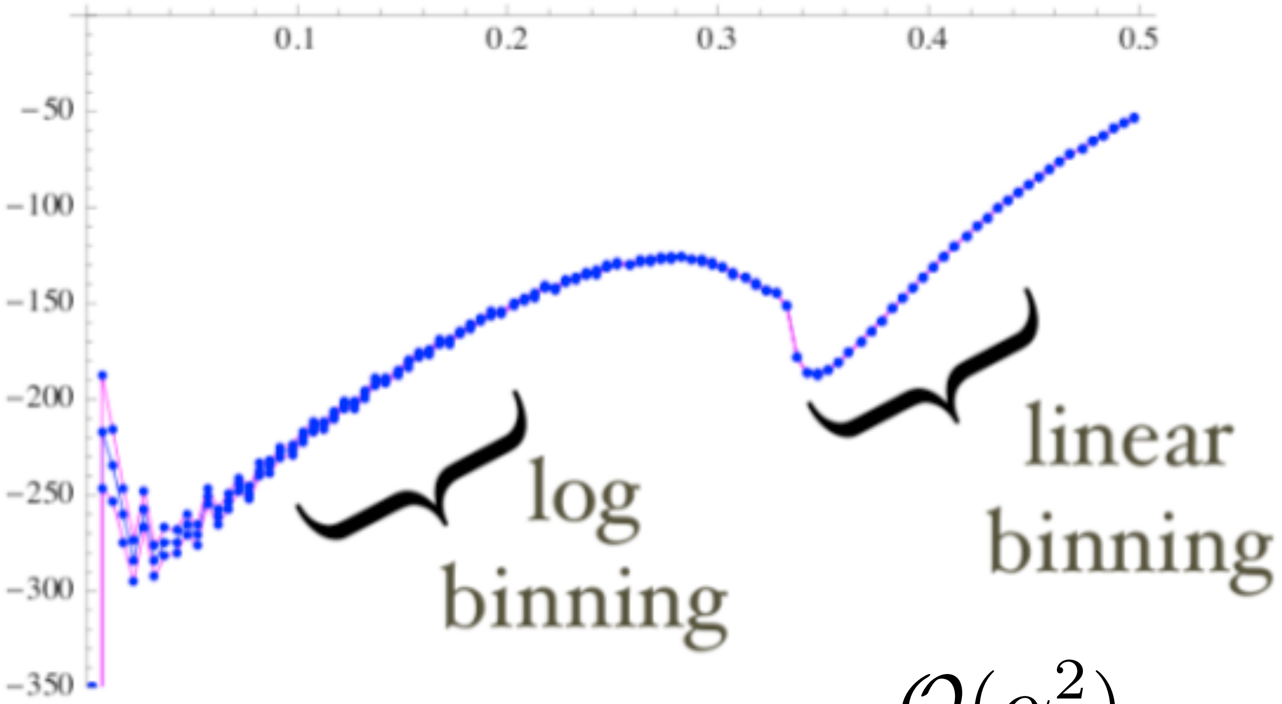
Non Singular partonic cross section

$$\frac{d\hat{\sigma}_{ns}}{d\tau}\left(\tau, \frac{\mu_{ns}}{Q}\right) = \sigma_0^I \; e^{2\frac{\delta(R)}{Q}} \; \frac{\partial}{\partial\tau} f^I\left(\tau, \frac{\mu_{ns}}{Q}\right)$$

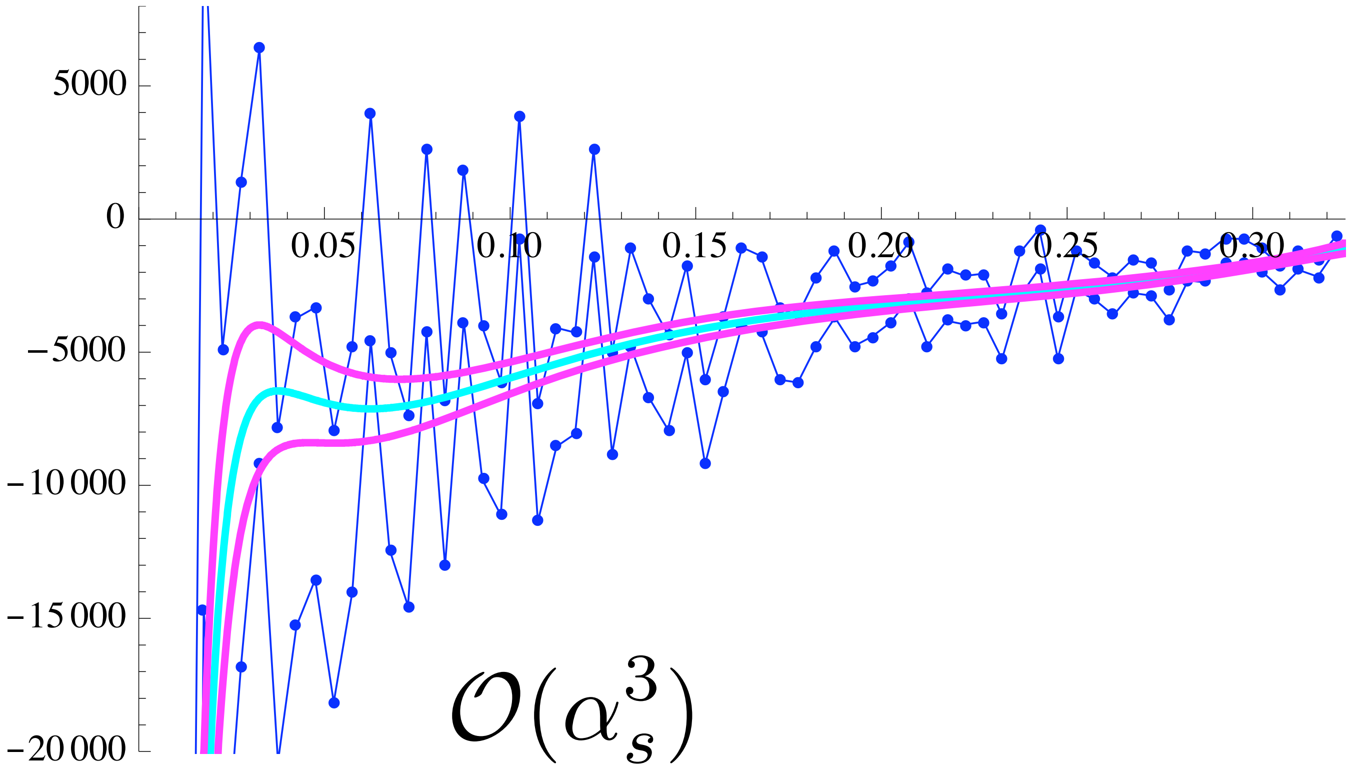
$$\left.\frac{d\hat{\sigma}}{d\tau}\right|_{\text{fixed order}} = \left.\frac{d\hat{\sigma}}{d\tau}\right|_{\text{SCET expanded}} + \frac{d\hat{\sigma}_{ns}}{d\tau}$$

NNLO, $\mathcal{O}(\alpha_s^3)$ in QCD known numerically Gehrmann et al.

and independently S. Weinzierl



$\mathcal{O}(\alpha_s^2)$



$\mathcal{O}(\alpha_s^3)$

$f_2(\tau) + \epsilon_2\,\delta_2(\tau)$

$f_3(\tau) + \epsilon_3\,\delta_3(\tau)$

Non Singular partonic cross section

$$\frac{d\hat{\sigma}_{ns}}{d\tau}\left(\tau, \frac{\mu_{ns}}{Q}\right) \quad = \sigma_0^I \; e^{2\frac{\delta(R)}{Q}} \; \frac{\partial}{\partial\tau} f^I\left(\tau, \frac{\mu_{ns}}{Q}\right)$$

$$\frac{d\hat{\sigma}}{d\tau}\Big|_{\text{fixed order}} = \frac{d\hat{\sigma}}{d\tau}\Big|_{\text{SCET expanded}} + \frac{d\hat{\sigma}_{ns}}{d\tau}$$

NNLO, $\mathcal{O}(\alpha_s^3)$ in QCD known numerically Gehrmann et al.

and independently S. Weinzierl

$$f_2(\tau) + \epsilon_2\,\delta_2(\tau) \qquad\qquad f_3(\tau) + \epsilon_3\,\delta_3(\tau)$$

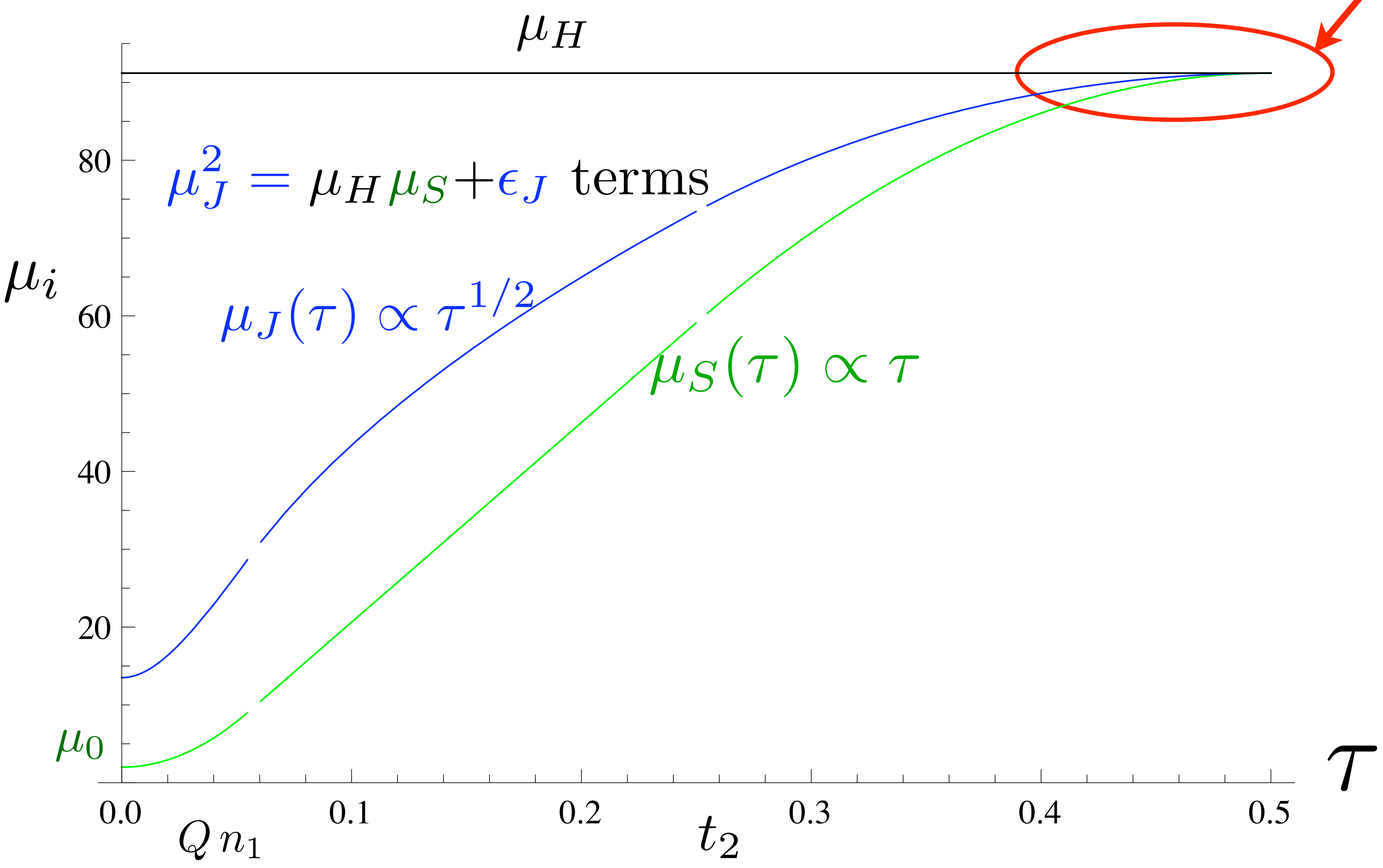
Total Cross Section Constraints:

- reduces uncertainties $\delta_2(\tau),\ \delta_3(\tau)$
- $f_2(\tau), f_3(\tau)$ depend on $h_3 + j_3 + s_3$

In our theory uncertainty estimate we vary $\epsilon_2, \epsilon_3, \mu_{ns}$

Profile Functions

Peak	Tail	Multijet
$\mu_H \sim Q$	$\mu_H \sim Q$	$\mu_H = \mu_J = \mu_S \sim Q$
$\mu_J \sim \sqrt{Q\,\Lambda_{QCD}}$	$\mu_J \sim Q\sqrt{\tau}$	
$\mu_S \geq \Lambda_{QCD}$	$\mu_S \sim Q\tau$	



Scales must be equal for multijet region

$$\frac{\mathrm{d}\hat{\sigma}_{\rm s}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\rm ns}}{\mathrm{d}\tau} = \frac{d\hat{\sigma}_{\rm FO}}{d\tau}$$

Parameters varied in scan

$\mu_0 \quad n_1 \quad t_2$

$\epsilon_J \quad \epsilon_h = \mu_H/Q$

n_s

Profile functions implement scale dependence.

Nonperturbative Corrections

$$S_\tau(k,\mu) = \frac{1}{N_c}\langle 0 \big| \mathrm{tr}\, \overline{Y}_{\bar n} Y_n\, \delta(k - i\partial_\tau) Y_n^\dagger \overline{Y}_{\bar n}^\dagger \big| 0\rangle$$

↑ soft Wilson lines

$$i\partial_\tau \equiv \theta(i\bar n\cdot\partial - in\cdot\partial)\, in\cdot\partial + \theta(in\cdot\partial - i\bar n\cdot\partial)\, i\bar n\cdot\partial$$

The soft function factorizes as

$$S_\tau(k,\mu) = \int dk'\ S_\tau^{\rm part}(k-k',\mu)\ S_\tau^{\rm mod}(k')$$

Hoang, Stewart
Ligeti, Stewart, Tackmann

fixed order pertubative expression

nonperturbative model soft function. Exp. small tail

In peak region we use a complete basis for $S_\tau^{\rm mod}$

From an OPE analysis, we obtain $S_\tau(k,\mu_S) = S_\tau^{\rm part}(k,\mu_S) - 2\overline{\Omega}_1 \dfrac{d\, S_\tau^{\rm part}}{dk}(k,\mu_S) + \cdots$

This generalizes the OPE result

(tree level matching)

$$S_\tau(k,\mu_S) = \delta(k) - 2\Omega_1\, \delta'(k) + \cdots$$

Lee, Sterman
Korchemsky, Sterman

Renormalon subtraction

In $\overline{MS}$, $S_\tau^{\rm part}$ has an $\mathcal{O}(\Lambda_{QCD})$ renormalon

In $\overline{MS}$, also $\overline{\Omega}_1$ has an $\mathcal{O}(\Lambda_{QCD})$ renormalon

Hoang, Stewart

Solved introducing a gap parameter Δ: $S_\tau^{\rm mod}(k) \to S_\tau^{\rm mod}(k-2\Delta)$

and writing

$$\Delta = \bar{\Delta}(R,\mu_S) + \delta(R,\mu_S)$$

renormalon free

$$\delta(R,\mu_S) = e^{\gamma_E} R\Big[-0.849\,\frac{\alpha_s(\mu_S)}{4\pi}\,\log\frac{\mu_S}{R} + \cdots\Big]$$

Renormalon subtraction

In $\overline{MS}$, $S_\tau^{\rm part}$ has an $\mathcal{O}(\Lambda_{QCD})$ renormalon

In $\overline{MS}$, also $\overline{\Omega}_1$ has an $\mathcal{O}(\Lambda_{QCD})$ renormalon

Hoang, Stewart

Solved introducing a gap parameter Δ: $S_\tau^{\rm mod}(k) \to S_\tau^{\rm mod}(k-2\Delta)$

and writing $\Delta = \bar{\Delta}(R,\mu_S) + \delta(R,\mu_S)$

so,

$$S_\tau(k,\mu) = \int dk' \left[e^{-2\delta \frac{\partial}{\partial k}} S_\tau^{\rm part}(k-k',\mu)\right] S_\tau^{\rm mod}(k' - 2\overline{\Delta})$$

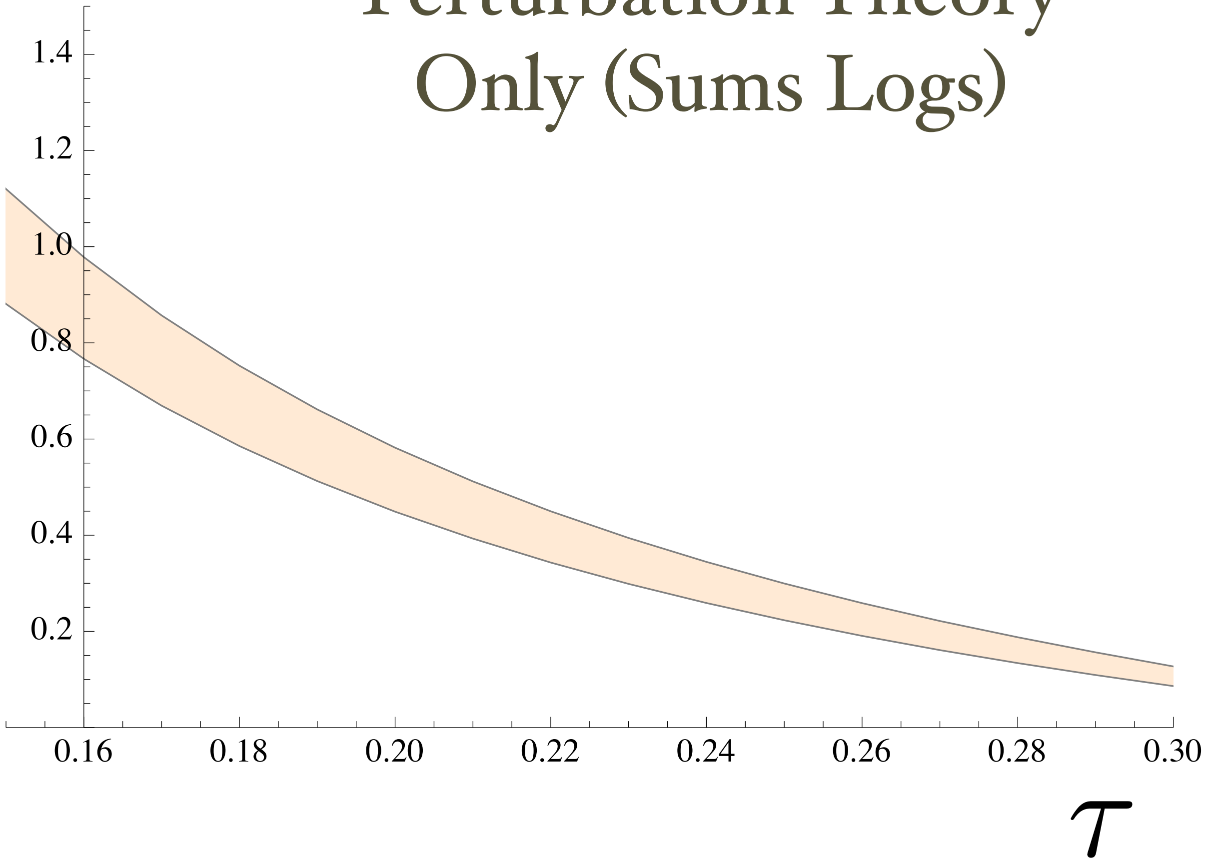
renormalon free perturbative non perturbative

To avoid large logarithms, we need $R \sim \mu_S$

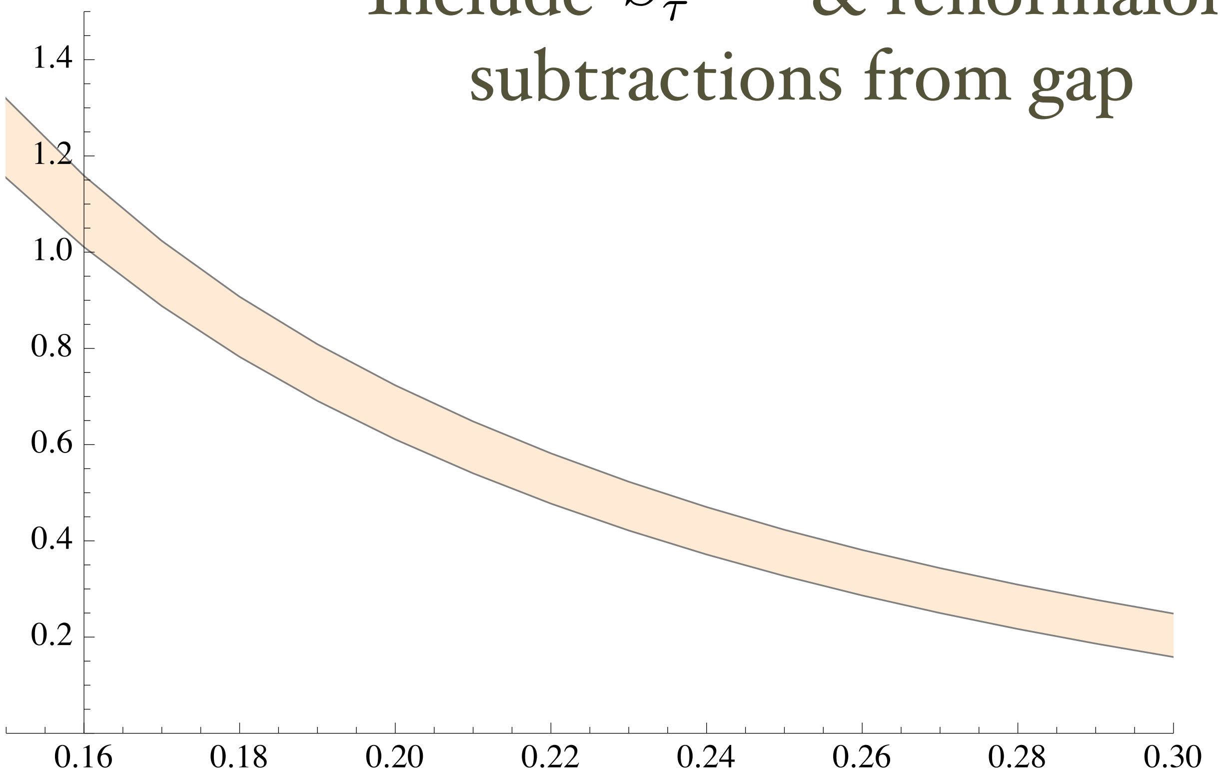
Tail Predictions with Scan over Theory Uncertainties

NLL′

$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$



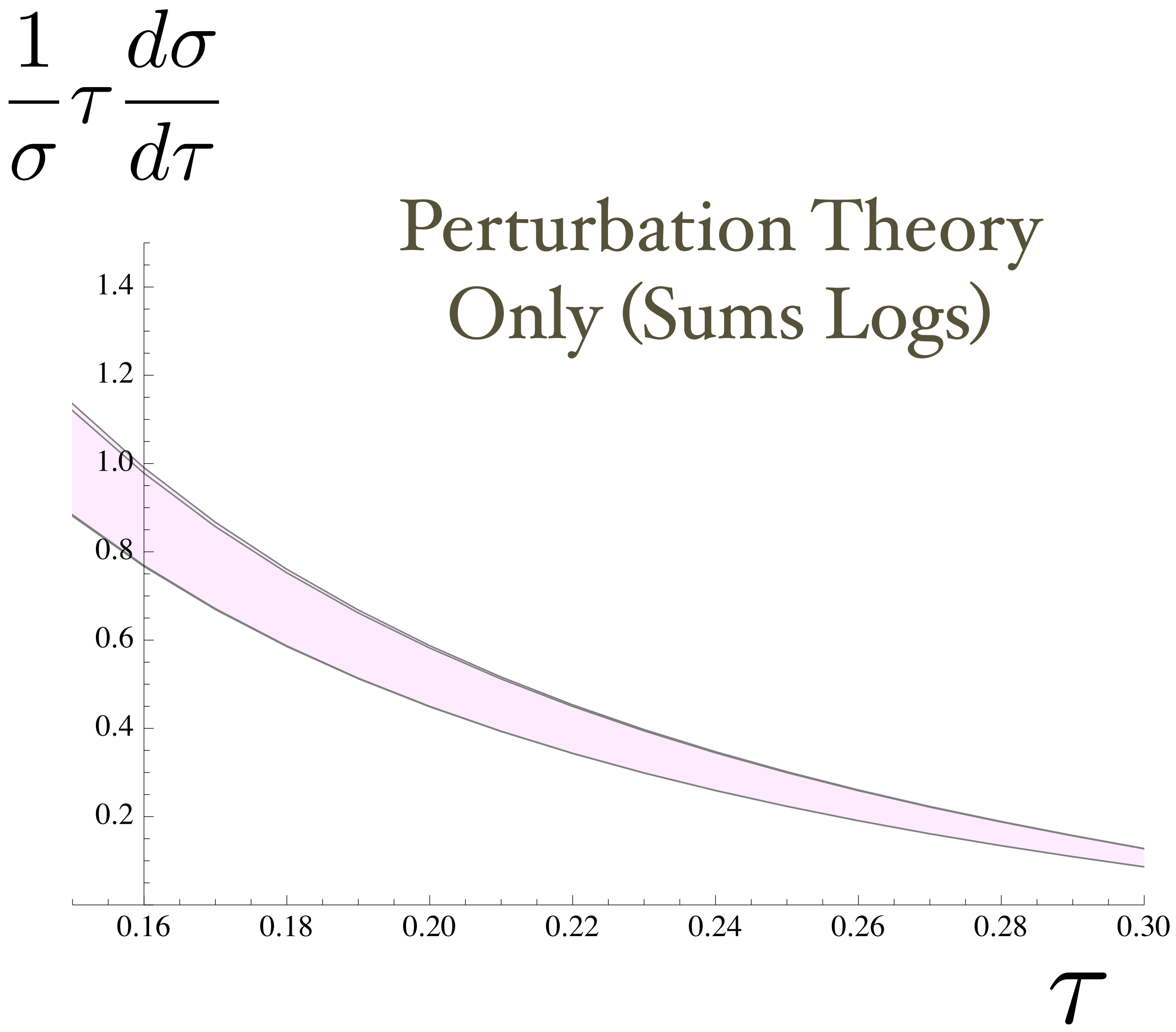
Perturbation Theory Only (Sums Logs)

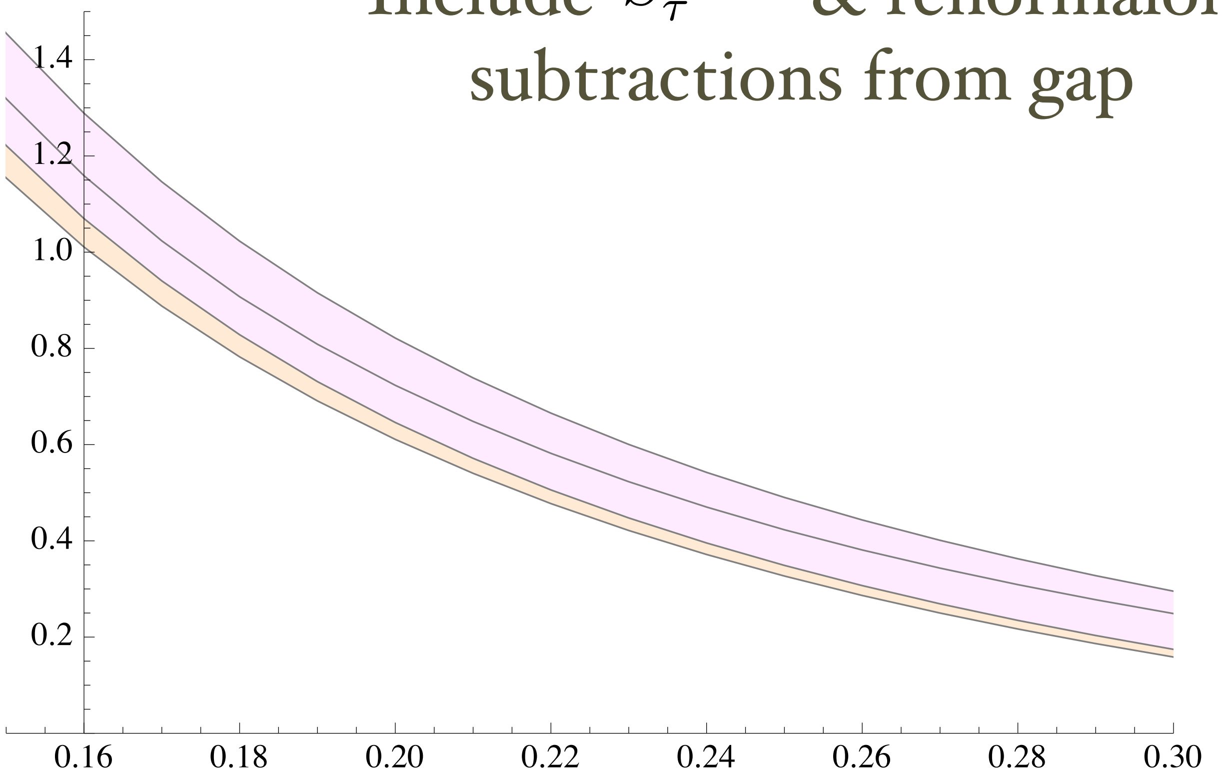


Include S_τ^{mod} & renormalon subtractions from gap

Tail Predictions with Scan over Theory Uncertainties

NLL' NNLL


Perturbation Theory Only (Sums Logs)

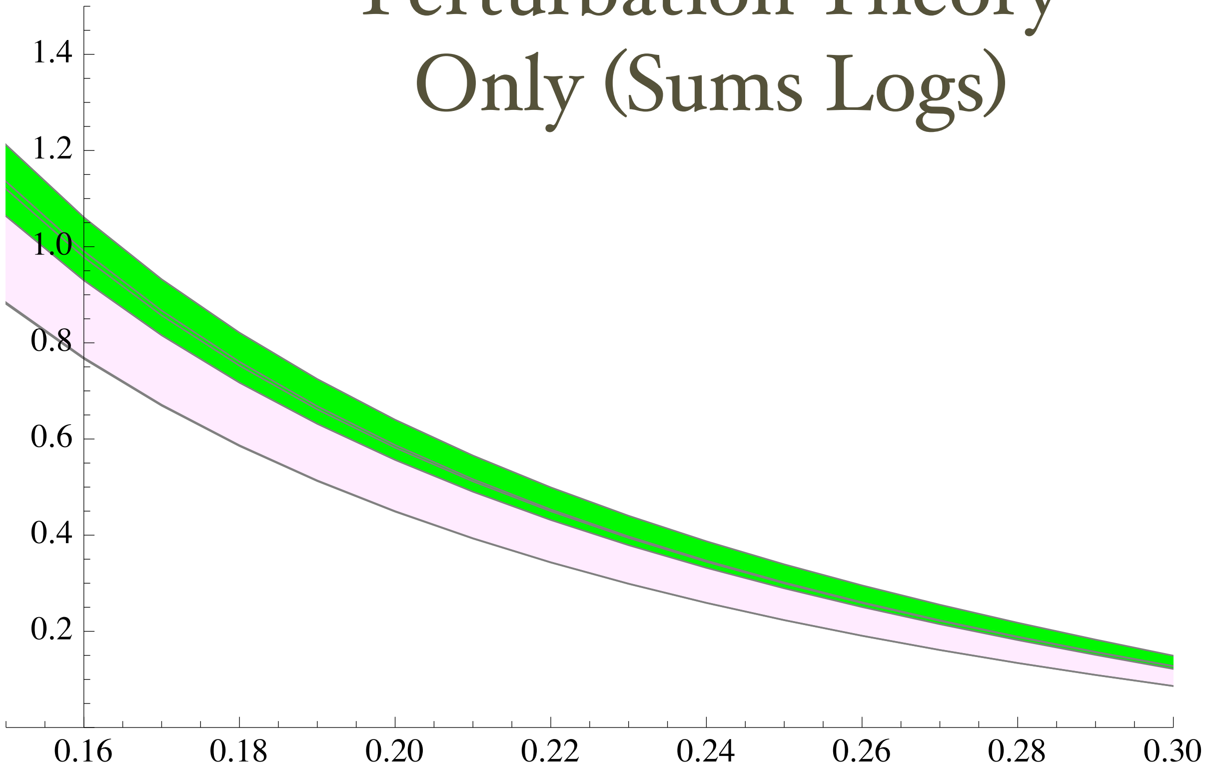

Include S_τ^{mod} & renormalon subtractions from gap

Tail Predictions with Scan over Theory Uncertainties

NLL' NNLL NNLL'

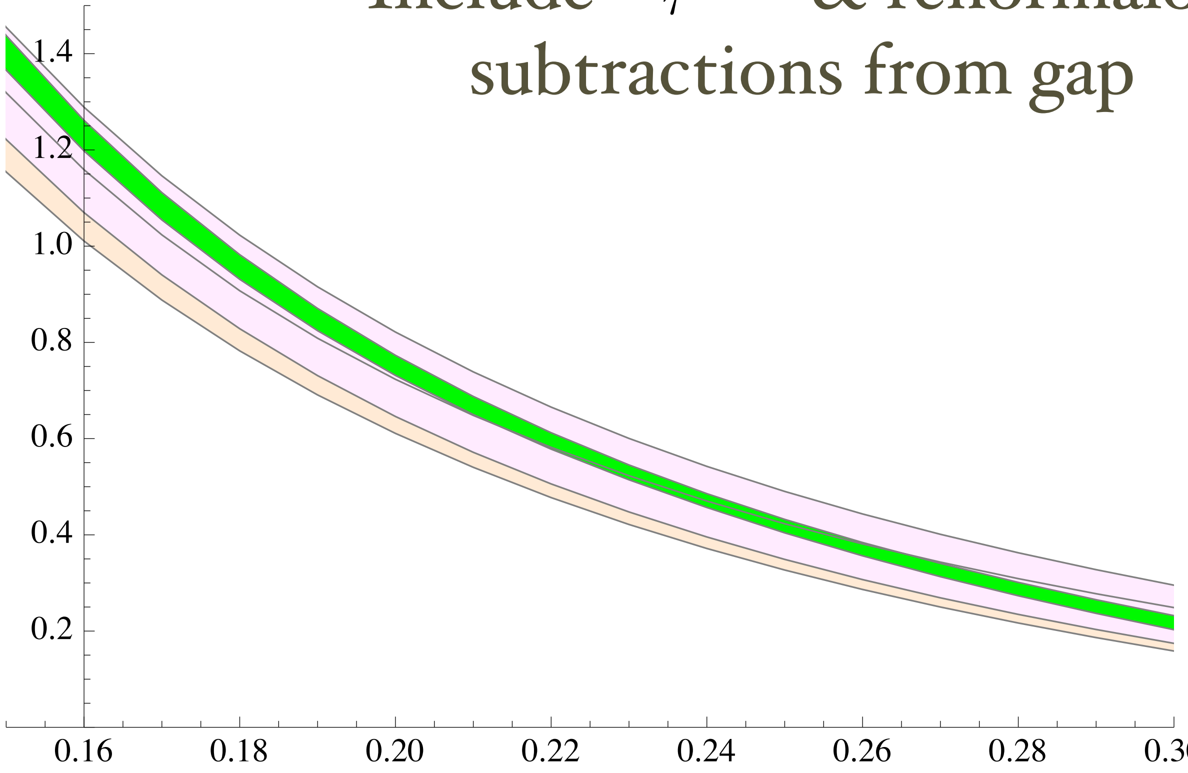
$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$

Perturbation Theory Only (Sums Logs)



τ

Include S_τ^{mod} & renormalon subtractions from gap



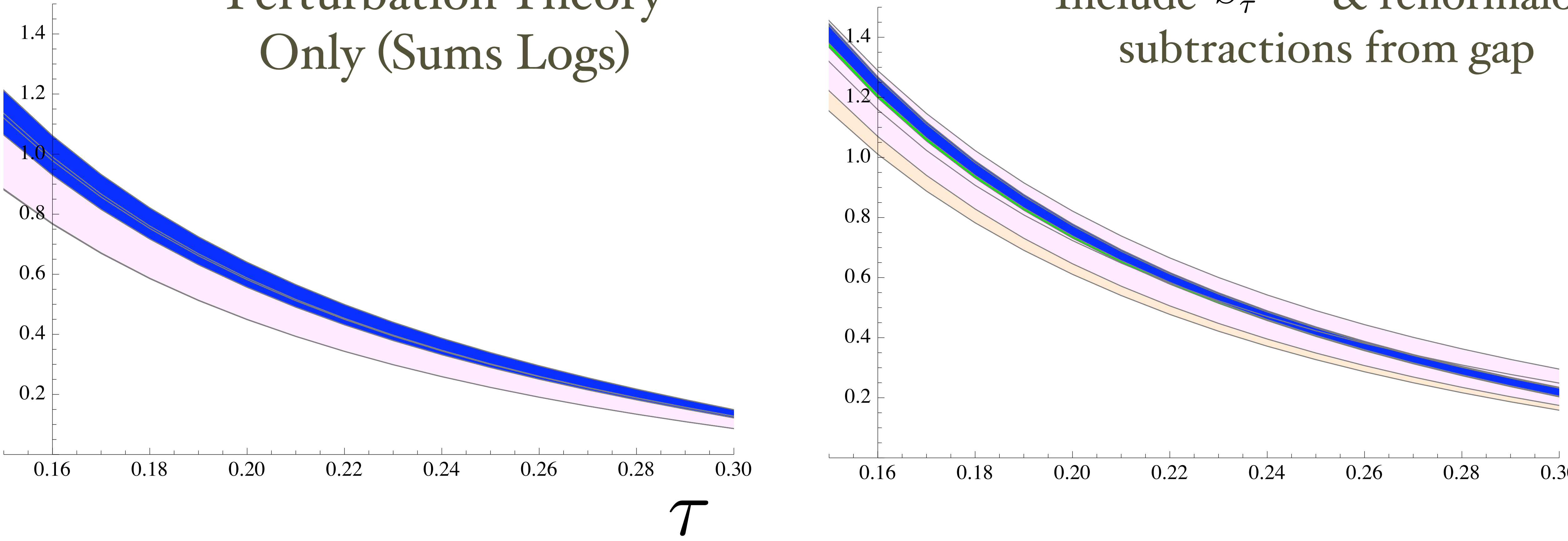
Tail Predictions with Scan over Theory Uncertainties

NLL' NNLL NNLL' $\mathrm{N^3LL}$

$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$

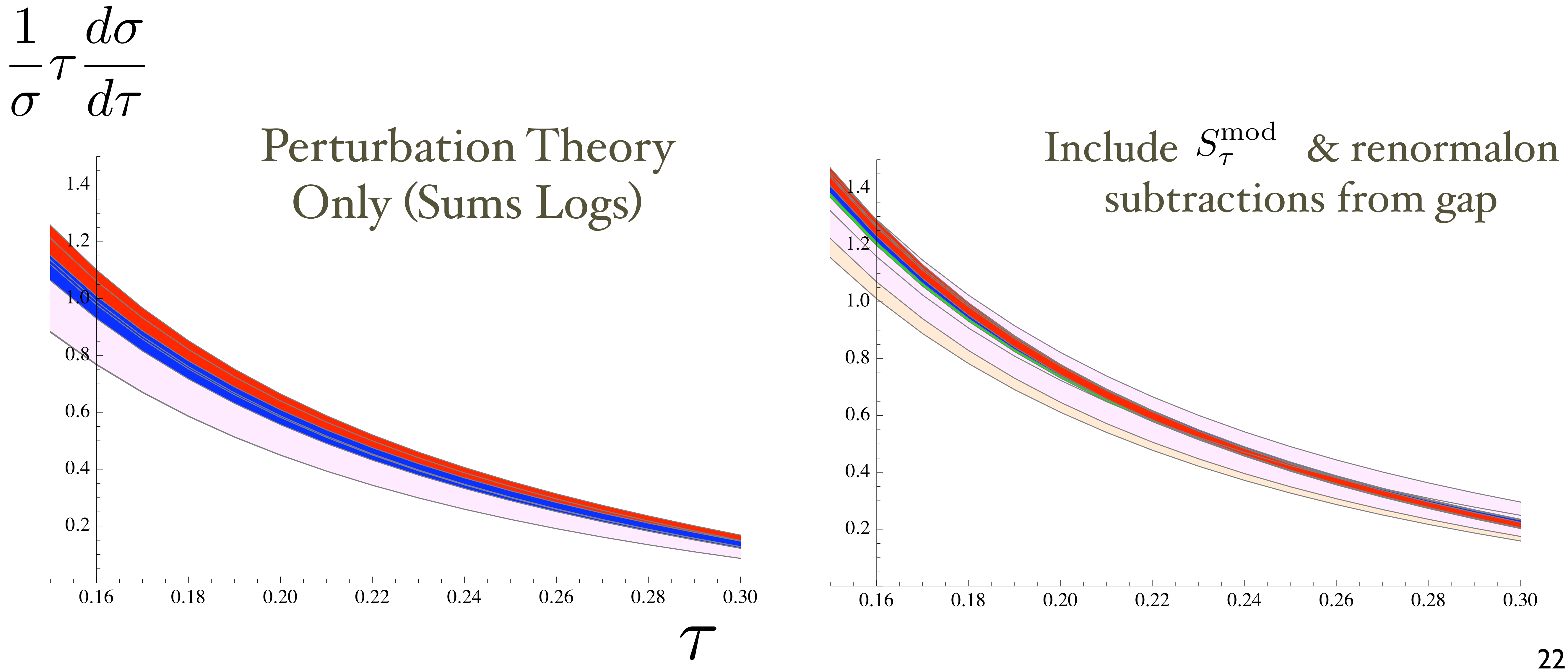
Perturbation Theory Only (Sums Logs)

Include $S_\tau^{\rm mod}$ & renormalon subtractions from gap



Tail Predictions with Scan over Theory Uncertainties

NLL' NNLL NNLL' $\mathrm{N^3LL}$ $\mathrm{N^3LL}'$



Perturbation Theory Only (Sums Logs)

Include S_τ^{mod} & renormalon subtractions from gap

Include b-mass effects in Factorization Thm: $(\sim 2\%\text{ effect})$

$$\frac{\Delta\, d\hat{\sigma}^b}{d\tau} = \frac{d\hat{\sigma}^{NNLL}_{\rm massive}}{d\tau} - \frac{d\hat{\sigma}^{NNLL}_{\rm massless}}{d\tau}$$

- at this order it effects only the jet function and τ limits
- use SCET massive fact. thm

Fleming, Hoang,Mantry,Stewart

- charm quarks are much smaller effect

Include QED effects in Factorization Thm: $(\sim 2\%\text{ effect})$

- count $\alpha \sim \alpha_s^2$, include only final state radiation
- include $\mathcal{O}(\alpha_s^2\,\alpha)$ corrections to QCD β-function
- include one-loop QED corrections to $H_Q,\, J_\tau,\, S_\tau$

Include axial anomaly contribution $(\sim 1\%\text{ effect})$

- affects H_Q^{ua}, H_Q^{da} at $\mathcal{O}(\alpha_s^2)$

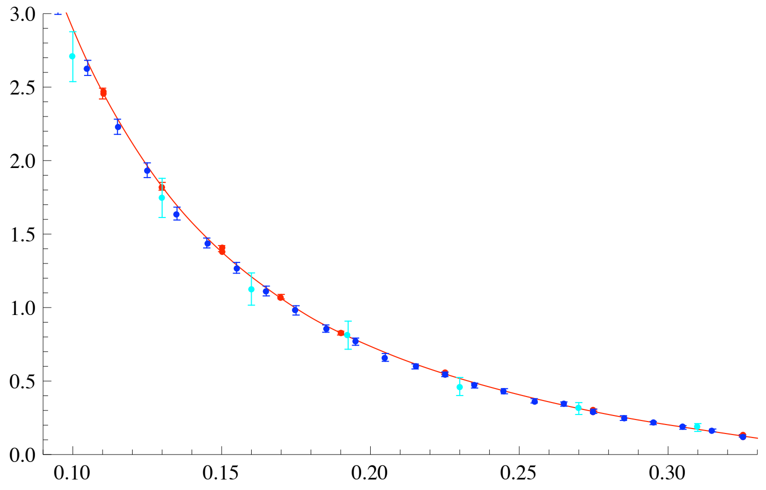
$$H_Q^a = H_Q^v + H^{\rm singlet} \qquad f^{da}(\tau,1) = f^v(\tau,1) + \frac{\alpha_s^2}{4\pi^2}\, f_{\rm singlet}\Big(\tau, \frac{Q^2}{4\, m_t^2}\Big)$$

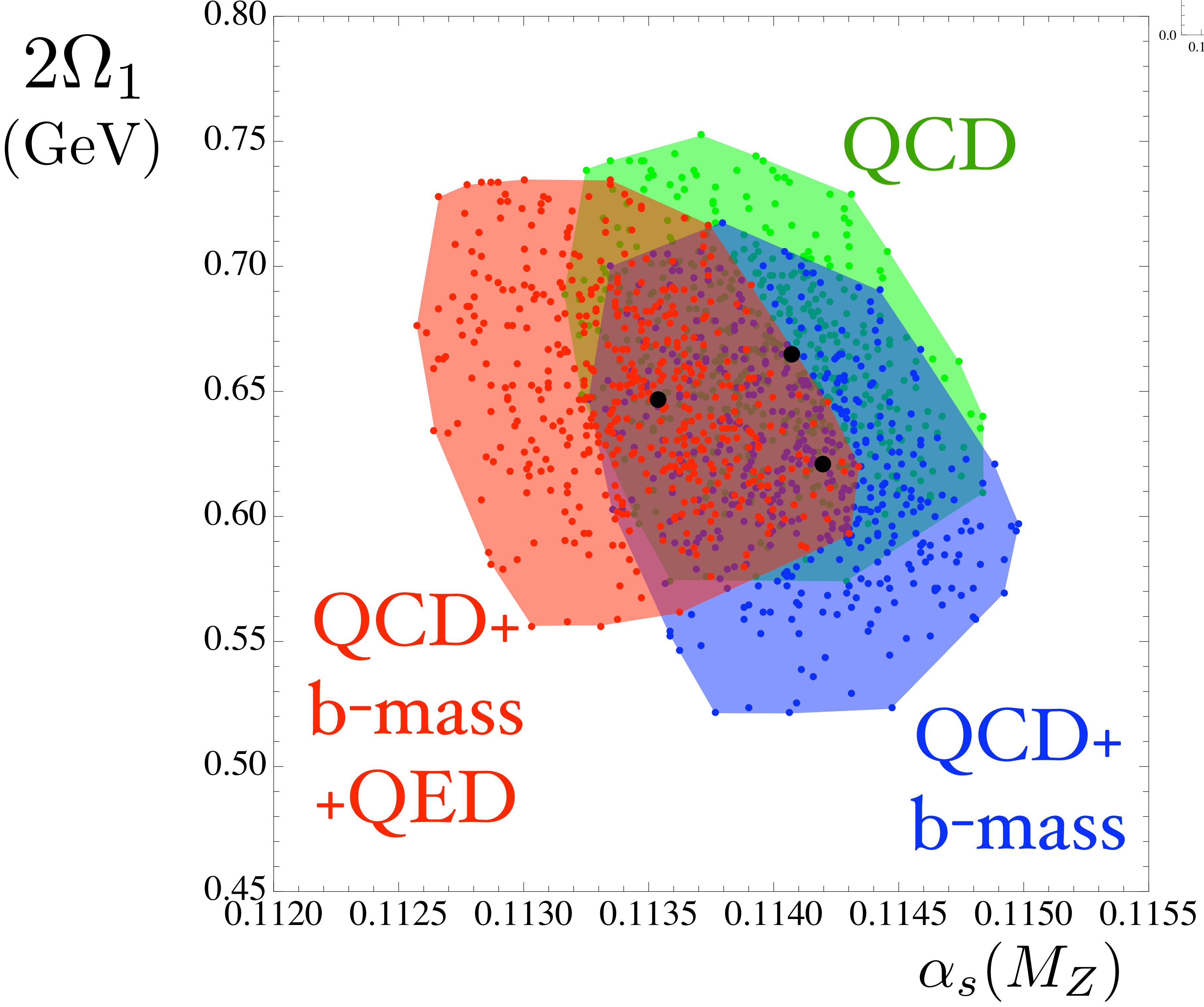
- due to large top-bottom mass splitting

Kniehl, Kuhn
Hagiwara, Kuruma, Yamada

Theory Scan Results

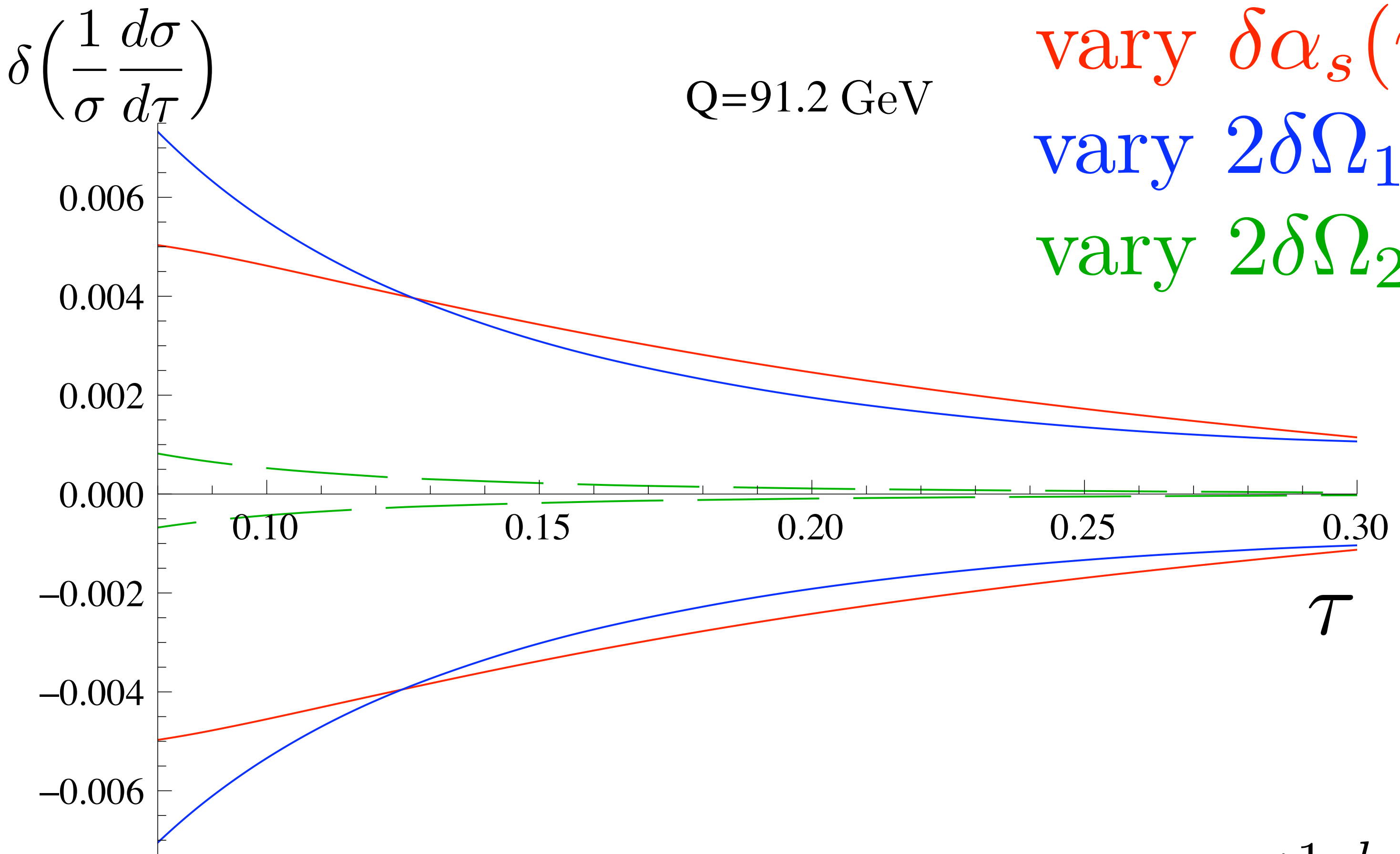
N^3LL' with/without mass & QED





Global Fit

Degeneracy: $\alpha_s(m_Z)$ versus Ω_1



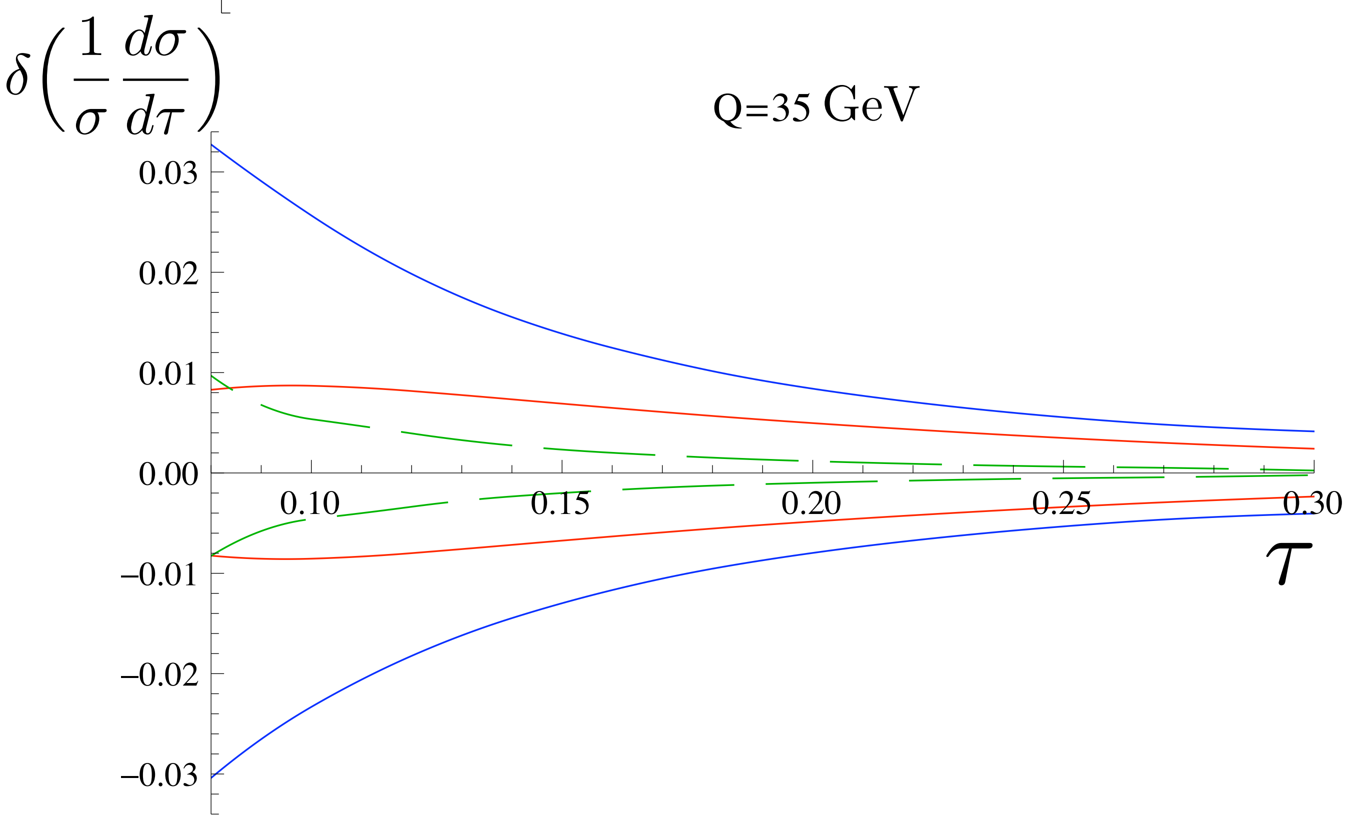
vary $\delta\alpha_s(m_Z) = \pm 0.001$

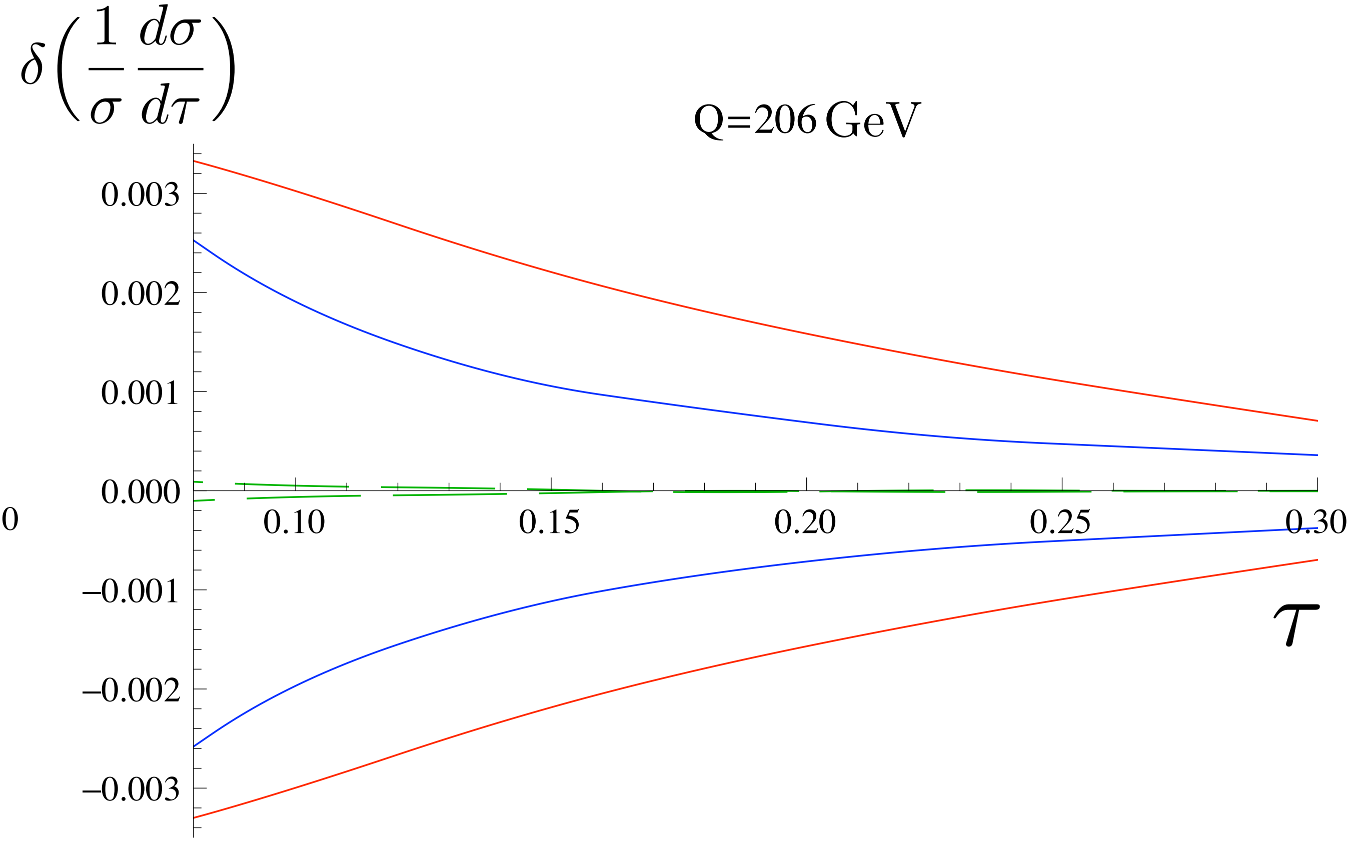
vary $2\delta\Omega_1 = \mp 0.1$ GeV

vary $2\delta\Omega_2$

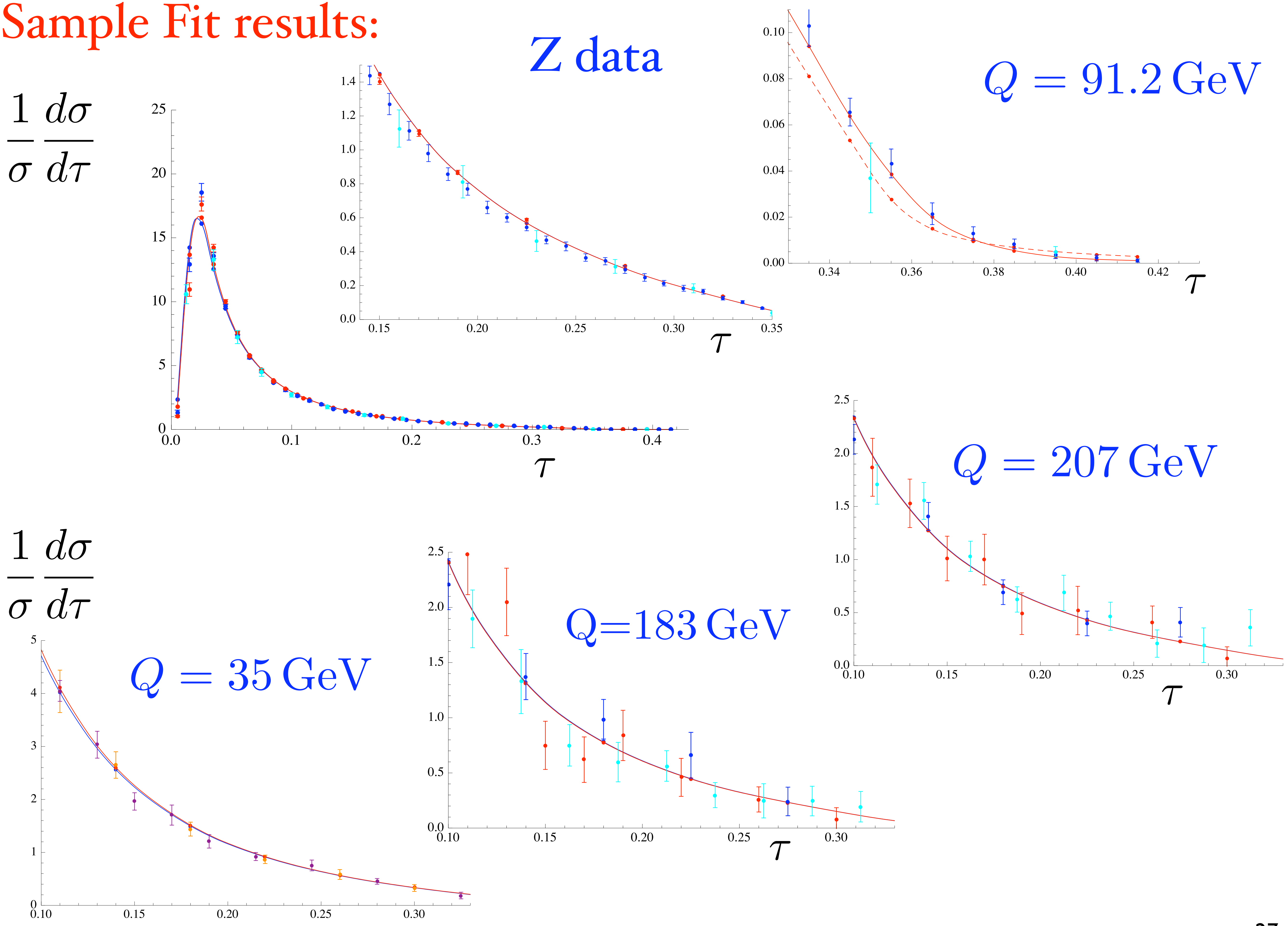
} degenerate for a single Q

Resolved by fitting multiple Q's





Sample Fit results:



Z data

$Q = 91.2\ \mathrm{GeV}$

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$$

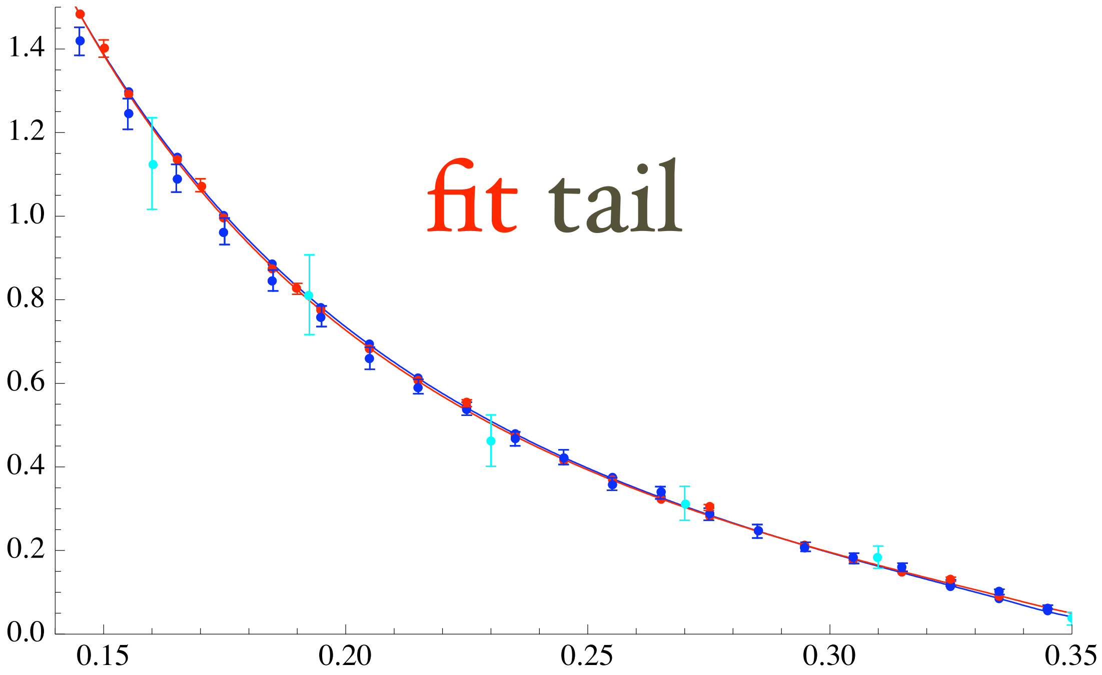
$Q = 207\ \mathrm{GeV}$

$Q\!=\!183\ \mathrm{GeV}$

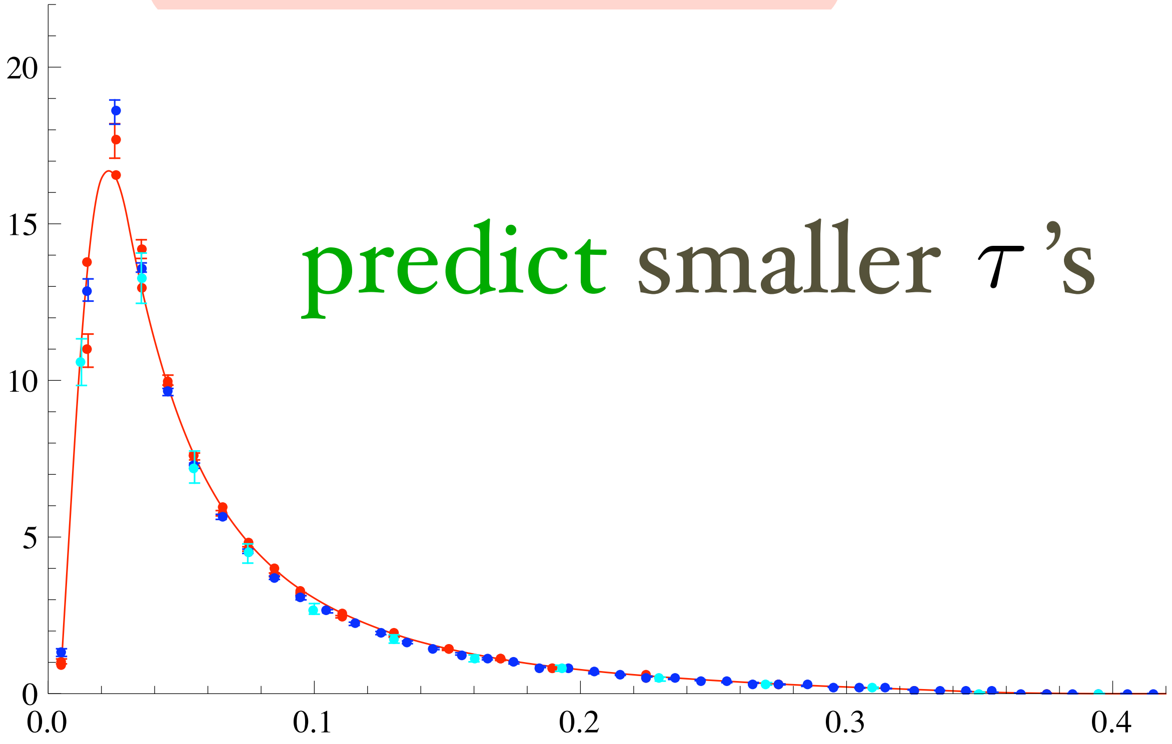
$Q = 35\ \mathrm{GeV}$

A Tail Fit

For τ in the tail region we can safely do a two parameter fit: $\{\alpha_s(m_Z), \Omega_1\}$

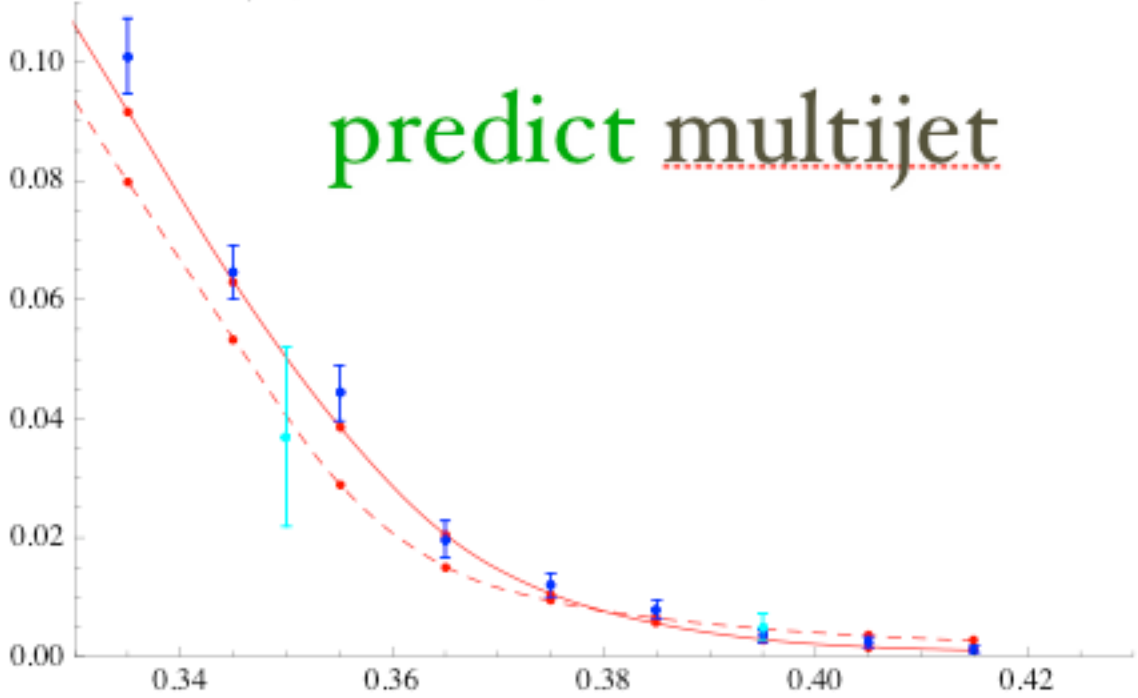


fit tail



predict smaller τ's

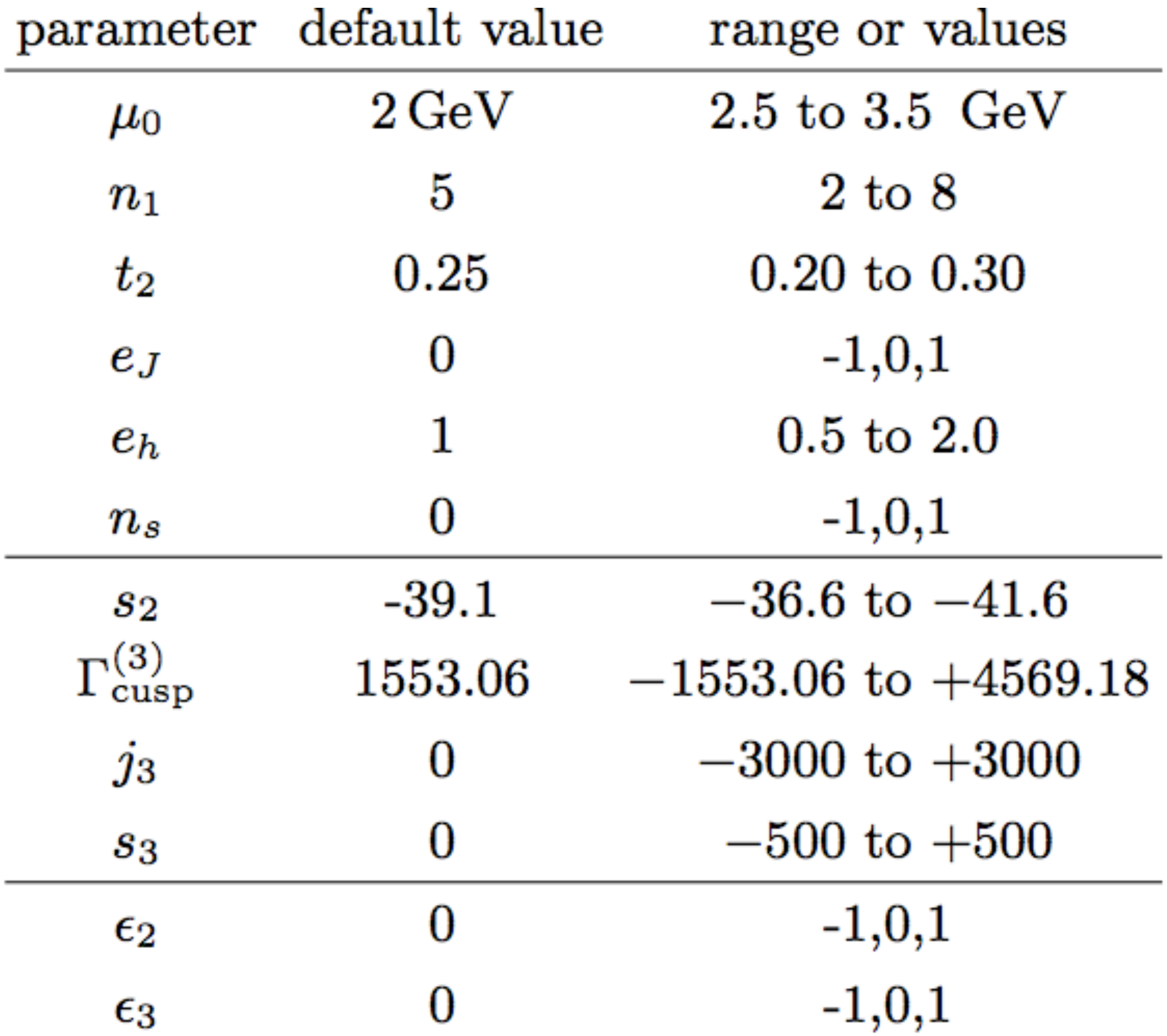
dσ/dt, Dashed=N3LL, Solid=N3LL', same fit coeffs.



predict multijet

Theory Uncertainty

We do a flat scan over unknown theory parameters, fitting each time and take the range of central values



parameter	default value	range or values
μ_0	2 GeV	2.5 to 3.5 GeV
n_1	5	2 to 8
t_2	0.25	0.20 to 0.30
e_J	0	-1,0,1
e_h	1	0.5 to 2.0
n_s	0	-1,0,1
s_2	-39.1	-36.6 to -41.6
$\Gamma^{(3)}_{\rm cusp}$	1553.06	-1553.06 to $+4569.18$
j_3	0	-3000 to $+3000$
s_3	0	-500 to $+500$
ϵ_2	0	-1,0,1
ϵ_3	0	-1,0,1

Profile Functions

$h_3 = 8998.05$

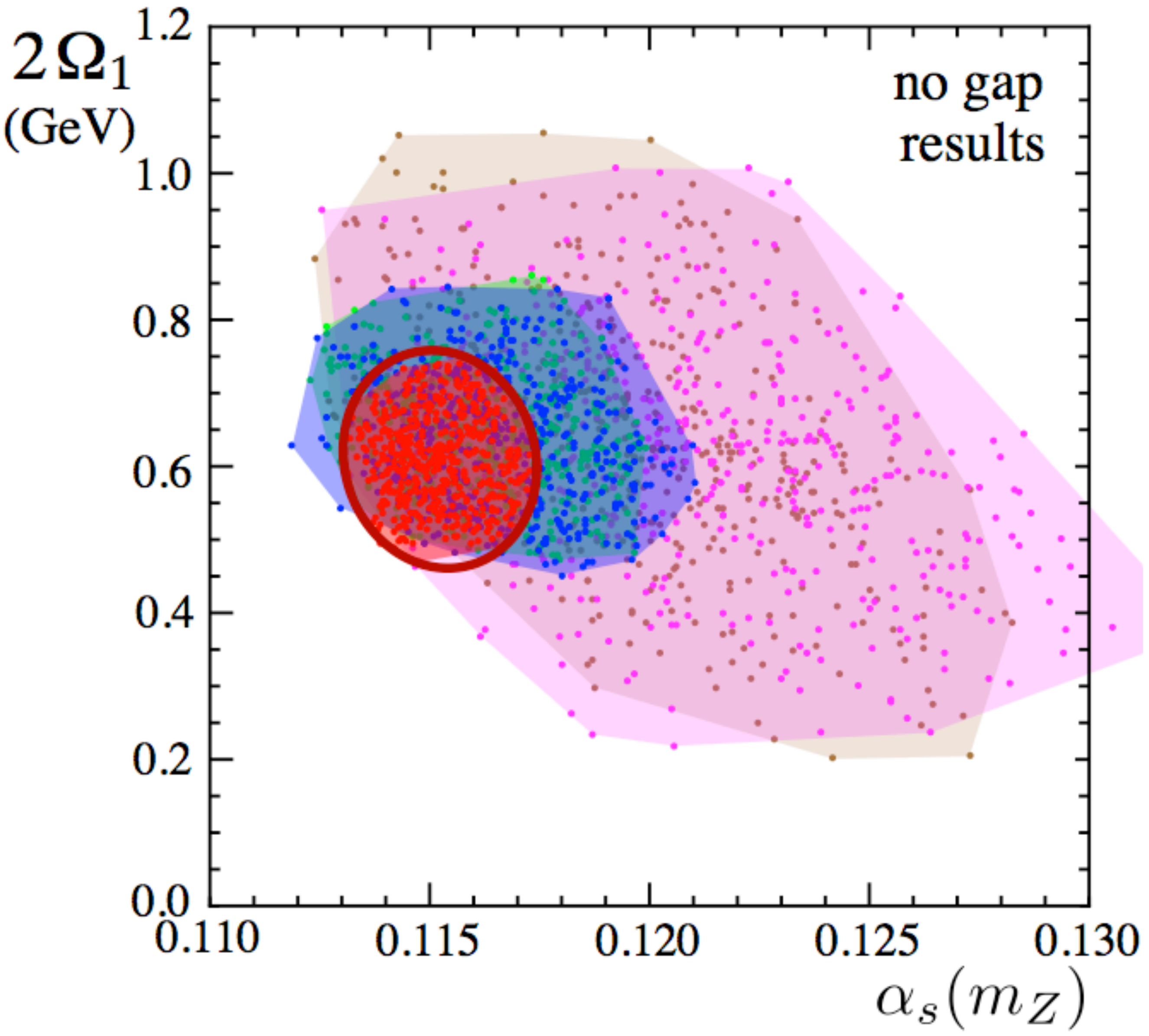
$\}$ Padè approximants

non singular stat. error

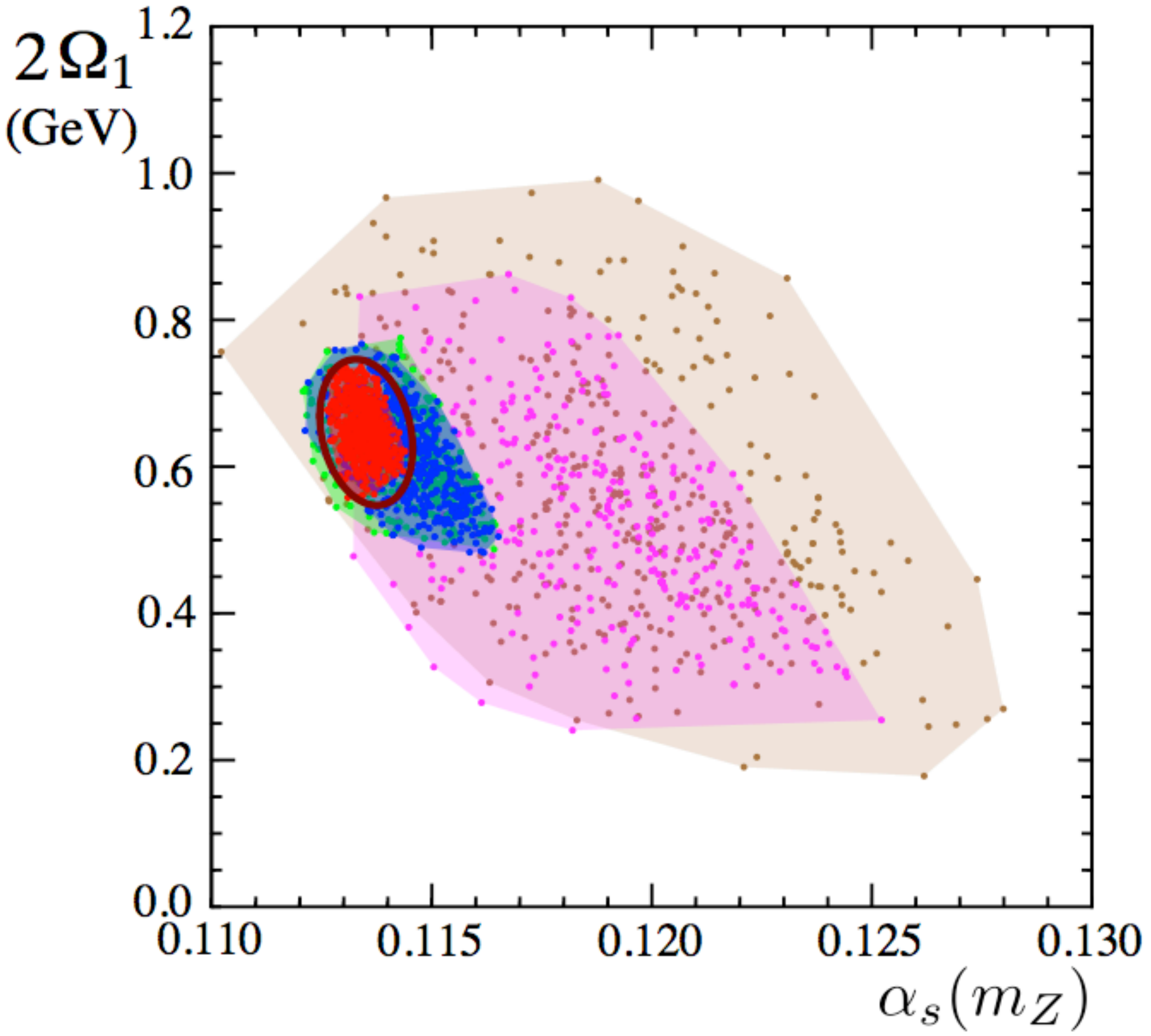
Theory Scan Results

NLL$'$, NNLL, NNLL$'$, N^{3}LL, N^{3}LL$'$

Perturbation Theory,
Sums Logs + add S_τ^{mod}



Include S_τ^{mod} & renormalon subtractions from gap

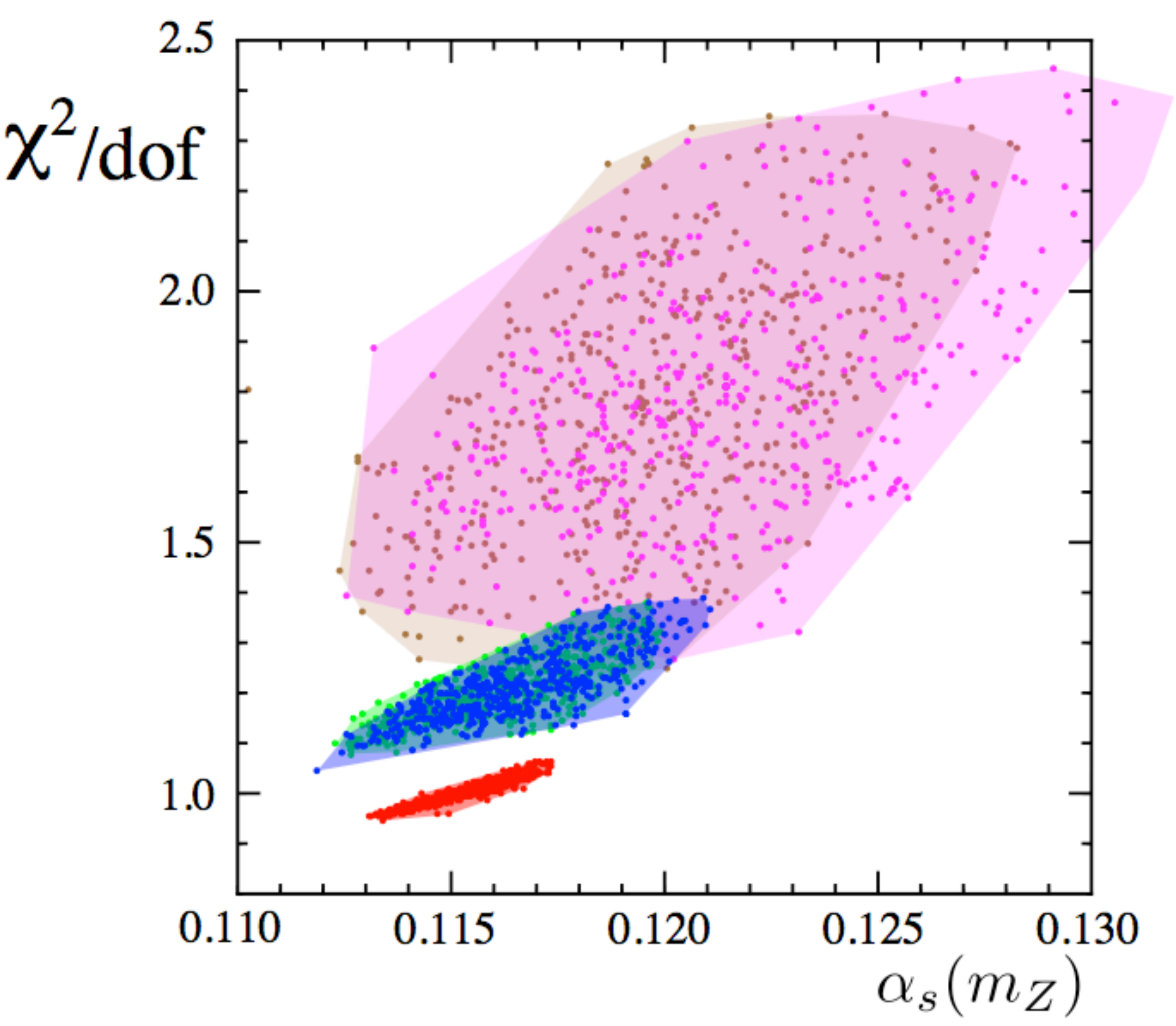


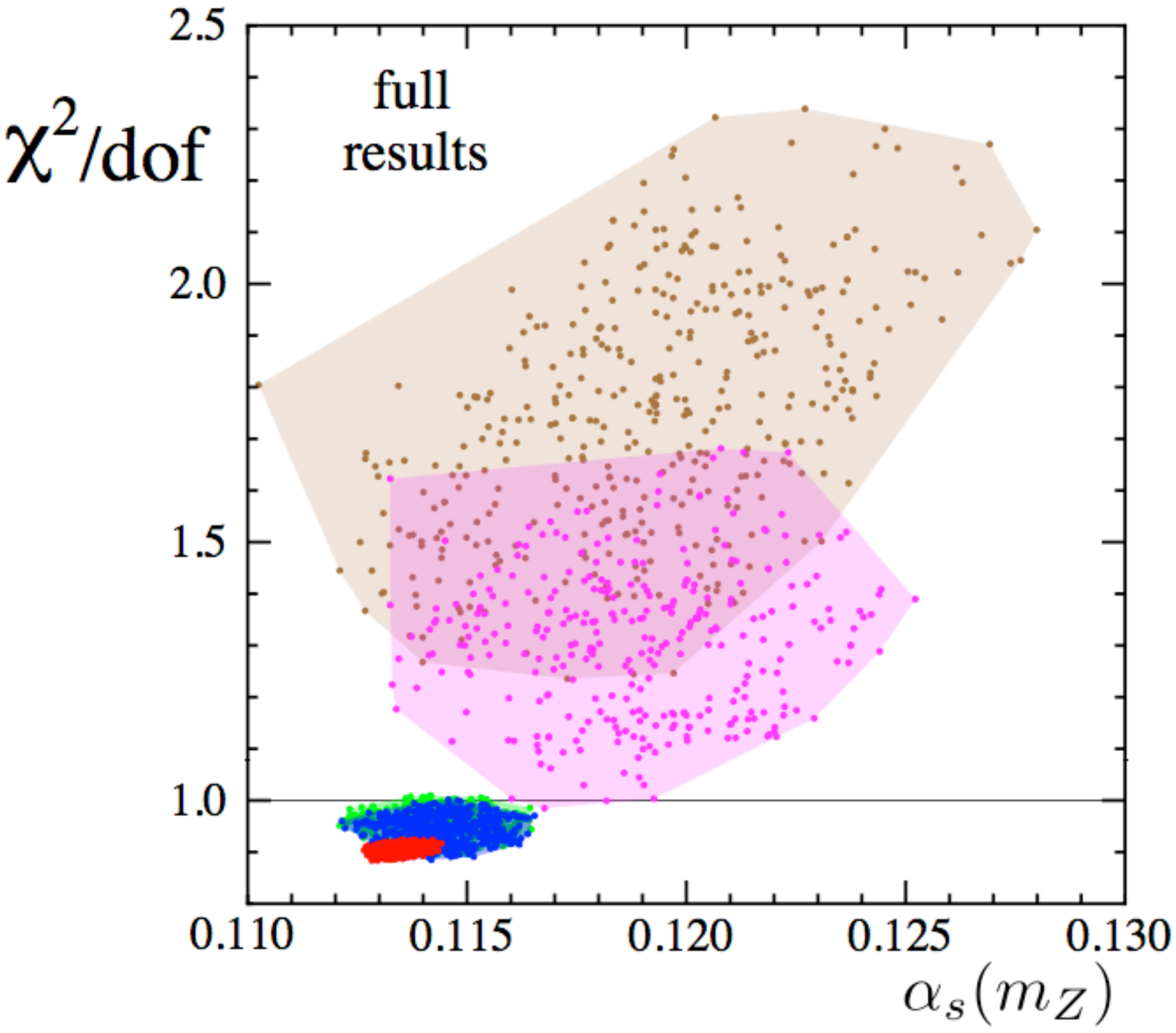
Theory Scan Results

NLL$'$, NNLL, NNLL$'$, N^{3}LL, N^{3}LL$'$

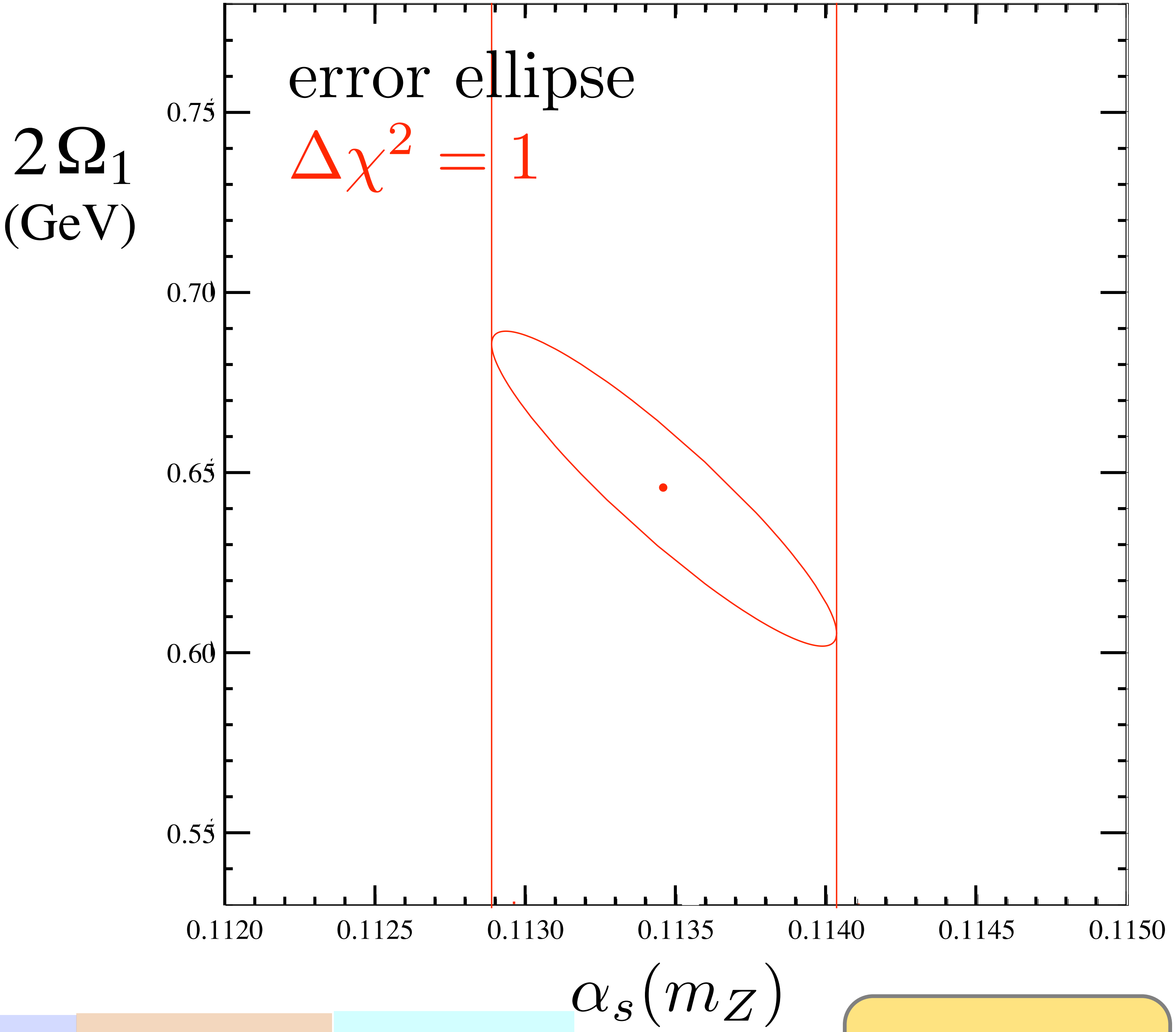
Perturbation Theory,
Sums Logs + add $S_\tau^{\rm mod}$

Include $S_\tau^{\rm mod}$ & renormalon
subtractions from gap





Tail Fit Result



we use LEP working group's corr. model for syst.errors:

$$\frac{\chi^2}{dof} = \frac{439.7}{487-2} = 0.907$$

$$\alpha_s(m_Z) = 0.1135 \pm 0.0002 \pm 0.0005 \pm 0.0009$$

expt. error

hadronization error

pert.error

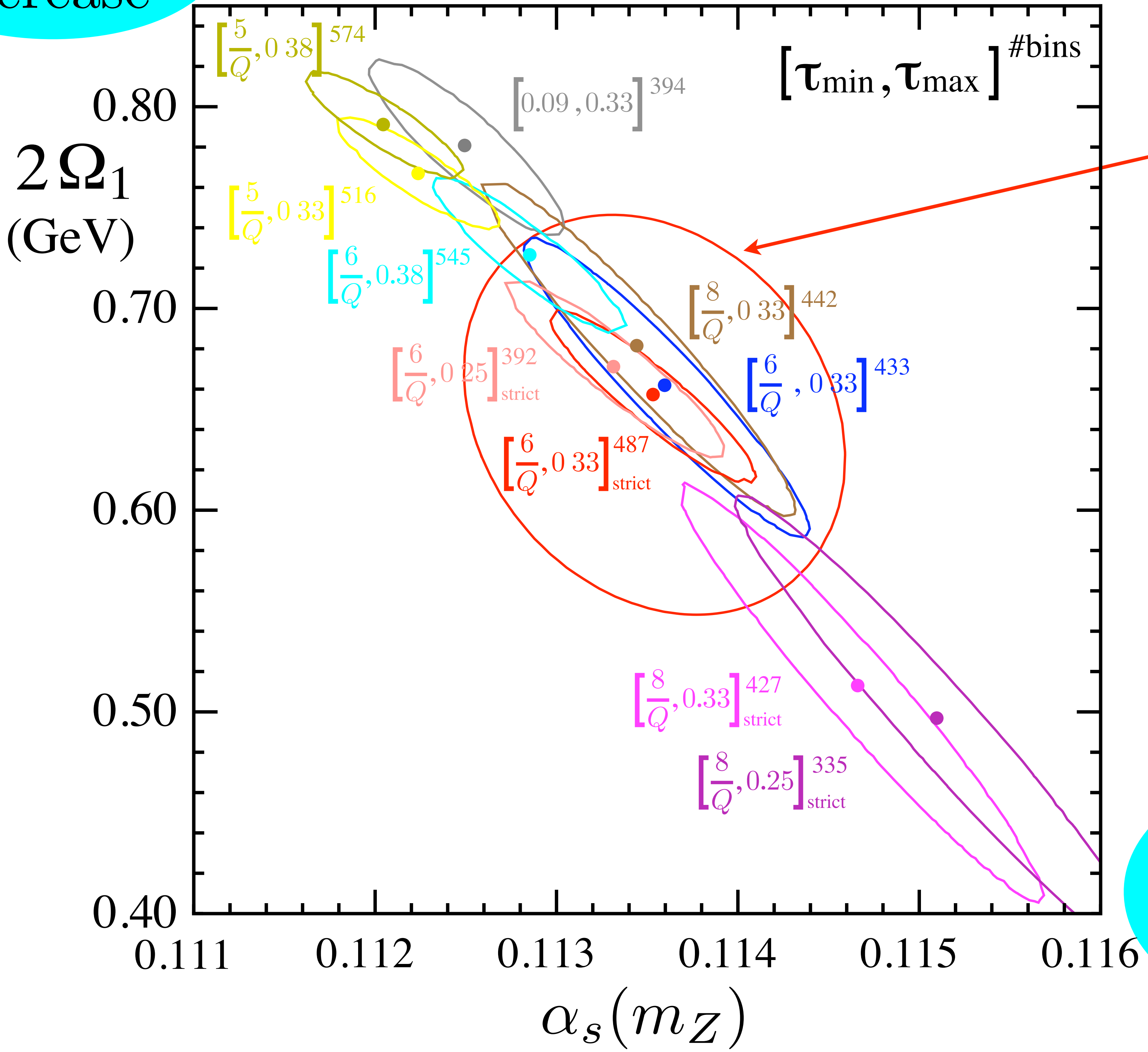
± 0.0003
Pert error
Band Method

comparison to Becher & Schwartz fit

$$\alpha_s(m_Z) = 0.1172 \pm 0.0010(stat) \pm 0.0008(sys) \pm 0.0012(had) \pm 0.0012(pert)$$

Two Parameter Fits for Different Cuts on Data

Ω_2 effects increase



All errors

all other ellipses do not include pert. error

statistical errors increase

Cross Checks and Comparisons

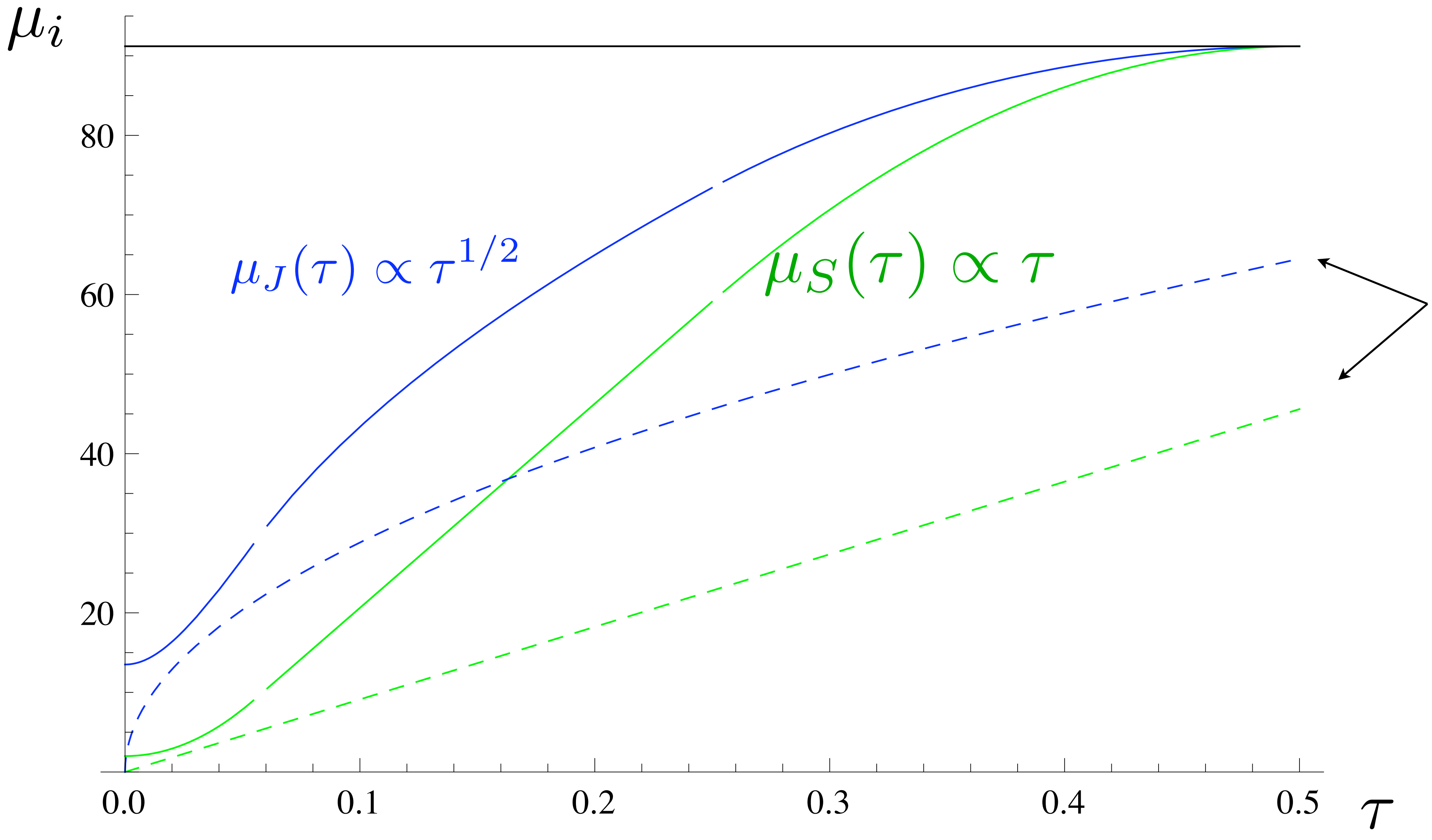
Comparison with Becher, Schwartz

- Turn off: power corrections & gap subtractions

  QED, b-mass, axial singlet

  systematic errors in χ^2
- set $h_3 = 0$

- Tune the profile functions to reproduce their scale dependence



BS profile

Comparison with Becher, Schwartz

Differences: treatment of nonsingular terms

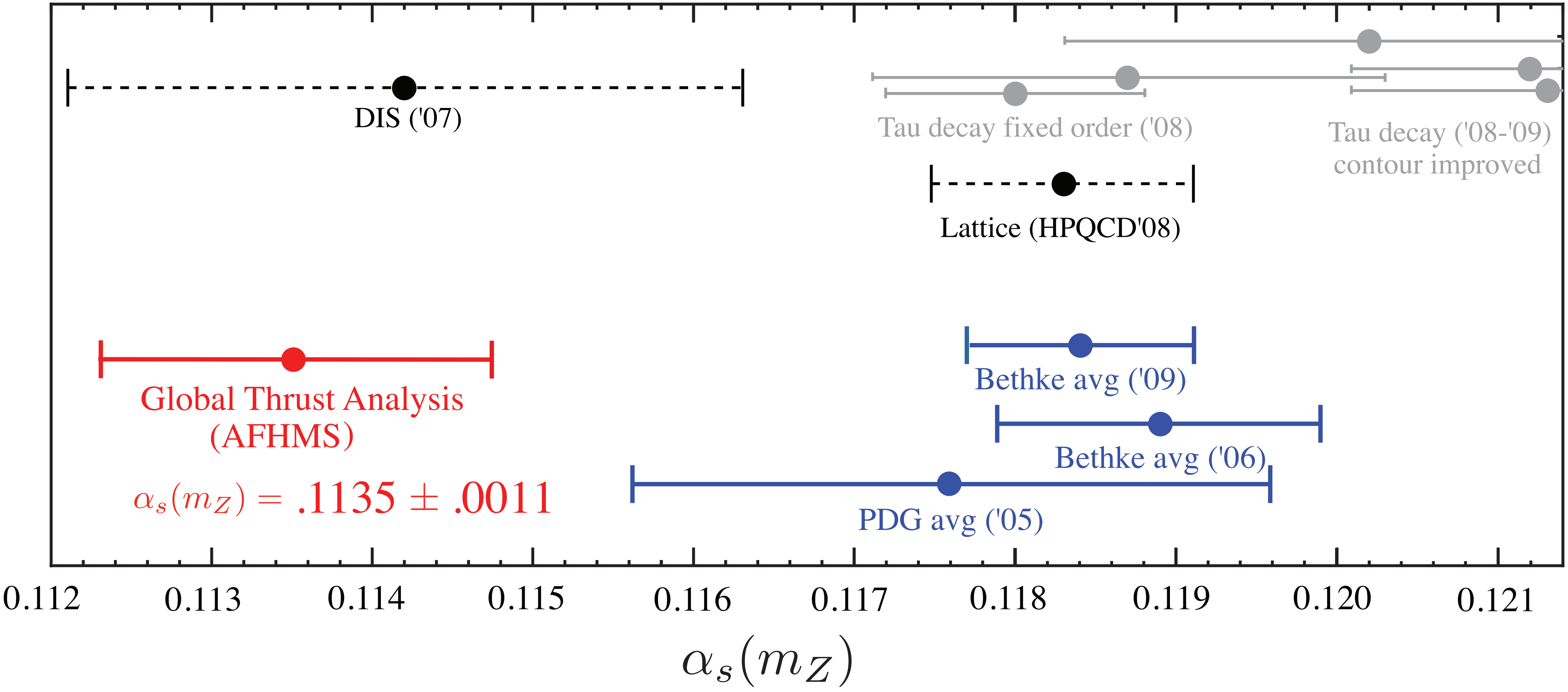
Our profile function leads to higher $\alpha_s(M_Z)$

Experiment	Energy	BS	Our reproduction of BS profile	Our default profile
ALEPH	91.2 GeV	0.1168(1)	0.1175	0.1226
ALEPH	133 GeV	0.1183(37)	0.1192	0.1239
ALEPH	161 GeV	0.1263(70)	0.1275	0.1332
ALEPH	172 GeV	0.1059(80)	0.1064	0.1091
ALEPH	183 GeV	0.1160(43)	0.1170	0.1208
ALEPH	189 GeV	0.1203(22)	0.1219	0.1264
ALEPH	200 GeV	0.1175(23)	0.1186	0.1227
ALEPH	206 GeV	0.1140(23)	0.1153	0.1188
OPAL	133 GeV	0.1165(38)	0.1180	0.1222

~ 0.001 →

~ 0.005 →

Compare to other Methods:



results from jets differ by 3.5σ from lattice results

Summary

- we have shown that in tail region, non perturbative effects are encoded in the first moment, Ω_1 , of the soft function

- treatment the of renormalon subtraction is a substantial part of high precision analyses of $\alpha_s(m_Z)$ and Ω_1

- use of actual value of h_3 decreased perturbative uncertainties

- we performed a global tail fit on all available thrust data obtaining

$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\rm exp} \pm (0.0005)_{\rm hadr} \pm (0.0009)_{\rm pert}$$

- as promised, we have provided a determination of $\alpha_s(m_Z)$ with an accuracy comparable with the lattice result!