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Subleading Jet Functions in Inclusive B Decays

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Outline

- Introduction
- Subleading Jet Functions in Inclusive B Decays
- Outlook and Conclusions

Introduction

Motivation

- Improve precision of Charmless Inclusive B decays
  - $\Gamma(\bar{B} \to X_s \gamma)$ is a probe of new physics
  - $\bar{B} \to X_u \, l \, \bar{\nu}$ is used for precision measurment of $|V_{ub}|$
- Test of SCET
  - EFT should allow us to calculate power corrections
  - New features that arise only at subleading power
- Relevant for other processes
  - Subleading jet functions

    = Discontinuity of time ordered products of collinear fields
  - Relevant for hard QCD processes e.g. $x \to 1$ DIS

Factorization at Leading Power

- Experimental cuts force charmless inclusive B decay spectra

  to the end point region:

  - $\bar{B} \to X_u \, l \, \bar{\nu}$: $P_X^2 < M_D^2$
  - $\bar{B} \to X_s \gamma$: $E_\gamma > E_{\rm cut}$

  where $P_X^2 \sim m_b \, \Lambda_{\rm QCD}$

- At the end point region, spectra of

  - $\bar{B} \to X_u \, l \, \bar{\nu}$
  - $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s \gamma$

  obey a **leading power** factorization formula

  (Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01)

$$\bar{B} \to X_s \gamma : \qquad d\Gamma_s^{77} \sim H_s \cdot J \otimes S + \ldots$$

$$\bar{B} \to X_u \, l \, \bar{\nu} : \qquad d\Gamma_u \sim H_u \cdot J \otimes S + \ldots$$

- S (leading) shape function, non-perturbative
- H_i and J calculable using PT in α_s

Factorization at Leading Power

- Progress in the perturbative calculation:

$$\bar{B} \to X_s \gamma : \qquad d\Gamma_s^{77} \sim H_s \cdot J \otimes S + ...$$
$$\bar{B} \to X_u \, l \, \bar{\nu} : \qquad d\Gamma_u \sim H_u \cdot J \otimes S + ...$$

- 2005: H_s calculated at $\mathcal{O}(\alpha_s^2)$

  (Melnikov, Mitov ’05, ...)

- 2006: J calculated at $\mathcal{O}(\alpha_s^2)$

  (Becher, Neubert ’06)

- 2008: H_u calculated at $\mathcal{O}(\alpha_s^2)$

  (Bonciani, Ferroglia ’08; Asatrian, Greub, Pecjak ’08;
  Beneke, Huber, Li ’08; Bell ’08)

- Current status: leading power $H \cdot J \otimes S$ at $\mathcal{O}(\alpha_s^2)$

Subleading Shape Functions

- 2004: Using SCET, study of **one** type of power corrections

  subleading shape functions (subleading "twist")

  for $X_u \, l \, \bar{\nu}$ and $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s \gamma$

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \ldots$$

  (K.S.M. Lee, Stewart '04; Bosch, Neubert, GP '04; Beneke, Campanario, Mannel, Pecjak '04)

  Supersedes earlier studies

- The subleading shape function s_i are non perturbative

  known at tree level: $\sum_i H \cdot J \otimes s_i$ at $\mathcal{O}(\alpha_s^0)$

Kinematical Power Corections

- In absence of proper factorization for α_s/m_b terms

  "kinematical corrections" are incorparated in phenomenological anlysis

- Kinematical = (Partonic - LO corrections) $\otimes$ LO SF

- Example: $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s\,\gamma$

$$\frac{d\Gamma}{dE_\gamma} = -\frac{G_F^2\alpha}{4\pi^4} E_\gamma^3 |V_{tb}V_{ts}^*|^2\, \overline{m}_b^2\, |C_{7\gamma}^{\rm eff}|^2\, W(P_+)$$

$$W^{\rm Kin.} = \frac{1}{m_b}\frac{C_F\,\alpha_s(\mu)}{4\pi}\int d\omega\,\theta(\omega{+}n{\cdot}p)\left[32\,\ln\frac{\omega+n\cdot p}{m_b}+30\right] S(\omega) + \mathcal{O}(\alpha_s^2)$$

Status at the end of 2008

- Status at the end of 2008

$$\bar{B} \to X_s \gamma : \qquad d\Gamma_s^{77} \sim H_s \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \ldots$$

$$\bar{B} \to X_u \, l \, \bar{\nu} : \qquad d\Gamma_u \sim H_u \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \ldots$$

Leading power $H \cdot J \otimes S$ at $\mathcal{O}(\alpha_s^2)$

Subleading power $\sum_i H \cdot J \otimes s_i$ at $\mathcal{O}(\alpha_s^0)$

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Leading power $H \cdot J \otimes S$ at $\mathcal{O}(\alpha_s^2)$

Subleading power $\sum_i H \cdot J \otimes s_i$ at $\mathcal{O}(\alpha_s^0)$

- Time to think about α_s/m_b corrections!

Factorization beyond leading power:
Subleading Jet Functions
in Inclusive B Decays

Preliminary Remarks: Beyond Leading Power

$$d\Gamma \sim H \cdot J \otimes S + ...$$

- Beyond leading power naively expect:

$h_i \cdot J \otimes S$	$H \cdot j_i \otimes S$	$H \cdot J \otimes s_i$
Subleading	Subleading	Subleading
hard functions	jet functions	shape functions

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Subleading hard functions	Subleading jet functions	Subleading shape functions

- Hard function **always** $\mathcal{O}(1)$ quantities (depend on m_b)

$\Rightarrow$ Only subleading jet and shape functions

- Confirmed at $\mathcal{O}(\alpha_s)$ by analysis of momentum regions

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- Confirmed at $\mathcal{O}(\alpha_s)$ by analysis of momentum regions
- Subleading shape functions

  $H \cdot J \otimes s_i$ with H, J at $\mathcal{O}(\alpha_s^0)$

  known since 2004
- What about subleading jet functions?

Preliminary Remarks: Subleading Jet Functions

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \frac{1}{m_b} \sum_i H \cdot j_i \otimes S + \ldots$$

- Subleading jet functions are perturbative, what do they look like?
- For end point region $p^2 \sim m_b \Lambda_{\rm QCD}$

  Leading jet function $\quad J(p^2) = \delta(p^2) + \mathcal{O}(\alpha_s) \quad \Rightarrow J \sim \dfrac{1}{\Lambda_{\rm QCD}}$

- Subleading jet functions $j_i \sim 1$

  subleading jet function $\quad j_i(p^2) = \alpha_s \left[\text{const.} + \ln\left(\dfrac{p^2}{\mu^2}\right)\right] + \mathcal{O}(\alpha_s^2)$

- Note $H \cdot j_i \otimes S$ is both α_s and $\Lambda_{\rm QCD}/m_b$ suppressed

Preliminary Remarks: α_s/m_b Corrections

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b}\sum_i H \cdot J \otimes s_i + \frac{1}{m_b}\sum_i H \cdot j_i \otimes S + \ldots$$

- Two types of α_s/m_b corrections
  - $H \cdot J \otimes s_i$ with H or J calculated at $\mathcal{O}(\alpha_s)$
  - $H \cdot j_i \otimes S$ with H at $\mathcal{O}(\alpha_s^0)$ and j_i at $\mathcal{O}(\alpha_s)$
- Experimental improvement $\Rightarrow$ increase p^2

  Example: Lower cut on $E_\gamma \Rightarrow$ increase $p^2 = m_b(m_b - 2E_\gamma)$
- $j_i(p^2)$ become less power suppressed

$$j_i(p^2) = \alpha_s \left[\text{const.} + \ln\left(\frac{\mu^2}{p^2}\right)\right] + \mathcal{O}(\alpha_s^2)$$

- s_i remain power suppressed
- In other words:
  - s_i are hadronically suppressed
  - j_i are kinematically suppressed

Power Suppressed Currents I

- Enough of generalities! Let's calculate.
- $H \cdot j_i \otimes S$ is already α_s/m_b suppressed, can take H at tree level

  Recall H is a product of Wilson coefficients in matching QCD $\rightarrow$ SCET

  Enough to match at tree level, but to second order in $\sqrt{\Lambda_{\rm QCD}/m_b}$
- Can use known results e.g.

  Beneke, Chapovsky, Diehl, Feldmann '02

Power Suppressed Currents I

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  Enough to match at tree level, but to second order in $\sqrt{\Lambda_{\rm QCD}/m_b}$
- Can use known results e.g.

  Beneke, Chapovsky, Diehl, Feldmann '02
- Recall matching QCD $\to$ SCET: $\quad \bar{s}\,\Gamma\, b \to C\,\bar{\mathcal{X}}\,\Gamma\,\mathcal{H}, \quad \mathcal{X} = W^\dagger \xi^{(0)}$
- To get power suppression can add power suppressed
  - Collinear fields $\mathcal{A}_\perp,\ i\mathcal{D}_\perp,\ n\cdot\mathcal{A}$
  - Soft fields and their covariant derivatives $S^\dagger D_\mu h$
  - or both
- The power suppressed currents are...

Power Suppressed Currents II

- Only collinear suppression

$$J^{(0)} = \bar{\mathfrak{X}}\,\Gamma\left(S^\dagger h\right)_{x_-},$$

$$J^{(1)} = -\bar{\mathfrak{X}}\,\frac{\bar{\not{n}}}{2}\,\not{\mathcal{A}}_{\perp hc}\,\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}\,\Gamma\left(S^\dagger h\right)_{x_-} - \bar{\mathfrak{X}}\,\Gamma\,\frac{\not{n}}{2m_b}\,\not{\mathcal{A}}_{\perp hc}\left(S^\dagger h\right)_{x_-},$$

$$J^{(2)} = -\bar{\mathfrak{X}}\,\Gamma\frac{\not{n}}{2m_b}\,n\cdot\mathcal{A}_{hc}\left(S^\dagger h\right)_{x_-} - \bar{\mathfrak{X}}\,\Gamma\,\frac{1}{i\bar{n}\cdot\partial}\,n\cdot\mathcal{A}_{hc}\left(S^\dagger h\right)_{x_-}$$

$$-\bar{\mathfrak{X}}\,\Gamma\,\frac{1}{i\bar{n}\cdot\partial}\,\frac{(i\not{\mathcal{D}}_{\perp hc}\,\not{\mathcal{A}}_{\perp hc})}{m_b}\left(S^\dagger h\right)_{x_-} + \bar{\mathfrak{X}}\,\frac{i\overleftarrow{\not{\mathcal{D}}}_{\perp hc}}{m_b}\,\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}\,\frac{\bar{\not{n}}}{2}\,\Gamma\,\frac{\not{n}}{2}\,\not{\mathcal{A}}_{\perp hc}\left(S^\dagger h\right)_{x_-}$$

Power Suppressed Currents II

- Only collinear suppression

$$J^{(0)} = \bar{\mathfrak{X}}\,\Gamma\left(S^\dagger h\right)_{x_-},$$

$$J^{(1)} = -\,\bar{\mathfrak{X}}\,\frac{\bar{n}\!\!\!/}{2}\,\mathcal{A}\!\!\!/_{\perp hc}\,\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}\,\Gamma\left(S^\dagger h\right)_{x_-} - \bar{\mathfrak{X}}\,\Gamma\,\frac{n\!\!\!/}{2m_b}\,\mathcal{A}\!\!\!/_{\perp hc}\left(S^\dagger h\right)_{x_-},$$

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$$-\,\bar{\mathfrak{X}}\,\Gamma\,\frac{1}{i\bar{n}\cdot\partial}\,\frac{(i\mathcal{D}\!\!\!/_{\perp hc}\,\mathcal{A}\!\!\!/_{\perp hc})}{m_b}\left(S^\dagger h\right)_{x_-} + \bar{\mathfrak{X}}\,\frac{i\overleftarrow{\mathcal{D}\!\!\!/}_{\perp hc}}{m_b}\,\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}\,\frac{\bar{n}\!\!\!/}{2}\,\Gamma\,\frac{n\!\!\!/}{2}\,\mathcal{A}\!\!\!/_{\perp hc}\left(S^\dagger h\right)_{x_-}$$

- Soft suppression or both

$$J^{(1)}_{\text{not used}} = \bar{\mathfrak{X}}\,\Gamma\,x_\perp^\mu\left(S^\dagger D_\mu h\right)_{x_-} + \bar{\mathfrak{X}}\,\frac{\bar{n}\!\!\!/}{2}\,i\overleftarrow{\partial\!\!\!/}_\perp\,\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}\,\Gamma\left(S^\dagger h\right)_{x_-},$$

$$J^{(2)}_{\text{not used}} = \bar{\mathfrak{X}}\,\Gamma\left[\frac{n\cdot x}{2}\left(S^\dagger\bar{n}\cdot Dh\right)_{x_-} + \frac{x_\perp^\mu x_\perp^\nu}{2}\left(S^\dagger D_\mu D_\nu h\right)_{x_-} + \left(S^\dagger\,\frac{iD\!\!\!/}{2m_b}\,h\right)_{x_-}\right]$$

$$+\;\bar{\mathfrak{X}}\,\frac{\bar{n}\!\!\!/}{2}\,i\overleftarrow{\partial\!\!\!/}_\perp\,\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}\,\Gamma\,x_\perp^\mu\left(S^\dagger D_\mu h\right)_{x_-}$$

$$-\;\bar{\mathfrak{X}}\left(\frac{\bar{n}\!\!\!/}{2}\,\mathcal{A}\!\!\!/_{\perp hc}\,\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}\,\Gamma + \Gamma\,\frac{n\!\!\!/}{2m_b}\,\mathcal{A}\!\!\!/_{\perp hc}\right)x_\perp^\mu\left(S^\dagger D_\mu h\right)_{x_-}$$

What We Need To Do...

- Soft suppressed currents (and Lagrangian) were used to calculate $H \cdot J \otimes s_i$
- To calculate $H \cdot j_i \otimes S$ use the collinear suppressed currents
- "All" we need to do is to combine the various currents

$$
\begin{aligned}
J^{(0)} &= \bar{\mathfrak{X}}\,\Gamma\left(S^\dagger h\right)_{x_-}\,,\\
J^{(1)} &= -\,\bar{\mathfrak{X}}\,\frac{\not{\bar n}}{2}\,\not{\mathcal{A}}_{\perp hc}\,\frac{1}{i\bar n\cdot\overleftarrow{\partial}}\,\Gamma\left(S^\dagger h\right)_{x_-} - \bar{\mathfrak{X}}\,\Gamma\,\frac{\not{n}}{2m_b}\,\not{\mathcal{A}}_{\perp hc}\left(S^\dagger h\right)_{x_-}\,,\\
J^{(2)} &= -\,\bar{\mathfrak{X}}\,\Gamma\frac{\not{n}}{2m_b}\,n\cdot\mathcal{A}_{hc}\left(S^\dagger h\right)_{x_-} - \bar{\mathfrak{X}}\,\Gamma\,\frac{1}{i\bar n\cdot\partial}\,n\cdot\mathcal{A}_{hc}\left(S^\dagger h\right)_{x_-}\\
&\quad -\,\bar{\mathfrak{X}}\,\Gamma\,\frac{1}{i\bar n\cdot\partial}\,\frac{(i\not{\mathcal{D}}_{\perp hc}\not{\mathcal{A}}_{\perp hc})}{m_b}\left(S^\dagger h\right)_{x_-} + \bar{\mathfrak{X}}\,\frac{i\overleftarrow{\not{\mathcal{D}}}_{\perp hc}}{m_b}\,\frac{1}{i\bar n\cdot\overleftarrow{\partial}}\,\frac{\not{\bar n}}{2}\,\Gamma\,\frac{\not{n}}{2}\,\not{\mathcal{A}}_{\perp hc}\left(S^\dagger h\right)_{x_-}
\end{aligned}
$$

$$
\text{in}\quad W_{ij} = \frac{1}{\pi}\,\frac{1}{2M_B}\,\mathrm{Im}\,\langle\bar B|\,i\int d^4x\,e^{iq\cdot x}\,T\left\{J_i^\dagger(0)\,J_j(x)\right\}|\bar B\rangle
$$

- Can keep Γ in $J = \bar q\,\Gamma\,b$ arbitrary to include both

  $\bar B \to X_u\,l\,\bar\nu$ and $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar B \to X_s\gamma$

Definition of SJF

- Recall

$$\frac{\not{\bar{n}}}{2}\,\delta_{kl}\,J(p^2) = \frac{1}{\pi}\,\mathrm{Disc}\,i\int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\bar{\mathcal{X}}_l(x)\right\}|\Omega\rangle$$

- For SJF have two candidates

$$\begin{aligned} I &= \int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\left[\bar{\mathcal{X}}\,n\cdot\mathcal{A}_{hc}\right]_l(x)\right\}|\Omega\rangle \\ II &= \int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\left[n\cdot\mathcal{A}_{hc}\,\mathcal{X}\right]_l(0),\,\bar{\mathcal{X}}_k(x)\right\}|\Omega\rangle, \end{aligned}$$

Which is the SJF? I, II, $I+II$? (note $I \neq II^*$)

Definition of SJF

- Recall

$$\frac{\not{n}}{2}\,\delta_{kl}\,J(p^2) = \frac{1}{\pi}\,\text{Disc}\,i\int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\bar{\mathcal{X}}_l(x)\right\}|\Omega\rangle$$

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Which is the SJF? I, II, $I+II$? (note $I \neq II^*$)

- Direct calculation at $\mathcal{O}(\alpha_s)$: $I = II$, one loop accident?

Definition of SJF and PT Symmetry I

- Recall

$$\frac{\not{\bar{n}}}{2}\,\delta_{kl}\,J(p^2) = \frac{1}{\pi}\,\text{Disc}\,i\int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\bar{\mathcal{X}}_l(x)\right\}|\Omega\rangle$$

- For SJF have two candidates

$$\begin{aligned} I &= \int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\left[\bar{\mathcal{X}}\,n\cdot\mathcal{A}_{hc}\right]_l(x)\right\}|\Omega\rangle \\ II &= \int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\left[n\cdot\mathcal{A}_{hc}\,\mathcal{X}\right]_l(0),\,\bar{\mathcal{X}}_k(x)\right\}|\Omega\rangle, \end{aligned}$$

  Which is the SJF? I, II, $I+II$? (note $I \neq II^*$)

- Direct calculation at $\mathcal{O}(\alpha_s)$: $I = II$, one loop accident?

- Using the PT symmetry of the strong interaction

  we can show $I = II$

$$\begin{aligned} \frac{\not{\bar{n}}}{2}\delta_{kl}\,j_n(p^2) &\overset{\text{def.}}{=} \frac{1}{\pi}\,\text{Disc}\,i\int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\left[\bar{\mathcal{X}}\,n\cdot\mathcal{A}_{hc}\right]_l(x)\right\}|\Omega\rangle \\ &\overset{PT}{=} \frac{1}{\pi}\,\text{Disc}\,i\int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\left[n\cdot\mathcal{A}_{hc}\,\mathcal{X}\right]_l(0),\,\bar{\mathcal{X}}_k(x)\right\}|\Omega\rangle, \end{aligned}$$

Definition of SJF and PT Symmetry II

- Similarly using PT we can show that

$$\int d^4x\, e^{-ip\cdot x} \langle\Omega|\, T\Big\{ \Big[\mathcal{A}^{\mu}_{\perp hc}\, \mathfrak{X}\Big]_k(0)\,,\Big[\bar{\mathfrak{X}}\,\mathcal{A}^{\nu}_{\perp\, hc}\Big]_l(x)\Big\}|\Omega\rangle =$$

$$\bar{n}\cdot p\,\frac{\not{n}}{2}\,\frac{g_{\perp}^{\mu\nu}}{d-2}\,\delta_{kl}\,\mathcal{J}^{S}_{11}(p^2) + \bar{n}\cdot p\,\frac{\not{n}}{2}\,\gamma_5\,\frac{i\epsilon_{\perp}^{\mu\nu}}{d-2}\,\delta_{kl}\,\mathcal{J}^{A}_{11}(p^2)$$

and the SJFs are

$$j^{S}_{11}(p^2) = \frac{1}{\pi}\mathrm{Im}\left[i\,\mathcal{J}^{S}_{11}(p^2)\right] \quad \text{and} \quad j^{A}_{11}(p^2) = \frac{1}{\pi}\mathrm{Im}\left[i\,\mathcal{J}^{A}_{11}(p^2)\right]$$

How Many SJF?

- "All" we need to do is to combine the various currents

$$
\begin{aligned}
J^{(0)} &= \bar{\mathfrak{X}}\,\Gamma\left(S^\dagger h\right)_{x_-}, \\
J^{(1)} &= -\,\bar{\mathfrak{X}}\,\frac{\not\bar{n}}{2}\,\not\mathcal{A}_{\perp hc}\,\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}\,\Gamma\left(S^\dagger h\right)_{x_-} - \bar{\mathfrak{X}}\,\Gamma\,\frac{\not n}{2m_b}\,\not\mathcal{A}_{\perp hc}\left(S^\dagger h\right)_{x_-}, \\
J^{(2)} &= -\,\bar{\mathfrak{X}}\,\Gamma\,\frac{\not n}{2m_b}\,n\cdot\mathcal{A}_{hc}\left(S^\dagger h\right)_{x_-} - \bar{\mathfrak{X}}\,\Gamma\,\frac{1}{i\bar{n}\cdot\partial}\,n\cdot\mathcal{A}_{hc}\left(S^\dagger h\right)_{x_-} \\
&\quad -\,\bar{\mathfrak{X}}\,\Gamma\,\frac{1}{i\bar{n}\cdot\partial}\,\frac{(i\not\mathcal{D}_{\perp hc}\not\mathcal{A}_{\perp hc})}{m_b}\left(S^\dagger h\right)_{x_-} + \bar{\mathfrak{X}}\,\frac{i\overleftarrow{\not\mathcal{D}}_{\perp hc}}{m_b}\,\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}\,\frac{\not\bar{n}}{2}\,\Gamma\,\frac{\not n}{2}\,\not\mathcal{A}_{\perp hc}\left(S^\dagger h\right)_{x_-}
\end{aligned}
$$

$$\text{in}\quad W_{ij} = \frac{1}{\pi}\,\frac{1}{2M_B}\,\mathrm{Im}\,\langle\bar{B}|i\int d^4x\,e^{iq\cdot x}\,T\left\{J_i^\dagger(0)\,J_j(x)\right\}|\bar{B}\rangle$$

- A priori many possible combinations, but
  - If inverse derivative acts on **all** coll. fields no new SJF
  - 3-body T.O.P $\Rightarrow$ 1 SJF
  - 4-body T.O.P $\Rightarrow$ 2 SJF
- $T\left\{J^{(1)}\,J^{(1)}\right\} \Rightarrow$ 2 SJF: j_{11}^S, j_{11}^A
- $T\left\{J^{(0)}\,J^{(2)}\right\} \Rightarrow$ 6 SJF: $j_n, j_{n'}, j_K, j_G, j_S, j_A$

$j_{n'}$

$$\frac{1}{\bar{n}\cdot p}\,\frac{\not{n}}{2}\delta_{kl}\,j_{n'}(p^2) = \frac{1}{\pi}\,\mathrm{Disc}\,i\int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\left[\bar{\mathcal{X}}\,\frac{1}{i\bar{n}\cdot\partial}\,n\cdot\mathcal{A}_{hc}\right]_l(x)\right\}|\Omega\rangle$$

$$= \frac{1}{\pi}\,\mathrm{Disc}\,i\int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\left[n\cdot\mathcal{A}_{hc}\,\frac{1}{-i\bar{n}\cdot\overleftarrow{\partial}}\,\mathcal{X}\right]_l(0),\,\bar{\mathcal{X}}_k(x)\right\}|\Omega\rangle$$

j_G and j_K

$$\frac{\not{n}}{2}\delta_{kl}\,\mathcal{J}_K(p^2) = \int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\left[\bar{\mathcal{X}}\,\frac{1}{i\bar{n}\cdot\partial}\,W^\dagger\,(iD_{\perp hc})^2\,W\right]_l(x)\right\}|\Omega\rangle$$

$$= \int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\left[W^\dagger\,\left(i\overleftarrow{D}_{\perp hc}\right)^2\,W\,\frac{1}{-i\bar{n}\cdot\overleftarrow{\partial}}\,\mathcal{X}\right]_l(0),\,\bar{\mathcal{X}}_k(x)\right\}|\Omega\rangle,$$

$$\frac{i\epsilon_\perp^{\mu\nu}}{d-2}\,\frac{\not{n}}{2}\gamma_5\,\delta_{kl}\,\mathcal{J}_G(p^2) = \int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\left[\bar{\mathcal{X}}\,\frac{1}{i\bar{n}\cdot\partial}\,W^\dagger\,G^{\mu\nu}\,W\right]_l(x)\right\}|\Omega\rangle$$

$$= \int d^4x\,e^{-ip\cdot x}\langle\Omega|\,T\left\{\left[W^\dagger\,G^{\mu\nu}\,W\,\frac{1}{-i\bar{n}\cdot\overleftarrow{\partial}}\,\mathcal{X}\right]_l(0),\,\bar{\mathcal{X}}_k(x)\right\}|\Omega\rangle,$$

$$j_K(p^2) = \frac{1}{\pi}\mathrm{Im}\left[i\,\mathcal{J}_K(p^2)\right] \quad \text{and} \quad j_G(p^2) = \frac{1}{\pi}\mathrm{Im}\left[i\,\mathcal{J}_G(p^2)\right].$$

j_A and j_S

$$\int d^4x\, e^{-ip\cdot x} \langle\Omega|\, T\Big\{\mathcal{X}_k(0), \Big[\bar{\mathcal{X}}\, i\overleftarrow{\mathcal{D}}^{\mu}_{\perp hc} \frac{1}{i\bar{n}\cdot\overleftarrow{\partial}} \mathcal{A}^{\nu}_{\perp\, hc}\Big]_l(x)\Big\}|\Omega\rangle =$$

$$\frac{\not{n}}{2}\frac{g_\perp^{\mu\nu}}{d-2}\,\delta_{kl}\,\mathcal{J}_S(p^2) + \frac{\not{n}}{2}\gamma_5\frac{i\epsilon_\perp^{\mu\nu}}{d-2}\,\delta_{kl}\,\mathcal{J}_A(p^2)$$

$$j_S(p^2) = \frac{1}{\pi}\text{Im}\left[i\,\mathcal{J}_S(p^2)\right] \quad \text{and} \quad j_A(p^2) = \frac{1}{\pi}\text{Im}\left[i\,\mathcal{J}_A(p^2)\right]$$

Final Results

- After renormalization we find

$$W_{ij}^{\text{SJF}} = -\int d\omega \quad \Bigg[\quad j_n(p_\omega^2,\mu)\frac{2\tilde{T}_2}{m_b} + j_{n'}(p_\omega^2,\mu)\frac{\tilde{T}_1}{\bar{n}\cdot p} + j_K(p_\omega^2,\mu)\frac{\tilde{T}_1}{m_b} + j_G(p_\omega^2,\mu)\frac{\tilde{T}_4}{m_b}$$

$$+ \quad j_{11}^S(p_\omega^2,\mu)\left(\frac{\bar{n}\cdot p}{m_b^2}\tilde{T}_2 + \frac{\tilde{T}_3}{\bar{n}\cdot p} - \frac{\tilde{T}_5}{m_b} - \frac{\tilde{T}_6}{m_b}\right)$$

$$+ \quad j_{11}^A(p_\omega^2,\mu)\left(\frac{\bar{n}\cdot p}{m_b^2}\tilde{T}_7 + \frac{\tilde{T}_3}{\bar{n}\cdot p} - \frac{\tilde{T}_5}{m_b} - \frac{\tilde{T}_6}{m_b}\right)$$

$$+ \quad j_S(p_\omega^2,\mu)\left(\frac{\tilde{T}_5}{m_b} - \frac{\tilde{T}_6}{m_b}\right) + j_A(p_\omega^2,\mu)\left(-\frac{\tilde{T}_5}{m_b} + \frac{\tilde{T}_6}{m_b}\right)\Bigg]S(\omega),$$

  where $\tilde{T}_i$ are traces

- The one loop expressions for the subleading jet functions

  $j_{11}^S, j_{11}^A, j_n, j_{n'}, j_K, j_G, j_S, j_A$ are

One Loop Expressions

$$
\begin{aligned}
j^S_{11}(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\big(-1\big)+\mathcal{O}(\alpha_s^2)\\
j^A_{11}(p^2,\mu) &= 0+\mathcal{O}(\alpha_s^2)\\
j_n(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\left(5+4\ln\frac{\mu^2}{p^2}\right)+\mathcal{O}(\alpha_s^2)\\
j_{n'}(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\left(-6+6\ln\frac{\mu^2}{p^2}\right)+\mathcal{O}(\alpha_s^2)\\
j_K(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\left(-\frac{5}{2}-2\ln\frac{\mu^2}{p^2}\right)+\mathcal{O}(\alpha_s^2)\\
j_G(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\big(-1\big)+\mathcal{O}(\alpha_s^2)\\
j_S(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\left(-\frac{3}{2}\right)+\mathcal{O}(\alpha_s^2)\\
j_A(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\left(\frac{1}{2}\right)+\mathcal{O}(\alpha_s^2).
\end{aligned}
$$

What is the SJF Contribution to the Rate?

- $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s\,\gamma$

$$\frac{d\Gamma}{dE_\gamma} = -\frac{G_F^2\alpha}{4\pi^4} E_\gamma^3 |V_{tb}V_{ts}^*|^2\, \overline{m}_b^2\, |C_{7\gamma}^{\rm eff}|^2\, W(P_+)$$

$$\begin{aligned} W^{\,\rm SJF} = \int d\omega \quad \Bigg[& \quad \frac{1}{m_b}\Big(4 j_K(p_\omega^2,\mu) + 4 j_n(p_\omega^2,\mu) + 2 j_G(p_\omega^2,\mu)\Big) \\ + & \quad \frac{4}{\bar{n}\cdot p}\, j_{n'}(p_\omega^2,\mu) \\ + & \quad \frac{2\bar{n}\cdot p}{m_b^2}\Big(j_{11}^S(p_\omega^2,\mu) - j_{11}^A(p_\omega^2,\mu)\Big)\Bigg]\, S(\omega) \end{aligned}$$

- At the lowest order in α_s

$$W^{\,\rm SJF} = \int d\omega\, \frac{1}{m_b}\, \frac{C_F\alpha_s(\mu)}{4\pi}\, \theta(p_\omega^2) \left[32\, \ln\frac{\mu^2}{p_\omega^2} - 18\right] S(\omega) + \mathcal{O}(\alpha_s^2)$$

where

$$p_\omega^2 = \bar{n}\cdot p\,(n\cdot p + \omega)$$

What is the SJF Contribution to the Rate?

- $\bar{B} \to X_u \, l \, \bar{\nu}$

$$\frac{d^3\Gamma}{dP_+ \, dP_- \, dP_l} = \frac{G_F^2 |V_{ub}|^2}{16\pi^3} (M_B - P_+) \Big[(P_- - P_l)(M_B - P_- + P_l - P_+) \, \tilde{W}_1 + (M_B - P_-)(P_- - P_+) \, \frac{\tilde{W}_2}{2} + (P_- - P_l)(P_l - P_+) \tilde{W}_{\rm comb} \Big]$$

$$\tilde{W}_1^{\rm SJF} = -\int d\omega \left[\frac{1}{\bar{n}\cdot p} \left(2 j_{n'}(p_\omega^2, \mu) + j_{11}^S(p_\omega^2, \mu) + j_{11}^A(p_\omega^2, \mu) \right) + \frac{1}{m_b} \left(2 j_K(p_\omega^2, \mu) + j_G(p_\omega^2, \mu) \right) \right] S(\omega)$$

$$\tilde{W}_2^{\rm SJF} = -\int d\omega \, \frac{2}{\bar{n}\cdot p} \left[j_{11}^S(p_\omega^2, \mu) + j_{11}^A(p_\omega^2, \mu) \right] S(\omega)$$

$$\tilde{W}_{\rm comb}^{\rm SJF} = -\int d\omega \left[\left(\frac{4}{m_b} - \frac{2}{\bar{n}\cdot p} \right) \left(j_{11}^S(p_\omega^2, \mu) + j_{11}^A(p_\omega^2, \mu) \right) + \frac{2}{\bar{n}\cdot p} \left(j_n(p_\omega^2, \mu) - j_G(p_\omega^2, \mu) \right) \right] S(\omega)$$

What is the SJF Contribution to the Rate?

- At the lowest order in α_s

$$\tilde{W}_1^{\rm SJF} = -\int d\omega \frac{C_F\alpha_s(\mu)}{4\pi}\,\theta(p_\omega^2)\left[\frac{1}{\bar{n}\cdot p}\left(12\ln\frac{\mu^2}{p_\omega^2}-11\right)-\frac{1}{m_b}\left(4\ln\frac{\mu^2}{p_\omega^2}+6\right)\right]S(\omega)$$

$$\tilde{W}_2^{\rm SJF} = \int d\omega \frac{C_F\alpha_s(\mu)}{4\pi}\,\theta(p_\omega^2)\,\frac{2}{\bar{n}\cdot p}\,S(\omega)$$

$$\tilde{W}_{\rm comb}^{\rm SJF} = -\int d\omega \frac{C_F\alpha_s(\mu)}{4\pi}\,\theta(p_\omega^2)\left[\frac{1}{\bar{n}\cdot p}\left(8\ln\frac{\mu^2}{p_\omega^2}+14\right)-\frac{4}{m_b}\right]S(\omega)$$

- If using “BLNP” approach where

$$y=\frac{P_- - P_+}{M_B - P_+}$$

Some of the subleading terms are absorbed into the leading order formula.

- Only $\tilde{W}_1$ changes

$$\tilde{W}_{1,\,\rm BLNP}^{\rm SJF} = -\int d\omega \frac{C_F\alpha_s(\mu)}{4\pi}\,\theta(p_\omega^2)\left[\frac{1}{\bar{n}\cdot p}\left(12\ln\frac{\mu^2}{p_\omega^2}-15\right)-\frac{1}{m_b}\left(4\ln\frac{\mu^2}{p_\omega^2}+2\right)\right]S(\omega)$$

How is the new analysis related to the old one?

- Demonstrate using $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s\, \gamma$

  Similar results for $\bar{B} \to X_u\, l\, \bar{\nu}$

- Two terms at subleading power:
  - "Hadronic"

$$W^{\,\mathrm{SSF}} = \frac{2}{m_b} \int d\omega\, \delta(n\cdot p + \omega) \Big[\omega\, S(\omega) - s(\omega) + t(\omega) - u(\omega) + v(\omega) + \pi\alpha_s\, f_u(\omega) + \pi\alpha_s\, f_v(\omega)\Big] + \mathcal{O}(\alpha_s)$$

  - "Kinematic"

$$W^{\mathrm{Kin.}} = \frac{1}{m_b}\frac{C_F\,\alpha_s(\mu)}{4\pi} \int d\omega\, \theta(\omega{+}n{\cdot}p) \left[32\, \ln \frac{\omega + n\cdot p}{m_b} + 30\right] S(\omega) {+} \mathcal{O}(\alpha_s^2)$$

- Subleading shape function contribution

$$W^{\,\mathrm{SJF}} = \frac{1}{m_b}\frac{C_F \alpha_s(\mu)}{4\pi} \int d\omega\ \theta(\omega{+}n{\cdot}p) \left[32\, \ln \frac{\mu^2}{m_b(\omega + n\cdot p)} - 18\right] S(\omega) {+} \mathcal{O}(\alpha_s^2)$$

- SJF contribution looks similar to "kinematic"

  but argument of log and constant are different...

How is the puzzle resolved?

- "Kinematic" correction is derived from incorrect matching

$$W^{\text{Kin.}} = \frac{1}{m_b}\frac{C_F\,\alpha_s(\mu)}{4\pi}\int d\omega\,\theta(\omega{+}n{\cdot}p)\left[32\,\ln\frac{\omega+n\cdot p}{m_b}+30\right]S(\omega){+}\mathcal{O}(\alpha_s^2)$$

- The correct splitting of log and constant (Conjecture by Neubert '07)

$$32\ln\frac{\omega+n\cdot p}{m_b}+30 \quad = \quad 32\,\ln\frac{\mu^2}{m_b(\omega+n\cdot p)}-18 \quad + \quad 32\,\ln\frac{(\omega+n\cdot p)^2}{\mu^2}+48$$

$\downarrow$ Subleading Jet Functions

$\downarrow$ Subleading Shape Functions

- The second part is **already** included in the SSF contribution
- Confirmed twice
  - Analysis of regions
  - 1-loop parton expressions for SSF + SJF analysis
- Including both "Kinematic" and "Hadronic" leads to **double counting**

What is the Bottom Line?

- The correct subleading power term is the sum of

$$W^{\,\mathrm{SSF}} = \frac{2}{m_b}\int d\omega\,\delta(n\cdot p+\omega)\Big[\omega\,S(\omega,\mu) - s(\omega,\mu) + t(\omega,\mu) - u(\omega,\mu) + v(\omega,\mu)$$
$$+ \quad \pi\alpha_s\, f_u(\omega,\mu) + \pi\alpha_s\, f_v(\omega,\mu)\Big] + \mathcal{O}(\alpha_s)$$

and

$$W^{\,\mathrm{SJF}} = \frac{1}{m_b}\,\frac{C_F\alpha_s(\mu)}{4\pi}\int d\omega\;\theta(\omega{+}n{\cdot}p)\left[32\,\ln\frac{\mu^2}{m_b(\omega+n\cdot p)} - 18\right] S(\omega){+}\mathcal{O}(\alpha_s^2)$$

and **not** as is currently done

- Need to modify treatment and modeling of SSF

  to account for their non zero one loop contribution

  Only ωS and u need to be modified

- New analysis not implemented yet

- Although α_s and $1/m_b$ suppressed, effect can be non-negligible

  e.g. constant change from $+30$ to -18

Note on Renormalization

- Some SJF need renormalization at $\mathcal{O}(\alpha_s)$

$$
\begin{aligned}
j_n^{\rm bare}(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s}{4\pi}\cdot 4\left(\frac{1}{\epsilon}+\frac{5}{4}+\ln\frac{\mu^2}{p^2}\right)+\mathcal{O}(\alpha_s^2)\\
j_{n'}^{\rm bare}(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s}{4\pi}\cdot 6\left(\frac{1}{\epsilon}-1+\ln\frac{\mu^2}{p^2}\right)+\mathcal{O}(\alpha_s^2)\\
j_K^{\rm bare}(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s}{4\pi}\cdot(-2)\left(\frac{1}{\epsilon}+\frac{5}{4}+\ln\frac{\mu^2}{p^2}\right)+\mathcal{O}(\alpha_s^2).
\end{aligned}
$$

- At one loop can define a new SJF to renormalize "against"

$$
j_0(p^2)=\theta(p^2)+\mathcal{O}(\alpha_s)\stackrel{?}{=}\int^{p^2}dp'^{\,2}\,J(p'^{\,2},\mu)
$$

- More generally, expect basis to contain multi local SJF

  Example: instead of bi-local

$$
\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\left[\bar{\mathcal{X}}\,n\cdot\mathcal{A}_{hc}\right]_l(x)\right\}|\Omega\rangle
$$

  need multi local

$$
\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,n\cdot\mathcal{A}^{hc}{}_{mn}(y),\,\bar{\mathcal{X}}_l(x)\right\}|\Omega\rangle
$$

  finite in general but divergent for $y\to x$ or $y\to 0$

Note on Renormalization

- Ren. of leading order bi-local operators is well known

$$\phi(\xi) \quad \text{F.T.} \quad \text{of} \quad \langle P|\bar{\psi}(0)\, \not{n}\, \psi(tn)|P\rangle$$

$$S(\omega) \quad \text{F.T.} \quad \text{of} \quad \langle \bar{B}|h(0)\, h(tn)|\bar{B}\rangle$$

$$J(p^2) \quad \text{F.T.} \quad \text{of} \quad \langle \Omega|\, T\left\{\mathcal{X}_k(0),\, \bar{\mathcal{X}}_l(x)\right\}|\Omega\rangle$$

- Ren. of (2^{nd} order in SCET) multi-local operators
  - Subleading jet functions (GP '09)
  - Subleading shape functions (Trott, Williamson '05)
  - Twist 4 operators

  still an open problem

- But, Ren. of (1^{st} order in SCET) multi-local operators is known
  - Twist 3 chiral odd (Belitsky, Müller '97)
  - Heavy-to-light form factors (Hill, Becher, Lee, Neubert '04)

Outlook and Conclusions

$\bar{B} \to X_s\gamma$ beyond $Q_{7\gamma} - Q_{7\gamma}$

- So far considered only $Q_{7\gamma}$ contribution to $\bar{B} \to X_s\gamma$

  But Q_1 and Q_{8g} contribution also important

$$\begin{aligned} Q_{7\gamma} &= \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu} F^{\mu\nu}(1+\gamma_5)b \\ Q_{8g} &= \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu} G^{\mu\nu}(1+\gamma_5)b \\ Q_1^q &= (\bar{q}b)_{V-A}(\bar{s}q)_{V-A} \quad (q=u,c) \end{aligned}$$

- Including Q_1 and Q_{8g} changes factorization **dramatically**

  beyond LO in $1/m_b$: See Michael Benzke's talk

- In particular introduces more SJF
  - $Q_1 - Q_1$: known SJF

$$j_{11}^s \sim \text{Disc F.T.} \langle\Omega| T\left\{\mathcal{A}^\mu_{\perp hc}\,\mathfrak{X},\, \bar{\mathfrak{X}}\mathcal{A}^{\perp\, hc}_\mu\right\}|\Omega\rangle$$

  - $Q_8 - Q_8$: New SJF

$$j_{88} \sim \text{Disc F.T.} \langle\Omega| T\left\{ i\bar{n}\cdot\partial\mathcal{A}^\mu_{\perp hc}\frac{1}{i\bar{n}\cdot\partial}\,\mathfrak{X},\, \bar{\mathfrak{X}}\frac{1}{i\bar{n}\cdot\overleftarrow{\partial}}\, i\bar{n}\cdot\partial\mathcal{A}^{\perp\, hc}_\mu\right\}|\Omega\rangle$$

SJF Beyond Flavor Physics

- Subleading jet functions are TOP of collinear fields

- Subleading jet functions can arise outside of flavor physics

  1. For example, $x \to 1$ region of deep inelastic scattering
     More specifically, j_{11}^{S} appear in factorization formula for longitudinal structure function F_L

SJF Beyond Flavor Physics

- Subleading jet functions are TOP of collinear fields

- Subleading jet functions can arise outside of flavor physics

1. For example, $x \to 1$ region of deep inelastic scattering
   More specifically, j^S_{11} appear in factorization formula for
   longitudinal structure function F_L

2. For example, α_s extraction from thrust?
   Vicent Mateu talk at SCET 2009
   Kinematical $=$ (Partonic - LO corrections) $\otimes$ LO SF
   but Kinematical $=$ SJF $+$ SSF
   Need SJF $\otimes$ LO SF, not Kinematical
   Is this a problem?

Conclusions

- Factorization in the end-point region for

  $\bar{B} \to X_u\, l\, \bar{\nu}$ and $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s\, \gamma$

$$d\Gamma \sim \overbrace{H \cdot J \otimes S + \frac{1}{m_b} \sum_i H_0 \cdot J_0 \otimes s_i}^{\text{known}} + \overbrace{\frac{1}{m_b} \sum_i H_0 \cdot j_i \otimes S}^{\text{new}} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$

- Future directions:
  - Improve Precision on $|V_{ub}|$
  - Application to non-flavor physics