Quark fragmentation within an identified jet

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SCET Workshop, Ringberg Castle, April 2010

Outline

- Single inclusive hadron production: the fragmentation function $D_j^h(z)$

- Fragmenting jet function $\mathcal{G}_j^h(s,z)$ in SCET factorization formulae when the jet invariant mass is measured

- Case study: $\bar{B} \to X\pi\ell\bar{\nu}$, where $(X\pi)_u$ is a jet produced by a u-quark and the pion fragments from the jet

- Relations between $\mathcal{G}_j^h(s,z)$ the jet function $J_j(s)$ (pert.) and $D_j^h(z)$(non-pert.)

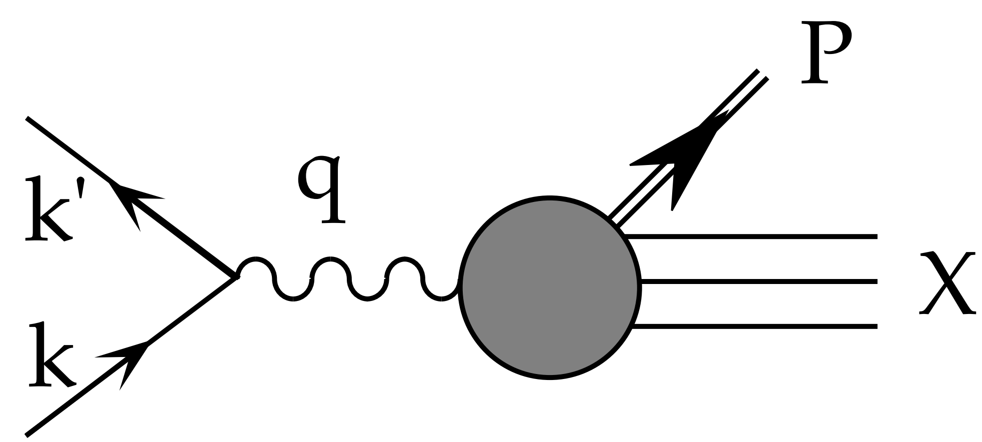
M.P. and I. W. Stewart, PRD 81 (2010), arXiv:0911.4980

A. Jain, M.P. and W. J. Waalewijn, in preparation

Single inclusive hadron production

e.g. : $e^+ e^- \to h\ X$

$q^2 > 0\,,\ \nu = P \cdot q$



$$d\sigma \sim L^{\alpha\beta} W_{\alpha\beta}\, \frac{d^3 P}{(2\pi)^3 2E}$$

$$W_{\alpha\beta} = \frac{1}{4\pi} \int d^4\xi\, e^{iq\cdot\xi} \sum_X \langle 0|J_\alpha(\xi)|hX\rangle\langle hX|J_\beta(0)|0\rangle$$

vs. $$W^{\rm DIS}_{\alpha\beta} = \frac{1}{4\pi} \int d^4\xi\, e^{iq\cdot\xi} \sum_X \langle P|J_\alpha(\xi)|X\rangle\langle X|J_\beta(0)|P\rangle \quad \to \text{OPE}$$

Factorization and fragmentation

- Factorization proofs to all orders in α_s , at leading power, for processes in which all Lorentz invariants are large and comparable, except masses (i.e. $q^2, \nu \to \infty$ in $e^+ e^- \to hX$)

  Collins, Soper, Sterman

- Separation between long- and short-distance contributions:

$$\frac{1}{\sigma_0} \frac{d\sigma^h}{dz}(e^+e^- \to h\,X) = \sum_{i=u,\bar{u},d,g\dots} \int_z^1 \frac{dx}{x}\, C_i\left(\frac{E_{\rm cm}^2}{\mu^2}, \frac{z}{x}, \alpha_S(\mu)\right) D_i^h(x,\mu) + \dots$$

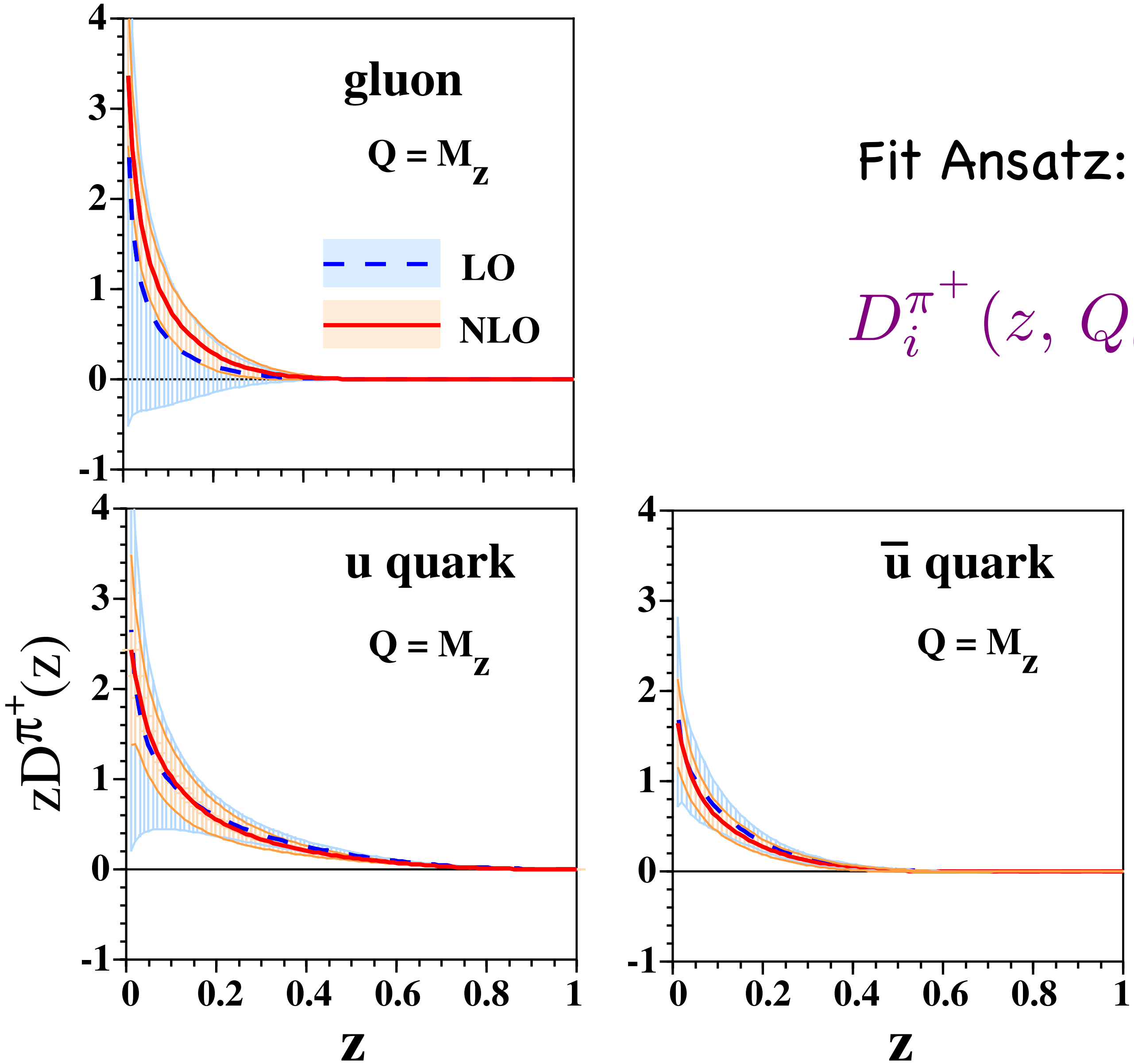
z is the fraction hadron/parton large light-cone momentum component

- The fragmentation function $D_i^h(z)$ is non-perturbative but universal

→ constraints on model parameters from phenomenology

Pion fragmentation from phenomenology

$$\frac{1}{\sigma_0}\frac{d\sigma^{\pi^+}}{dz}(e^+e^- \to \pi^+ X) = \sum_{i=u,\bar{u},d,g\ldots}\int_z^1 \frac{dx}{x}\, C_i\left(\frac{E_{\rm cm}^2}{\mu^2}, \frac{z}{x}, \alpha_S(\mu)\right) D_i^{\pi^+}(x,\mu)$$



Fit Ansatz:

$$D_i^{\pi^+}(z,\, Q_0^2 = 1\,\mathrm{GeV}^2) = N_i^{\pi^+}\, z^{\alpha_i^{\pi^+}}\, (1-z)^{\beta_i^{\pi^+}}$$

$$z = \frac{2}{\sqrt{s}}\, E_h^{\rm CM}$$

CERN, DESY & SLAC data

Hirai et al. (2007)

Definition of fragmentation function

Collins and Soper (1982)

$n^\mu = (1,0,0,1), \quad \bar{n}^\mu = (1,0,0,-1), \quad p^+ = n\cdot p, \quad p^- = \bar{n}\cdot p$ (large)

$$D_q^h(z) = z\int \frac{\mathrm{d}x^+}{4\pi}\, e^{ik^- x^+/2}\, \frac{1}{4N_c}\, \mathrm{Tr} \sum_X \langle 0|\not{\bar{n}}\, \Psi(x^+,0,0_\perp)|Xh\rangle\langle Xh|\bar{\Psi}(0)|0\rangle\Big|_{p_h^\perp=0}$$

- Gauge-invariance: $\Psi(x^+,0,0_\perp)$ contains a Wilson line of gluon fields
- Boost invariance: D is a function of $z = p_h^-/k^-$
- $|Xh\rangle$ is jet-like but $D(z)$ does not carry information about jet features

what amounts to the measurement of m^2_{Xh} ?

Case study: semi-inclusive B-decay

- Weak transition $b \to u \ell \bar{\nu}_\ell$: single jet X_u initiated by u-quark

- Inclusive decay $\bar{B} \to X_u \ell \bar{\nu}_\ell$: factorization analysis involving jet functions

  Korchemsky and Sterman (1994)

  many people in the audience ...

- Semi-inclusive vs. inclusive:

$$X_u \longrightarrow (Xh)_u \,, \quad m_h \ll m_B \quad \text{(e.g. pion)}$$

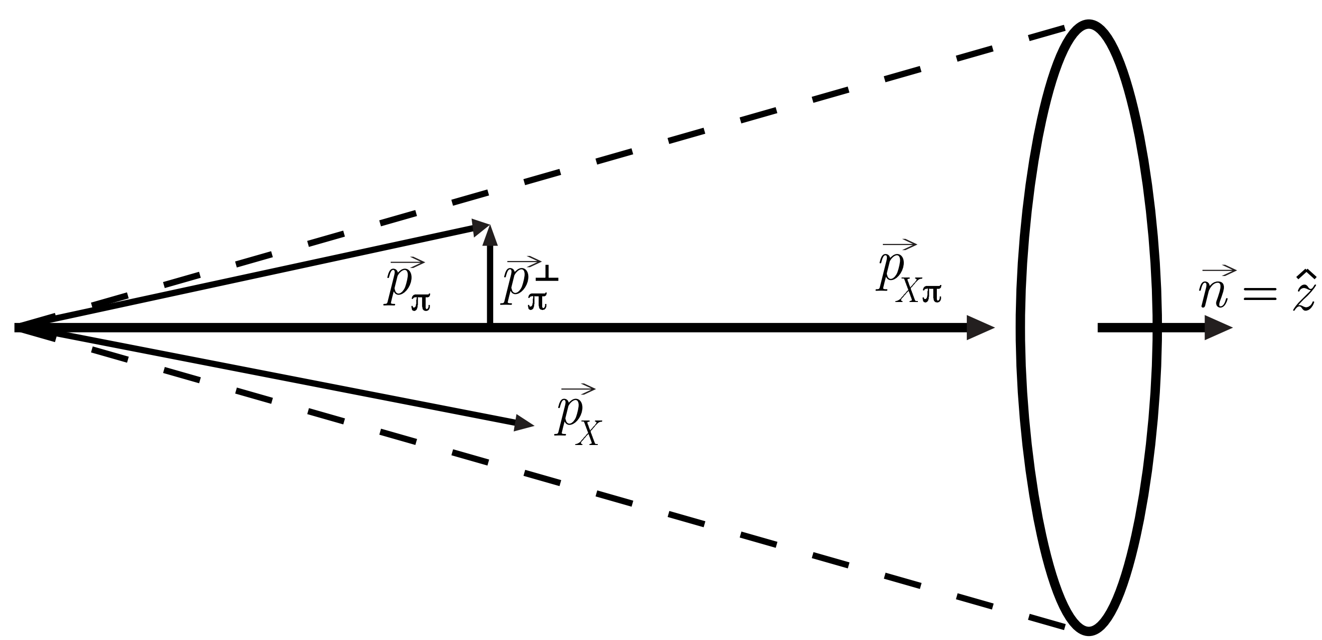
- In $\bar{B} \to X_u \ell \bar{\nu}_\ell$ the spectrum for $m^2_{X_u}$ is available

  BaBar and Belle collaborations

Kinematics

$\bar{B} \to X\pi\ell\bar{\nu}$: six independent kinematic variables in the B rest frame

$$\frac{d^6\Gamma}{dm^2_{X\pi}\, dq^2\, dE_\ell\, dp_\pi{}^x\, dp_\pi{}^y\, dp_\pi{}^z} \propto \frac{G_F^2\, |V_{ub}|^2}{(2\pi)^3\, 2\sqrt{\vec{p}_\pi^{\,2} + m_\pi^2}}\, L^{\alpha\beta}\, W_{\alpha\beta}$$



$$q^\perp = p^\perp_{X\pi} = 0\,, \qquad p_\pi^\perp \neq 0$$

$$\{p^+_{X\pi},\, p^-_{X\pi},\, p^+_\pi,\, p^-_\pi,\, E_\ell,\, \phi_\ell\}$$

$$p^+_{X\pi} \ll p^-_{X\pi}\,, \qquad p^+_\pi \ll p^-_\pi$$

Hadronic tensor and decay rates

In full QCD, in the B rest frame ($p_B^\mu = m_B v^\mu$) :

$$W_{\mu\nu} = \frac{1}{4\pi m_B} \int d^4x\, e^{-i\,q\cdot x} \sum_X \langle \bar{B} | J_\mu^{u\,\dagger}(x) | X\pi \rangle \, \langle X\pi | J_\nu^u(0) | \bar{B} \rangle$$

$$\begin{aligned} W_{\alpha\beta} = & -g_{\alpha\beta}\, W_1 + v_\alpha v_\beta\, W_2 - i\,\epsilon_{\alpha\beta\mu\nu}\, v^\mu q^\nu\, W_3 + q_\alpha q_\beta\, W_4 + (v_\alpha q_\beta + v_\beta q_\alpha)\, W_5 \\ & + (v_\alpha p_{\pi\,\beta} + v_\beta p_{\pi\,\alpha})\, W_6 - i\epsilon_{\alpha\beta\mu\nu}\, p_\pi^\mu q^\nu\, W_7 - i\,\epsilon_{\alpha\beta\mu\nu}\, v^\mu p_\pi^\nu\, W_8 + p_{\pi\,\alpha} p_{\pi\,\beta}\, W_9 \\ & + (p_{\pi\,\alpha} q_\beta + p_{\pi\,\beta} q_\alpha)\, W_{10} \end{aligned}$$

$$W_i = W_i(p_{X\pi}^+,\, p_{X\pi}^-,\, p_\pi^+,\, p_\pi^-)$$

Hadronic tensor and decay rates

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$$W_{\mu\nu} = \frac{1}{4\pi m_B} \int d^4x\, e^{-i\,q\cdot x} \sum_X \langle \bar{B}|J_\mu^{u\,\dagger}(x)|X\pi\rangle \,\langle X\pi|J_\nu^u(0)|\bar{B}\rangle$$

Integrating $d^6\Gamma$ over E_ℓ and ϕ_ℓ :

$$\frac{d^4\Gamma}{dp_{X\pi}^+\, dp_{X\pi}^-\, dp_\pi^-\, dp_\pi^+} = \frac{G_F^2\,|V_{ub}|^2}{128\pi^5}\Big(K_1\, W_1 + K_2\, W_2 + K_6\, W_6 + K_9\, W_9\Big)$$

$$K_1 = (m_B - p_{X\pi}^-)(m_B - p_{X\pi}^+)(p_{X\pi}^- - p_{X\pi}^+)^2,$$

$$K_2 = \frac{1}{12}(p_{X\pi}^- - p_{X\pi}^+)^4,$$

$$K_6 = \frac{1}{6}\,(p_{X\pi}^- - p_{X\pi}^+)^3\,\left[m_B(p_\pi^- - p_\pi^+) + p_\pi^+ p_{X\pi}^- - p_\pi^- p_{X\pi}^+\right],$$

$$K_9 = \frac{1}{12}(p_{X\pi}^- - p_{X\pi}^+)^2\Big\{\left[p_\pi^+(m_B - p_{X\pi}^-) + p_\pi^-(m_B - p_{X\pi}^+)\right]^2 - 4m_\pi^2(m_B - p_{X\pi}^-)(m_B - p_{X\pi}^+)\Big\}$$

SCET factorization

- $(X\pi)_u$ collinear jet: $p^\mu_{X\pi} \sim (\Lambda_{\rm QCD}, m_b, \sqrt{m_b\,\Lambda_{\rm QCD}}) = m_b(\lambda^2, 1, \lambda) \sim p^\mu_X$
- The pion fragments from the jet: $p^\mu_\pi \sim (\Lambda^2_{\rm QCD}/m_b, m_b, \Lambda_{\rm QCD})$, $p^2_\pi \ll m^2_{X\pi}$
- Matching the $b \longrightarrow u$ QCD current onto SCET operators at a scale $\sim m_b$:

$$J^\nu_u(x) = e^{i\mathcal{P}\cdot x - i m_b v\cdot x} \sum_{j=1}^{3} \sum_\omega C_j(\omega)\, {J^\nu_{uj}}^{(0)}(\omega)$$

Bauer, Fleming, Pirjol, Stewart (2001)

where

$$\mathcal{P}^\mu = n^\mu \bar{\mathcal{P}}/2 + \mathcal{P}^\mu_\perp\,, \qquad {J^\nu_{uj}}^{(0)}(\omega) = \bar{\chi}_{n,\omega}\,\Gamma^\nu_j\,\mathcal{H}_v,$$

$$\bar{\chi}_{n,\omega} \equiv \left(\bar{\xi}_n\, W_n\right) \delta_{\omega,\bar{\mathcal{P}}^\dagger}\,, \qquad \mathcal{H}_v \equiv Y^\dagger h_v$$

SCET factorization

The hadronic tensor at leading order in SCET:

same steps as in the inclusive case

cf. Lee and Stewart (2005)

$$W^{(0)}_{\mu\nu} = \frac{1}{4\pi}\int d^4x\, e^{-ir\cdot x} \sum_{j,j'=1}^{3}\sum_{\omega,\omega'} C_{j'}(\omega')\, C_j(\omega)\, \delta_{\omega',\bar{n}\cdot p}$$

$$\times \sum_X \langle \bar{B}_v|\, \big[\bar{\mathcal{H}}_v\, \bar{\Gamma}_{j'\mu}\, \chi_{n,\omega',0_\perp}\big](x)|X\pi\rangle\langle X\pi|\, [\bar{\chi}_{n,\omega}\, \Gamma_{j\nu}\, \mathcal{H}_v](0)|\bar{B}_v\rangle$$

with

$$r^\mu = \bar{n}^\mu r^+/2\,, \qquad r^+ = m_b - q^+ \sim \lambda^2\,, \qquad \chi_{n,\omega',0_\perp} \equiv \delta_{\omega,\bar{\mathcal{P}}}\, \delta_{0,\mathcal{P}_\perp}\big(W_n^\dagger\, \xi_n\big)$$

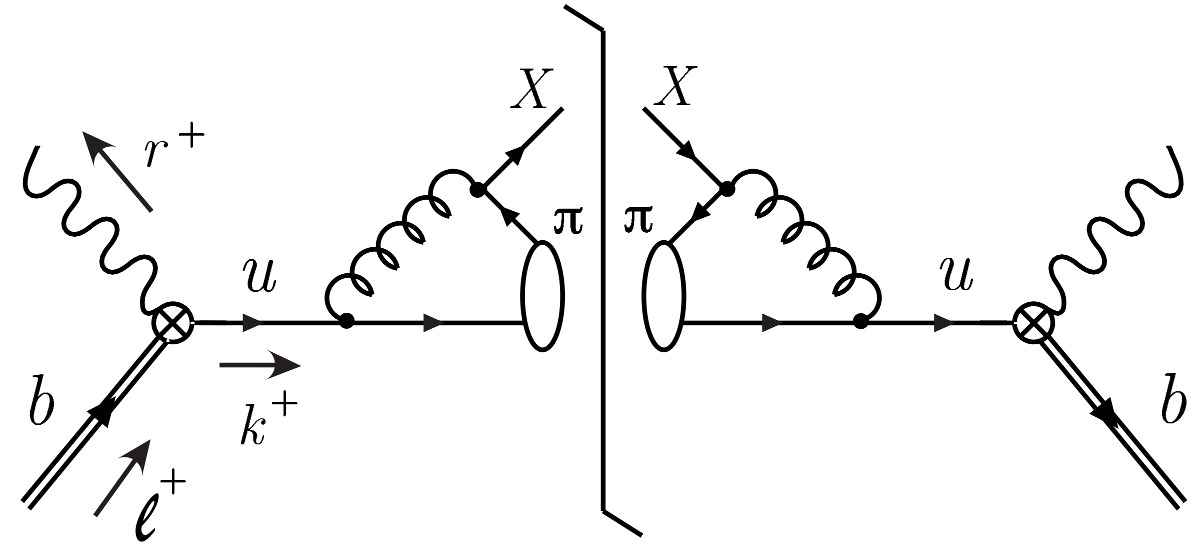
- Group usoft and collinear fields by a Fierz transformation:

$$\big[\bar{\mathcal{H}}_v\, \bar{\Gamma}_{j'\mu}\, \chi_{n,\omega',0_\perp}\big](x)\, [\bar{\chi}_{n,\omega}\, \Gamma_{j\nu}\, \mathcal{H}_v](0) = (-1)\Big[\bar{\mathcal{H}}_v(x)\bar{\Gamma}_{j'\mu}\frac{\not{n}}{2}\Gamma_{j\nu}\mathcal{H}_v(0)\Big]\Big[\bar{\chi}_{n,\omega}(0)\frac{\not{\bar{n}}}{4N_c}\chi_{n,\omega',0_\perp}(x)\Big]$$

$$+\ldots$$

SCET factorization

Collinear:



$$\frac{1}{4N_c}\,\mathrm{Tr}\sum_X \bar{n}\!\!\!/\,\langle 0|\chi_{n,\omega',0_\perp}(x)|X\pi\rangle\langle X\pi|\bar{\chi}_{n,\omega}(0)|0\rangle =$$

$$= 2\,\delta_{\omega,\omega'}\,\delta(x^+)\,\delta^2(x_\perp)\,\omega\int\frac{dk^+}{2\pi}\,e^{-ik^+x^-/2}\,\bar{\mathcal{G}}^\pi_u\left(k^+\omega,\,\frac{p_\pi^-}{\omega},\,p_\pi^+p_\pi^-\right)$$

Fragmenting jet function

$$p^- = m_b - q^- = m_b - m_B + p^-_{X\pi} = p^-_{X\pi} - \bar{\Lambda} + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{m_b}\right) \Rightarrow \quad z = \frac{p_\pi^-}{\omega} \simeq \frac{p_\pi^-}{p^-_{X\pi}}$$

Usoft:

$$f(l^+) = \frac{1}{2}\int\frac{dx^-}{4\pi}\,e^{-ix^-l^+/2}\,\langle\bar{B}_v|\bar{h}_v(\tilde{x})Y(\tilde{x},0)h_v(0)|\bar{B}_v\rangle\,, \qquad \tilde{x}^\mu = \bar{n}\cdot x\,n^\mu/2$$

SCET factorization

Projecting out the scalar functions $W_i = W_{\mu\nu}\, P_i^{\mu\nu}$:

$$W_i^{(0)} = \frac{h_i(\mu)}{\pi}\, p_{X\pi}^- \int_0^{p_{X\pi}^+} dk^+\, \bar{\mathcal{G}}_u^\pi\left(k^+ p_{X\pi}^-,\, \frac{p_\pi^-}{p_{X\pi}^-},\, p_\pi^+ p_\pi^-,\, \mu\right) S(p_{X\pi}^+ - k^+,\, \mu)$$

$$= \frac{h_i(\mu)}{\pi}\, p_{X\pi}^- \int_0^{p_{X\pi}^+} dk'^+\, \bar{\mathcal{G}}_u^\pi\left(p_{X\pi}^-(p_{X\pi}^+ - k'^+),\, \frac{p_\pi^-}{p_{X\pi}^-},\, p_\pi^+ p_\pi^-,\, \mu\right)\, S(k'^+,\, \mu)$$

with

$$h_i = \sum_{j,j'=1}^{3} C_{j'}(m_b, p_{X\pi}^-, \mu)\, C_j(m_b, p_{X\pi}^-, \mu)\, \mathrm{Tr}\left[\frac{P_v}{2}\, \bar{\Gamma}_{j'\mu}^{(0)}\, \frac{\not{n}}{2}\, \Gamma_{j\nu}^{(0)}\right] P_i^{\mu\nu}, \qquad S(p) \equiv f(\bar{\Lambda} - p)$$

$$\rightarrow \quad \frac{d^4\Gamma}{dp_{X\pi}^+\, dp_{X\pi}^-\, dp_\pi^-\, dp_\pi^+} = \Gamma_0\, H(m_B, p_{X\pi}^-, p_{X\pi}^+, \mu)\, p_{X\pi}^-$$
$$\times \int_0^{p_{X\pi}^+} dk'^+\, \bar{\mathcal{G}}_u^\pi\left(p_{X\pi}^-(p_{X\pi}^+ - k'^+),\, \frac{p_\pi^-}{p_{X\pi}^-},\, p_\pi^+ p_\pi^-,\, \mu\right) S(k'^+, \mu)$$

Fragmenting jet function vs. jet function

Consider $\dfrac{d^2\Gamma}{dm^2_{X\pi}\,dz}$ which involves $\mathcal{G}_u^\pi(k^+\omega,\,z,\mu) \equiv \omega\int dp_\pi^+\;\bar{\mathcal{G}}_u^\pi(k^+\omega,\,z,\,p_\pi^+p_\pi^-,\,\mu)$:

$$\frac{1}{4N_c}\,\mathrm{Tr}\int dp_\pi^+ \sum_X \bar{n}\!\!\!/\,\langle 0|\chi_{n,\omega',0_\perp}(x)|X\pi\rangle\langle X\pi|\bar\chi_{n,\omega}(0)|0\rangle =$$

$$= 2\,\delta_{\omega,\omega'}\,\delta(x^+)\,\delta^2(x_\perp)\int\frac{dk^+}{2\pi}\,e^{-ik^+x^-/2}\,\mathcal{G}_u^\pi\left(k^+\omega,\,\frac{p_\pi^-}{\omega}\right)$$

cf. with

$$\frac{1}{4N_c}\,\mathrm{Tr}\sum_{X_u}\langle 0|\bar{n}\!\!\!/\,\chi_n(x)|X_u\rangle\langle X_u|\bar\chi_{n,\omega,0_\perp}(0)|0\rangle = \delta(x^+)\,\delta^2(x_\perp)\,\omega\int dk^+\,e^{-ik^+x^-/2}\,J_u(\omega k^+)$$

$$\longrightarrow\quad \sum_{h\in\mathcal{H}_u}\int dz\,\mathcal{G}_j^h\,(k^+p^-_{Xh},z,\mu) = 2\,(2\pi)^3\,J_j(k^+p^-_{X_u},\mu)$$

Replacement rule to obtain SCET factorization formulae for semi-inclusive processes !

$$J_j(k^+\omega)\longrightarrow\frac{1}{2\,(2\pi)^3}\,\mathcal{G}_j^h(k^+\omega,z)\;dz$$

Relation with fragmentation function D

In the SCET notation:

$$D_q^\pi\Big(\frac{p_\pi^-}{\omega},\mu\Big) = \frac{1}{z}\sum_{p_{\pi,\ell}^\perp}\int \mathrm{d}^2 p_{\pi,r}^\perp \,\frac{1}{4N_c}\,\mathrm{Tr}\sum_X \bar{n}\!\!\!/\,\langle 0|[\delta_{\omega,\bar{\mathcal{P}}}\,\delta_{0,\mathcal{P}_\perp}\,\chi_n(0)]|X\pi\rangle\langle X\pi|\bar{\chi}_n(0)|0\rangle$$

vs.

$$\mathcal{G}_q^\pi\Big(k^+\omega,\frac{p_\pi^-}{\omega},\mu\Big) = \frac{1}{2\pi p_\pi^-}\sum_{p_{\pi,\ell}^\perp}\int \mathrm{d}^2 p_{\pi,r}^\perp \int \mathrm{d}^4x\, e^{ik^+x^-/2}$$

$$\times\frac{1}{4N_c}\,\mathrm{Tr}\sum_X \bar{n}\!\!\!/\,\langle 0|[\delta_{\omega,\bar{\mathcal{P}}}\,\delta_{0,\mathcal{P}_\perp}\,\chi_n(x)]|X\pi\rangle\langle X\pi|\bar{\chi}_n(0)|0\rangle$$

By performing an OPE ($\mathrm{SCET}_{\mathrm{I}}$ to $\mathrm{SCET}_{\mathrm{II}}$ matching):

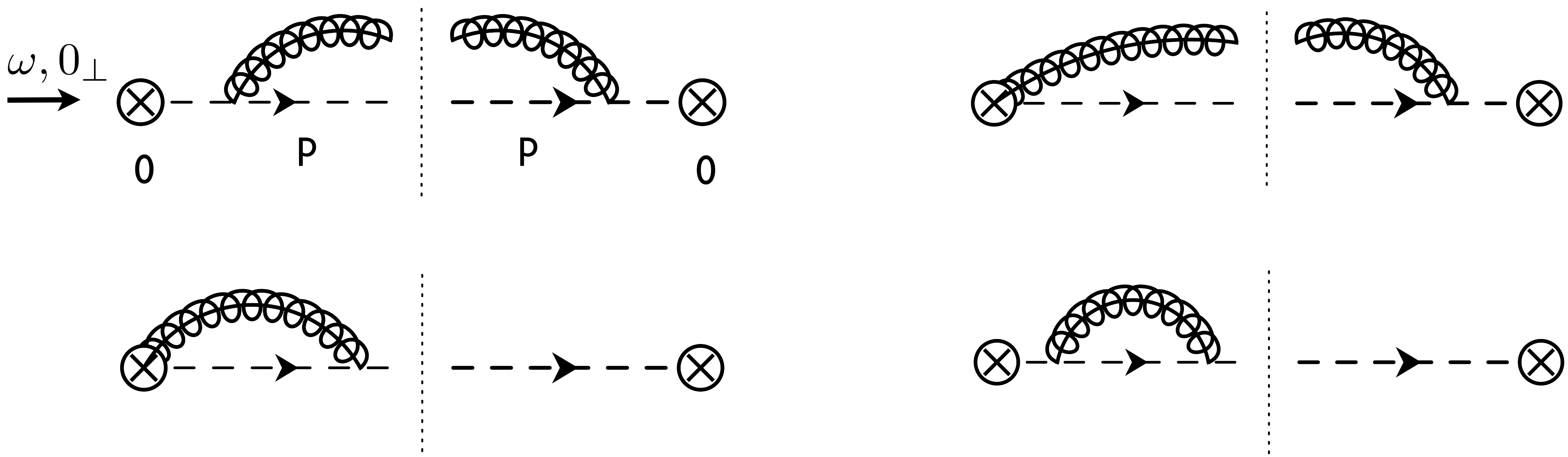
$$\mathcal{G}_i^\pi(s,\,z,\mu) = \sum_{j=u,d,g,\bar{u}\dots}\int_z^1 \frac{dx}{x}\,\mathcal{J}_{ij}\Big(s,\,\frac{z}{x},\mu\Big)\,D_j^\pi(x,\mu)\,\Big[1+\mathcal{O}\Big(\frac{\Lambda_{\mathrm{QCD}}^2}{s}\Big)\Big]$$

A. Jain, M.P. and W. J. Waalewijn, in preparation

The partonic fragmentation function

$$\hat{D}_q^q\left(z = \frac{p^-}{\omega}, \mu\right) = \frac{1}{z} \sum_{p^\perp_{\pi,\ell}} \int d^2 p^\perp_{\pi,r} \frac{1}{4N_c} \text{Tr} \sum_X \bar{\not{n}} \langle 0 | [\delta_{\omega,\bar{\mathcal{P}}}\, \delta_{0,\mathcal{P}_\perp}\, \chi_n(0)] | X q_n(p) \rangle \langle X q_n(p) | \bar{\chi}_n(0) | 0 \rangle$$

- at tree level: $\hat{D}_q^q(z) = \delta(1-z)$
- at one loop (Feynman gauge):



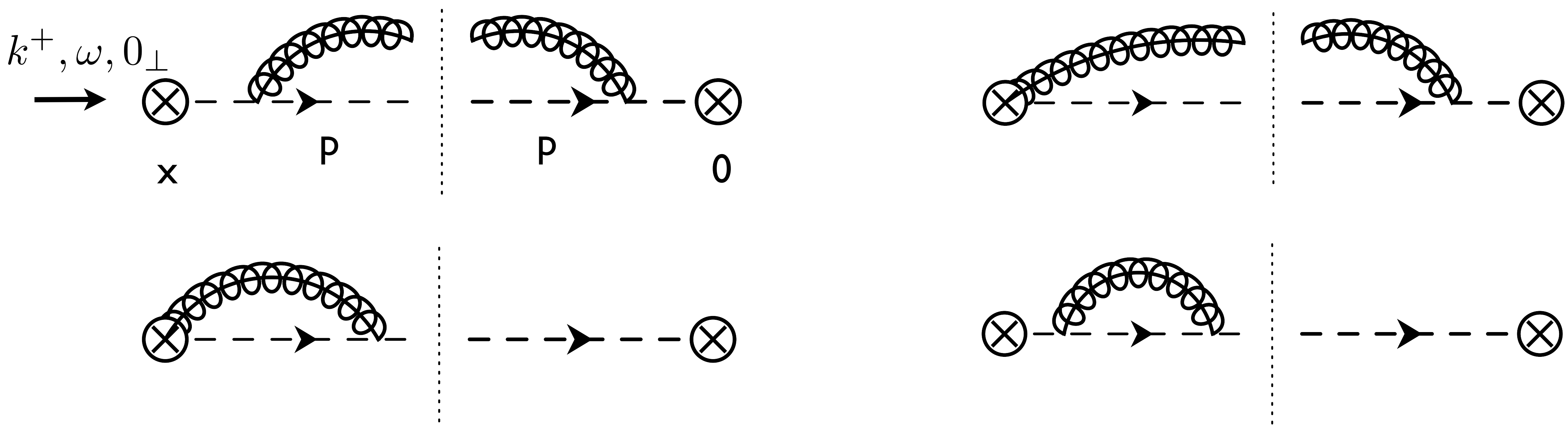
using dim. reg. and $\overline{\text{MS}}$ scheme:

$$\gamma^D_{qq}(z,\mu) = \frac{\alpha_s(\mu)}{\pi}\theta(1-z)\theta(z) P_{qq}(z)\,, \qquad \gamma^D_{qg}(z,\mu) = \frac{\alpha_s(\mu)}{\pi}\theta(1-z)\theta(z) P_{gq}(z)$$

The partonic fragmenting jet function

$$\hat{\mathcal{G}}_q^q\left(s = k^+\omega, \frac{p^-}{\omega}, \mu\right) = \frac{1}{2\pi p^-} \sum_{p^\perp_{\pi,\ell}} \int \mathrm{d}^2 p^\perp_{\pi,r} \int \mathrm{d}^4 x\, e^{ik^+x^-/2}$$
$$\times \frac{1}{4N_c} \mathrm{Tr} \sum_X \bar{n}\!\!\!/ \langle 0|[\delta_{\omega,\bar{\mathcal{P}}}\, \delta_{0,\mathcal{P}_\perp}\, \chi_n(x)]|X q_n(p)\rangle\langle X q_n(p)|\bar{\chi}_n(0)|0\rangle$$

- at tree level: $\hat{\mathcal{G}}_q^q(s,z,\mu) = 2(2\pi)^3\delta(s)\delta(1-z)$
- at one loop (Feynman gauge):



- check: anomalous dimension of $\mathcal{G}_q$ = anomalous dimension of the quark jet function J_q
- evolution changes only virtuality (no quark/gluon mixing, no change in z)

The matching coefficients up to one loop

$$\mathcal{G} = \mathcal{J} \otimes D$$

- at tree level: $\mathcal{J}_{qq}^{(0)}(s,z,\mu) = 2(2\pi)^3\delta(s)\delta(1-z)$, $\mathcal{J}_{qg}^{(0)} = 0$

- at one loop: checked that the IR divergences in $\mathcal{G}$ and D match up and

$$\mathcal{J}_{qq}^{(1)}(s,z,\mu) = \mathcal{G}_q^{q,(1)}(s,z,\mu) - 2(2\pi)^3\delta(s)\, D_q^{q,(1)}(z,\mu)$$
$$\mathcal{J}_{qg}^{(1)}(s,z,\mu) = \mathcal{G}_q^{g,(1)}(s,z,\mu) - 2(2\pi)^3\delta(s)\, D_q^{g,(1)}(z,\mu)$$

$$\mathcal{J}_{qq}^{(1)}(s,z,\mu) = \frac{\alpha_s C_F}{2\pi}\,\theta(1-z)\theta(z)\left\{\frac{2}{\mu^2}\left(\frac{\theta(s/\mu^2)\ \ln s/\mu^2}{s/\mu^2}\right)_+\delta(1-z) + \frac{1}{\mu^2}\left(\frac{\theta(s/\mu^2)}{s/\mu^2}\right)_+\frac{1+z^2}{(1-z)_+} + \right.$$
$$\left.\delta(s)\left[\ln z\left(\frac{1+z^2}{1-z}\right)_+ + 2z\left(\frac{\log(1-z)}{1-z}\right)_+ - \frac{\pi^2}{6}\delta(1-z) + (1-z)(1+\ln(1-z))\right]\right\}$$

$$\mathcal{J}_{qg}^{(1)}(s,z,\mu) = 2(2\pi)^2\alpha_s C_F\,\theta(1-z)\theta(z)\left\{\frac{2-2z+z^2}{z}\left[\frac{1}{\mu^2}\left(\frac{\theta(s/\mu^2)}{s/\mu^2}\right)_+ + \delta(s)\ln(z(1-z))\right] + z\,\delta(s)\right\}$$

D from differential B-decay rates

If $\omega \in [0, \Delta]$ with $\Delta \gg \Lambda_{\rm QCD}$ — Ligeti, Stewart and Tackmann (2008)

$$S(\omega) = \int_0^\infty d\omega' \underbrace{C_0(\omega - \omega')}_{\text{pert.}} \underbrace{F(\omega')}_{\text{non-pert.}}$$

- $C_0(p^+_{X\pi} - k^+ - \omega') = C_0(p^+_{X\pi} - k^+) - \omega'\, C_0'(p^+_{X\pi} - k^+) + \ldots$
- $\int_0^\infty d\omega'\, F(\omega') = 1$ and $\int_0^\infty d\omega'\, \omega'^{\,n}\, F(\omega') = \mathcal{O}(\Lambda^n_{\rm QCD})$

Hence, if $p^{+\,\rm min}_{X\pi} \gg \Lambda_{\rm QCD}$, $p^{+\,\rm max}_{X\pi} \ll p^-_{X\pi}$, at leading-order in $\Lambda_{\rm QCD}/p^+_{X\pi}$ -expansion:

$$\frac{d^2\Gamma^{\rm \ cut}}{dm^2_{X\pi}\, dz} = \Gamma_0 \sum_{j=u,\bar{u},d,g\ldots} \int_z^1 \frac{dx}{x} D^\pi_j(x,\mu) \left[\int_{m^2_{X\pi}/m_B}^{m_{X\pi}} dp^+_{X\pi} \frac{m^2_{X\pi}}{(p^+_{X\pi})^2} H\left(m_B, \frac{m^2_{X\pi}}{p^+_{X\pi}}, p^+_{X\pi}, \mu\right) \times \int_0^{p^+_{X\pi}} dk^+ \mathcal{J}_{uj}\left(k^+ \frac{m^2_{X\pi}}{p^+_{X\pi}}, \frac{z}{x}, \mu\right) C_0(p^+_{X\pi} - k^+, \mu) \right]$$

$$\hat{H}_{uj}\left(m_B, m^2_{X\pi}, \frac{z}{x}, \mu\right)$$ perturbative

Conclusions

- Derived leading-order SCET factorization formulae for differential decay rates in $\bar{B} \to X h \ell \bar{\nu}$ in the endpoint region with h light, energetic and fragmenting from a u-quark jet whose invariant mass is measured

- Defined a fragmenting jet function $\mathcal{G}_j^h(k^+\omega, z)$ which incorporates information about the invariant mass of the jet, at variance with the standard fragmentation function $D_j^h(z)$

- The relation between $\mathcal{G}_j^h(k^+\omega, z)$ and the jet function $J_j(s)$ leads to a simple replacement rule to obtain SCET factorization formulae for semi-inclusive processes with jet fragmentation from the corresponding inclusive case

- The matching of $\mathcal{G}_j^h(k^+\omega, z)$ onto $D_j^h(z)$ opens up the possibility to measure fragmentation functions in new processes, like B-decays

Additional slides

Further LO SCET factorization formulae

- $\bar{B} \to X K \gamma$:

$$\frac{d^2\Gamma}{dE_\gamma\, dz} = \frac{\Gamma_{0\,s}\, m_b}{(2\pi)^3} H_s(p^+_{XK},\mu) \int_0^{p^+_{XK}} dk^+ \mathcal{G}^K_s(k^+ m_b, z, \mu)\, S(p^+_{XK} - k^+, \mu)$$

$$= \frac{\Gamma_{0\,s}\, m_b}{(2\pi)^3} H_s(p^+_{XK},\mu) \sum_j \int_0^{p^+_{XK}} dk^+ \int_z^1 \frac{dx}{x} \mathcal{J}_{sj}\left(k^+ m_b, \frac{z}{x}, \mu\right) D^K_j(x,\mu)\, S(p^+_{XK} - k^+, \mu)$$

- $e^+e^- \to (\text{dijets}) + h$:

$$\frac{d^3\sigma}{dM^2\, d\bar{M}^2\, dz} = \frac{\sigma_0}{2(2\pi)^3} H_{2\text{jet}}(Q,\mu) \int_{-\infty}^{+\infty} dl^+ dl^- \Big[\mathcal{G}^h_q(M^2 - Ql^+, z, \mu)\, J_{\bar{n}}(\bar{M}^2 - Ql^-, \mu) + J_n(M^2 - Ql^+, \mu)\, \mathcal{G}^h_{\bar{q}}(\bar{M}^2 - Ql^-, z, \mu)\Big] S_{2\text{jet}}(l^+, l^-, \mu)$$

$$= \frac{\sigma_0}{2(2\pi)^3} H_{2\text{jet}}(Q,\mu) \times \sum_j \int_{-\infty}^{+\infty} dl^+\, dl^- \int_z^1 \frac{dx}{x} \Big[\mathcal{J}_{qj}\Big(M^2 - Ql^+, \frac{z}{x}, \mu\Big) J_{\bar{n}}(\bar{M}^2 - Ql^-, \mu) + J_n(M^2 - Ql^+, \mu)\, \mathcal{J}_{\bar{q}j}\Big(\bar{M}^2 - Ql^-, \frac{z}{x}, \mu\Big)\Big] D^h_j(x,\mu)\, S_{2\text{jet}}(l^+, l^-, \mu)$$