Non-local $1/m_b$ corrections to $\bar{B} \to X_s\gamma$

Michael Benzke

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

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In collaboration with S. J. Lee, M. Neubert, G. Paz

Abstract

What is this talk about?

The decay rate of $\bar{B} \to X_s\gamma$ is considered. Subleading contributions in the $\frac{1}{m_b}$ expansion obey a new factorization formula which introduces a new type of soft function, that is not perturbatively calculable. Their effect on the partially integrated rate is estimated.

Based on arXiv:1003.5012

Motivation

- The radiative decay $\bar{B} \to X_s\gamma$ is a loop level effect in the Standard Model

  $\rightarrow$ Probe for new physics

- To eliminate experimental background a lower cut on the photon energy is introduced

  $\rightarrow$ Theoretical analysis must be performed in this **endpoint region**

  $\rightarrow$ Jet X_s has a large energy $\mathcal{O}(m_b)$ but small invariant mass $\mathcal{O}(\sqrt{m_b\Lambda_{\mathrm{QCD}}})$

- The appropriate effective field theories are therefore SCET (hc,$\overline{\text{hc}}$,s) and HQET

  C. W. Bauer et al. '01

  $\rightarrow$ Expansion in small parameter $\lambda \sim \frac{\Lambda_{\mathrm{QCD}}}{m_b}$

Subleading Shape Functions

- 2004: Using SCET, study of **one** type of power corrections

  subleading shape functions (subleading "twist")

  for $X_u\, l\, \bar{\nu}$ and $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s \gamma$

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + ...$$

  (K.S.M. Lee, Stewart '04; Bosch, Neubert, GP '04; Beneke, Campanario, Mannel, Pecjak '04)

  Supersedes earlier studies

- The subleading shape function s_i are non perturbative

  known at tree level: $\sum_i H \cdot J \otimes s_i$ at $\mathcal{O}(\alpha_s^0)$

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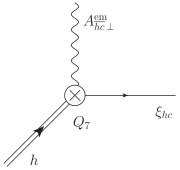
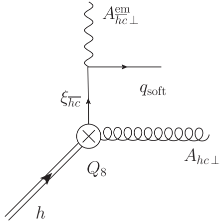
Calculation

- At leading power in $\frac{1}{m_b}$ only one SCET operator contributes



- At subleading power multiple diagrams are possible, e.g.



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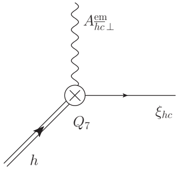

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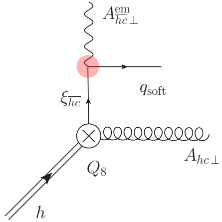


SCET-Lagrangian insertion $\rightarrow$ suppressed by another $\sqrt{\lambda}$

Calculation

- Decay rate $\sim$ amplitude squared
- Leading power contribution to the decay rate:

  Interference of Q_7 with Q_7
- The Rate **factorizes** at leading power

  $d\Gamma^{\mathrm{LO}} \sim H \cdot J \otimes S$

  Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01
- The hard function H and the jet function J are perturbative quantities, the shape function S is non-perturbative

  The leading order shape function S is related to the measured photon spectrum

Calculation at subleading power

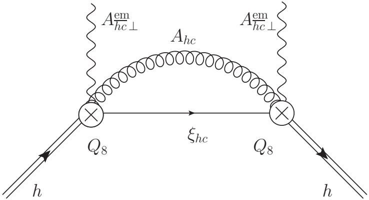
- Subleading contributions originating from the interference $Q_7 - Q_7$
  $\rightarrow$ Gil Paz' talk
- But other operators of the weak effective Hamiltonian also contribute
- Some of them are suppressed by small Wilson coefficients or CKM elements

Important Operator combinations

$Q_1 - Q_7$, $Q_7 - Q_8$ and $Q_8 - Q_8$
($Q_1 - Q_1$ and $Q_1 - Q_8$ only appear at $\mathcal{O}\left(\frac{1}{m_b^2}\right)$)

Calculation at subleading power

- Distinguish contributions by their factorization properties
- 1. **Direct** photon contributions



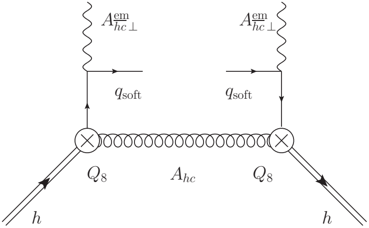
$\rightarrow \frac{\alpha_s}{m_b}$ in endpoint region

- Factorizes $\quad d\Gamma \sim H \cdot j \otimes S$

  with the subleading jet function j

Calculation at subleading power

- 2. **Resolved** photon contributions



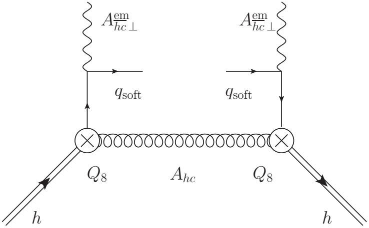
$\longrightarrow \frac{\Lambda}{m_b}$

(in this case *double resolved*)

- Factorizes $\quad d\Gamma^{\rm res} \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$
- The $\bar{J}$ are new jet functions corresponding to the uncut hard-collinear propagators

Calculation at subleading power

- 2. **Resolved** photon contributions



$\longrightarrow \frac{\Lambda}{m_b}$

- The subleading shape function s is a non-local HQET matrix element
- It cannot be separately extracted from the photon spectrum
- It is non-local in two light-cone directions

  $\longrightarrow$ No OPE even in integrated rate

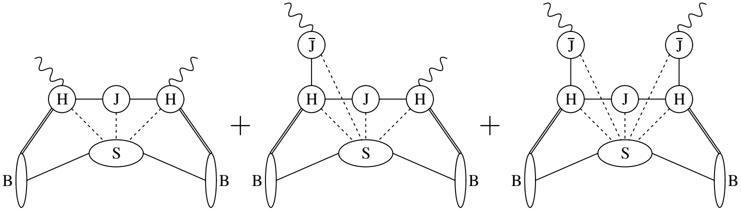
  $\longrightarrow$ Hadronic uncertainty

Factorization Formula

- In the endpoint region the rate factorizes

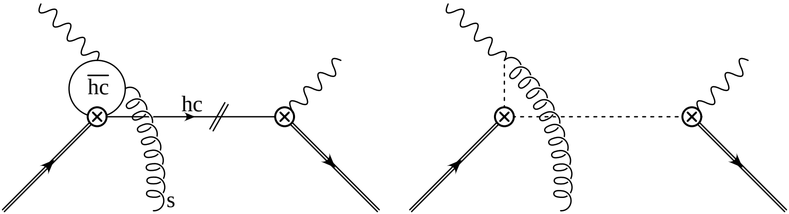
$$
\begin{aligned}
d\Gamma \quad \sim \quad & H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i \\
& + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right)
\end{aligned}
$$

- Visualize as



- Consider all relevant combinations in turn

Interference of $Q_1 - Q_7$



- Non-perturbative contribution to total rate $\mathcal{O}\left(\frac{1}{m_c^2}\right)$ (Voloshin '96)

$$\frac{d\Gamma^{\text{res}}}{dE_\gamma} \sim \frac{1}{m_b}\int d\omega\, \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\varepsilon}\left[1 - F\left(\frac{m_c^2}{2E_\gamma\omega_1}\right)\right] g_{17}(\omega,\omega_1)$$

$$g_{17}(\omega,\omega_1) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t} \frac{1}{M_B} \langle \bar{B}|\bar{h}(\mathbf{tn}) \ldots G_s^{\alpha\beta}(\mathbf{r\bar{n}}) \ldots h(\mathbf{0})|\bar{B}\rangle$$

- Expand F for $m_c \sim m_b \rightarrow$ Reproduce Voloshin term in total rate $\sim \frac{-\lambda_2}{9m_c^2}$

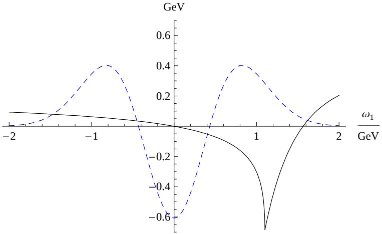
Interference of $Q_1 - Q_7$

- Estimate the possible contribution to the partial rate $\Gamma(E_0) = \int_{E_0} dE_\gamma \frac{d\Gamma}{dE_\gamma}$
- PT invariance implies that all considered subleading shape functions are real
- Moment constraints

$$\int d\omega \int d\omega_1 g_{17}(\omega,\omega_1) = 2\lambda_2$$
$$\int d\omega\omega \int d\omega_1 g_{17}(\omega,\omega_1) = -\rho_2$$

- Symmetry

$$\int d\omega g_{17}(\omega,\omega_1) = \int d\omega g_{17}(\omega,-\omega_1)$$

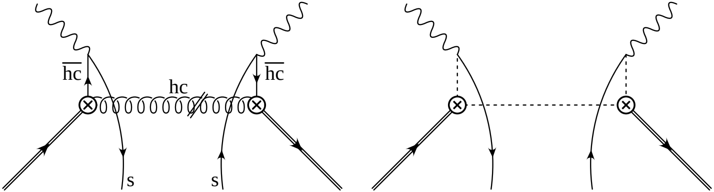


- Model function

$$h_{17}(\omega_1) = \int d\omega g_{17}(\omega,\omega_1) = \frac{2\lambda_2}{\sqrt{2\pi}} \frac{\omega_1^2-\Lambda^2}{\sigma^2-\Lambda^2} e^{-\frac{\omega_1^2}{2\sigma^2}}$$

$\rightarrow -1.7\ldots 4.0\,\%$ non-perturbative uncertainty due to $Q_1 - Q_7$ contribution (possible correction to partonic rate)

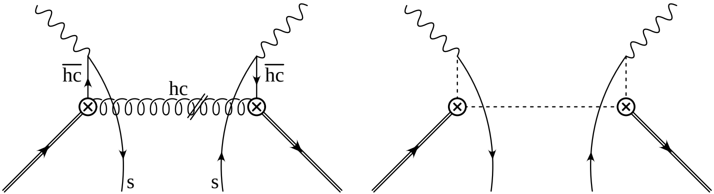
Interference of $Q_8 - Q_8$



$$\frac{d\Gamma^{\rm res}}{dE_\gamma} \sim \frac{e_s^2 8\pi\alpha_s}{m_b} \int d\omega\, \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\varepsilon} \int \frac{d\omega_2}{\omega_2 - i\varepsilon} g_{88}(\omega, \omega_1, \omega_2)$$

$$g_{88}(\omega, \omega_1, \omega_2) = \frac{1}{M_B} \langle \bar{B} | \bar{h}(\mathbf{tn}) \ldots s(\mathbf{tn} + \mathbf{u\bar{n}}) \bar{s}(\mathbf{r\bar{n}}) \ldots h(\mathbf{0}) | \bar{B} \rangle_{\rm F.T.}$$

Interference of $Q_8 - Q_8$



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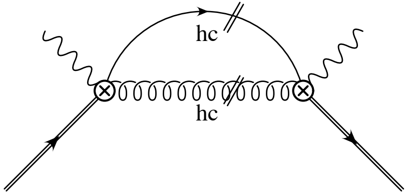
- Important subtlety

Interference of $Q_8 - Q_8$



$$\frac{d\Gamma^{\mathrm{res}}}{dE_\gamma} \sim \frac{e_s^2 8\pi\alpha_s}{m_b} \int d\omega\, \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\varepsilon} \int \frac{d\omega_2}{\omega_2 - i\varepsilon} g_{88}(\omega, \omega_1, \omega_2)$$

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- Consider scale dependence of direct contribution

$$\frac{d\Gamma^{\mathrm{dir}}}{dE_\gamma} \sim \frac{e_s^2 C_F \alpha_s}{4\pi m_b} \int d\omega \left(2 \ln \frac{m_b(\omega + p_+)}{\mu^2} + 1 \right) S(\omega)$$

Interference of $Q_8 - Q_8$

- How does the scale dependence cancel?
- Consider the asymptotic form of g_{88} for $\omega_{1,2} \gg \Lambda_{\rm QCD}$

$$g_{88}(\omega,\omega_1,\omega_2) \to \frac{C_F}{(4\pi)^{2-\epsilon}} \frac{\theta(\omega_1)\omega_1^{1-\epsilon}}{\Gamma(1-\epsilon)} \delta(\omega_1-\omega_2) \int_\omega d\omega'\, S(\omega')(\omega'-\omega)^{-\epsilon} + \dots$$

- Convolution integral is UV divergent!

  $\to$ Introduce cutoff and consider high and low momentum part of the convolution separately

$$\frac{d\Gamma^{\rm res}}{dE_\gamma} \sim \frac{e_s^2 8\pi\alpha_s}{m_b} \int d\omega\, \delta(\omega+p_+) \int^\Lambda \frac{d\omega_1}{\omega_1+i\varepsilon} \int^\Lambda \frac{d\omega_2}{\omega_2-i\varepsilon} g_{88}(\omega,\omega_1,\omega_2)$$
$$- \frac{e_s^2 C_F 8\pi\alpha_s}{2\pi^2 m_b} \int d\omega \left(\ln\frac{\Lambda(\omega+p_+)}{\mu^2} + 2\right) S(\omega)$$

$\to$ Scale dependence cancels

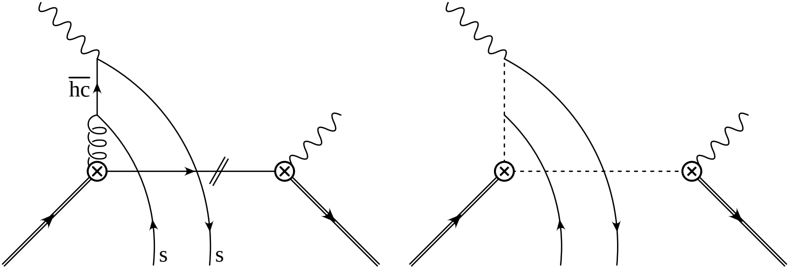
Interference of $Q_8 - Q_8$ - Numerical Estimate

- This time no moment constraint for g_{88}

  $\rightarrow$ Assume that the convolution of jet and shape function yields a value of $\mathcal{O}(\Lambda_{\rm QCD})$
- But suppressed by e_s^2

  $\rightarrow -0.3 \ldots 1.9\,\%$ non-perturbative uncertainty due to $Q_8 - Q_8$ contribution

Interference of $Q_7 - Q_8$



$$\frac{d\Gamma^{\rm res}}{dE_\gamma} \sim \frac{\alpha_s}{m_b} \int d\omega\, \delta(\omega + p_+) \int d\omega_1 \int \frac{d\omega_2}{\omega_1 - \omega_2 + i\varepsilon}$$

$$\left[\left(\frac{1}{\omega_1 + i\varepsilon} + \frac{1}{\omega_2 - i\varepsilon} \right) g_{78}^{(1)}(\omega, \omega_1, \omega_2) - \left(\frac{1}{\omega_1 + i\varepsilon} - \frac{1}{\omega_2 - i\varepsilon} \right) g_{78}^{(5)}(\omega, \omega_1, \omega_2) \right]$$

$$g_{78}^{(i)}(\omega, \omega_1, \omega_2) = \frac{1}{M_B} \langle \bar{B} | \bar{h}(\mathbf{tn}) \ldots h(\mathbf{0}) \sum_q e_q \bar{q}(\mathbf{r\bar{n}}) \ldots q(\mathbf{u\bar{n}}) | \bar{B} \rangle_{\rm F.T.}$$

Interference of $Q_7 - Q_8$

- Due to the sum over flavors it is possible to roughly estimate the effects through vacuum insertion approximation

  Lee, Neubert, Paz '06

$$\int d\omega\, g_{78}^{(i)}(\omega, \omega_1, \omega_2)|_{\mathrm{VIA}} = -e_{\mathrm{spec}} \frac{f_B^2 M_B}{8} \left(1 - \frac{1}{N_c^2}\right) \phi^B(-\omega_1) \phi^B(-\omega_2)$$

$\rightarrow -2.8 \ldots -0.3\,\%$ non-perturbative uncertainty due to $Q_7 - Q_8$ contribution

Interference of $Q_7 - Q_8$

- Alternatively it is possible to relate the effect to the measured isospin asymmetry
- Wigner-Eckart Theorem

$$\langle \bar{B}|\bar{h}(tn)\dots h(0)\sum_q e_q\bar{q}(r\bar{n})\dots q(u\bar{n})|\bar{B}\rangle$$
$$= \frac{1}{6}\Lambda_0 \pm \frac{1}{2}\Lambda_1 = \frac{1}{6}(\Lambda_0 - \Lambda_1) + e_{\rm spec}\Lambda_1$$

- Isospin asymmetry $\Delta_{0-} \sim \Lambda_1$; averaged rate $\Gamma^{\rm avg} \sim \Lambda_0$
- $SU(3)$ symmetry $\rightarrow \Lambda_0 = \Lambda_1$

  M. Misiak '09

  $\rightarrow -4.4\dots5.6\,\%$ non-perturbative uncertainty for an $SU(3)$ breaking of $30\,\%$
- Can be reduced by improved measurement of isospin asymmetry

Summary - The Numbers

Operators	Effect
$Q_1 - Q_7$	$-1.7 \ldots 4.0\,\%$
$Q_8 - Q_8$	$-0.3 \ldots 1.9\,\%$
$Q_7 - Q_8^{\mathrm{VIA}}$	$-2.8 \ldots -0.3\,\%$
$Q_7 - Q_8^{\mathrm{exp}}$	$-4.4 \ldots 5.6\,\%$

Summary

- In order to estimate the non-perturbative uncertainty in the $\bar{B} \to X_s\gamma$ branching ratio the subleading order of the power expansion in $\frac{1}{m_b}$ must be considered
- At this order the decay rate obeys a new factorization formula

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b}\sum H \cdot j_i \otimes S + \frac{1}{m_b}\sum H \cdot J \otimes s_i + \frac{1}{m_b}\sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b}\sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$

- Effects of the new non-local matrix elements can only be estimated
- A careful consideration of everything we know yields a non-perturbative uncertainty of $\pm 5\,\%$ to the partial decay rate
- Open questions:
  · CP violation due to new jet functions
  · Effect of the SSF on the spectrum (determination of HQET parameters)
- **Thank you for your attention!**