Effective field theory approach to threshold resummation for heavy coloured particles

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(Based on M.Beneke, P.Falgari, CS, arXiv:0907.1443 [hep-ph], NPB828:69 (2010);

arXiv:1001.4621 [hep-ph], arXiv:1001.4627 [hep-ph], (RADCOR09) and work in progress

M.Beneke, M.Czakon, P.Falgari, A.Mitov, CS arXiv:0911.5166 [hep-ph])

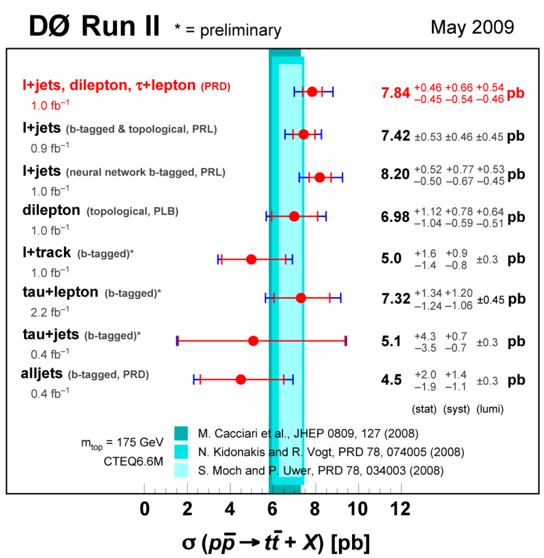
Pair production of heavy coloured particles at Tevatron/LHC

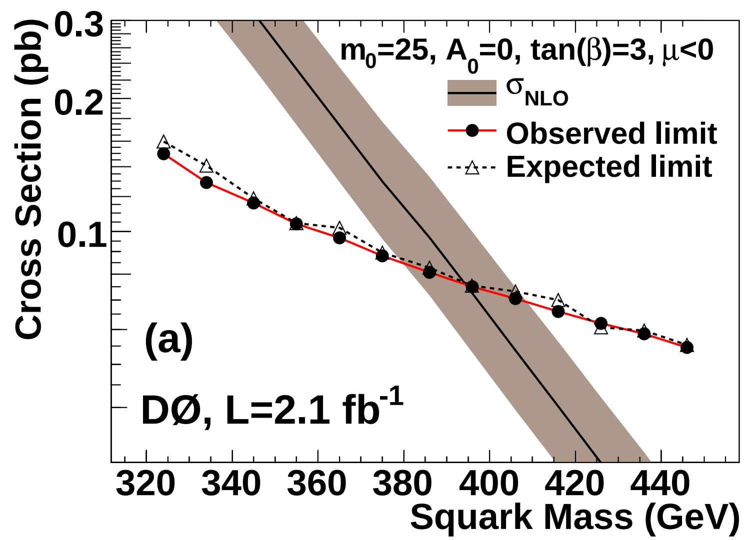
$$N(K_1)N'(K_2) \to H(p_1)H'(p_2) + X$$

- N, N': pp, $p\bar{p}$; HH': top-quarks, squarks, gluinos...

Precise knowledge of total cross sections:

- top-quarks: sensitivity on mass, constraining gluon PDFs
- new particles: Exclusion bounds, model discrimination,...





NLO corrections **enhanced** for $\beta = \sqrt{1 - \frac{(M_H+M_{H'})^2}{\hat{s}}} \to 0$

$$\hat{\sigma}^{(1)}_{pp' \to HH'} = \hat{\sigma}^{(0)}_{pp' \to HH'} \, \alpha_s \left[\underbrace{a \log^2(8\beta^2) + b \log(8\beta^2)}_{\text{"threshold logarithms"}} + \underbrace{c \frac{1}{\beta}}_{\text{"Coulomb singularity"}} + \dots \right]$$

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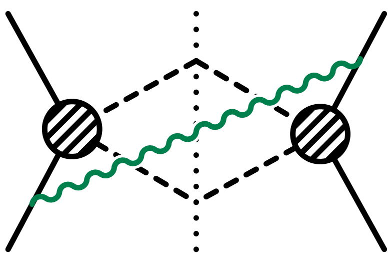
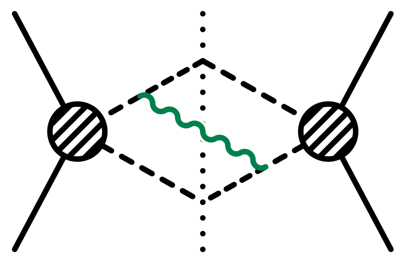
Soft corrections: (Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et.al. 98, . . .)



$\Rightarrow \alpha_s \; \log^2(8\beta^2)$



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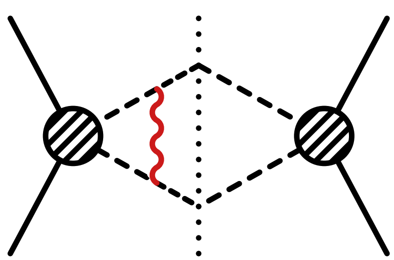
Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,. . .)



$\Rightarrow \alpha_s \, \frac{1}{\beta}$

Aim: Resummation of partonic cross section at threshold

$$\hat{s} \sim (M_H + M_{H'})^2 \equiv 4M^2$$

$\Rightarrow$ Heavy particles nonrelativistic

Alternative approach: partonic threshold for invariant mass $\hat{s} \sim M^2_{HH'}$ (Ahrens et.al. 09)

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EFT to show **factorization** into hard, soft and Coulomb functions

$$\hat{\sigma}_{pp' \to HH'}|_{\hat{s} \to 4M^2} = H_{ij} \otimes W_{ij} \otimes J$$

- Momentum space threshold resummation (Becher, Neubert 06)
- simultaneous summation of Coulomb corrections in J

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Application to top quark, squark and gluino production

- Diagonal colour bases for $3 \otimes 3$, $3 \otimes \bar{3}$, $3 \otimes 8$, $8 \otimes 8$
- NLL results for squark-antisquark production
- $\mathcal{O}(\alpha_s^4)$ threshold expansion of $t\bar{t}$ cross section

Parametric representation of cross section near threshold:

$$\hat{\sigma}_{pp'} \quad \propto \quad \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \ldots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \Big\{1\,\text{(LL,NLL)}; \alpha_s, \beta\,\text{(NNLL)}; \ldots\Big\}\,,$$

- Counting of soft logs usually defined by exponential only (Bonciani et.al. 98)
- Compared to $e^- e^+ \to t\bar{t}$ at threshold: N^{n}LL$\Leftrightarrow$ N^{n-1}LL$|_{e^- e^+ \to t\bar{t}}$
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Fixed order expansion contains all terms of the form

LL: $\alpha_s \left\{\frac{1}{\beta}, \ln^2\beta\right\}; \quad \alpha_s^2 \left\{\frac{1}{\beta^2}, \frac{\ln^2\beta}{\beta}, \ln^4\beta\right\}; \ldots,$

NLL: $\alpha_s \ln\beta; \quad \alpha_s^2 \left\{\frac{\ln\beta}{\beta}, \ln^3\beta\right\}; \ldots$

NNLL: $\alpha_s\;; \quad \alpha_s^2 \left\{\frac{1}{\beta}, \ln^{2,1}\beta\right\}; \ldots$

Matching of scattering amplitude (for S-wave production)

$$\mathcal{A}_{pp'\to HH'X} = \sum_i C^{(i)}_{\{\alpha\}}(M,\mu)\, c^{(i)}_{\{a\}}\, \langle HH'X|\phi_{c;a_1\alpha_1}\phi_{\bar{c};a_2\alpha_2}\psi^{\dagger}_{a_3\alpha_3}\psi'^{\dagger}_{a_4\alpha_4}|pp'\rangle_{\mathrm{EFT}}$$

- $\psi^\dagger$, $\psi'^\dagger$: non-relativistic fields that create H and H'
  ⇒ (P)NRQCD
- ϕ_c ($\phi_{\bar{c}}$): collinear (anti-collinear) fields that destroy p and p'
  ⇒ SCET
- α_i: spin, a_i: colour indices, $c^{(i)}_{\{a\}}$: colour basis
- only (u)soft hadronic final states X for threshold kinematics

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Collinear and nonrelativistic fields only connected by (u)soft gluons
$\Rightarrow$ Soft-gluon decoupling field redefinition (Bauer, Pirjol, Stewart 01)

$$\xi_c(x) = S_n(x_-)\xi_c^{(0)}(x) \qquad S_n(x) = \mathrm{P}\exp\left[ig_s\int_{-\infty}^0 dt\, n\cdot A_s^a(x+nt)T^a\right]$$

LO NRQCD Lagrangian for particles H, H' in representations R, R':

$$\mathcal{L}_{\text{PNRQCD}} = \psi^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi'$$
$$+ \int d^3\vec{r} \left[\psi^\dagger \mathbf{T}^{(R)a} \psi \right](\vec{r}) \left(\frac{\alpha_s}{r} \right) \left[\psi'^\dagger \mathbf{T}^{(R')a} \psi' \right](0) \,,$$

with $D_s^0 = \partial^0 - i g_s A_s^0(x_0, \vec{0})$.

$\mathcal{L}$ for arbitrary R, R' invariant under gauge transformations $U(x_0)$:

$$U^{(R)\dagger} \mathbf{T}^{(R)a} U^{(R)} = U_{ab}^{(8)} \mathbf{T}^{(R)b}$$

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Decoupling for heavy particle fields:

$$\psi^\dagger(x) = \psi^{(0)\dagger}(x) S_v^{(R)\dagger}(x_0), \quad S_v^{(R)\dagger}(x) = \mathrm{P} \exp\left[ig_s \int_0^\infty ds \, v \cdot A^a(x+vs) \mathbf{T}^{(R)a} \right]$$

same $v = (1, \vec{0})$ for both heavy particles at threshold

Works at leading order in PNRQCD (⇒ Higher orders see below)

Apply soft-gluon decoupling to matrix element:

$$\langle H H' X|\phi_{c1}\,\phi_{\bar{c}1}\,\psi_3^\dagger\,\psi_4'^\dagger|pp'\rangle \Rightarrow \langle H H'|\psi_3^{(0)\dagger}\psi_4'^{(0)\,\dagger}|0\rangle\ \langle 0|\phi_{c1}^{(0)}\phi_{\bar{c}2}^{(0)}|pp'\rangle\ \ \langle X|S_n^1 S_{\bar{n}}^2 S_v^{3\dagger} S_v^{4\dagger}|0\rangle$$

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Inserting into formula for σ, summing over complete set of $|X\rangle \ldots$

$$\hat{\sigma}_{pp'}(\hat{s},\mu) = \sum_{i,i'} H_{ii'}(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(\sqrt{\hat{s}} - 2M - \frac{\omega}{2}) W_{ii'}^{R_\alpha}(\omega,\mu)$$

Irreducible representations $R \otimes R' = \sum_{R_\alpha} R_\alpha$ e.g. $3 \otimes \overline{3} = 1 \oplus 8$.

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Potential function:

$$J_{\{a\}}(q) = \int d^4z e^{iq\cdot z} \langle 0 | [\psi_{a_1}^{(0)} \psi_{a_2}^{'(0)}](z) [\psi_{a_3}^{(0)\dagger} \psi_{a_4}^{'(0)\dagger}](0) | 0\rangle \quad = \sum_{R_\alpha} P_{\{a\}}^{R_\alpha} J_{R_\alpha}(q)$$

- P^{R_α}: projector on irrep R_α
- Given by Coulomb Green function $J_{R_\alpha}(E) = 2\text{Im} G_C^{R_\alpha}(0,0,E)$

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Soft function:

$$W_{ii'}^{R_\alpha}(\omega) = \int \frac{dz_0}{4\pi}\, e^{i\omega z_0/2} \langle 0 | \overline{\mathsf{T}}[S_v^4 S_v^3 c^{i'*} S_{\bar{n}}^{1\dagger} S_n^{2\dagger}](0) P^{R_\alpha} \mathsf{T}[S_n^1 S_{\bar{n}}^2 c^i S_v^{3\dagger} S_v^{4\dagger}](x_0) | 0 \rangle$$

Subleading PNRQCD and SCET interactions:

$$\psi^\dagger \vec{x}\cdot \vec{E}_{us}(x_0,0)\psi'^\dagger\,, \quad \bar{\xi}\left(x_\perp^\mu n_-^\nu\, W_c\, gF_{\mu\nu}^{\rm us} W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi \,\ldots$$

Soft gluons not decoupled by field redefinitions.

(Higher order interactions can be treated as perturbation to LO Lagrangian ⇒ factorized cross section with higher order soft and potential functions $\hat{\sigma} = \sum_a H^{(a)} \otimes W^{(a)} \otimes J^{(a)}$)

Potentially relevant at NNLL in soft ⊗ potential corrections :

$$\frac{\alpha_s}{\beta} \times \alpha_s \beta \log\beta \sim \alpha_s^2 \log\beta$$

$\sigma_{\bf tot}$: effects vanish at NNLL! (Beneke, Czakon,Falgari, Mitov, CS 09)

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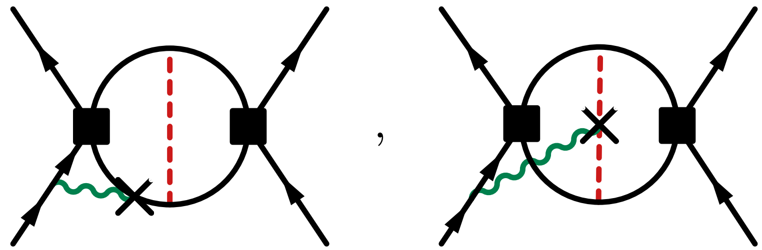
Subleading collinear corrections:

Collinear momenta of incoming partons: $(\lambda\sim\beta^2\ll 1)$

$$k_-\sim M,\ k_+\sim M\lambda,\ k_\perp\sim M\sqrt{\lambda}$$

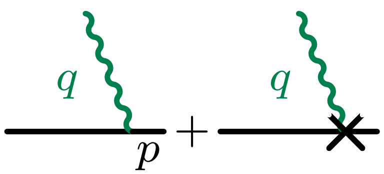
$\Rightarrow$ No correction $\sim\beta$ for incoming partons with $k_\perp=0$

Subleading soft $\otimes$ collinear corrections:



Related to three-parton colour correlations in IR singularities of amplitudes (Ferroglia et.al. 09)

Expansion of Hg_sH vertex: $\left(p = Mv + r,\ (r^0, \vec{r}) \sim (\lambda, \sqrt{\lambda}),\ q \sim \lambda\right)$:



$$: \quad \frac{v^\mu}{r^0+q^0-\frac{\vec{r}^2}{2M}} + \left[\frac{\vec{r}/M}{r^0+q^0-\frac{\vec{r}^2}{2M}} - v^\mu\,\frac{\vec{r}\cdot\vec{q}}{M}\left(\frac{1}{r^0+q^0-\frac{\vec{r}^2}{2M}}\right)^2\right] + \mathcal{O}(\beta^2)$$

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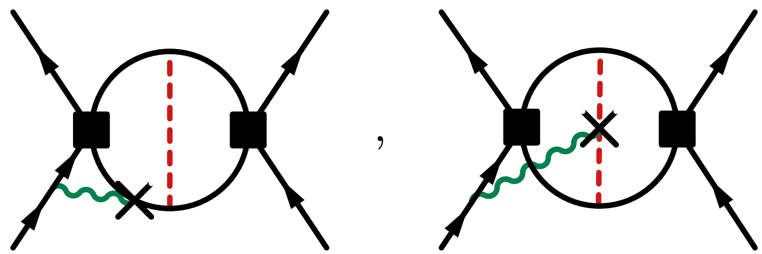

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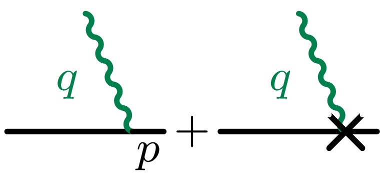


$$\frac{v^\mu}{r^0 + q^0 - \frac{\vec{r}^2}{2M}} + \left[\frac{\vec{r}/M}{r^0 + q^0 - \frac{\vec{r}^2}{2M}} - v^\mu \, \frac{\vec{r}\cdot\vec{q}}{M} \left(\frac{1}{r^0 + q^0 - \frac{\vec{r}^2}{2M}} \right)^2 \right] + \mathcal{O}(\beta^2)$$

$\Rightarrow$ Integrals of the form ($\vec{v} = 0$ in partonic CMS, no $\vec{q}$ in denominator)

$$\int d^D q \prod_i d^D r_i \, \frac{\{(\vec{k}_- \cdot \vec{r}_i)\ ,\ (\vec{q} \cdot \vec{r}_i)\}}{F(q^0, (k_- \cdot q), r_i^0, (\vec{r}_i + \vec{r}_j)^2)} = 0$$

Vanish since no external potential 3-momentum available!

Colour representations for process $pp' \to HH'$:

$$r \otimes r' = \sum_\alpha r_\alpha \, , \qquad R \otimes R' = \sum_{R_\alpha} R_\alpha$$

Physical picture: (Bonciani et.al. 98)

soft radiation off total colour charge C_{R_α} of HH' system

Construction of colour basis (Beneke, Falgari, CS 09)

- form pairs $P_i = (r_\alpha, R_\beta)$ of equivalent representations, e.g.

$$gg \to \tilde{q}\bar{\tilde{q}} \quad 8 \otimes 8 \to 3 \otimes \bar{3} : \qquad P_i \in \{(1,1),\ (8_S, 8),\ (8_A, 8)\}$$

- construct basis tensors from Clebsch Gordan coefficients:

$$c^{(i)}_{\{a\}} = \frac{1}{\sqrt{\mathrm{dim}(r_\alpha)}} C^{r_\alpha}_{\alpha a_1 a_2} C^{R_\beta *}_{\alpha a_3 a_4} \qquad \text{e.g. } c^{(3)}_{\{a\}} = \frac{i}{\sqrt{12}} f^{a_2 \alpha a_1} T^{\alpha}_{a_3 a_4}$$

⇒ Bases for all squark/gluino production processes $\tilde{q}\bar{\tilde{q}}$, $\tilde{q}\tilde{q}$, $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{g}$

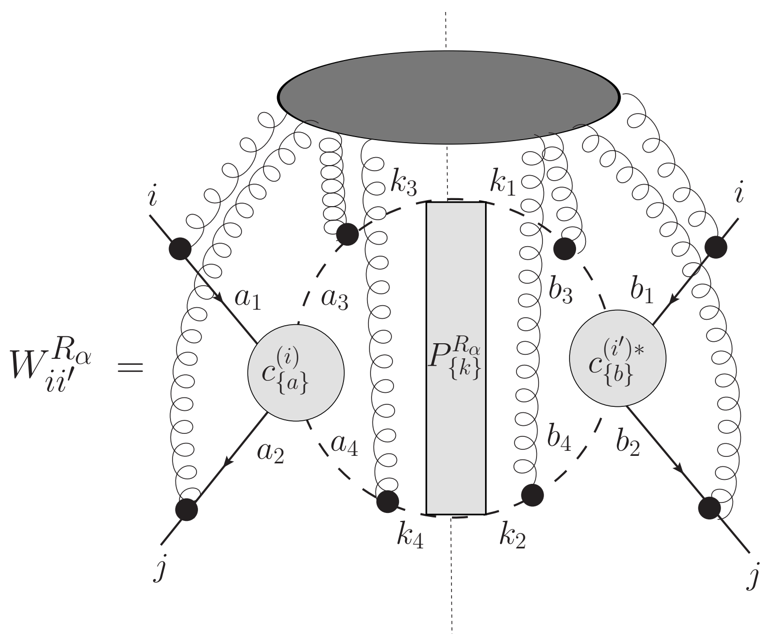
(equivalent to explicit one-loop results; Kidonakis/Sterman 97; Kulesza/Moytka 08, Beenakker et.al. 09)

Diagonalization of soft function:

$$W^{R_\alpha}_{ii'}(z_0) = \langle 0|\overline{\mathrm{T}}[S^4_v S^3_v c^{i'*} S^{1\dagger}_{\bar{n}} S^{2\dagger}_n](0) P^{R_\alpha} \mathrm{T}[S^1_n S^2_{\bar{n}} c^i S^{3\dagger}_v S^{4\dagger}_v](x_0)|0\rangle$$

Basis tensors and projectors from Clebsch-Gordan coefficients:

$$P^{R_\alpha} = C^{R_\alpha *} C^{R_\alpha}, \qquad c^{(i)} = C^{r_\alpha} C^{R_\beta *} \qquad (P_i = (r_\alpha, R_\beta), r_\alpha \sim R_\beta)$$



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Combine Wilson lines: $C^{R_\beta} S^3_v S^4_v = S^{R_\beta}_v C^{R_\beta}$

(Works only since both heavy particles have same velocity!)

$\Rightarrow$ reduce to soft function for single final-state particle in R_α:

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(Works only since both heavy particles have same velocity!)

$\Rightarrow$ reduce to soft function for single final-state particle in R_α:

$$W^{R_\alpha}_{ii'}(z_0) = \langle 0|\overline{\mathrm{T}}[S^{R_\alpha}_v C^{r_{\alpha'}*} S^{1\dagger}_{\bar n} S^{2\dagger}_n](0) \mathrm{T}[S^1_n S^2_{\bar n} C^{r_\alpha} S^{R_\alpha \dagger}_v](x_0)|0\rangle\, \delta_{R_\beta, R_{\beta'}}$$

$\Rightarrow$ soft function automatically block diagonal, e.g. $8 \otimes 8 \to 3 \otimes \bar{3}$

$P_i, P_{i'} = (1,1)$: single element $\qquad (8_S, 8),\ (8_A, 8) : 2 \times 2$ matrix

no $8_S/8_A$ interference (Bose symm.) $\Rightarrow$ $W^{R_\alpha}_{ii'}$ diagonal!

Evolution equations from reduction to $2 \to 1$ process

(adequate up to NNLL: scale independent potential function, no three parton correlations):

Hard function (from Becher/Neubert 09)

$$\frac{d}{d\ln\mu} H_i(M,\mu) = \left(\gamma_{\text{cusp}}(C_r + C_{r'}) \ln\left(\frac{4M^2}{\mu^2}\right) + 2(\underbrace{\gamma^r + \gamma^{r'}}_{\text{as for Drell-Yan/Higgs}} + \gamma^{R_\alpha}_{H,s})\right) H_i^S(M,\mu).$$

Soft anomalous dimension: (agrees with Czakon, Mitov, Sterman 09)

$$\gamma^{R_\alpha}_{H,s} = \frac{\alpha_s}{4\pi}(-2C_{R_\alpha}) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_{R_\alpha}\left[-C_A\left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3\right) + \frac{40}{18} n_f\right] + \mathcal{O}(\alpha_s^3).$$

(using Becher, Neubert 09; Korchemsky Radyushkin 92, Kidonakis 09)

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(using Becher, Neubert 09; Korchemsky Radyushkin 92, Kidonakis 09)

Soft function from scale independence of $\sigma = f_1 \otimes f_2 \otimes H \otimes W \otimes J$

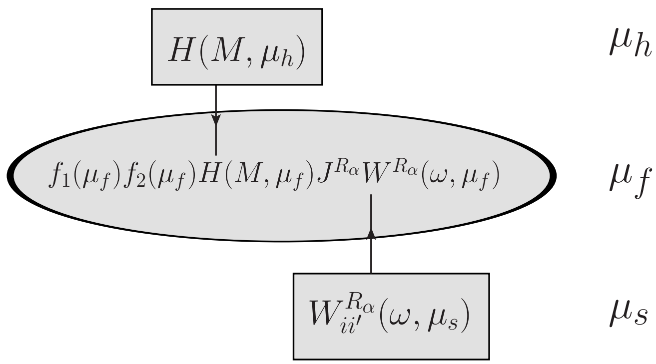
$$\frac{d}{d\log\mu} W_i^{R_\alpha}(z^0,\mu) = \left(2\gamma_{\text{cusp}}(C_r + C_{r'}) \log\left(\frac{iz_0\mu e^{\gamma_E}}{2}\right) - 2(\gamma_{H.s}^{R_\alpha} + \gamma_s^r + \gamma_s^{r'})\right) W_i^{R_\alpha}(z^0,\mu)$$

(same form as for Drell-Yan: Korchemsky, Marchesini 92)

Resummation of threshold logs

- Solution to RGE in momentum space (Becher/Neubert 06)
- evolve hard function from $\mu_h \sim Q \sim 2M$ to μ_f
- evolve soft function from μ_s to μ_f
- solution in Mellin-space corresponds to $\mu_s = Q/N$,$\mu_h = \mu_f$

(Korchemsky/Marchesini 92, Manohar 03, Becher/Neubert/Xu 07)



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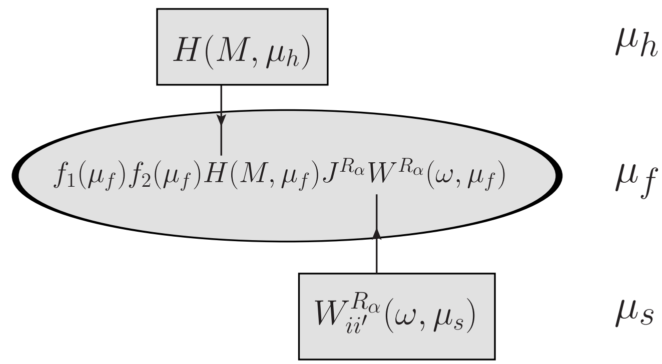

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(Korchemsky/Marchesini 92, Manohar 03, Becher/Neubert/Xu 07)

Coulomb resummation

NLL: use LO Coulomb-Green function $D_1 = -C_F$, $D_8 = \frac{1}{2N_C}$

$$J^{R_\alpha(0)}(E) \sim \text{Im}\left\{\sqrt{-\frac{E}{m_{\tilde{q}}}} - D_{R_\alpha}\alpha_s(\mu_C)\left[\frac{1}{2}\ln\left(-\frac{4m_{\tilde{q}}E}{\mu^2}\right) + \psi\left(1+\frac{\alpha_s(\mu_C)D_{R_\alpha}}{2\sqrt{-E/(m_{\tilde{q}})}}\right)\right]\right\}$$

Scale independent by itself $\Rightarrow$ evaluate at μ_C

Squark-antisquark production at NLL

Recent higher order results for squark-antiquark production:

- NLL soft gluon resummation in Mellin space (Kulesza/Motyka 08, Beenakker et.al. 09)
- Partial NNLO threshold approximation (Langenfeld/Moch 09)
- Coulomb resummation (Kulesza/Motyka 09)
  Bound state effects (Hagiwara/Yokoya 09)

Combined NLL soft/Coulomb resummation (Beneke, Falgari, CS)

- NLL solutions for H, W, LO resummed J
- Simplified setup: equal squark masses, exclude stop
- Matching to fixed order NLO results (Beenakker, Höpker, Spira, Zerwas 96, PROSPINO (Plehn et.al.), Langenfeld, Moch 09)

$$\hat{\sigma}_{pp'}^{\text{match}}(\hat{s}, \mu_f) = \left[\hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s}, \mu_f) - \hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s}, \mu_f)|_{\text{NLO}}\right] + \hat{\sigma}_{pp'}^{\text{NLO}}(\hat{s}, \mu_f)$$

Soft scale

- Soft corrections suggest $\mu_s \sim M\beta^2 \Rightarrow$ Landau pole problem

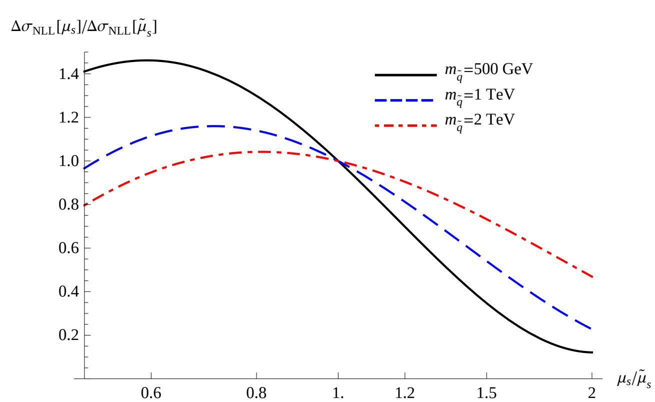
$\Rightarrow$ Default choice: (Becher, Neubert, Xu 07)

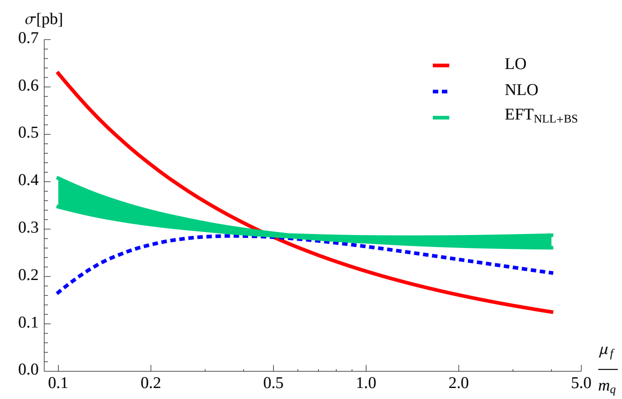
fixed $\tilde{\mu}_s$ that minimizes soft corrections to hadronic σ

$$(\tilde{\mu}_s(m_{\tilde{q}}=500\text{GeV}) = 240\text{GeV}, \dots \tilde{\mu}_s(m_{\tilde{q}}=2\text{TeV}) = 450\text{GeV})$$

Hard scale: $\tilde{\mu}_h = 2m_{\tilde{q}}$

Coulomb scale: $\mu_C = \max\{2m_{\tilde{q}}\beta, C_F m_{\tilde{q}}\alpha_s(\mu_C)\}$





($\sqrt{s} = 14$ TeV, MSTW08NLO, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$)

($m_{\tilde{q}} = 1$ TeV, $\mu_s^0/2 < \mu_s < 2\mu_s^0$)

Impact of higher order corrections

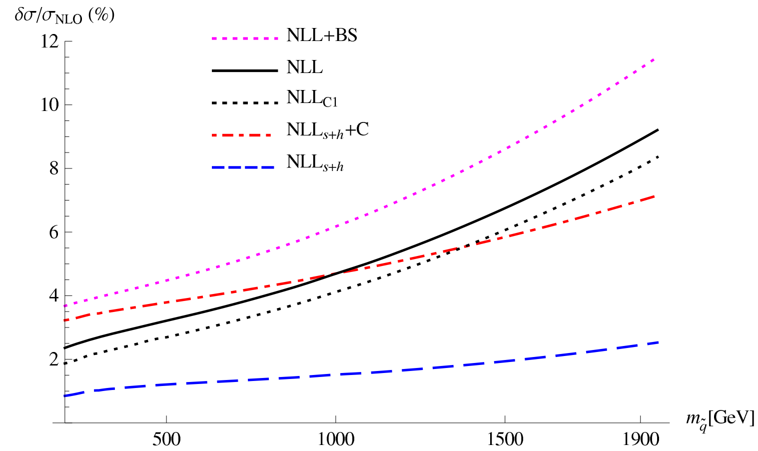
$\mathbf{NLL_{s+h}}$: resummation of H and W

C: Coulomb resummation

$\mathbf{NLL_{C1}}$: first Coulomb $\otimes$ res. soft

NLL: full Coulomb $\otimes$ res. soft

BS Bound state effects



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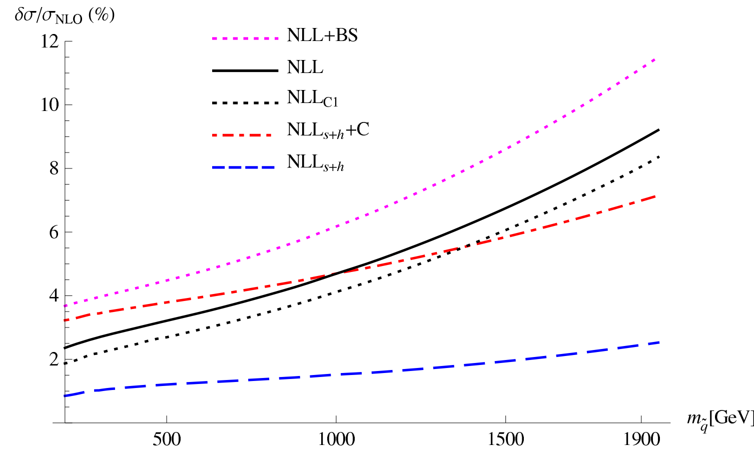


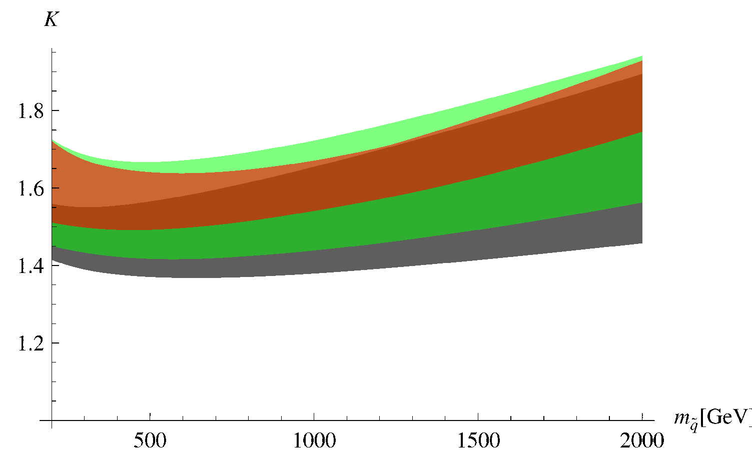
($\sqrt{s} = 14$ TeV, $m_{\tilde g}/m_{\tilde q} = 1.25$)

Scale uncertainty

NLO $\frac{m_{\tilde q}}{2} < \mu_f < m_{\tilde q}$

$\mathrm{NLL_{s+h}}$, **NLL** independent variation $\frac{m_{\tilde q}}{2} < \mu_f < 2m_{\tilde q}$, $m_{\tilde q} < \mu_h < 4m_{\tilde q}$, $\frac{\mu_s^0}{2} < \mu_s < 2\mu_s^0$

$\Rightarrow$ significant reduction only for combined resummation!



All threshold enhanced $\mathcal{O}(\alpha_s^2)$ terms (Beneke, Czakon, Falgari, Mitov, CS 09)

(Partial results: (Langenfeld)/Moch/Uwer 08/09)

- LL-NNLL soft corrections:

$$\sim \alpha_s^2(c_{\mathsf{LL}}^{(2)} \ln^4\beta + c_{\mathsf{NLL}}^{(2)}\ln^3\beta + c_{\mathsf{NNLL,2}}^{(2)}\ln^2\beta + \underbrace{c_{\mathsf{NNLL,1}}^{(2)}\ln\beta}_{\text{2-loop }\gamma_{H,s}})$$

- mixed Coulomb/soft, hard corrections

$$\sim \frac{\alpha_s}{\beta}\alpha_s(c_{\mathsf{LL}}^{(1)}\ln\beta^2 + c_{\mathsf{NLL}}^{(1)}\ln\beta + c + \underbrace{H^{(1)}}_{\text{process dependent}})$$

- 2nd Coulomb, NLO Coulomb/non-Coulomb potentials:

$$\alpha_s^2\,(\frac{c_{C^2}}{\beta^2} + \frac{1}{\beta}(c_{\mathsf{C,0}}^{(2)} + c_{\mathsf{C,1}}^{(2)}\log\beta) + \underbrace{c_{\mathsf{n\text{-}C}}^{(2)}\ln\beta}_{\text{spin-dependent}})$$

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Numerical impact of $\mathcal{O}(\alpha_s^2)$ correction on $t\bar{t}$ production at LHC:

	$\Delta\sigma_{\text{NNLO}} \sim 55$pb	($\approx 6\%$ of NLO)
correct $\gamma_{H,s}$ compared to LMU :	$\Delta\sigma \sim 6$pb	$(< 1\%)$
correct (α_s^2/β) terms :	$\Delta\sigma \sim 15$pb	$(\approx 2\%)$

Summary

EFT approach to $\log\beta$, β^{-n} resummation for $\sigma_{\rm tot}$

- use SCET+NRQCD to factorize soft and Coulomb gluons
- $\log\beta$ resummation from momentum space solution to RGEs
- subleading soft interactions not relevant at NNLL

Colour structure of soft function

- diagonal basis to all orders
- two-loop soft anomalous dimension

Application to squark-antisquark production

- combined Soft and Coulomb resummation
- reduced μ-dependence for $m_{\tilde{q}} > 500$ GeV, soft resummation alone not sufficient
- total corrections $4-10\%$ for $m_{\tilde{q}} = 300$ GeV-2 TeV

Threshold expansion to $\mathcal{O}(\alpha_s^2)$ of $t\bar{t}$ cross section

Bonus slides

Why perform resummation?

Resummation required for

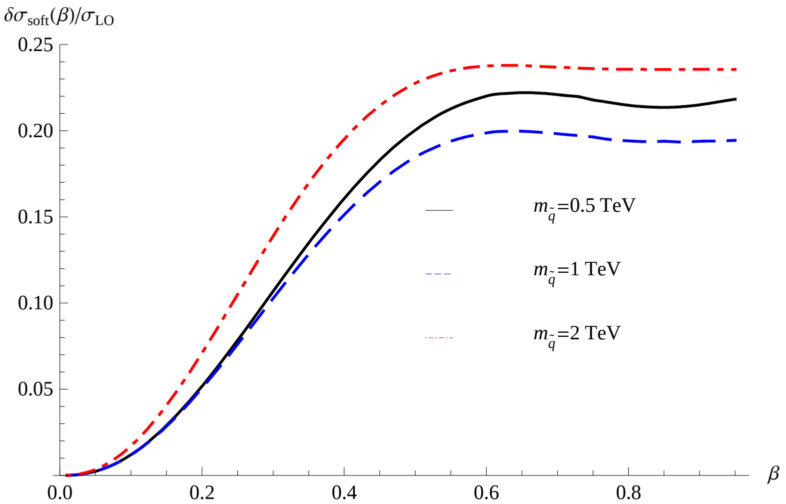
$$\alpha_s \log(\beta) \sim 1$$

Example: $\tilde{q}\bar{\tilde{q}}$-production

dominant contribution from

$$\beta > 0.2 \qquad \Rightarrow |\alpha_s \log \beta| \lesssim 0.2$$

$\Rightarrow$ resummation not mandatory



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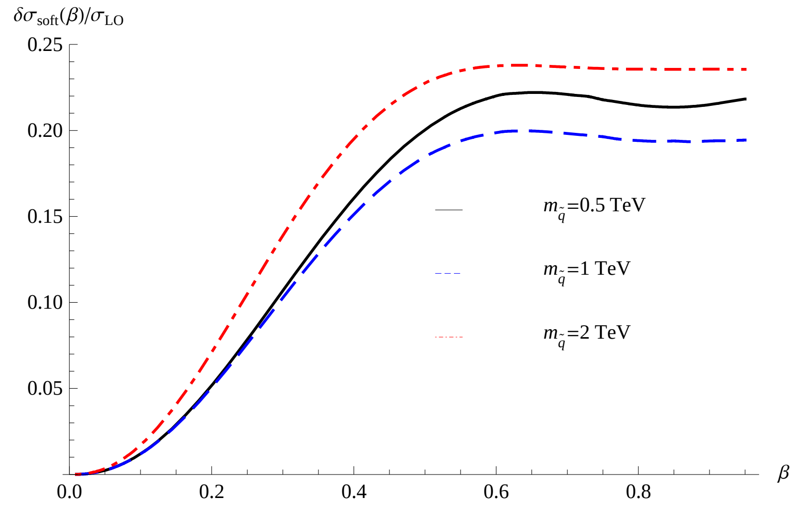
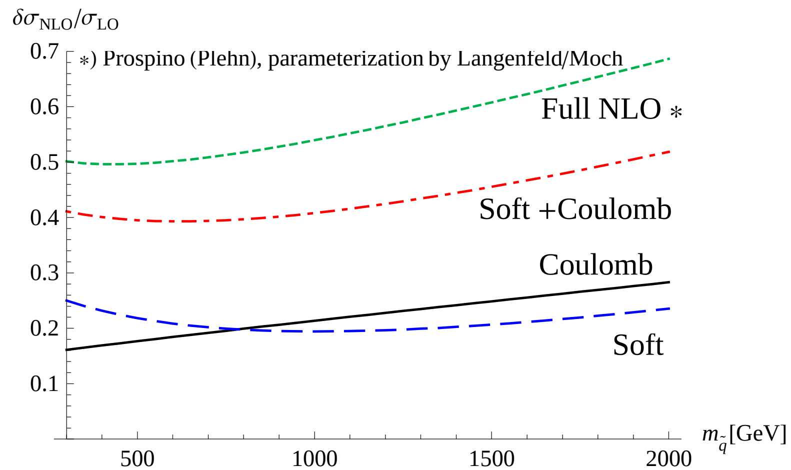


Is it useful?

- threshold terms bulk of NLO also for $\beta > 0.1$
- predict higher order terms
- reduce scale dependence

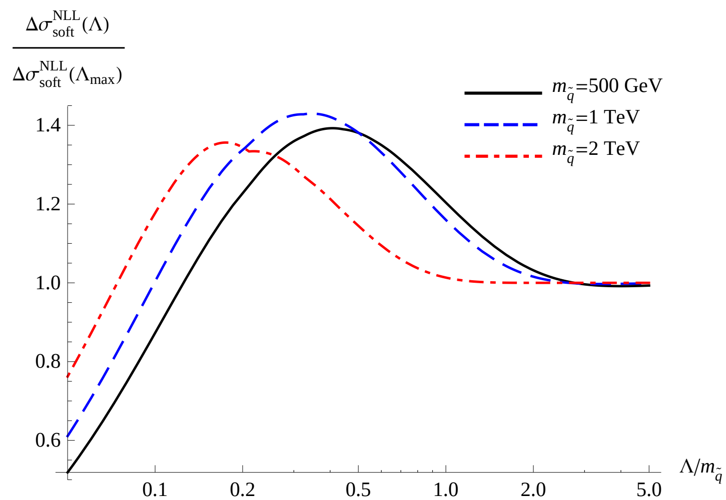


Factorization $\hat{\sigma} = H J \otimes W$ only valid near threshold

$\Rightarrow$ introduce cutoff $E = \sqrt{\hat{s}} - 2M < \Lambda$:

$$\Delta\hat{\sigma}^{\text{NLL}}_{pp'}(\hat{s},\Lambda) = \left[\hat{\sigma}^{\text{NLL}}_{pp'}(\hat{s}) - \hat{\sigma}^{\text{NLL}(1)}_{pp'}(\hat{s})\right]\theta(\Lambda - E)$$

Dependence of hadronic corrections on Λ:



Default choice: use $\Delta\hat{\sigma}^{\text{NLL}}_{pp'}$ for all $\hat{s}$ without cutoff

Hadronic squark anti-squark production:

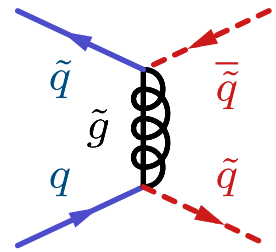
two partonic subprocesses: $q_i\bar{q}_j \to \tilde{q}_i\overline{\tilde{q}}_j$, $\quad gg \to \tilde{q}_i\overline{\tilde{q}}_j$

Matching to EFT:

$$\mathcal{A}_{pp'\to HH'X} = \sum_i C^{(i)}_{\{\alpha\}}(M,\mu)\, c^{(i)}_{\{a\}}\, \langle HH'X|\phi_{c;a_1\alpha_1}\phi_{\bar{c};a_2\alpha_2}\psi^{\dagger}_{a_3\alpha_3}\psi'^{\dagger}_{a_4\alpha_4}|pp'\rangle_{\mathrm{EFT}}$$

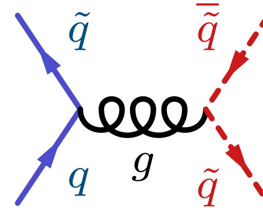
Example $q\bar{q}$ channel:

$$i\mathcal{A}^{(0)}_{q_i\bar{q}_j\to\tilde{q}_{iL}\overline{\tilde{q}}_{jR}}|_{\hat{s}=4m^2_{\tilde{q}}} = -\frac{i2g_s^2 m_{\tilde{g}}}{m^2_{\tilde{q}}+m^2_{\tilde{g}}} T^b_{a_3a_1}T^b_{a_2a_4}\bar{v}(m_{\tilde{q}}\bar{n})\left(\tfrac{1-\gamma^5}{2}\right)u(m_{\tilde{q}}n)$$



s-channel singlet/Octet colour basis

$$c^{(1)}_{\{a\}} = \frac{1}{N_c}\delta_{a_1a_2}\delta_{a_3a_4}, \qquad c^{(2)}_{\{a\}} = \frac{1}{\sqrt{2}}T^{\beta}_{a_2a_1}T^{\beta}_{a_3a_4}$$



$\Rightarrow$ Short-distance coefficients:

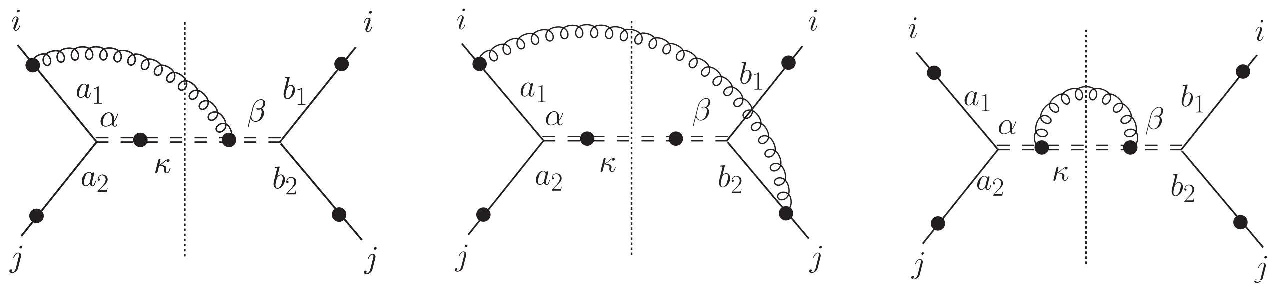
$$C^{(1)}_{\{\alpha\}} = (-C_F)\frac{4\pi\alpha_s m_{\tilde{g}}}{m^2_{\tilde{q}}+m^2_{\tilde{g}}}\left(\tfrac{1-\gamma^5}{2}\right)_{\alpha_1\alpha_2}, \qquad C^{(2)}_{\{\alpha\}} = \sqrt{\frac{C_F}{2N_C}}\frac{4\pi\alpha_s m_{\tilde{g}}}{m^2_{\tilde{q}}+m^2_{\tilde{g}}}\left(\tfrac{1-\gamma^5}{2}\right)_{\alpha_1\alpha_2}$$

Solution to RGE in position space:

$$W_i^{R_\alpha}(z^0,\mu_f) = \exp\left[-2S(\mu_s,\mu_f) + 2a_{\gamma_i^W}(\mu_s,\mu_f)\right]\left(\frac{iz_0\mu_s e^{\gamma_E}}{2}\right)^{2a_{\Gamma^r+\Gamma^{r'}}(\mu_s,\mu_f)} W_i^{R_\alpha}(z^0,\mu_s)$$

$$S(\mu_s,\mu) = -\int_{\alpha_s(\mu_s)}^{\alpha_s(\mu)} d\alpha_s \frac{\Gamma^r_{\text{cusp}}(\alpha_s)+\Gamma^{r'}_{\text{cusp}}(\alpha_s)}{2\beta(\alpha_s)} \int_{\alpha_s(\mu_h)}^{\alpha_s} \frac{d\alpha_s'}{\beta(\alpha_s')}, \quad a_\gamma(\mu_s,\mu) = -\int_{\alpha_s(\mu_s)}^{\alpha_s(\mu)} d\alpha_s \frac{\gamma(\alpha_s)}{\beta(\alpha_s)}.$$

One loop result (For octet final state Idilbi/Kim/Mehen 09)



$$W_{ii'}^{(1)R_\alpha}(L,\mu) = \delta_{ii'}\left[(C_r + C_{r'})\left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon}L + L^2 + \frac{\pi^2}{6}\right) + 2C_{R_\alpha}\left(\frac{1}{\epsilon} + L + 2\right)\right]$$

with $L = 2\log\left(\frac{iz_0\mu e^{\gamma_E}}{2}\right)$, $(\mathbf{T}^{(R)a}\mathbf{T}^{(R)a})_{a_1a_2} = C_R\,\delta_{a_1a_2}$.

Nonvanishing imaginary part of singlet Coulomb Green function below thrheshold:

$$J_1^{(0)\mathrm{bound}}(E) = 2\sum_{n=1}^{\infty} \delta(E+E_n)\left(\frac{m_{\tilde{q}}\alpha_s(\mu_C)C_F}{2n}\right)^3 \qquad E<0$$

with the bound state energies

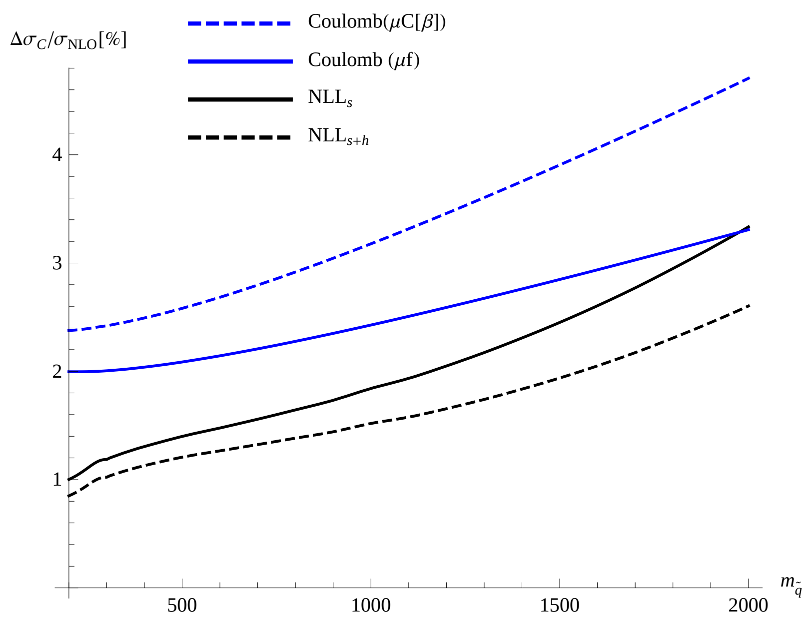
$$E_n = \frac{m\alpha_s^2 C_F^2}{4n^2}$$

Correction to cross section:

$$\Delta\sigma_{\mathrm{bound}} = \sum_{pp'}\sum_{n} \mathcal{L}_{pp'}(z_n,\mu_f)\hat{\sigma}^{(0)}_{pp'(1)}(s z_n)\frac{\pi}{m_{\tilde{q}}^2}\sqrt{\frac{z_n}{s}}\left(\frac{m_{\tilde{q}}\alpha_s(\mu_C)C_F}{n}\right)^3$$

with $z_n = (2m_{\tilde{q}} + E_n)^2/s$.

Good agreement with independent calculation using Mellin-space resummation for appropriate choice of scales ($\mu_h = \mu_C = \mu_f$)



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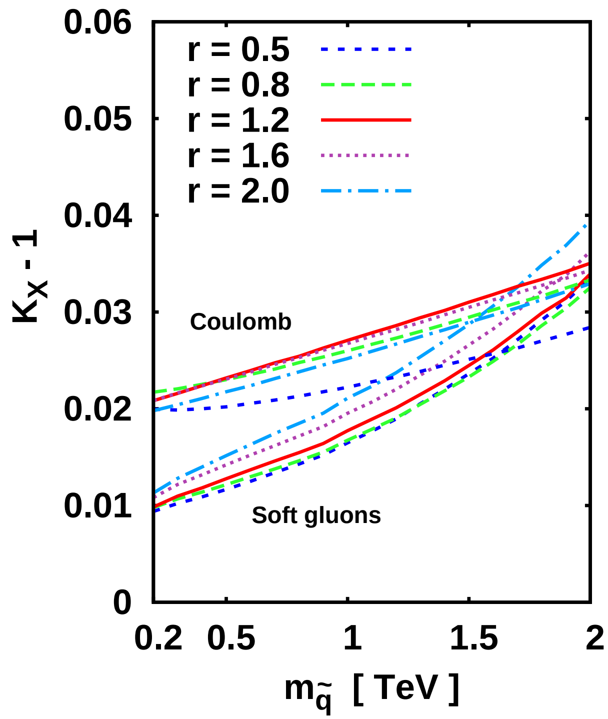

(Kulesza, Motyka 09)