Soft gluon k_T-resummation and total cross-sections in the asymptotic limit

With A. Grau.R. M. Godbole. Y.N. Srivastava

G. Pancheri

INFN Frascati National Laboratories

[and visiting MIT CTP]

SCET, Ringberg, April 9, 2010

Large distances: total cross-section as choice phenomenological observable

Total cross-sections A+ B → all

- Large Distances dominate most of the scattering process
- Data are available over a wide energy range

$$\sqrt{s} \sim 5\ GeV \div 50\ TeV$$

Revisiting k_T-resummation

Very large b-values require going into the Infrared region (IR)

↓

- Revisit k_t -resummation and extend soft gluon integration down to to the InfraRed region (IR)

- We exploit the IR limit with an ansatz inspired by the Richardson potential

Revisit soft k_t - Resummation

$\mathbf{K}_\perp$ Overall transverse momentum carried by all soft gluons emitted in a given process

$$d^2P(\mathbf{K}_\perp) = d^2\mathbf{K}_\perp \frac{1}{(2\pi)^2} \int d^2\mathbf{b}\; e^{-i\mathbf{K}_\perp\cdot\mathbf{b} - h(b,E)}$$

Enforces momentum Conservation
Between overall and all the soft
gluons emitted through various
independent processes

$$h(b,E) = \int d^3\bar{n}(k)[1 - e^{i\mathbf{k}_t\cdot\mathbf{b}}]$$

Process dependent single soft gluon distribution,

The single soft gluon integral
h(b,E)

[Dokshitzer, Dyakonov, Troyan, Parisi,Petronzio 1978-79]

$$h(b,E) = c_0(\mu,b,E) + \Delta h(b,E),$$

$$\Delta h(b,E) = \frac{16}{3}\int_{\mu}^{E} \frac{\alpha_s(k_t^2)}{\pi}[1 - J_o(bk_t)]\frac{dk_t}{k_t}\ln\frac{2E}{k_t}.$$

$$J_0(bk_t) \approx 0 \quad k_t > 1/b$$

$$\approx \frac{16}{3}\int_{1/b}^{E} \frac{\alpha_s(k_t^2)}{\pi}\frac{dk_t}{k_t}\ln\frac{2E}{k_t}$$

$$N_f = 4$$

Upon exponentiation $e^{-h_{eff}(b,E)} = [\frac{ln(1/b^2\Lambda^2)}{ln(E^2/\Lambda^2)}]^{(16/25)ln(E^2/\Lambda^2)}$

Our proposal for probing the IR

Use the full integration range

$$h(b,E) = \frac{16}{3}\int_0^E \frac{\alpha_s(k_t^2)}{\pi}\frac{dk_t}{k_t}\log\frac{2E}{k_t}[1-J_0(k_t b)]$$

with a singular but integrable expression for $\alpha_s(k_t^2)$ inspired by the Richardson potential for a dressed gluon that exhibits confinement

$$\alpha_s(k_t^2) \xrightarrow[k_t^2 \ll \Lambda^2]{} \tilde{\alpha}_s(k_t^2) = \frac{B}{(k_t^2/\Lambda^2)^p}$$

N.B. p<1 for integral to converge

Our singular coupling with p< 1 allows to perform the soft k_t integral down to $k_t \approx 0$

$$h(b,E) = \frac{16}{3}\int_0^E \frac{\alpha_s(k_t^2)}{\pi}\frac{dk_t}{k_t}\log\frac{2E}{k_t}[1-J_0(k_t b)]$$

At very large distances

$$1 - J_0(bk_t) \approx (1/4)\ b^2 k_t^2 \qquad k_t b << 1$$

$$b > \frac{1}{\Lambda} > \frac{1}{E}$$

$$h(b,E,\Lambda) = constant\ (b^2\Lambda^2)^p\left[2\ln(2Eb) + \tfrac{1}{1-p}\right] + \ double\ logs$$

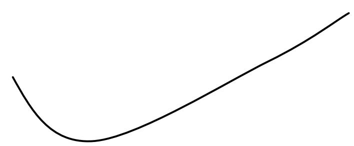
Comparing DDT vs. a soft k_t integral with singular coupling in the IR

$$e^{-h(b,E)}\Big|_{DDT} = e^{-c_0(\mu,b,E)}\left[\frac{\ln(1/b^2\Lambda^2)}{\ln(E^2/\Lambda^2)}\right]^{(16/25)\ln(E^2/\Lambda^2)}$$

$$e^{-h(b,E)}\Big|_{ours} = e^{-(b\bar{\Lambda})^{2p}}$$

How to use our ansatz: study x-sections in asymptotic energy region

- What makes the cross-section rise?



- What makes the cross-section rise within the limits imposed by the Froissart bound?

Outline of argument

- In an eikonal mini-jet model, soft gluon k_t-resummation down to zero gluon momenta, can reduce the strong power-like rise due to increasing number of low-x gluon collisions [aka minijet cross-section].

- We use our phenomenological expression for the soft gluon k_t spectrum which is singular but integrable in the infrared

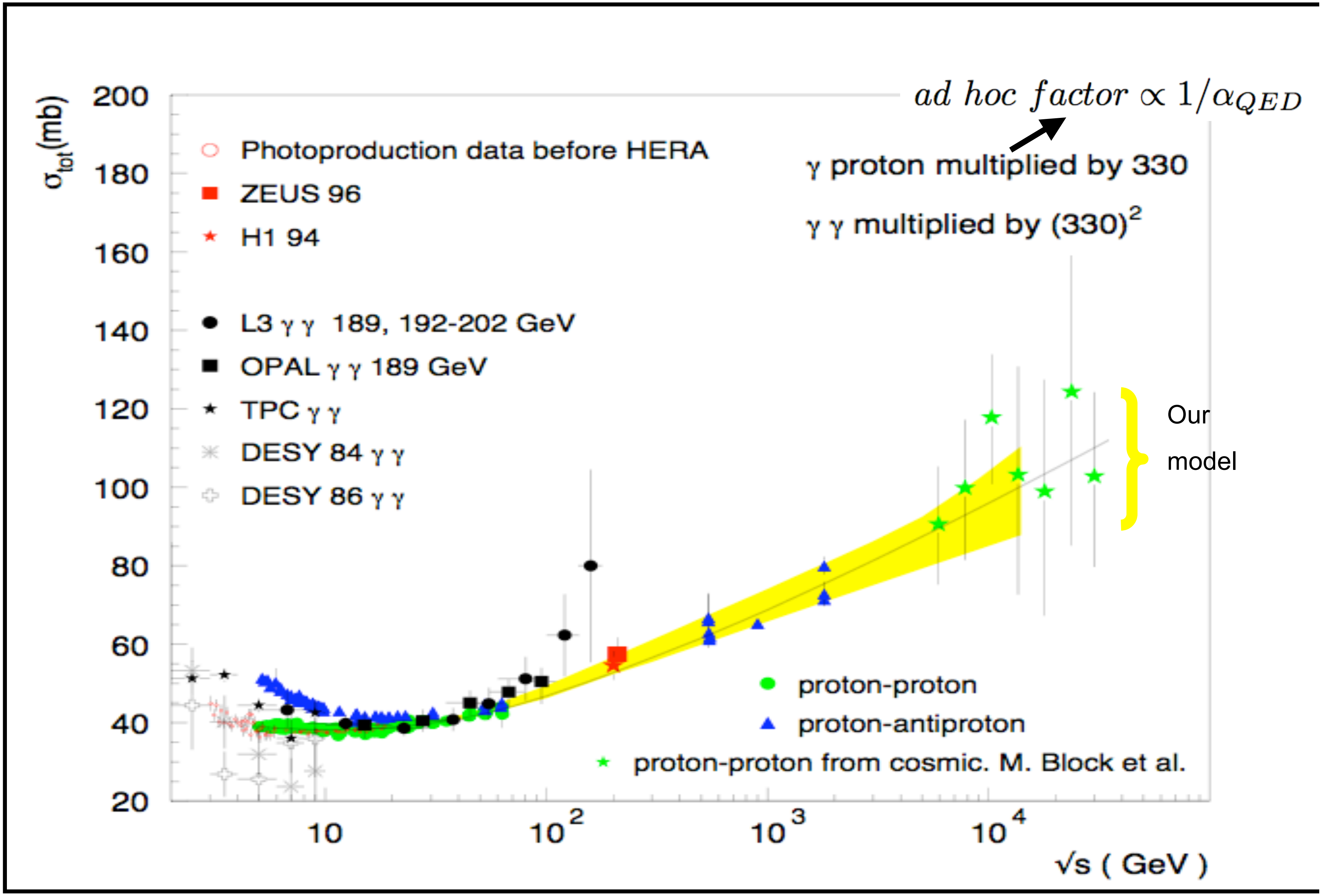
- We construct an explicit model to link confinement and the behaviour dictated by the Froissart bound:

$$\sigma_{total} < [\ln s]^{1/p} \qquad V(r) \simeq r^{2p-1}$$

- For 1/2 <p<1 and neglecting $\ln \ln s$ terms,

$$\sigma_{total} \lesssim C \ln^2 s$$

Total cross-section data for protons and photons



For total x-section one needs a model: Eikonal representation

$$\sigma_{elastic} = \int d^2\vec{b}|1 - e^{i\chi(b,s)}|^2$$

Use Optical Theorem

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\Im m\chi(b,s)} cos\Re e\chi(b,s)]$$

$$\sigma_{total\ \ inelastic} = \int d^2\mathbf{b}[1 - e^{-2\Im m\chi(b,s)}]$$

- Models for inelastic collisions can give
- Then use $\Re e\chi \approx 0$

A simple model to include QCD mini-jets and implement resummation in total cross-sections

$$P_{all\ inelastic\ collisions}(\{n(b,s)\}) =$$

$$\sum_{n=1} \frac{\bar{n}(b,s)^n}{n!} exp[-\bar{n}(b,s)] = 1 - exp[-\bar{n}(b,s)]$$

$$\bar{n}(b,s) = 2\Im m\chi(b,s) \approx n_{soft} + n_{hard-minijets}$$

$$n_{hard-minijets}(b) \approx A(b,s)\sigma_{jet}(s, p_{tmin})$$

Mini-jets drive the rise of σ_{total}

$$n_{hard-minijets}(b) \approx A(b,s)\sigma_{jet}(s,p_{tmin})$$

$$\sigma_{\rm jet}^{AB}(s,p_{tmin}) = \int_{p_{tmin}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \times \sum_{i,j,k,l} f_{i|A}(x_1,p_t^2) f_{j|B}(x_2,p_t^2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t}$$

$$p_{tmin} \sim 1 \div 2\ GeV$$

DGLAP evoluted PDF

Parton-parton x-sections:

$$parton_i + parton_j \rightarrow parton_k(p_t) + parton_l(-p_t)$$

$$\sigma_{jet}(s, p_{tmin}) \approx s^{\epsilon} \qquad \epsilon \simeq 0.3$$

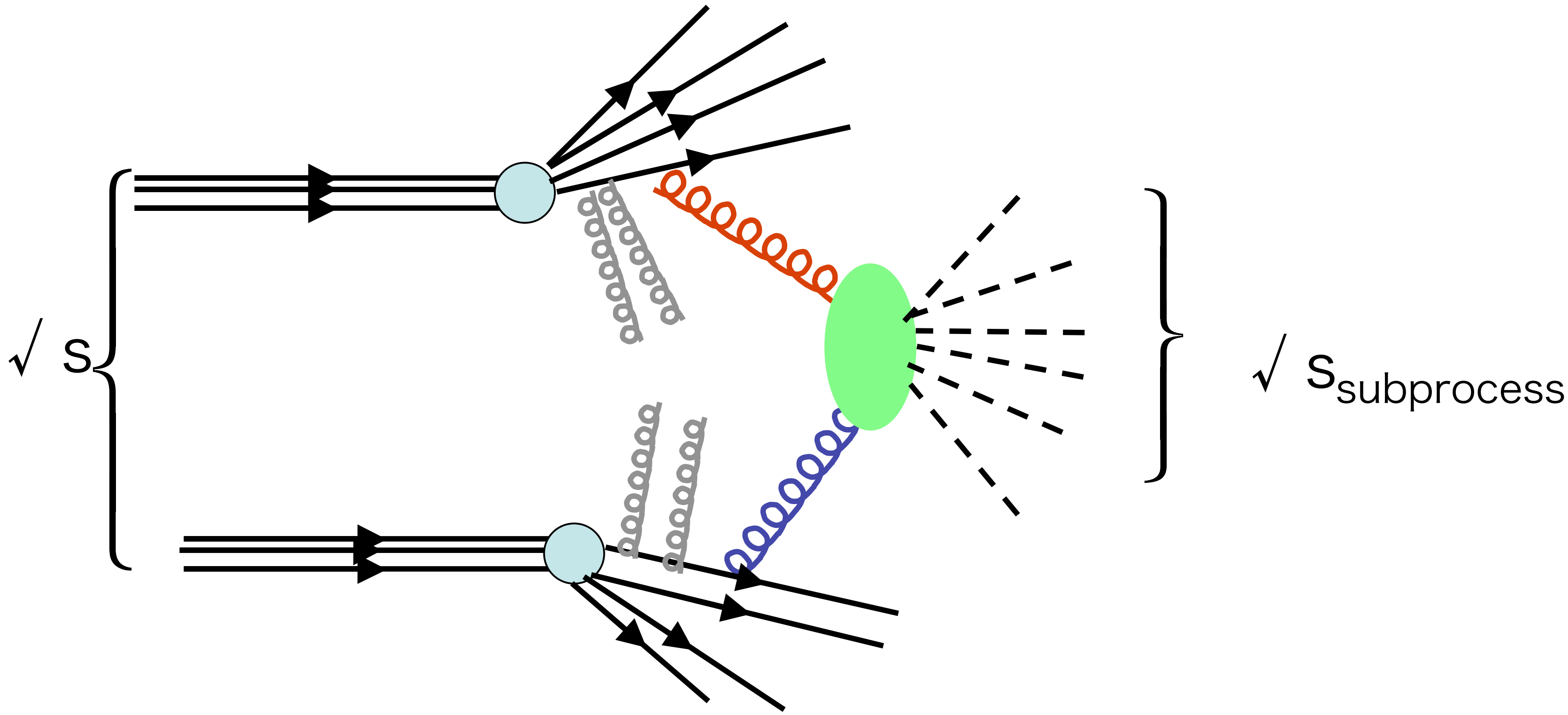
The Hard component of scattering
assumed to be responsible
for the rise of the total cross-section

rises much too fast

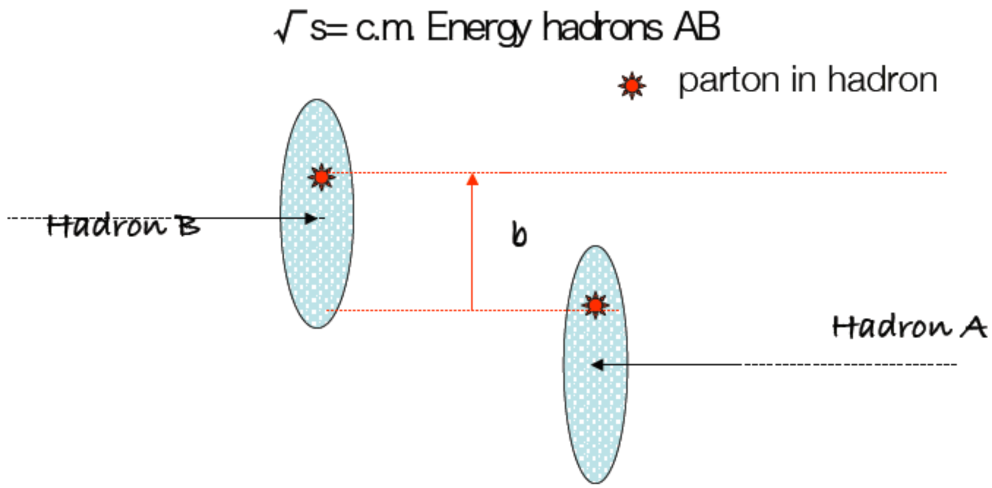
and

Violates the Froissart bound

One component missing in the mini-jet picture is soft gluon emission from the initial state to break the collinearity and reduce the parton-parton cross-section



The impact parameter distribution in our model is F-transform of ISR soft K_T distribution



$$A_{BN}(b,s) = N \int d^2\mathbf{K}_\perp \, e^{-i\mathbf{K}_\perp \cdot \mathbf{b}} \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b} \; e^{-h(b,q_{max})}}$$

$$h(b,q_{max}) = \frac{16}{3}\int_0^{q_{max}} \frac{\alpha_s(k_t^2)}{\pi}\frac{dk_t}{k_t}\log\frac{2q_{\max}}{k_t}[1-J_0(k_t b)]$$

The role of Soft gluon k_t-resummation in our model for total x-sections

- We implement soft gluon resummation of ISR and change the fast rise of mini-jet cross-sections into a smooth behaviour

- how is this possible?

- By exploiting the IR limit, as we showed before

σ_{total} AND THE LARGE-S LIMIT

$$2\Im m\chi = n_{soft} + n_{hard-minijets} \qquad Re\chi \approx 0$$

$$\sigma_{total} = 2\int d^2\vec{b}[1 - e^{-n_{soft}-n_{hard-minijets}}]$$

$$n_{hard-minijets}(b) \approx A(b,s)\sigma_{jet}(s,p_{tmin}) \qquad >> n_{soft}$$

$$A(b,s) \propto e^{-(bq)^{2p}} \qquad \sigma_{jet} \sim s^{\varepsilon}$$

Ultra-soft gluons effects bring cut-off in b-space

Mini-jets increase with energy

$$\sigma_{total} \to 2\pi\int db^2[1 - e^{-C(s)e^{-(bq)^{2p}}}]$$

$$C(s) = (s/s_0)^{\varepsilon}\sigma_1$$

At very large energy: from power law to log behaviour

$$\sigma_T(s) \approx \frac{2\pi}{p}\frac{1}{\bar{\Lambda}^2}\int_0^\infty du u^{1/p-1}[1-e^{-C(s)e^{-u}}]$$

$u = (\bar{\Lambda} b)^{2p}$ $\qquad$ $I(u,s) = 1 - e^{-C(s)e^{-u}}$ has the limits

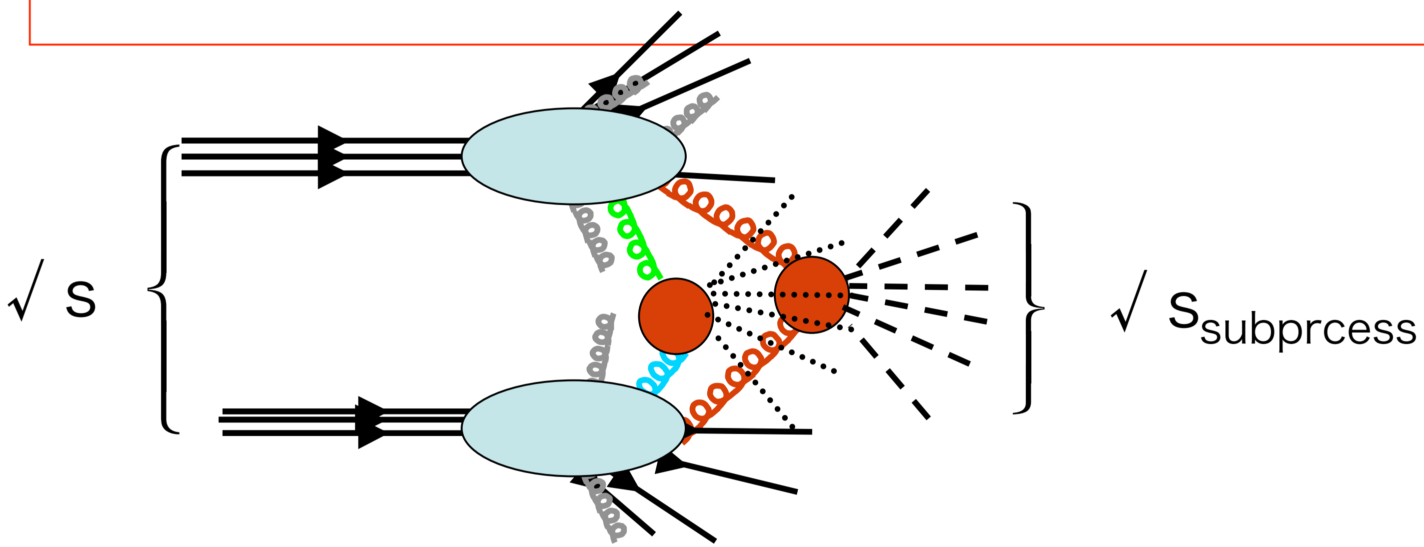
$I(u,s) \rightarrow 1$ at $u = 0$

$I(u,s) \rightarrow 0$ as $u = \infty$

$$\sigma_T \approx \frac{2\pi}{\bar{\Lambda}^2}[\varepsilon \ln \frac{s}{s_0}]^{1/p}$$

- $\sim \ln^2 s \qquad p = 1/2$
- $\sim \ln s \qquad p = 1$

How to implement this model for σ_{TOTAL}



- Eikonal representation (unitarity and multiple scattering)
- QCD minijets drive the rise
- Soft Gluon k_t-resummation (ISR) in the infrared main new ingredient of this model

Scales in this model

- $p_t \geq p_{tmin} \approx 1 \div 2\ GeV$

  lower cut-off for QCD mini-jets

- $k_t \leq q_{max} \approx p_{tmin}$ for single soft gluons [to be resummed]

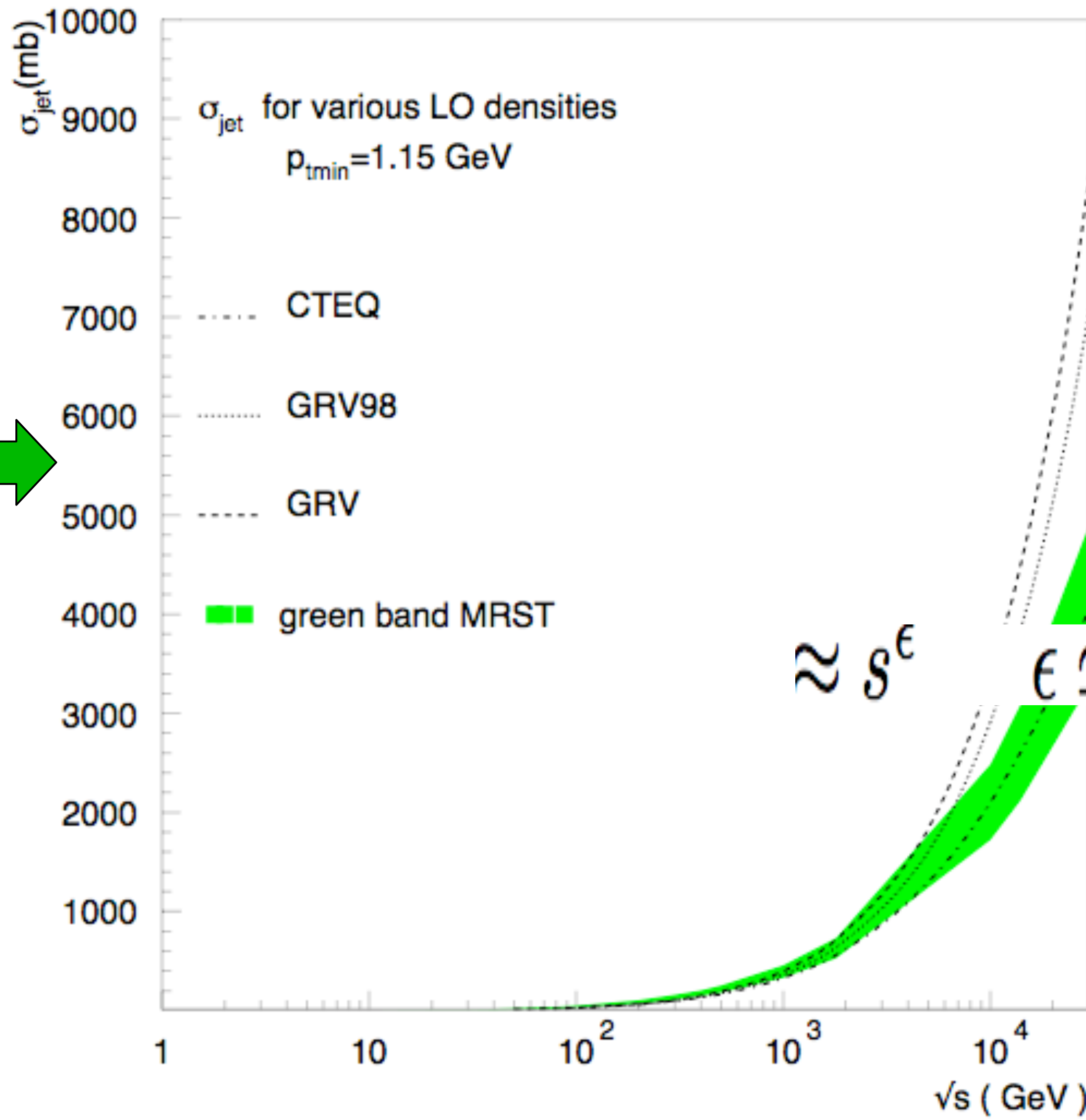
- $k_t \leq \Lambda \approx \Lambda_{QCD}$ for ultra soft gluons

1. Choose PDFs and p_{tmin}

and calculate

$\sigma_{jet}(s, p_{tmin})$

HARD COMPONENT OF SCATTERING RESPONSIBLE FOR THE RISE OF THE TOTAL CROSS-SECTION

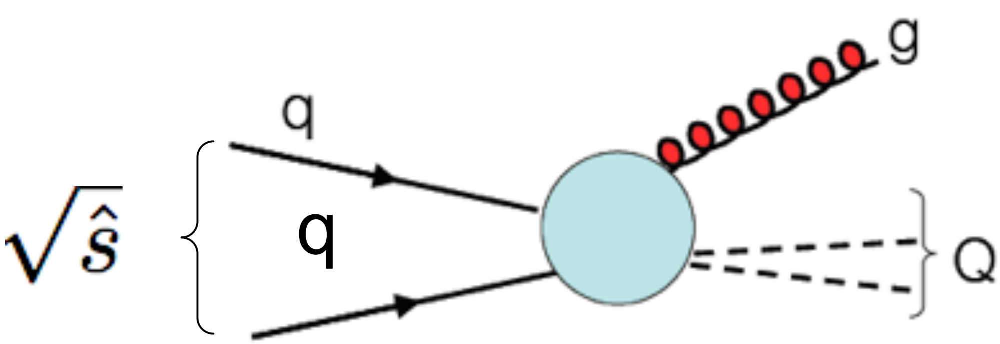


$\approx s^{\epsilon} \quad \epsilon \simeq 0.3$

2. Calculate $A(b,s) \sim e^{-h(b,q_{max})}$

q_{max} single soft gluon Integration limit

THE SINGLE SOFT GLUON INTEGRATION LIMIT CAN BE OBTAINED FROM KINEMATICS

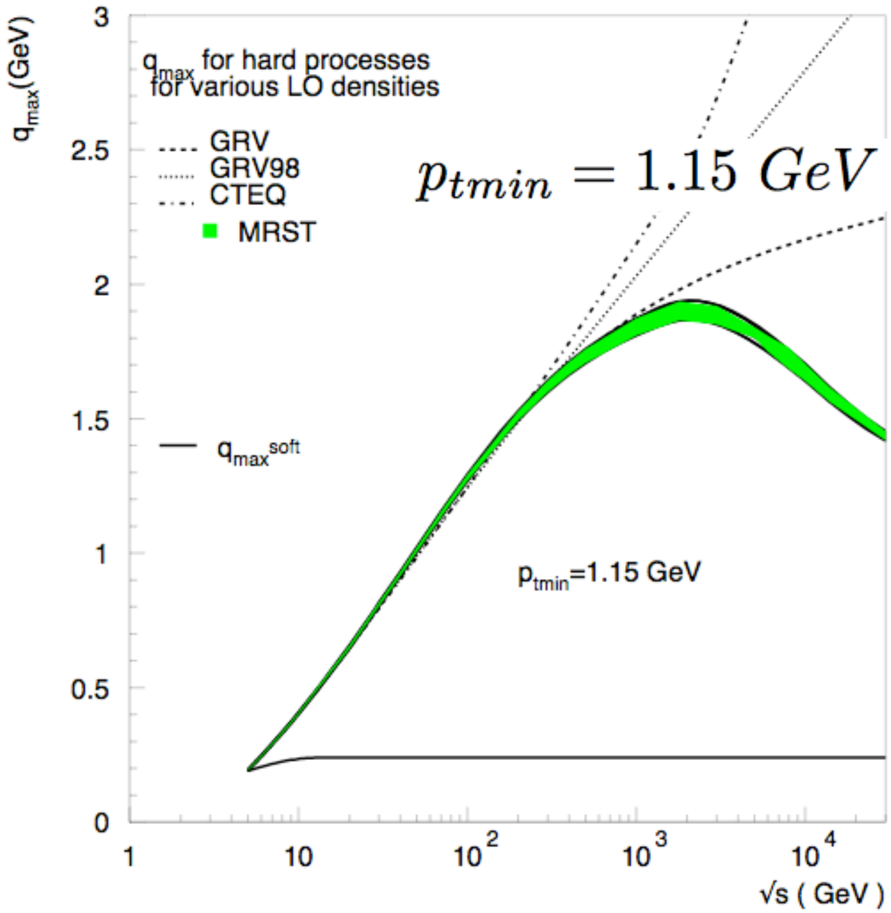


$$q_{max} = \frac{\sqrt{\hat{s}}}{2}\left(1 - \frac{Q^2}{\hat{s}}\right)$$

Calculate

$$q_{max}(s) = \frac{\sqrt{s}}{2} \frac{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i|A}(x_1) \int \frac{dx_2}{x_2} f_{j|B}(x_2) \sqrt{x_1 x_2} \int_{z_{min}}^{1} dz(1-z)}{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i|A}(x_1) \int \frac{dx_2}{x_2} f_{j|B}(x_2) \int_{z_{min}}^{1} (dz)}$$

A new energy parameter, the single soft gluon Integration limit Is introduced

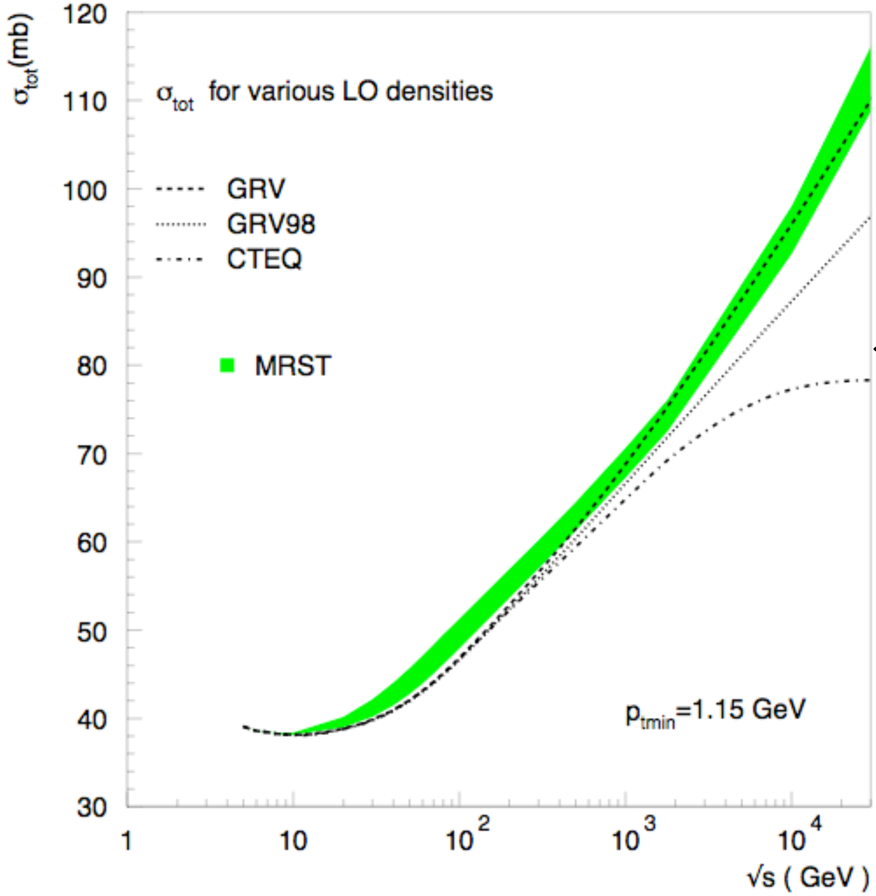


Average over same PDF as for sigma_{jet}

3.

Eikonalize with Re χ (b)=0

p_{tmin} =1.15 GeV and a chosen set of low energy parameters



GRV, MRST densities

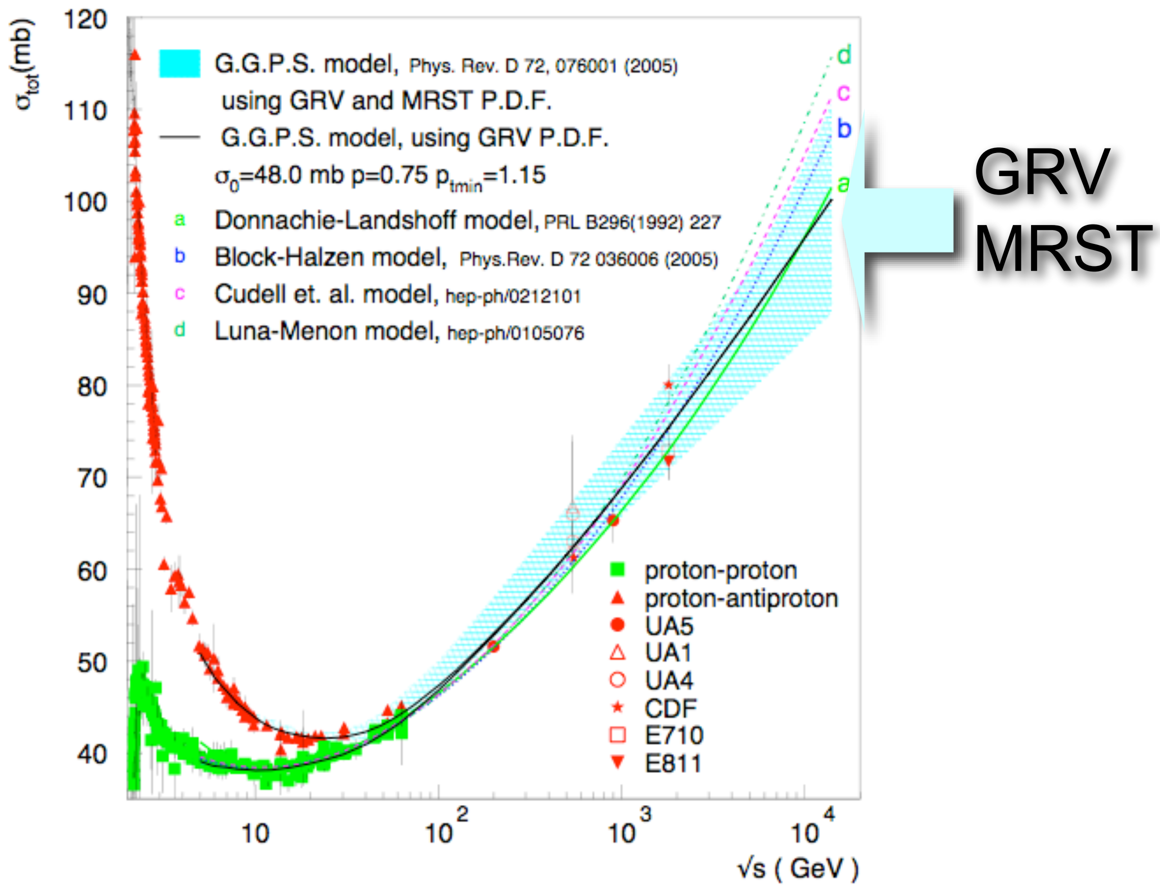
CTEQ densities

Results for the model for protons

BN model was developed from

1. QCD minijets to drive the rise (1985)
2. Soft gluon resummation to tame the rise (1996)

A.Grau,R.Godbole, GP,Y.N.Srivastava
PLB 1996, PRD1999,PRD 2005, PLB 2008,EPJC2009, PLB2009



Conclusions

- We have constructed a model in which soft gluon k_t-resummation and InfraRed (IR) gluons are linked to the energy dependence of total hadronic cross-sections in the very high energy limit.
- The model for the total cross-section embeds QCD mini-jets in the eikonal representation and exploits infrared gluons to change the rapidly rising mini-jet cross-sections into the observed high energy smooth behaviour.
- In this model, consistency of the energy dependence of the cross-section with the Froissart bound is directly related to the behaviour of the coupling of gluons to quarks in the infrared(IR) region.
- Our predictions for the asymptotic behaviour are shown to be related to the ansatz that the IR behaviour of the very soft gluon kt-distribution follows an inverse power law, in agreement with a Richardson-type potential model.
- Phenomenological applications to proton and photon total cross-sections support this model.

The IR region

- Dokshitzer, Dyakonov, Troyan, Parisi,Petronzio 1978-79

$$h(b,E) = c_0(\mu,b,E) + \Delta h(b,E) \quad (1)$$

$$\Delta h(b,E) = \frac{16}{3}\int_{\mu}^{E} \frac{\alpha_s(k_t)}{\pi}[1 - J_o(bk_t)]\frac{dk_t}{k_t} \ln\frac{2E}{k_t} \quad (2)$$

- Dropping the J_0, i.e. the IR region

$$e^{-h_{eff}(b,E)} = [\frac{ln(1/b^2\Lambda^2)}{ln(E^2/\Lambda^2)}]^{(16/25)ln(E^2/\Lambda^2)}$$

Richardson potential

- $$V(r) \to \mathcal{F}ourier[\tilde{V}(Q^2)]$$

such as to include

- Asymptotic freedom
- Linear quark confinement $V(r) \sim const \times r$

$$V(Q^2) = \frac{4}{3}\frac{\alpha_s(Q^2)}{Q^2} \to \quad \tilde{V}(Q^2) = \frac{12\pi}{33-2N_f}\frac{1}{Q^2}\frac{1}{\ln[1+Q^2/\Lambda^2]}$$

$$\sim \frac{1}{(Q^2)^2} \qquad Q^2/\Lambda^2 << 1$$

[Richardson inspired] Power law behaviour for the infrared region

$$\hat{V}(Q,p) = (\frac{K}{Q^2})\tilde{\alpha}_s(Q^2) = (\frac{K}{Q^2})[\frac{B}{(Q^2/\Lambda^2)^p}]$$

- For very large r

$$V(r,p) \sim_{r\to\infty} (r\Lambda)^{2p-1}$$

- p = 1 linearly rising
- p = 1/2 rising like ln r
- p = 0 Coulomb potential

We use a functional form for the coupling of ultra-soft gluons to the quark current inspired by the confining part of the Richardson potential

$$\tilde{\alpha}_s(k_t^2) = \frac{B}{(k_t^2/\Lambda^2)^p} \qquad k_t^2 \ll \Lambda^2$$

p=1 linearly rising Richardson potential for $k_t << \Lambda$
p < 1 for soft gluon integral to converge
p > 1/2 for a confining potential

Our phenomenological choice to match with higher

$$\alpha_s(k_t^2) = \frac{12\pi}{33 - 2N_f} \frac{p}{\ln[1 + p(\frac{k_t}{\Lambda})^{2p}]}$$

$k_t^2 \gg \Lambda^2 \qquad p \lesssim 1$

Usual one loop AF expression

Revisiting the soft gluon integral

Use the full integration range

$$h(b,E) = \frac{16}{3}\int_0^E \frac{\alpha_s(k_t^2)}{\pi}\frac{dk_t}{k_t}\log\frac{2E}{k_t}[1-J_0(k_t b)]$$

And compare two different phenomenological choices

singular but integrable α_s

$$\alpha_s(k_t^2) = \frac{12\pi}{33-2N_f}\frac{p}{\log[1+(k_t^2/\Lambda^2)^p]}$$

$$\to_{k_t\to 0} \left(\frac{\Lambda}{k_t}\right)^{2p}$$

frozen α_s

$$\alpha_s(k_t^2) = \frac{12\pi}{33-2N_f}\frac{1}{\log[a^2+(k_t^2/\Lambda^2)^2]}$$

$$\to_{k_t\to 0} \ constant$$

Approximatex expression for the case α_s singular

For very large distances $b > 1/\Lambda > 1/E$

$$h(b,E,\Lambda) = \frac{2c_F}{\pi} \times \qquad (1)$$

$$\left[\bar{b}\frac{b^2\Lambda^{2p}}{2}\int_0^{\frac{1}{b}} \frac{dk}{k^{2p-1}} \ln\frac{2E}{k} + 2\bar{b}\Lambda^{2p}\int_{\frac{1}{b}}^{N_p\Lambda} \frac{dk}{k^{2p+1}} \ln\frac{E}{k} + \bar{b}\int_{N_p\Lambda}^{E} \frac{dk}{k} \frac{\ln\frac{E}{k}}{\ln\frac{k}{\Lambda}}\right]$$

$$= \frac{2c_F}{\pi}\left[\frac{\bar{b}}{8(1-p)}(b^2\Lambda^2)^p\left[2\ln(2Eb) + \frac{1}{1-p}\right] + \frac{\bar{b}}{2p}(b^2\Lambda^2)^p\left[2\ln(Eb) - \frac{1}{p}\right] + \frac{\bar{b}}{2pN_p^{2p}}\left[-2\ln\frac{E}{\Lambda N_p} + \frac{1}{p}\right] + \bar{b}\ln\frac{E}{\Lambda}\left[\ln\frac{\ln\frac{E}{\Lambda}}{\ln N_p} - 1 + \frac{\ln N_p}{\ln\frac{E}{\Lambda}}\right]\right]$$

Frozen α_s gives nothing new of course

$$
\begin{aligned}
b > \frac{1}{a\Lambda} &> \frac{1}{M} \\
h(b,M,\Lambda) &= (constant)\ln(2Mb) + \ double \ \ logs
\end{aligned}
$$

$$
\begin{aligned}
h(b,M,\Lambda) = \frac{2c_F}{\pi}\Bigg\{ &\frac{\bar{\alpha}_s}{8}[1+2\ln(2Mb)] + 2\bar{\alpha}_s[\ln(Mb)\ln(a\Lambda b) - \frac{1}{2}\ln^2(a\Lambda b)] + \\
&\bar{b}\left[\ln\frac{M}{\Lambda}\ln\frac{\ln\frac{M}{\Lambda}}{\ln a} - \ln\frac{M}{a\Lambda}\right]\Bigg\}
\end{aligned}
$$

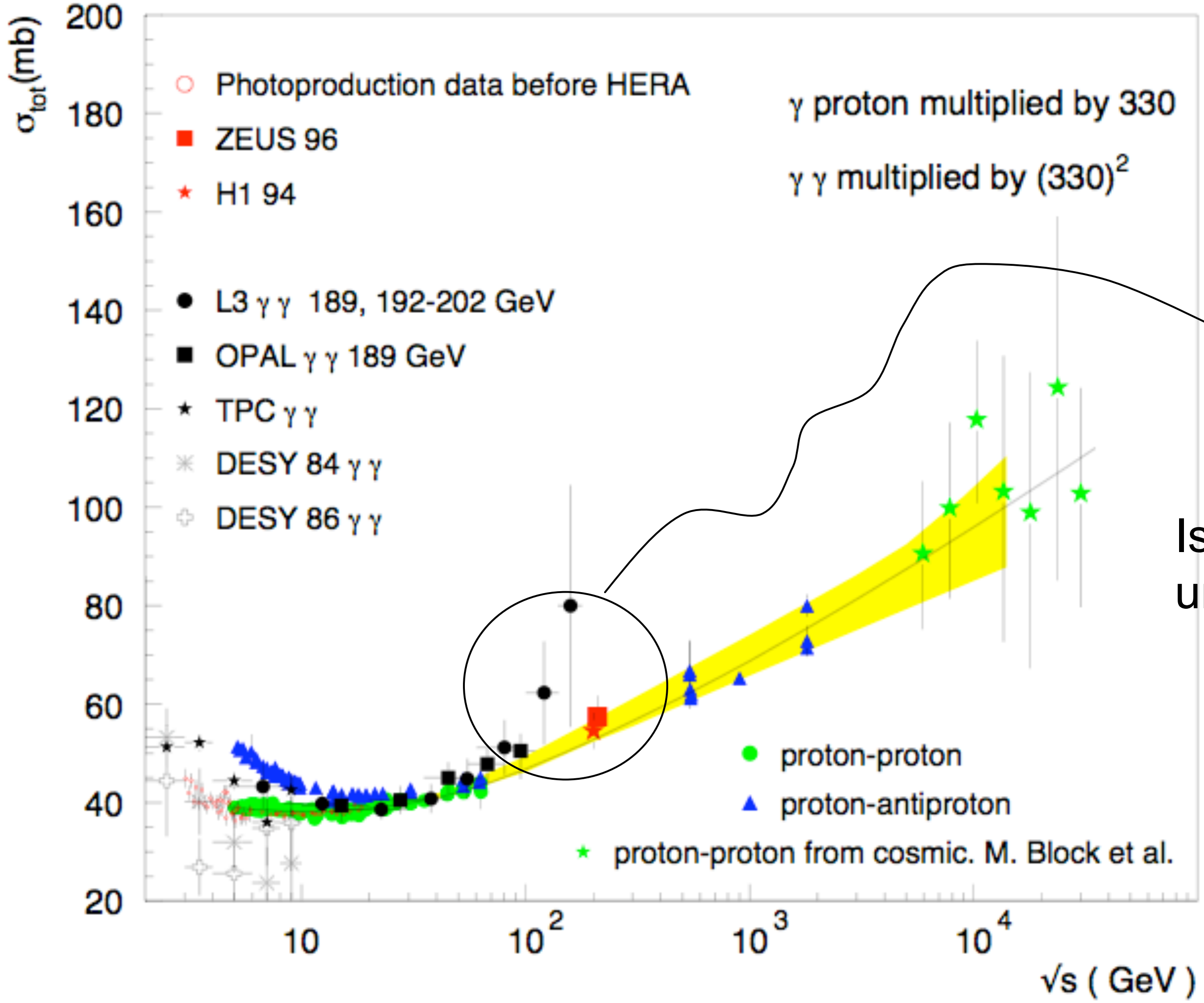
$$\bar{\alpha}_s = 12\pi/(33-2N_f)\ln(a^2)$$

Singular α_s gives a power law behavior

At very large distances $$b > \frac{1}{\Lambda} > \frac{1}{E}$$

$$h(b,E,\Lambda) = constant\ (b^2\Lambda^2)^p \left[2\ln(2Eb) + \tfrac{1}{1-p}\right] + double\ logs$$

Some questions about total cross-sections



How will the γ-cross -sections evolve in TeV region?

Is there a universal slope?

Particle Data Group 2008 fit

$$\sigma_{ab,\bar{a}b} = Z^{ab} + \ln^2(\frac{s}{s_0}) + Y_1^{ab}(\frac{s_1}{s})^{\eta_1} \pm Y_2^{ab}(\frac{s_1}{s})^{\eta_2}$$

$$\eta_1 \sim \eta_2 \sim 0.5$$

decreasing

Cudell, J. R. and others,Phys. Rev.D65, 2002,074024,hep-ph/0107219

$$\sigma_{tot} \approx \ln^2 s \quad at \quad high \quad energy$$

- Reflects a geometrical picture
- Saturates the Froissart bound

Eikonal model

$$\sigma_{elastic} = \int d^2\vec{b}|1 - e^{i\chi(b,s)}|^2$$

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\Im m\chi(b,s)} cos \Re e\chi(b,s)]$$

$$\sigma_{total\ inelastic} = \int d^2\mathbf{b}[1 - e^{-2\Im m\chi(b,s)}]$$

Models for inelastic collisions can give $\Im m\chi$

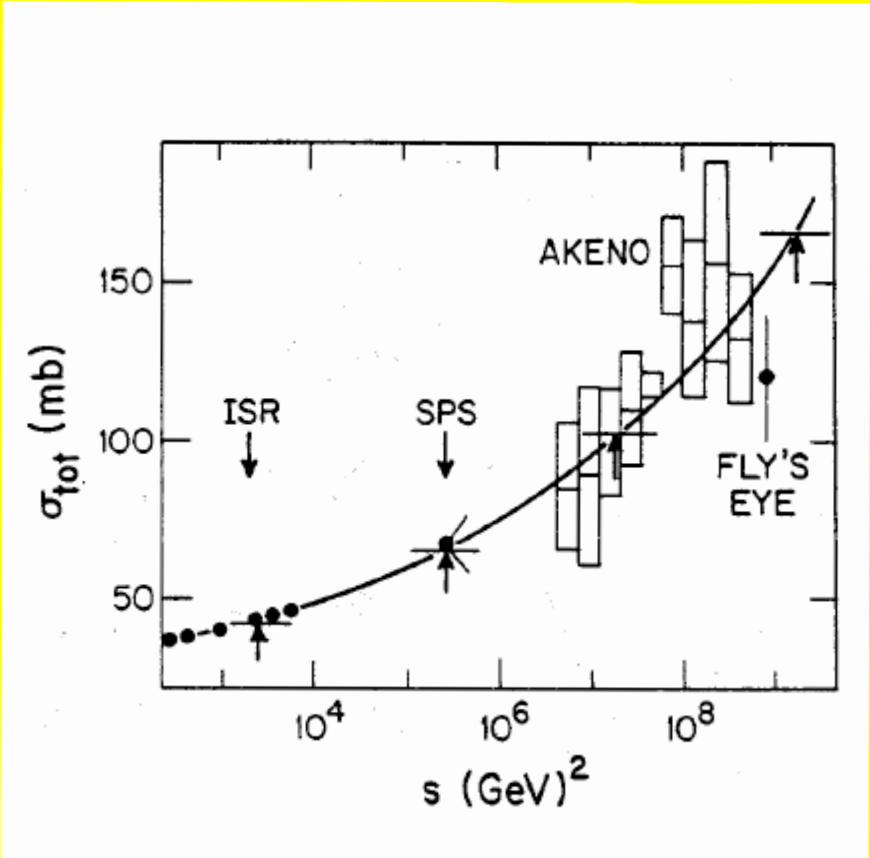
$$\sigma_{inelastic} = \int d^2\vec{b}\ P(b)_{all\ inelastic\ collisions}$$

The first mini-jet model

Soft Hard Scattering in the TeV Range. T.K. Gaisser, F. Halzen

Phys.Rev.Lett.54:1754,1985.

$$\sigma_{tot} = \sigma_0 + \sigma_{jet}(p_{T\,min})$$



Problems

- No unitarity
- energy-dependent ad hoc p_{tmin}
- Was later unitarized using σ_{jet} as driving the high energy term in an eikonal

Eikonal mini-jet models:

$$n_{hard-minijets}(b) \approx A(b,s)\sigma_{jet}(s, p_{tmin})$$

Average number of collisions at given energy and impact parameter

- $n(b,s) = n_{soft}(b,s) + n_{hard}(b,s)$

- $n_{soft/hard}(b,s) = A_{BN}^{soft/hard}(b,s)\sigma_{soft/hard}(s)$

b and s need not be factorized

Jet cross-sections at LO

Using current PDF's :

GRV, MRST, CTEQ

$$\sigma_{\text{jet}}^{AB}(s, p_{tmin}) =$$

$$\int_{p_{tmin}}^{\sqrt{s/2}} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \times$$

$$\sum_{i,j,k,l} f_{i|A}(x_1) f_{j|B}(x_2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t}.$$

Soft Gluon k_T-resummation from the beginning

$$d^4P(K) = \sum_{n_k} P\{n_k, \bar{n}_k\}\delta^4(K - \sum_k n_k k)d^4K$$

Poisson distributions

Energy-momentum Conservation induces IR real and virtual cancellation

Going to the continuum and to the k_t-variable

$$d^2P(\mathbf{K}_\perp) = d^2\mathbf{K}_\perp \int \frac{d^2\mathbf{b}}{(2\pi)^2}\, e^{-i\mathbf{K}_\perp\cdot\mathbf{b} - h(b,E)} \qquad (1)$$

$$h(b,E) = \int d^3\bar{n}_k[1 - e^{i\mathbf{k}_\perp\cdot\mathbf{b}}] \qquad (2)$$

$$h(b,E) = \frac{16}{3}\int_0^E \frac{\alpha_s(k_t^2)}{\pi}\frac{dk_t}{k_t}\log\frac{2E}{k_t}[1 - J_0(k_t b)] \qquad (3)$$

How soft gluons in the eikonal lead to saturation and restore the Froissart bound

At very large energies

$$\bar{\sigma}_{tot} \approx 2\pi \int_0^\infty (db^2)[1 - e^{-n_{hard}(b,s)/2}]$$

$$n_{hard}(b,s) = \sigma_{jet}(s) A_{hard}(b,s)$$

Mini-jets

IR gluons

$$\sigma_{jet}(s) \approx (\frac{s}{s_0})^\epsilon \sigma_1$$

$$A_{hard} \approx e^{-(bq)^{2p}}$$

$$n_{hard} = 2C(s/s_0)^\varepsilon e^{-(bq)^{2p}}$$

Dropping the J_0, i.e. the IR region

- Is acceptable as long as
  - No singularity is present in the IR region
  - Moderate b and relatively large k_t- values are involved - as in W-pt or Drell-Yan

Altarelli et al.

- May not be a good approximation if
  - A singularity is present
  - Very large b-values and small k_t are involved as in total x-section

By dropping the J_0 one deliberately ignores the IR region

- This is acceptable as long as
  - No singularity is present in the IR region Parisi Petronzio 1979
  - Moderate b and relatively large k_t- values are involved - as in W-p_t or Drell-Yan **[S. Ellis and J. Stirling 1980, …Altarelli et al. 1984…]**

- May not be a good approximation if
  - A singularity is present
  - Very large b-values and small k_t are involved as in

total x-section

Our proposal

- Use the full integration range

$$h(b,E) = \frac{16}{3}\int_0^E \frac{\alpha_s(k_t^2)}{\pi}\frac{dk_t}{k_t}\log\frac{2E}{k_t}[1 - J_0(k_t b)]$$

- With a singular but integrable α_s

What is the efect of a singular α_s on resummation?

$$h(b,E,\Lambda) = \frac{2c_F}{\pi} \times \qquad (1)$$

$$\left[\bar{b}\frac{b^2\Lambda^{2p}}{2}\int_0^{\frac{1}{b}} \frac{dk}{k^{2p-1}} \ln\frac{2E}{k} + 2\bar{b}\Lambda^{2p}\int_{\frac{1}{b}}^{N_p\Lambda} \frac{dk}{k^{2p+1}} \ln\frac{E}{k} + \bar{b}\int_{N_p\Lambda}^{E} \frac{dk}{k}\frac{\ln\frac{E}{k}}{\ln\frac{k}{\Lambda}}\right]$$

$$= \frac{2c_F}{\pi}\left[\frac{\bar{b}}{8(1-p)}(b^2\Lambda^2)^p\left[2\ln(2Eb) + \frac{1}{1-p}\right] + \right.$$

$$\frac{\bar{b}}{2p}(b^2\Lambda^2)^p\left[2\ln(Eb) - \frac{1}{p}\right] + \frac{\bar{b}}{2pN_p^{2p}}\left[-2\ln\frac{E}{\Lambda N_p} + \frac{1}{p}\right] +$$

$$\left.\bar{b}\ln\frac{E}{\Lambda}\left[\ln\frac{\ln\frac{E}{\Lambda}}{\ln N_p} - 1 + \frac{\ln N_p}{\ln\frac{E}{\Lambda}}\right]\right]$$

QCD Mini-jets

- The challenge of the mini-jet model for total cross-section is
  - To have unitarity implemented
  - to connect with current phenomenology with DGLAP evoluted PDFs, like
    - GRV
    - MRST
    - CTEQ
    - ….
  - To descrive both the early rise and asymptotic smooth behaviour

Singular α_s and resummation

- We chose p<1 for the gluon integrated spectrum to be finite and see which value of p fits the total cross-section

$$h(b,E) = \frac{16}{3}\int_0^E \frac{\alpha_s(k_t^2)}{\pi}\frac{dk_t}{k_t}\log\frac{2E}{k_t}[1-J_0(k_t b)]$$

Heisenberg geometrical picture for the total nucleon-nucleon cross-section with emission of a pion cloud

$$\sigma_{total} = \pi b_{max}^2$$

If pion wave function has a limited extension ~ $e^{-b_{max} m_\pi}$

$$\sigma_{total} \approx \frac{\pi}{m_\pi^2} \ln^2 \frac{\sqrt{s}}{< E_0 >}$$

$< E_0 >$: average energy of single pion in emitted pion cloud

$< E_0 >$ Constant

σ_{total} $\ln^2 s$

$< E_0 >$ ~ $\sqrt{s}$

σ_{total} ~ constant

Infrared gluons tame low-x gluon-gluon scattering (mini-jets) and restore the Froissart bound

$$\sigma_{tot}(s) \approx 2\pi \int_0^\infty db^2 [1 - e^{-C(s)e^{-(b\bar{\Lambda})^{2p}}}]$$

$$\sigma_{tot}(s) \to [\varepsilon \ln(s)]^{(1/p)}$$

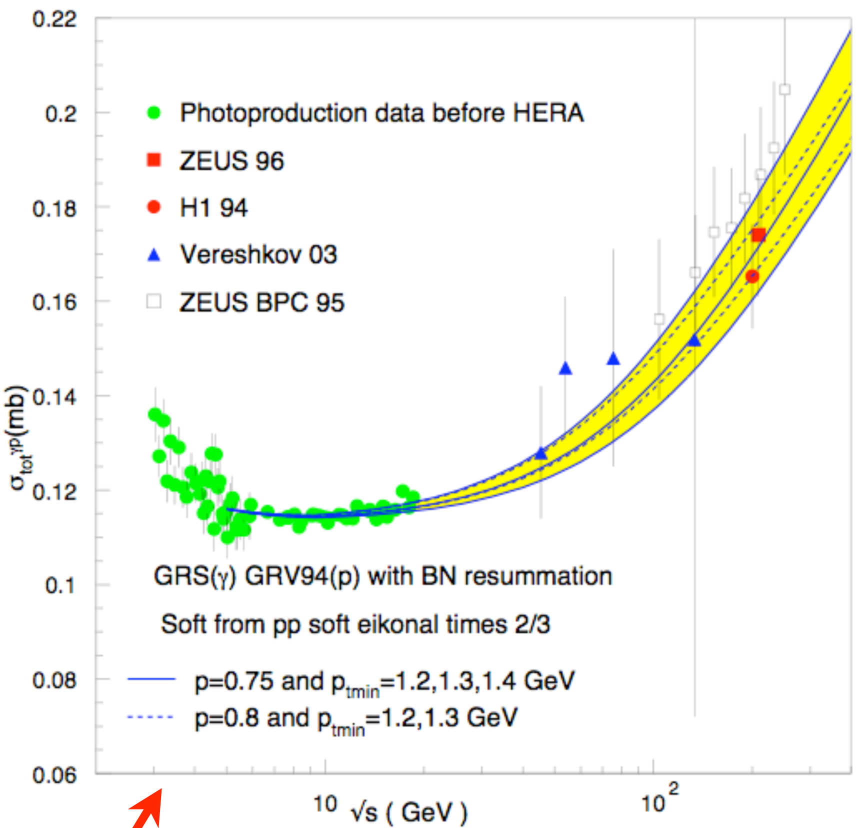
$$\frac{1}{2} < p < 1$$

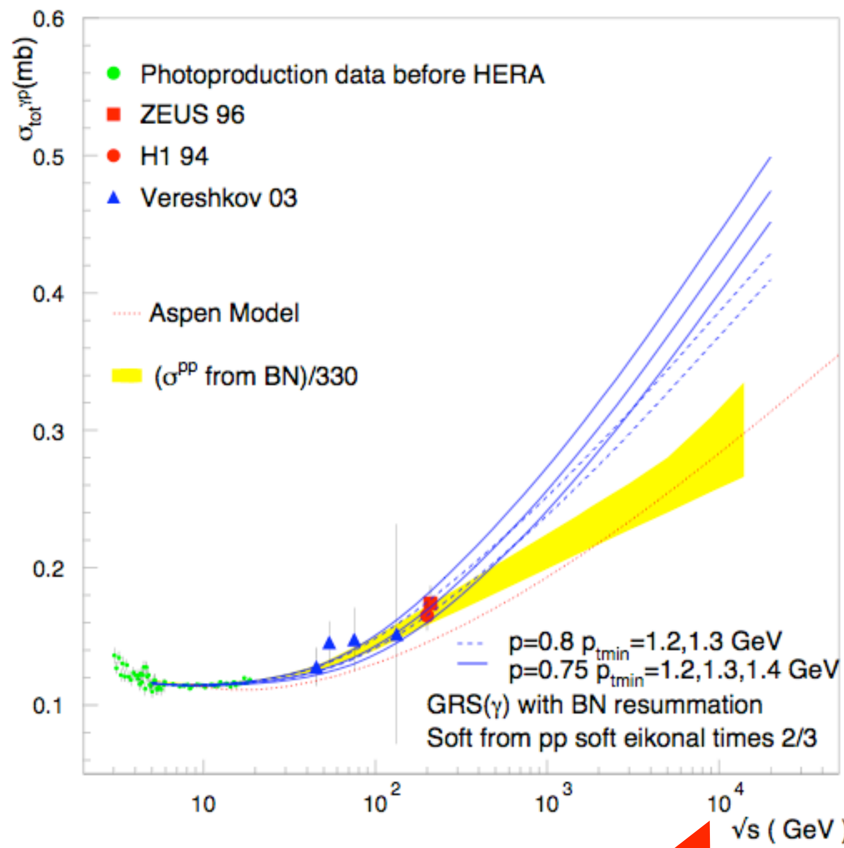
Extension to photon processes

$$\sigma_{total}^{\gamma p} = P_{had} \int d^2\mathbf{b}[1 - e^{-n^{\gamma p}()b,s}]$$

$$P_{had} = \Sigma_{\rho,\omega,\phi} \frac{4\pi\alpha}{f_V^2}$$

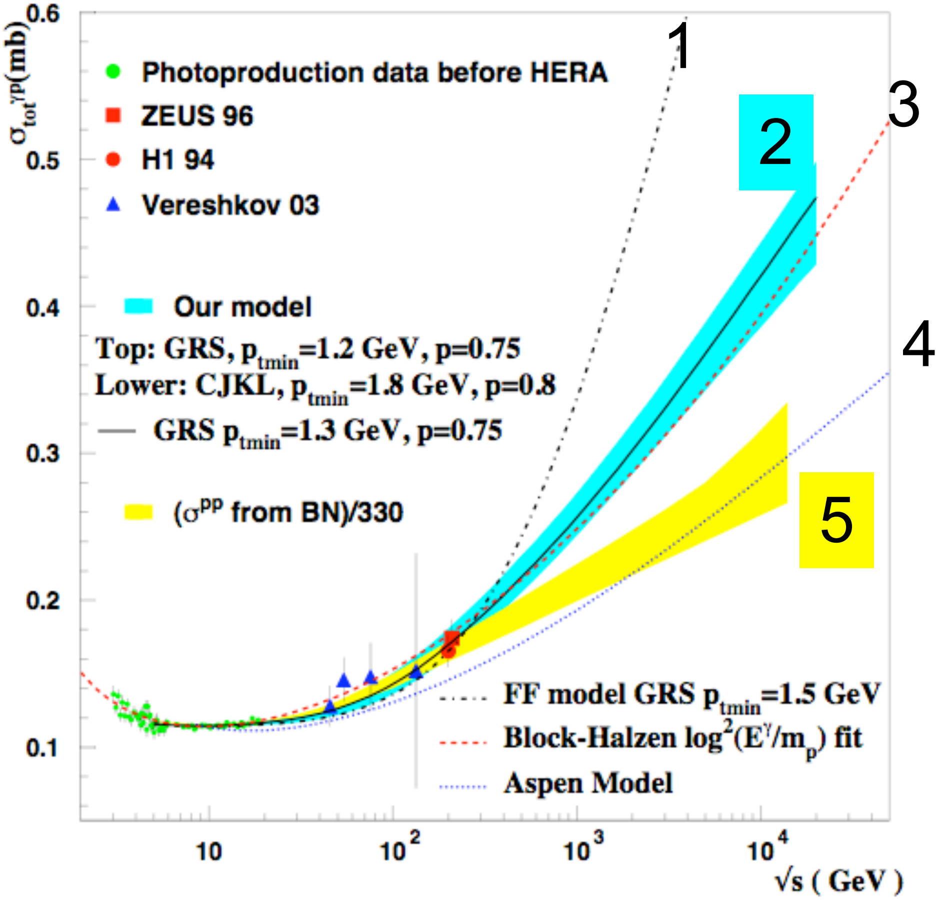
- Parametrize the low energy part
- Use PDFs for photons: GRS, CJKL
- Apply SGR as for protons





- Fix parameters from present data
- Extend model to TeV region
- Compare with factorization models (as suggested by PDG)

Predictions for γp from some current models and fits



1. Form factor and mini-jets

2. BN-model:soft gluons and minijets
Godbole, Grau,GP, Srivastava

3. Block-Halzen fit

4. Aspen Model
Block,Gregores,Halzen,GP,

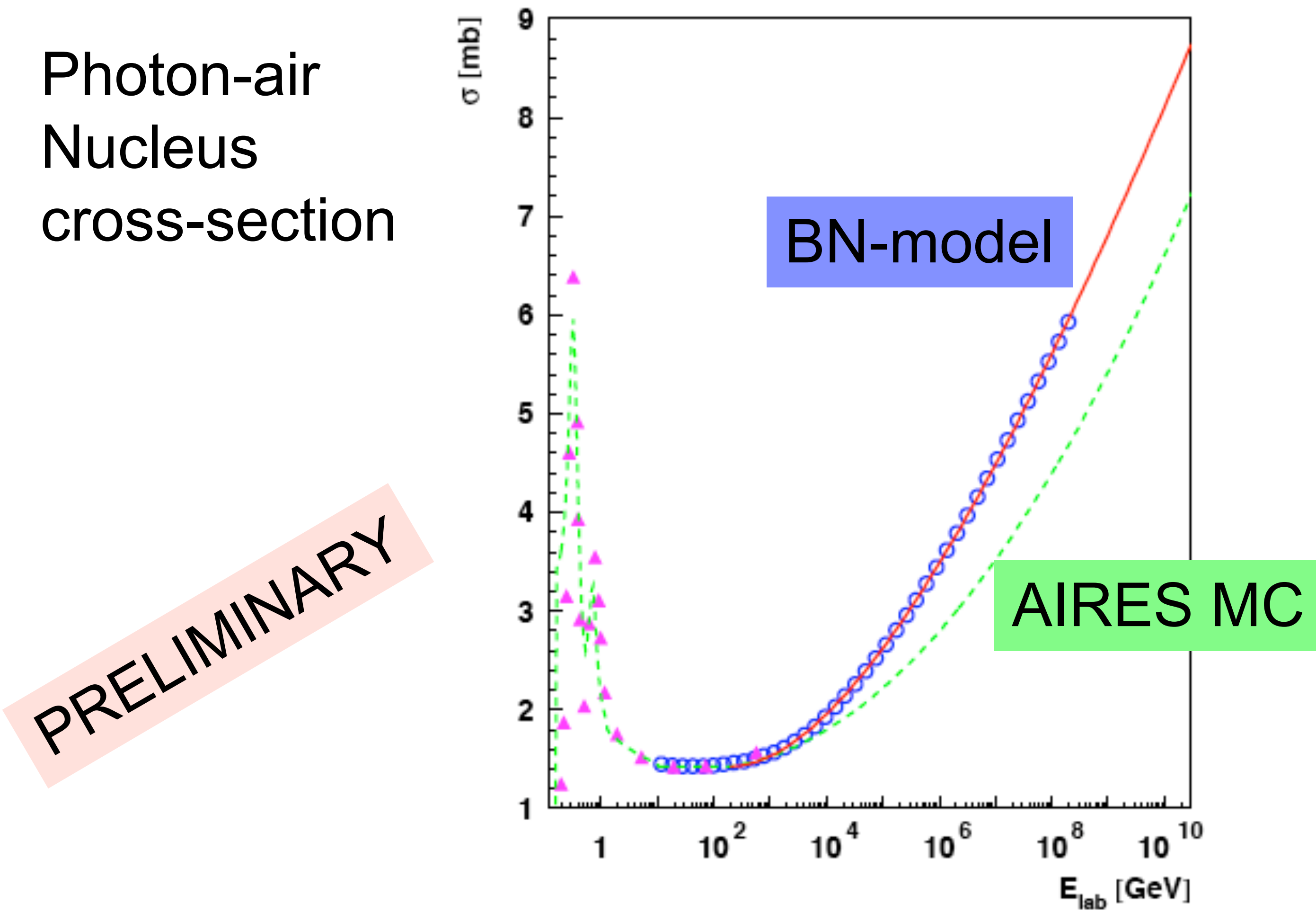
5. BN-model: factorization from pp

Godbole,Grau,GP,Srivastava, arXiv:0812.1065

Submitted to ICRC 2009

S. J. SCIUTTO *et al.* PHOTON SHOWER DEVELOPMENT

Photon-air Nucleus cross-section

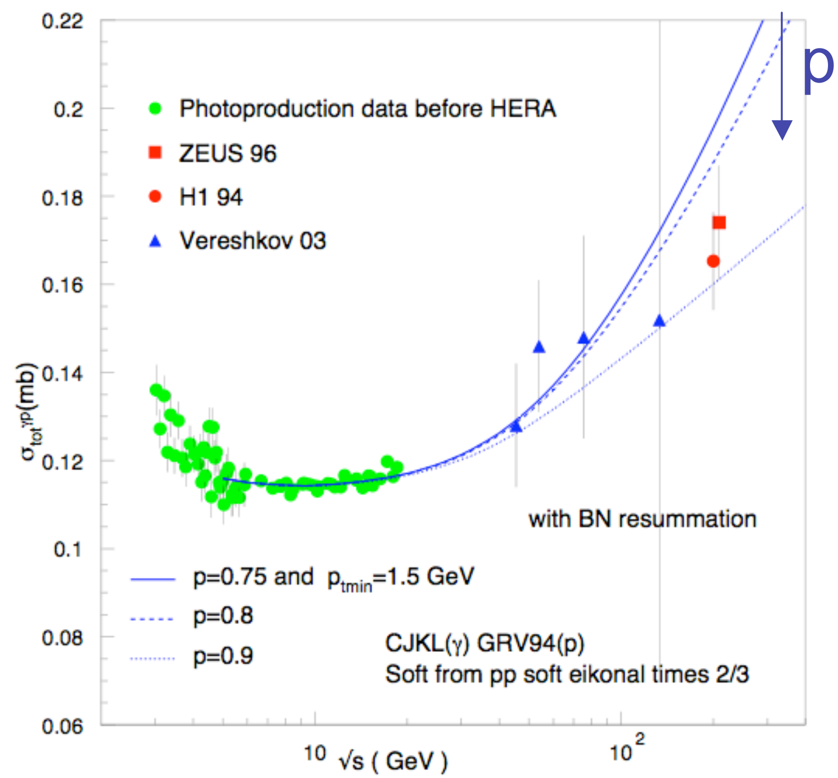


Photon Lab energy

BN model dependence from IR parameter p

- As p~1, x-section flattens more and more

- Smaller p_{tmin} ⟶ steeper rise: use larger p



Conclusions

- In the BN model for σ_{total} at very high energy

Soft Gluon Resummation with singular, integrable α_s softens the mini-jet rise

- Extension to γ-p gives results higher than factorization by a factor~ 2 at large TeV

- Embedded into γ-air, result is higher than present AIRES code results by ~ 20%

We use a Functional form inspired by a confining potential like the Richardson potential α_s

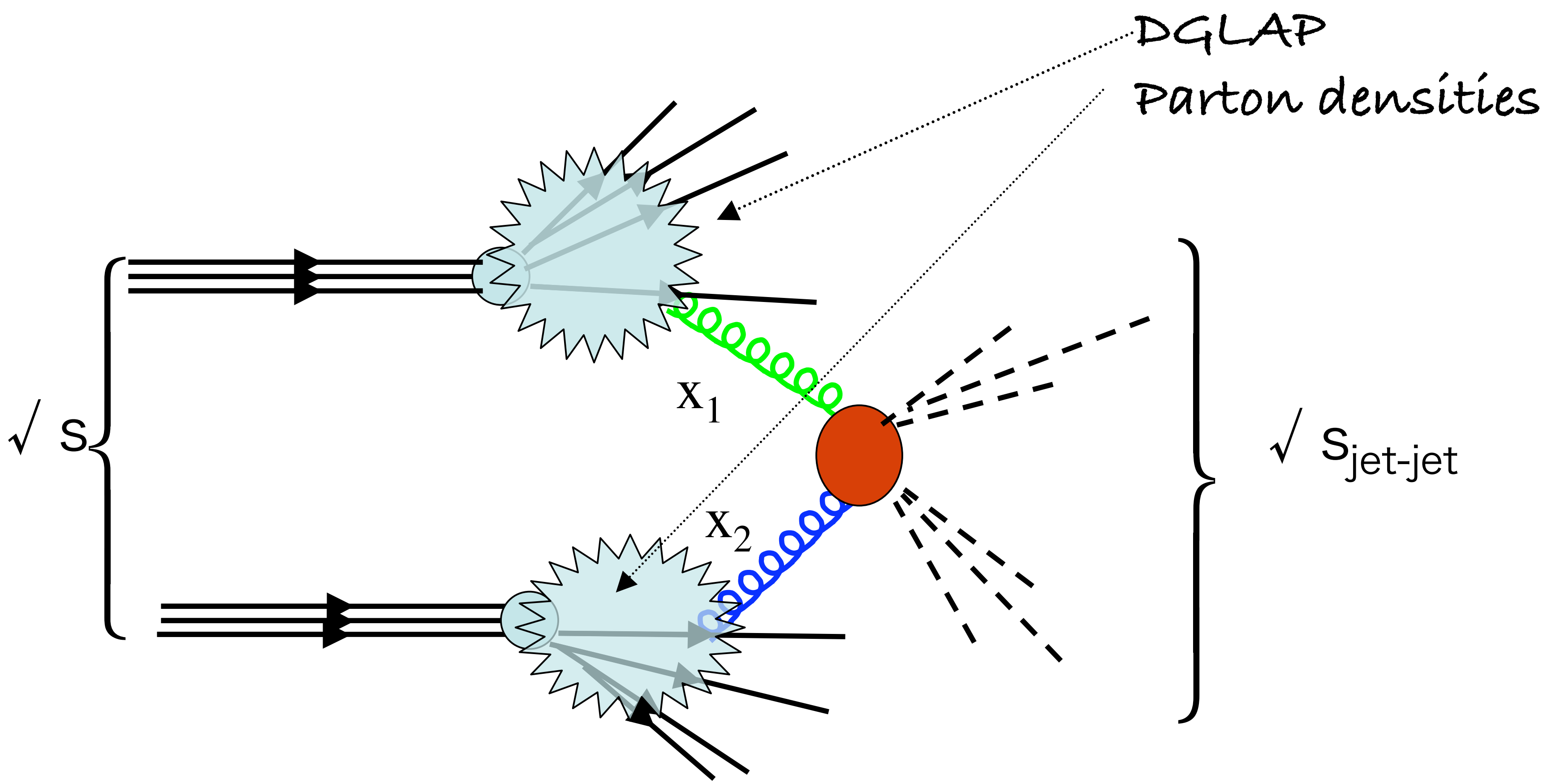
$$Q^2 << \Lambda^2 \quad \alpha_s(Q^2) = \frac{B}{(Q^2/\Lambda^2)^p}$$

- For $Q^2 >> \Lambda^2$ Usual AF expression
- Our phenomenological choice for all Q^2

$$\alpha_s(k_\perp) = \frac{12\pi}{(33-2N_f)} \frac{p}{\ln[1+p(\frac{k_\perp}{\Lambda})^{2p}]}$$

Hard component of scattering is responsible for the rise of the total cross-section

Cline,Halzen,Luthe 1972- Gaisser, Halzen 1985- G.P., Srivastava 1985



Outline

- Soft gluon k_t resummation: a model for ultra-soft gluons and large b-distribution in high energy collisions

- A model for total cross-sections:
  - Minijets
  - Soft gluon k_t resummation and Froissart bound
  - Phenomenology for proton [and photon] cross-sections