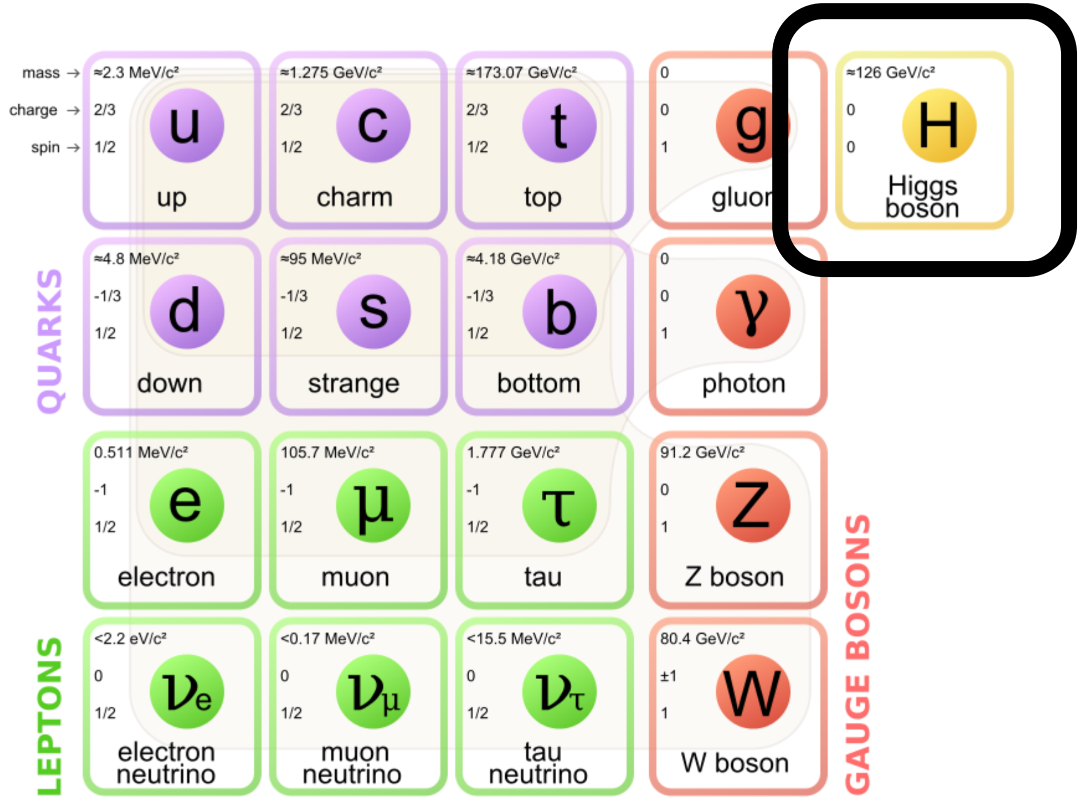
GG->ZZ PRODUCTION AT TWO LOOPS WITH FULL TOP-MASS DEPENDENCE

BAKUL AGARWAL

(WORK IN COLLABORATION WITH ANDREAS VON MANTEUFFEL)

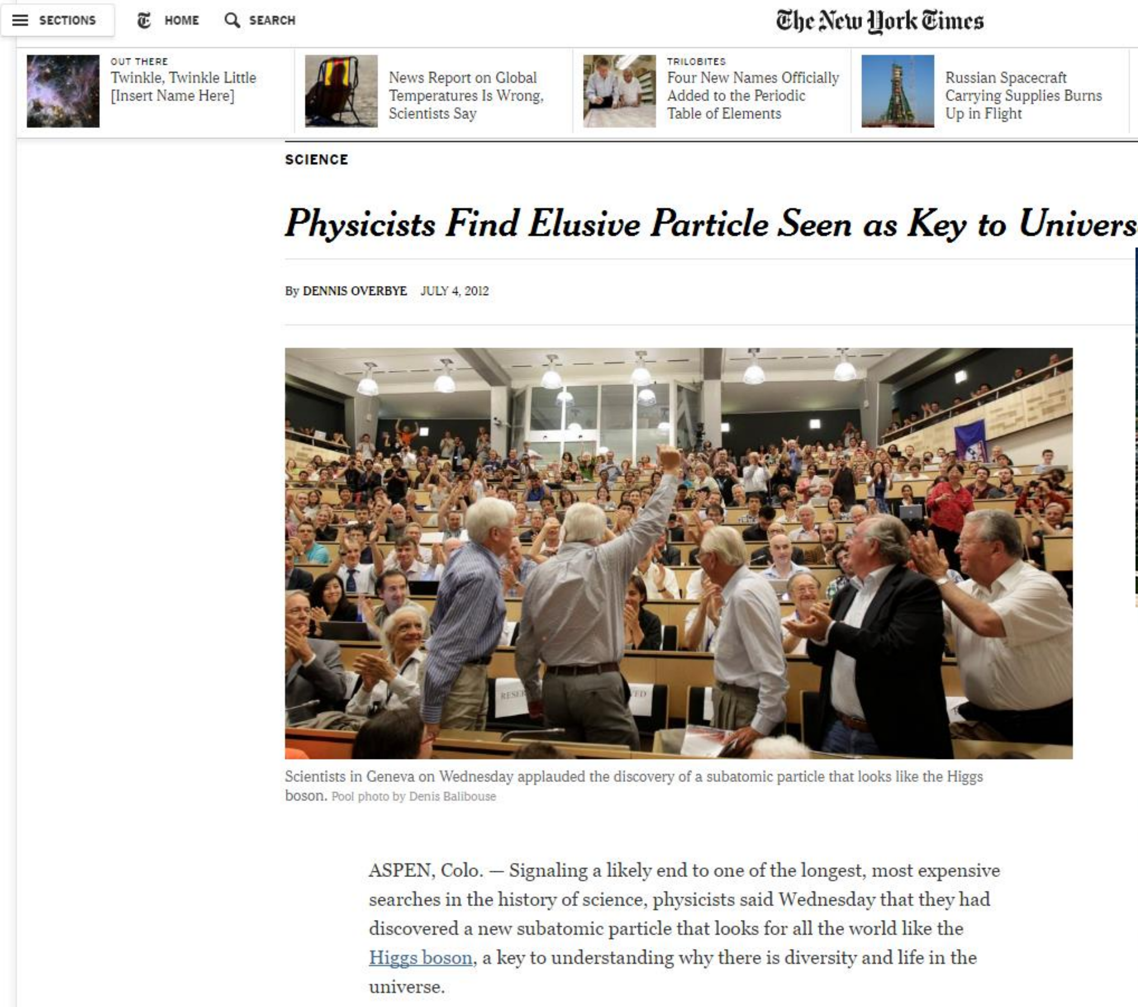
MAX-PLANCK-INSTITUT FÜR PHYSIK - 17/06/2019

THE STANDARD MODEL



- The theory of "almost everything"
- Describes all known fundamental interactions, except gravity
- Phenomena not accounted for : Dark matter, (Dark energy?), CP violation – matter-antimatter asymmetry
- Highly accurate predictions
- Two kinds of particles :-
  - Fermions : "Matter" particles
  - Bosons : Force carriers

THE HIGGS BOSON



SECTIONS HOME SEARCH

The New York Times

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TRILOBITES
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TRILOBITES
Study Suggests 3.2 Million-Year-Old Lucy Spent a Lot of Time in Trees

A CONVERSATION WITH
C. Megan Urry, Peering Into Universe, Spots Bias on the Ground

SCIENCE

Physicists Find Elusive Particle Seen as Key to Universe

By DENNIS OVERBYE JULY 4, 2012

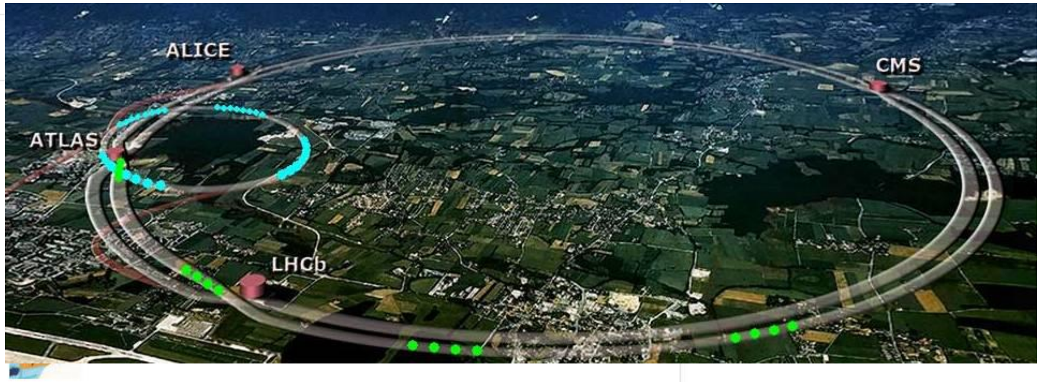
Scientists in Geneva on Wednesday applauded the discovery of a subatomic particle that looks like the Higgs boson. Pool photo by Denis Balibouse

ASPEN, Colo. — Signaling a likely end to one of the longest, most expensive searches in the history of science, physicists said Wednesday that they had discovered a new subatomic particle that looks for all the world like the Higgs boson, a key to understanding why there is diversity and life in the universe.

RECENT COMMENTS

Robert L. Oldershaw July 5, 2012
It seems to me that theoretical particle physics is more religion than science.If theories can avoid any predictions whatsoever (e.g.,...

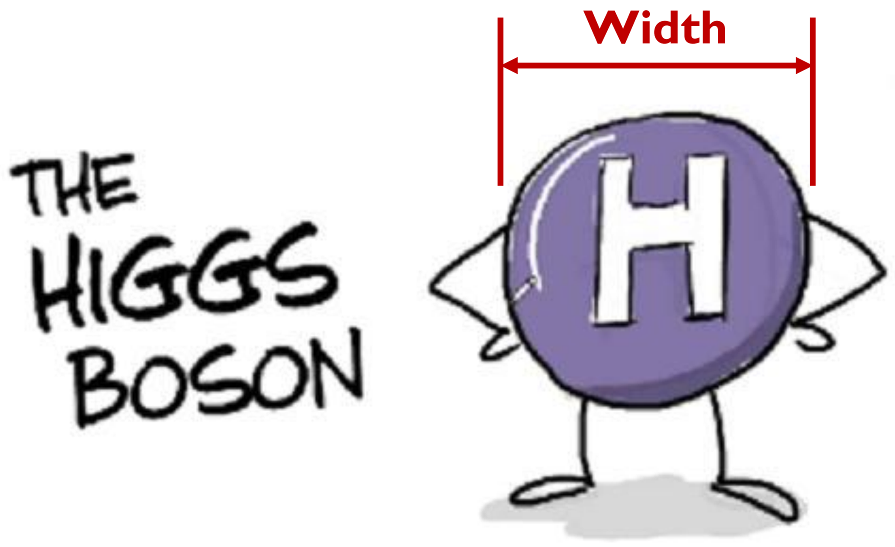
hansgolz July 5, 2012
CERN has proven nothing about nothing. This announcement and celebration seems to be more a desperate act to fake a finding just because...



THE HIGGS BOSON

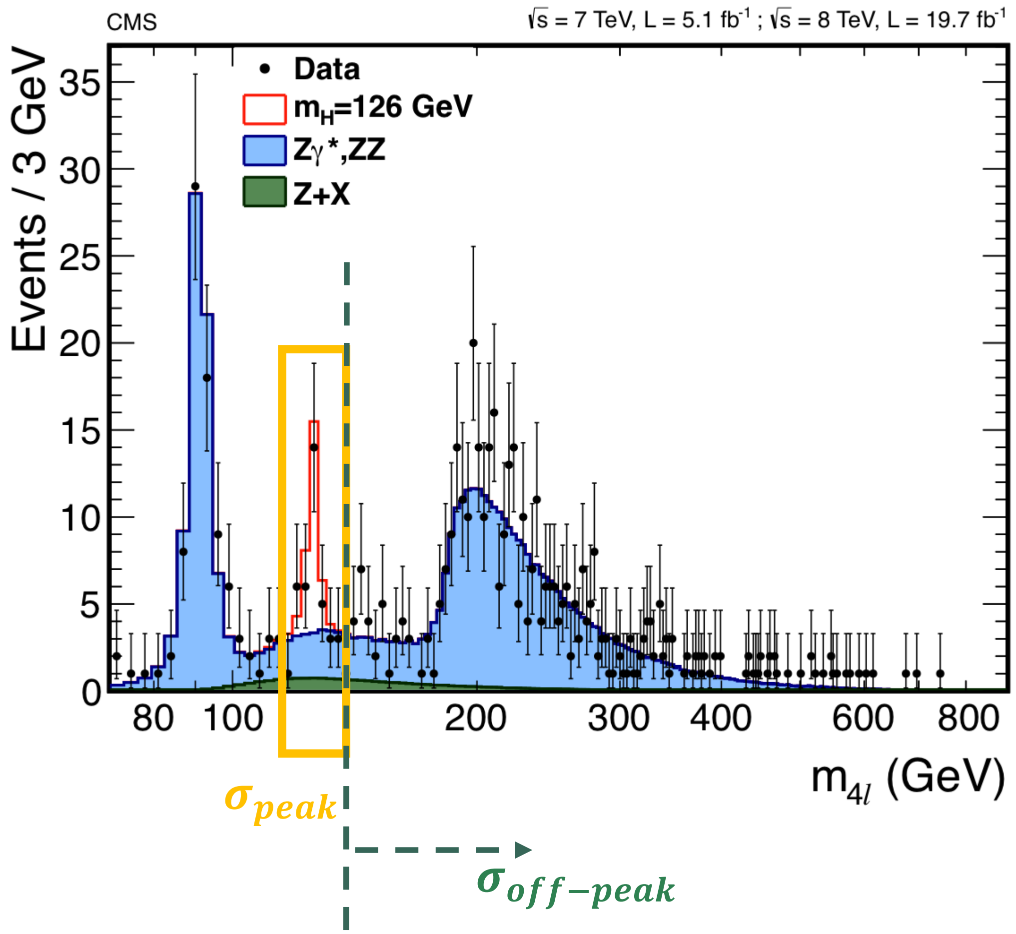
- Higgs couplings to SM particles
- Higgs potential
- Decay width

HIGGS WIDTH



- Higgs width predicted in SM : $\Gamma_H \sim 4.1$ MeV
- Important measurement. Deviation from SM value $\Rightarrow$ New Physics
- Too small to be measured at LHC. Detector resolution $\sim O(1)$ GeV
- Constrain using off-shell production. Proposed by F. Caola & K. Melnikov (arxiv:1307.4935)

HIGGS WIDTH



- $H \rightarrow ZZ^{*} \rightarrow 4l$

$$\frac{d\sigma}{dM_{4l}^{2}} \sim \frac{g_{Hgg}^{2} g_{HZZ}^{2}}{\left(M_{4l}^{2} - m_{H}^{2}\right)^{2} + m_{H}^{2}\Gamma_{H}^{2}}$$

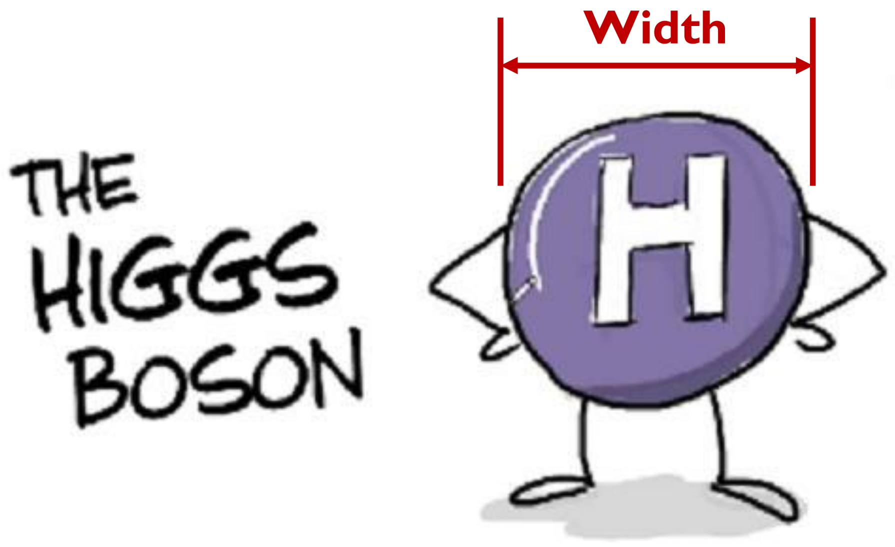
- Assume g_{Hgg}, g_{HZZ} scale linearly with ξ while Γ_H scales as ξ^4
- On-peak : $\sigma_{peak} \sim \frac{(\xi^2 g_{Hgg}^2)(\xi^2 g_{HZZ}^2)}{\xi^4 \Gamma_H} = \frac{g_{Hgg}^2 g_{HZZ}^2}{\Gamma_H}$

  unchanged

- Off-peak : $\sigma_{off-peak} \sim \xi^4 g_{Hgg}^2 g_{HZZ}^2$

  scales

HIGGS WIDTH

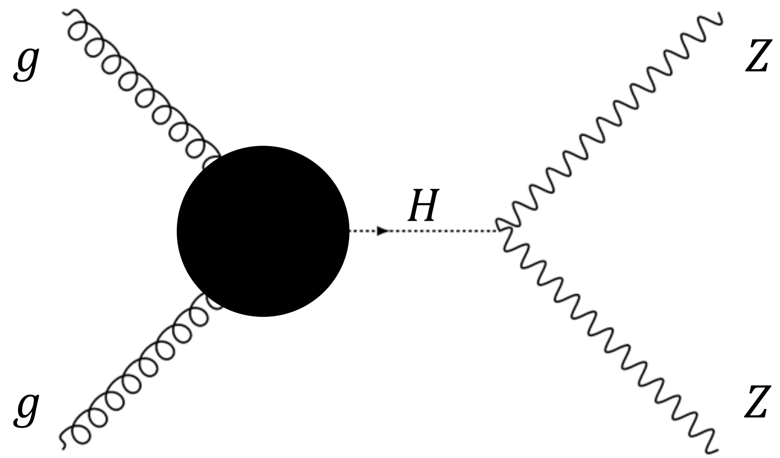


Current status :

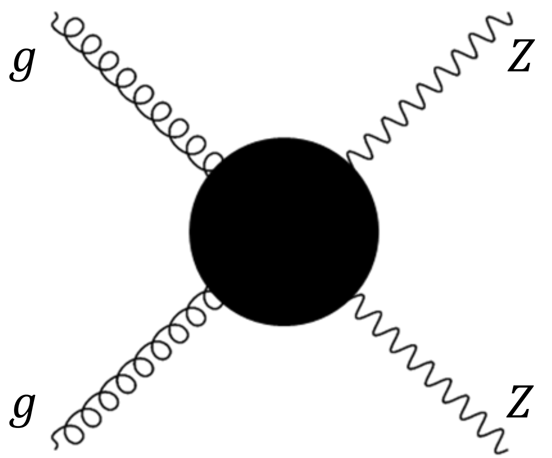
- ZZ channel - $\Gamma_H < 22$ MeV at 95% confidence level : CMS-HIG-14-002
- WW channel - $\Gamma_H < 26$ MeV at 95% confidence level
- Combined with ZZ analysis - $\Gamma_H < 13$ MeV at 95% confidence level : CMS-HIG-14-032
- ZZ channel - $\Gamma_H < 14.4$ MeV at 95% confidence level (arxiv:1808.01191)
- $\boxed{3.2^{+2.8}_{-2.2}\ \textbf{MeV}}$ from combined analysis gg->VV (arxiv:1901.00174)
- Direct constraints : CMS combined $H \to ZZ^* \to 4l$ & $H \to \gamma\gamma \Rightarrow \Gamma_H < 1.7$ GeV

HIGGS WIDTH

$$gg \to H \to ZZ^* \to 4l$$



Signal process



Continuum (Background) process

HIGGS WIDTH

$$gg \to H(\to ZZ)$$

- $gg \to H$ exact result known at NLO : M. Spira, A. Djouadi, D. Graudenz, P.M. Zerwas (arXiv:hep-ph/9504378)
- $gg \to H$ known at N3LO with infinite top mass approximation :

  C. Anastasiou et al (arXiv:1503.06056)

  B. Mistlberger (arXiv:1802.00833)

$$gg \to ZZ$$

- $gg \to ZZ$ exact result known at LO :

  E. N. Glover and J. J. van der Bij

  https://doi.org/10.1016/0550-3213(89)90262-9
- $gg \to ZZ$ NLO amplitude with massless quarks :

  A. von Manteuffel and L. Tancredi (arxiv:1503.08835)

  F. Caola, J. Henn, K. Melnikov, A. Smirnov & V. Smirnov (arxiv:1503.08759)
- $gg \to ZZ$ at NLO with expansion around heavy top limit

  F. Caola, M. Dowling, K. Melnikov, R. Röntsch, L. Tancredi (arxiv:1605.04610)
- NLO corrections to $gg \to ZZ$ around heavy top mass limit with Pade' approximants

  J. Campbell, R. Ellis, M. Czakon, S. Kirchner (arxiv:1605.01380)

HIGGS WIDTH

Importance of gg->ZZ :

- $\mathcal{O}$(10%) correction from off-shell production, Higgs-continuum interference very important (N. Kauer & G. Passarino, arxiv:1206.4803)
- gg -> ZZ @ LO very substantial to pp -> ZZ @ NNLO ~ 60% of the full NNLO correction, due to the large gg luminosity at the LHC (F. Cascioli, T. Gehrmann, M. Grazzini, S. Kallweit et al, arxiv:1405.2219)
- Expectation of large NLO K-factor : $\mathcal{O}$(40%-90%) increase from LO to NLO (F. Caola, K. Melnikov, R. Röntsch, L. Tancredi, arxiv:1509.06734)

HIGGS WIDTH

Limitations :

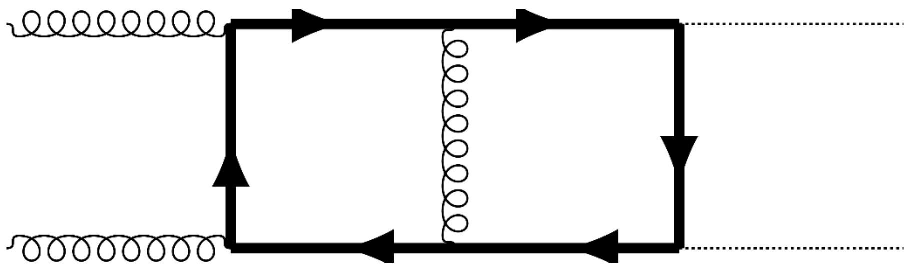
- Heavy top expansion breaks down around top quark threshold
- Equivalance theorem : At high energies, Longitudinal modes of gauge bosons ⇒ Goldstone bosons (coupling proportional to the mass of the fermion)
- Contribution from top quark loops at high invariant mass very significant

➢ Need an NLO calculation with full top mass dependence

HIGGS WIDTH

Similar calculations:

$gg \to HH$



- Same topologies
- Higgs is a scalar : rank 2 Lorentz tensor; rank 4 in ZZ production
- State of the art calculation done using purely numerical methods by S. Borowka, N. Greiner, G. Heinrich et al (arxiv:1608.04798)
- Incomplete reductions for the non-planar topologies, computed very difficult integrals numerically
- Using finite integrals very beneficial

$gg \to ZZ$ at 2-loops

- Construct the amplitude and decompose into sum of all possible Lorentz structures and their 'form factors'

$$\mathcal{A}^{\mu\nu\rho\lambda} = \sum p_i^{\mu}\, p_j^{\nu}\, p_k^{\rho}\, p_l^{\lambda}\, A_{ijkl} + ..$$

Virtual correction

- Solve linear system of equations to relate the 'form factors' to the original amplitude
- Use **Integration By Parts** identities to reduce the number of integrals to a basis set

New methods

- Rotate the basis integrals to a set of **finite integrals** ⇒ Much better behaved numerically

New methods

- **Evaluate** the finite integrals **numerically** using 'sector decomposition' (plus any needed improvements)

$gg \to ZZ$ at 2-loops

- Construct the amplitude and decompose into sum of all possible Lorentz structures and their 'form factors'

$$\mathcal{A}^{\mu\nu\rho\lambda} = \sum p_i^{\mu}\, p_j^{\nu}\, p_k^{\rho}\, p_l^{\lambda}\, A_{ijkl} + ..$$

- Solve linear system of equations to relate the 'form factors' to the original amplitude

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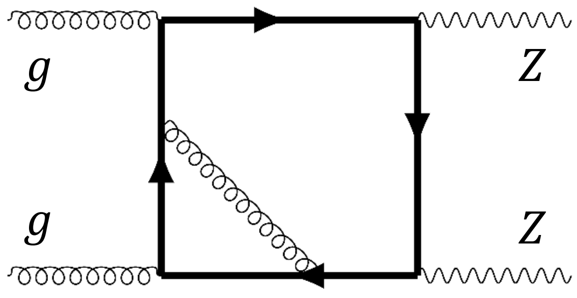
- Rotate the basis integrals to a set of **finite integrals** $\Rightarrow$ Much better behaved numerically

- **Evaluate** the finite integrals **numerically** using 'sector decomposition' (plus any needed improvements)

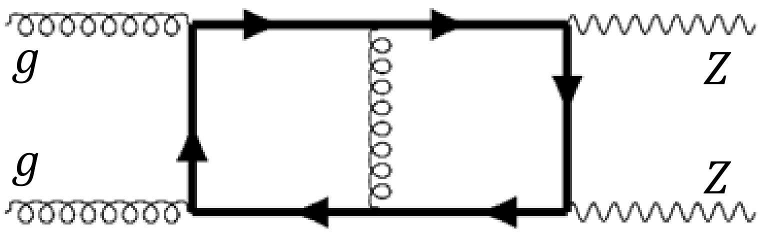
SETUP

- 166 Diagrams in total
- 48 diagrams vanish due to colour structure
- Need 4 different sets of propagators to cover all topologies : Integral families A, B, C, D

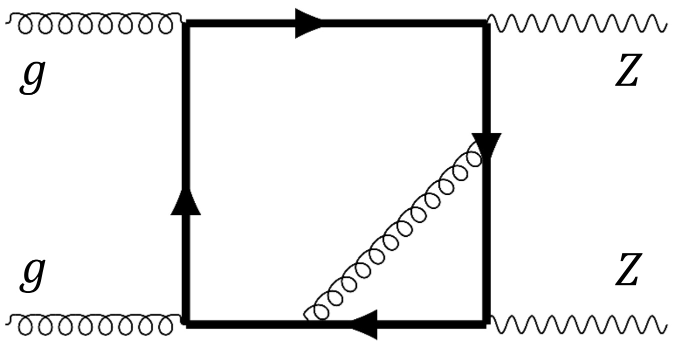
INTEGRAL FAMILIES & TOPOLOGIES



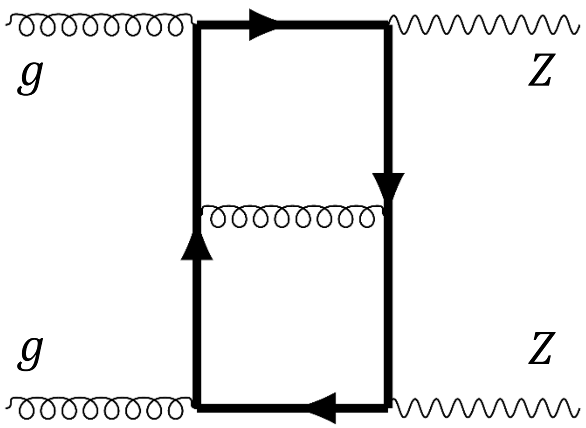
A - 319



A - 381

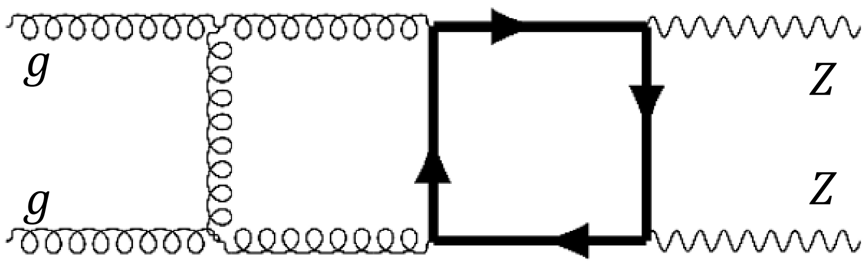


A - 415

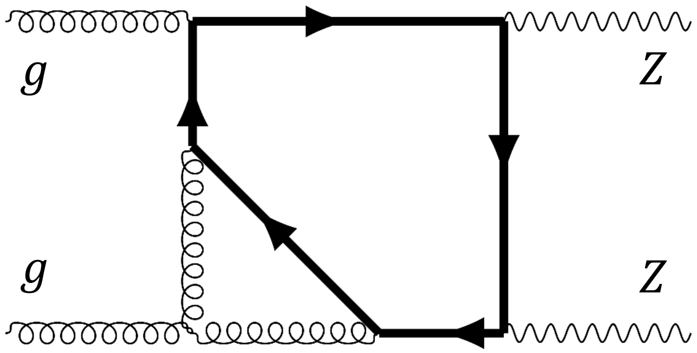


A - 446

INTEGRAL FAMILIES & TOPOLOGIES

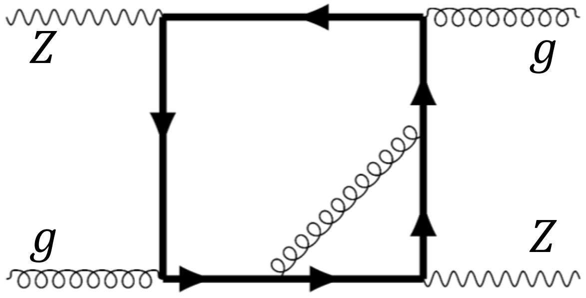


B - 471

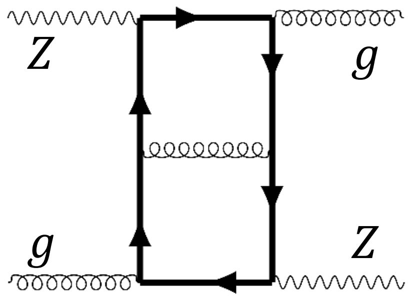


B - 499

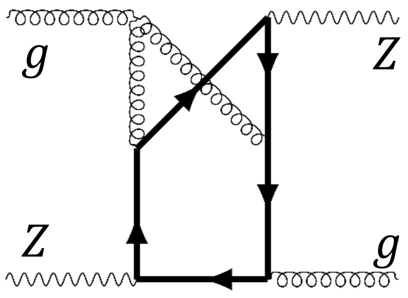
INTEGRAL FAMILIES & TOPOLOGIES



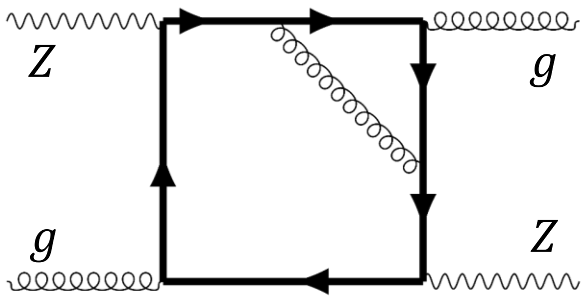
C - 223



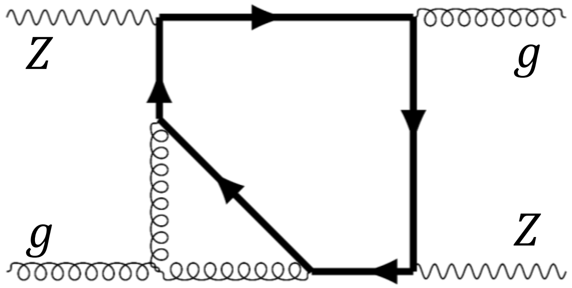
C - 247



C - 478

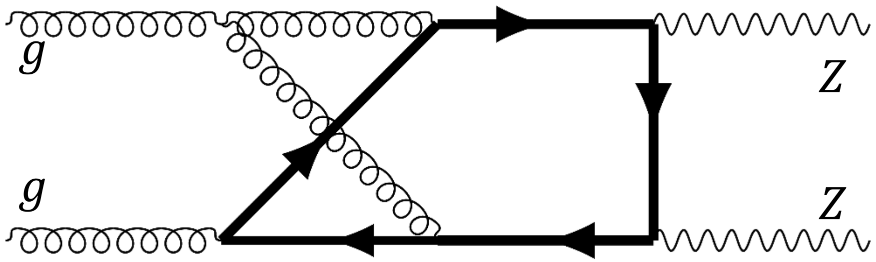


C - 239

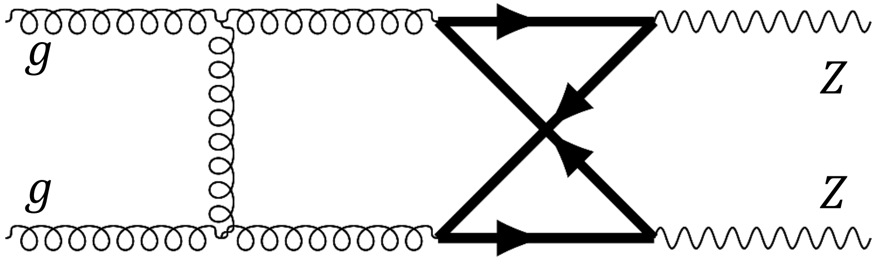


C - 415

INTEGRAL FAMILIES & TOPOLOGIES

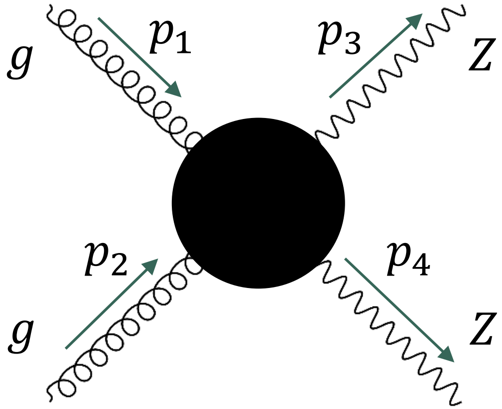


D - 251



D - 375

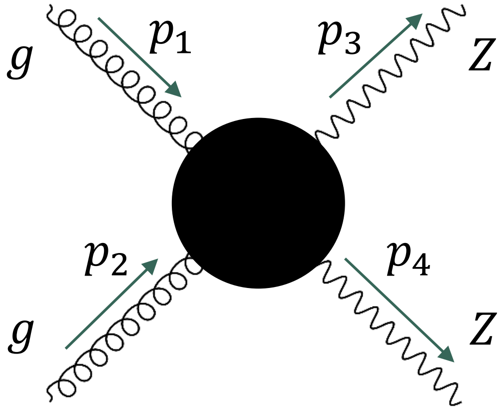
SETUP



- Amplitude : $\mathcal{M} = \mathcal{A}_{\mu\nu\rho\lambda}\, \epsilon_1^{\mu}\, \epsilon_2^{\nu}\, \epsilon_3^{\rho}\, \epsilon_4^{\lambda}$
- Can decompose the amplitude into 138 tensor structures :

$$
\begin{aligned}
= \quad & A_1\, g^{\mu\nu} g^{\rho\lambda} + A_2\, g^{\mu\rho} g^{\nu\lambda} + A_3\, g^{\mu\lambda} g^{\nu\rho} \\
+ & \sum_{i,j=1}^{3} \left(A_{1,i,j}\, g^{\mu\nu}\, {p_i}^{\rho}\, {p_j}^{\lambda} + A_{2,i,j}\, g^{\mu\rho}\, {p_i}^{\nu}\, {p_j}^{\lambda} + A_{3,i,j}\, g^{\mu\lambda}\, {p_i}^{\rho}\, {p_j}^{\nu} + A_{4,i,j}\, g^{\rho\nu}\, {p_i}^{\mu}\, {p_j}^{\lambda} + A_{5,i,j}\, g^{\rho\lambda}\, {p_i}^{\mu}\, {p_j}^{\nu} + A_{6,i,j}\, g^{\lambda\nu}\, {p_i}^{\rho}\, {p_j}^{\mu} \right) \\
+ & \sum_{i,j,k,l=1}^{3} A_{i,j,k,l}\, {p_i}^{\mu} {p_j}^{\nu} {p_k}^{\rho} {p_l}^{\lambda}
\end{aligned}
$$

SETUP



- Use transversality of gluons and gauge freedom to eliminate most of these:

$$\epsilon_1 . p_1 = \epsilon_2 . p_2 = 0 \quad \& \quad \epsilon_1 . p_2 = \epsilon_2 . p_1 = \epsilon_3 . p_3 = \epsilon_4 . p_4 = 0$$

- 20 tensor structures left :

$$\begin{aligned}
= \; & A_1\, g^{\mu\nu} g^{\rho\lambda} + A_2\, g^{\mu\rho} g^{\nu\lambda} + A_3\, g^{\mu\lambda} g^{\nu\rho} + (A_{1,1,1} - A_{1,1,3})\, g^{\mu\nu}\, {p_1}^{\rho}\, {p_1}^{\lambda} + (A_{1,1,2} - A_{1,1,3}) g^{\mu\nu}\, {p_1}^{\rho}\, {p_2}^{\lambda} + (A_{1,2,1} \\
& - A_{1,2,3}) g^{\mu\nu}\, {p_2}^{\rho}\, {p_1}^{\lambda} + (A_{1,2,2} - A_{1,2,3}) g^{\mu\nu}\, {p_2}^{\rho}\, {p_2}^{\lambda} + (A_{2,3,1} - A_{2,1,3})\; g^{\mu\rho}\, {p_3}^{\nu}\, {p_1}^{\lambda} + (A_{2,3,2} - A_{2,1,3})\; g^{\mu\rho}\, {p_3}^{\nu}\, {p_2}^{\lambda} \\
& + A_{3,1,3}\, g^{\mu\lambda} {p_1}^{\rho} {p_3}^{\nu} + A_{3,2,3}\, g^{\mu\lambda} {p_2}^{\rho} {p_3}^{\nu} + (A_{4,3,1} - A_{4,3,3}) g^{\rho\nu}\, {p_3}^{\mu}\, {p_1}^{\lambda} + (A_{4,3,2} - A_{4,3,3}) g^{\rho\nu}\, {p_3}^{\mu}\, {p_2}^{\lambda} \\
& + A_{5,3,3}\, g^{\rho\lambda}\, {p_3}^{\mu}\, {p_3}^{\nu} + A_{6,1,3}\, g^{\lambda\nu}\, {p_1}^{\rho}\, {p_3}^{\mu} + A_{6,2,3}\, g^{\lambda\nu}\, {p_2}^{\rho}\, {p_3}^{\mu} + (A_{3,3,1,1} - A_{3,3,1,3})\, {p_3}^{\mu} {p_3}^{\nu} {p_1}^{\rho} {p_1}^{\lambda} \\
& + +(A_{3,3,1,2} - A_{3,3,1,3}) {p_3}^{\mu} {p_3}^{\nu} {p_1}^{\rho} {p_2}^{\lambda} + (A_{3,3,2,1} - A_{3,3,2,3})\, {p_3}^{\mu} {p_3}^{\nu} {p_2}^{\rho} {p_1}^{\lambda} + +(A_{3,3,2,2} - A_{3,3,2,3}) {p_3}^{\mu} {p_3}^{\nu} {p_2}^{\rho} {p_2}^{\lambda}
\end{aligned}$$

SETUP

- Contract with each of the 20 tensor structures to relate form factors to the amplitude :

$$A_i = \mathcal{A}_{\mu\nu\rho\lambda} * P_i^{\mu\nu\rho\lambda}$$

$$= \sum_{j=1}^{20} A_j * T_{j,\mu\nu\rho\lambda} * P_i^{\mu\nu\rho\lambda}$$

Solve : $T_{j,\mu\nu\rho\lambda} * P_i^{\mu\nu\rho\lambda} = \delta_{ij}$ to obtain $P_i^{\mu\nu\rho\lambda}$

- Amplitude expressed in terms of these form factors after contraction
- Use FORM for the symbolic algebra

SETUP

- Total size of unreduced form factors : 2.8*20 GB, with the largest being ~50 MB
- Intermediate expressions in several gigabytes
- FORM code to perform the contraction and bringing the amplitude into the desired form
- Total of 29540 unreduced integrals; 281 master integrals

$gg \rightarrow ZZ$ at 2-loops

- Construct the amplitude and decompose into sum of all possible Lorentz structures and their 'form factors'

$$\mathcal{A}^{\mu\nu\rho\lambda} = \sum p_i^\mu \, p_j^\nu \, p_k^\rho \, p_l^\lambda \, A_{ijkl} + ..$$

- Solve linear system of equations to relate the 'form factors' to the original amplitude

- Use **Integration By Parts** identities to reduce the number of integrals to a basis set

- Rotate the basis integrals to a set of **finite integrals** $\Rightarrow$ Much better behaved numerically

- **Evaluate** the finite integrals **numerically** using 'sector decomposition' (plus any needed improvements)

INTEGRATION BY PARTS REDUCTION

- General scalar Feynman integral with **L-loops** and **N-edges** :

$$\mathrm{I}(\mathrm{a}_1..\mathrm{a}_\mathrm{N}) = \int d^D k_1..d^D k_L \prod_{i=1}^{N} \frac{1}{(q_i^2+m_i^2)^{a_i}}$$

Work in dimensional regularization to regulate the Ultraviolet/Infrared divergences appearing in the amplitude

$$D = 4 - 2\epsilon$$

p_i : *External momenta*
k_i : *Loop momenta*
q_i : *Momentum of the edge i*
m_i: *Mass of the edge i*
a_i : *Exponent of the propagator for the edge i*

INTEGRATION BY PARTS REDUCTION

- Integration by part identity:

$$0 = \int d^D k_1 .. d^D k_L \boxed{\frac{\partial}{\partial k_\mu} \; v_\mu \left(\prod_{i=1}^{N} \frac{1}{(q_i^2 + m_i^2)^{a_i}} \right)}$$

$v = \{p_i, k_i\}$

p_i : *External momenta*
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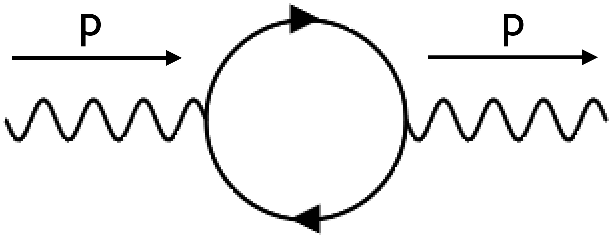
q_i : *Momentum of the edge i*
m_i: *Mass of the edge i*
a_i : *Exponent of the propagator for the edge i*

INTEGRATION BY PARTS REDUCTION



$$I(a_1, a_2) = \int d^D k \frac{1}{(k^2 - m^2)^{a_1}\left((k+p)^2 - m^2\right)^{a_2}}$$

IBP relations :

$$(D - 2a_1 - a_2)I(a_1, a_2) - 2a_1 m^2 \boxed{I(a_1 + 1, a_2)} - a_2(2m^2 - p^2)\boxed{I(a_1, a_2 + 1)} - a_2 I(a_1 - 1, a_2 + 1) = 0$$

$$(a_1 - a_2)I(a_1, a_2) + a_1 p^2 \boxed{I(a_1 + 1, a_2)} - a_1 I(a_1 + 1, a_2 - 1) + a_2 I(a_1 - 1, a_2 + 1) - a_2 p^2 \boxed{I(a_1, a_2 + 1)} = 0$$

- Integrals with doubled propagators don't usually appear in amplitudes
- Significantly larger system to reduce

BAIKOV REPRESENTATION

$$I(a_1,a_2) = \int d^D k \boxed{\frac{1}{(k^2-m^2)^{a_1}\left((k+p)^2-m^2\right)^{a_2}}}$$

Integrand depends only on k^2 & $k.p$
$\Rightarrow$ Instead of $D = 4-2\epsilon$ variables, only 2 appear in the integrand, rest can be integrated over

Change of variables : $k^2 - m^2 \rightarrow z_1$
$(k+p)^2 - m^2 \rightarrow z_2$

$$\Rightarrow I(a_1,a_2) = \frac{2}{(p^2)^{\frac{D}{2}-1}\,\pi^{\frac{1}{2}}\,\Gamma(\frac{D}{2}-\frac{1}{2})}\int dz_1 dz_2 \boxed{\frac{1}{z_1^{a_1}\,z_2^{a_2}}}\boxed{\left(p^2m^2-\frac{p^4}{2}-\frac{1}{2}(z_1^2+z_2^2-2z_1z_2+2z_2p^2)\right)^{\frac{D-3}{2}}}$$

Jacobian of the transformation

INTEGRATION BY PARTS REDUCTION

- General scalar Feynman integral in Baikov representation with **L-loops** and **N-edges** :

$$\mathrm{I}(\mathrm{a}_1..\mathrm{a}_N) = C U^{(D-L-E-1)/2} \int dz_1..dz_N \frac{1}{\prod_{i=1}^{N} {z_i}^{a_i}} P^{(D-L-E-1)/2}$$

z_i : *Baikov parameters*
P : *Baikov polynomial (depends on z_i in general)*
a_i : *Exponent of the propagator for the edge i*
C : *Constant from integrating over the solid angles*
U : *From the jacobian of transformation*

INTEGRATION BY PARTS REDUCTION

- IBPs in Baikov representation :

$$0 = \int dz_1..dz_N \sum_{i=1}^{N} \frac{\partial}{\partial z_i}\left(f_i(z_1,..,z_N)\ P^{(D-L-E-1)/2} \frac{1}{z_1^{a_1}..z_N^{a_N}} \right)$$

$$0 = \int dz_1..dz_N \sum_{i=1}^{N} \left(\frac{\partial f_i}{\partial z_i} + \underbrace{\boxed{\frac{D-L-E-1}{2P} f_i \frac{\partial P}{\partial z_i}}}_{\text{Dimension shifting term}} - \underbrace{\boxed{\frac{a_i f_i}{z_i}}}_{\text{Dots (doubled propagators)}} \right) P^{(D-L-E-1)/2}$$

- Impose following constraints :
  - No dimension shift –
  - No 'Doubled' propagators –

$$\boxed{\begin{gathered} \sum_{i=1}^{N} f_i \frac{\partial P}{\partial z_i} + g\ P = 0 \\ f_i \sim z_i \end{gathered}}$$

'Syzygy' constraints

SYZYGIES

$$\sum_{i=1}^{N} f_i \frac{\partial P}{\partial z_i} + g\, P = 0$$

- Explicit solutions known, pointed out by J. Boehm, A. Georgoudis, K. J. Larsen, H. Schoenemann, Y. Zhang arxiv:1805.01873
- Polynomials of degree 1 in z_i and kinematic invariants
- Very easy to construct

$$f_i \sim z_i$$

- $f_i's$ proportional to z_i to avoid doubled propagators

- Original strategy : Use $f_i = b_i z_i$ and substitute in the no dimension shift syzygy; solve the syzygy explicitly
- Zhang et al : Use Groebner bases methods to find the intersection between the sets of polynomials satisfying these two constraints
- Our method : Use explicit solutions for the no dimension shift syzygy to construct solutions also satisfying $f_i \sim z_i$

SYZYGIES

Singular https://www.singular.uni-kl.de/

- State-of-the-Art Public code for computer algebra; lot more powerful than Mathematica for such purposes
- Can almost use out of the box
- Provides all solutions to the syzygies
- Slow :
  - With Yang Zhang's inputs, able to solve for 6-line sectors
  - Solutions for 7-line sectors still unfeasible (nowhere close to finishing after ~40 hrs of CPU time)

New custom syzygy solver

- Custom implementation based on linear algebra to solve the syzygies
- Reduce the problem to row-reduction of a matrix - Use **Finred** for row-reduction
- Solutions only up to a requested 'degree' of polynomial
- Very fast : ~2 Hrs for 7-line sectors up to degree 5

COMPARISON

Conventional IBP reduction

- Setup : None
- Reduction :
  - ~1 yr of CPU time for family A, up to tensor rank 3 (tensor rank 4 needed)
  - Terabytes of disk space
  - Need special file system on the High Performance Computing Cluster at MSU due to file corruptions

New Syzygy based IBP reduction

New

- Setup : Generation of syzygies (Can be **parallelised**)
  - ~ 30 hrs CPU time (single core) for family A, B
  - ~ 50 hrs CPU time (single core) for family C, D
- Reduction :
  - ~ 120 hrs CPU time for family A, B
  - ~ 50 weeks of CPU time for family C
  - ~ 15 weeks of CPU time for family D
  - ➢ This is heavily parallelised

$gg \to ZZ$ at 2-loops

- Construct the amplitude and decompose into sum of all possible Lorentz structures and their 'form factors'

$$\mathcal{A}^{\mu\nu\rho\lambda} = \sum p_i^{\mu}\, p_j^{\nu}\, p_k^{\rho}\, p_l^{\lambda}\, A_{ijkl} + ..$$

- Solve linear system of equations to relate the 'form factors' to the original Feynman integral

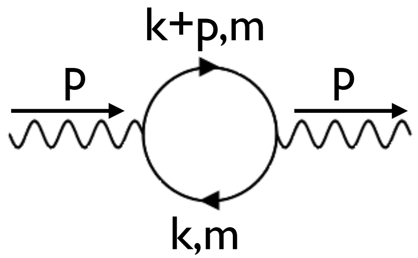
- Use **Integration By Parts** identities to reduce the number of integrals to a basis set

- Rotate the basis integrals to a set of **finite integrals** ⇒ Much better behaved numerically

- **Evaluate** the finite integrals **numerically** using 'sector decomposition' (plus any needed improvements

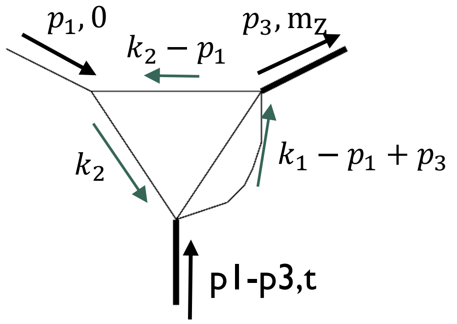
FINITE INTEGRALS

- Feynman integrals can have divergences
  - Ultraviolet divergence (UV) :



$$I = \int d^4k \frac{1}{(k^2 - m^2)\ \left((k+p)^2 - m^2\right)} \quad \sim \quad \int_{k\to\infty} \frac{dk}{k}$$

  - Infrared divergence (IR) :



$$I = \int d^4k_1 d^4k_2 \frac{1}{(k_2{}^2)\ (k_2 - p_1)^2\ (k_2 - k_1)^2\ (k_1 - p_1 + p_3)^2} \quad \sim \quad \int_{k_2\to p_1,\, p_1^2=0} \frac{dk}{k}$$

Dimensional regularisation : Use $D = 4 - 2\epsilon$ as the integration dimensions to extract explicit poles in ϵ

But :

- Can't expand the integrand around $\epsilon = 0$
- Can't numerically evaluate the integral

FINITE INTEGRALS

- Cannot expand around $\epsilon = 0$ inside the integrand

$$\int_0^1 dx\, dy \frac{1}{(x+y)^{2+\epsilon}}$$

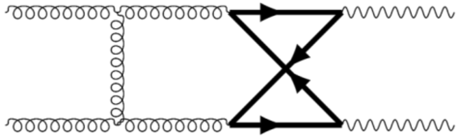
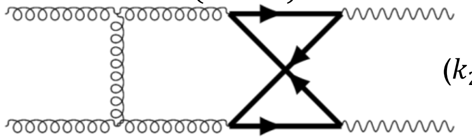
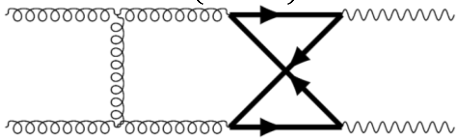
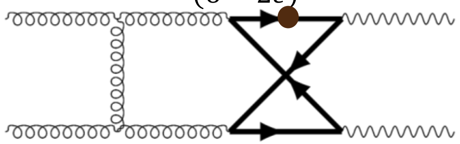
Overlapping singularities for $x \to 0, y \to 0$

- "Old standard" method to resolve singularities : Sector Decomposition
- Disentangle the overlapping singularities
- Public codes available : FIESTA 4, PYSECDEC

FINITE INTEGRALS

- Why use finite integrals?
  - Much better behaved numerically
  - Pole structure of the amplitude explicit

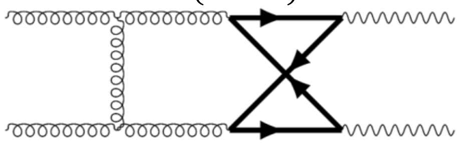
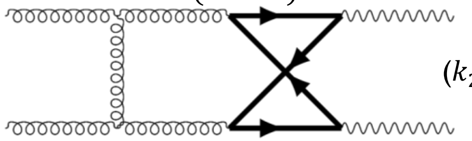
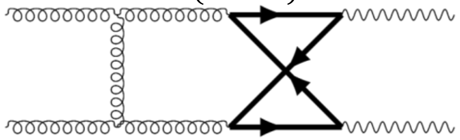
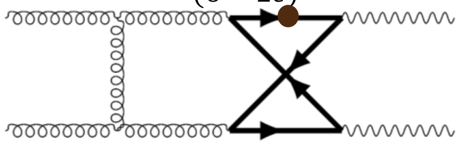
- How to get finite integrals?
  - Existence of a finite basis : A. von Manteuffel, E. Panzer & R. Schabinger arxiv:1411.7392
  - Reduze can generate finite integrals for any sector
  - Usually involves **dots** and **dimension shifts**

Integral	Rel.Err.	Timing(s)
$(4-2\epsilon)$ 	~1	123
$(4-2\epsilon)$  $(k_2^2-m_t^2)$	~5*10^-1	272
$(6-2\epsilon)$ 	~8*10^-4	81
$(6-2\epsilon)$ 	~2*10^-3	135

FINITE INTEGRALS

- Current prescription for finite integrals not enough
  - Not fast enough convergence
  - Reductions to such integrals very hard often e.g. integrals with up to **4 dots** required for computing the reductions to dimension shifted integrals

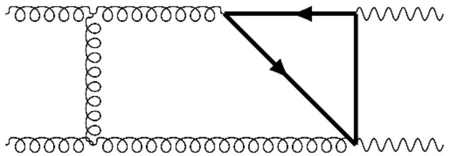
- Instead, use **linear combinations of divergent integrals** to produce finite integrals

Integral	Rel.Err.	Timing(s)
$(4-2\epsilon)$ 	~1	123
$(4-2\epsilon)$  $(k_2^2 - m_t^2)$	~5*10^-1	272
$(6-2\epsilon)$ 	~8*10^-4	81
$(6-2\epsilon)$ 	~2*10^-3	135

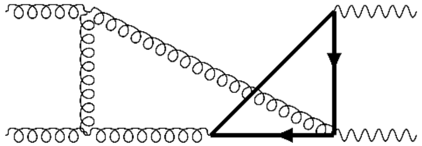
FINITE INTEGRALS

- Advantages:
  - Can write a custom integrator to evaluate such integrals much faster than available public codes : Initial tests suggest huge potential
  - Use integrals already appearing in the amplitude, often even as master integrals
  - Avoid computing reductions beyond those required for the amplitude

- Have a working code already; working on a more efficient implementation



$* \, (-s)$



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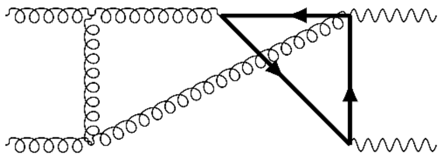
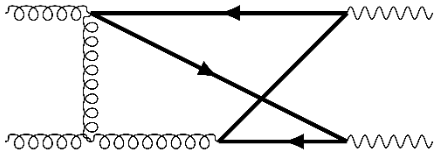
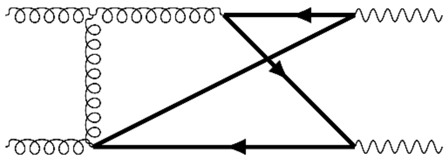


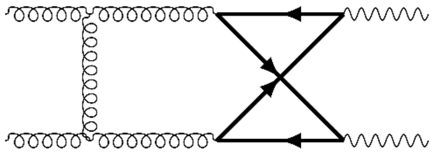
$* \, (s)$



$* \, (mz^2 - s - t)$

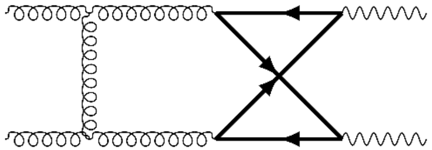


$* \, (mz^2 - t)$



$* \, s * (-mz^2 + s + t)$

$(k_2)^2 - m_t^2$



$* \, (-s)$

FINITE INTEGRALS

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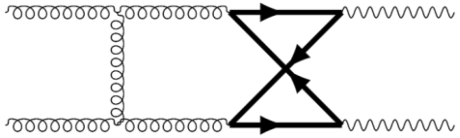
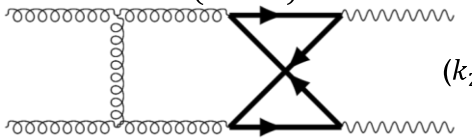
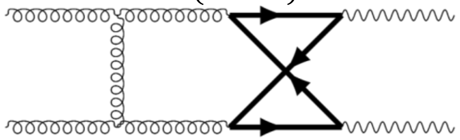
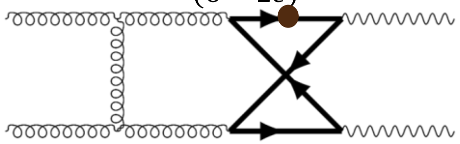
$(4-2\epsilon)$ 	~1	123
$(4-2\epsilon)$  $(k_2^2 - m_t^2)$	~5*10^-1	272
$(6-2\epsilon)$ 	~8*10^-4	81
$(6-2\epsilon)$ 	~2*10^-3	135
Linear combination	~2*10^-3	125

$gg \to ZZ$ at 2-loops

- Construct the amplitude and decompose into sum of all possible Lorentz structures and their 'form factors'

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- Solve linear system of equations to relate the 'form factors' to the original Feynman integral

- Use **Integration By Parts** identities to reduce the number of integrals to a basis set

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- **Evaluate** the finite integrals **numerically** using 'sector decomposition' (plus any needed improvements)

CONCLUSIONS

- Higher order calculations ever more important; need precision in theoretical predictions to match LHC data
- Great progress in the field of multiloop calculations
- Method of syzygies to construct smaller ibp systems very powerful
  - Can construct syzygies of other types, depending on the requirement
- Reductions for the amplitude to master integrals available
- Reductions for offshell Z-bosons still extremely challenging
- Exciting new method to construct finite integrals