Freeze out of relativistic particles: Limit on the neutrino mass

For details of freeze out: see lecture WS18/19 19.Nov 2019:
https://indico.mpp.mpg.de/event/6118/

In a nutshell:

Describe particles in thermal equilibrium and look at the evolution of the number of particles n_p in a co-moving volume, taking into account cosmological evolution (expansion) and number of degrees of freedom as function of temperature.

Entropy per co-moving volume is conserved $s \propto g^s_{eff} T^3$ with g^s_{eff} the effective number of degrees of freedom (function of T)

→ the particle density can be normalized to entropy: $Y \equiv \frac{n_p}{s}$.

→with cosmological evolution: $\dot{n}_p + 3Hn_p = s\dot{Y}$.

In thermal equilibrium:
Only annihilation and creation can occur: $p\bar{p} \leftrightarrow X\bar{X}$.
Here X denotes all possible states the particle can decay to →

$$\dot{n}_p + 3Hn_p = -\langle \sigma_{p\bar{p}\to X\bar{X}} |v| \rangle \left[n_p^2 - \left(n_p^{eq}\right)^2 \right]$$

or

$$\frac{x}{Y_{eq}} \frac{dY}{dx} = -\frac{\Gamma_A}{H} \left[\left(\frac{Y}{Y_{eq}} \right)^2 - 1 \right],$$

where $x = \frac{m}{T}$ with m an arbitrary energy scale usually chosen as the mass of the particle under consideration, $\Gamma_A = n_p^{eq} \langle \sigma_{p\bar{p}\to X\bar{X}} |v| \rangle$ describes the annihilation rate for the particle species under consideration (number density times thermally averaged annihilation cross-section for all available channels times the velocity) and $Y_{eq} = n_p^{eq}/s$ is the equilibrium number of particles n_p per co-moving volume.

→ "Effectiveness of annihilations": $\frac{\Gamma_A}{H}$ times a measure for deviation from thermal equilibrium equal to change of number density per commoving volume per temperature change temperature change (normalized to equilibrium number of particles)

For relativistic particle species (i.e. $x \gg 3$): $n_{rel} \propto T^3$

For relativistic particles: Freeze out occurs at $x_f \lesssim 3$.

We see that

$$Y_{eq,rel} = \frac{n_{eq,rel}}{s} \propto \frac{T^3}{s} \propto \frac{T^3}{T^3} \propto const.$$

is temperature independent, hence not changing in time as long as the particle is relativistic. Hence, the asymptotic value $Y(x \to \infty) \equiv Y_\infty$ is the equilibrium value at freeze out, can be calculated.

For the present relic mass density this translates into

$$\rho_{p,0} = s_0 Y_\infty m = 2970\, Y_\infty \left(\frac{m}{eV}\right) eV\ cm^{-3}$$

or in terms of critical density

$$\Omega_p h^2 = 0.078 \frac{g_{eff}}{g_*(x_f)} \left(\frac{m}{eV}\right).$$

This can be used to derive an upper limit on the mass density due to known neutrino species. We know $\Omega_0 h^2 \lesssim 1$. Using the decoupling temperature of neutrinos $T\sim$ few MeV this implies:

$$\Omega_{\nu\bar{\nu}} h^2 = \frac{m_\nu}{91.5\ eV}$$

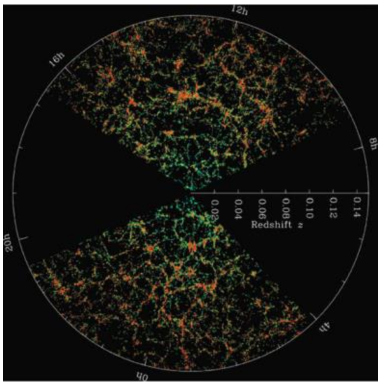
or

$$\sum m_{\nu\bar{\nu}} \lesssim 91.5\ eV.$$

Note that this solution is only very mildly dependent on the exact process of freeze out, due to the flatness of Y_{eq} as a function of T for $x_f \lesssim 3$.

Structure formation

The derivation of cosmological models is based on the cosmological principle: homogeneous and isotropic universe on cosmologically relevant distance scales $\frac{c}{H_0}$. Up to distance scale of $\sim 100\ Mpc$ there is large scale structure.



Distribution of galaxies derived from data of the Sloan Digital Sky Survey (SDSS).

→ How did observed structures form from tiny density fluctuations at decoupling of order $\frac{\Delta T}{T} \sim 10^{-5}$?

Evolution of universe is strongly coupled to content of universe!

The large scale structures are described by relative density contrast:

$$\delta(\vec{x},t) := \frac{\rho(\vec{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)} = \frac{\Delta\rho(\vec{r},t)}{\bar{\rho}(t)}$$

with $\bar{\rho}(t)$ the average density of the universe.

For a (matter dominated, i.e. $P = 0$) region with $\Delta\rho > 0$:

→ Gravitational field in this regions is stronger than average,

→ Hubble rate (expansion) in this region is slower,

→ the decrease of density due to expansion in this region is suppressed,

→ the density contrast $\delta(\vec{x},t)$ further increases.

For a region with $\Delta\rho < 0$ the opposite is the case, i.e. $\delta(\vec{x},t)$ decreases.

Growth of density fluctuations can be explained by the interplay of gravitation and expansion.

More quantitatively:

The content of the matter dominated universe can be described by hydrodynamics of a radiation free "cosmic fluid" made from dust → For distances $< \frac{c}{H_0}$ description by Newton's law of gravitation.

Using hydrodynamic equations for description of self-gravitating fluid:

Continuity equation: $\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$ (conservation of matter),

Euler equation: $\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{\vec{\nabla}\cdot\vec{P}}{\rho} - \vec{\nabla}\Phi$ (conservation of momentum),

Poisson equation: $\nabla^2\Phi = 4\pi G\rho$,

and taking into consideration cosmic expansion it can be shown that for small $\Delta\rho$ (linear perturbation theory):

$$\delta(\vec{x},t) = D(t)\cdot\tilde{\delta}(\vec{x})$$

with $\tilde{\delta}(\vec{x})$ a time independent relative density contrast. For $D(t)$ we find the differential equation

$$\ddot{D} + \frac{2\dot{a}}{a}\dot{D} - 4\pi G\bar{\rho}D = 0.$$

$D(t)$ is independent of location!

For the special case of a universe with $\Omega_m = 1$ and $\Omega_\Lambda = 0$ the equation can be solved analytically and one finds the solutions

$$D_+(t) = a(t) \qquad \text{and} \qquad D_-(t) = t^{-1}.$$

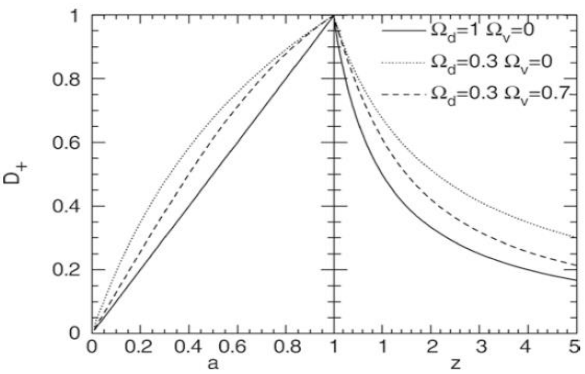
The latter would mean decreasing fluctuations, hence can be disregarded as non-realistic in the epoch from decoupling until now.

→ $\delta(\vec{x}, t) \propto a(t)$ Density contrast is growing linearly with scale factor!

Choosing (slightly) different input parameters changes evolution, but behavior is similar (see Fig.).

Taking this solution and the model used to derive it, one would expect that the density fluctuations in the CMB $z{\sim}1000$ were three orders of magnitude lower than they are now. At the present epoch: $\delta(\sim 2Mpc) \gg 1$ at the scale of galaxy clusters and $\delta(\sim 10Mpc) \sim 1$ at the scale of superclusters.

→ Expectation: $\delta(CMB){\sim}10^{-3}$.



Growth factor $D_+(t)$ as a function of a(t) (left) and z (rigt) for different cosmological models. For a critical matter dominated universe the growth of structures growth linearly with $a(t)$. The qualitative behavior of different realistic models is similar.

However, we observe $\delta(CMB) \cong 10^{-5}$, i.e. two orders of magnitude difference → This can be taken as a hint that visible matter (baryons) alone can not explain large scale structure as seen in galaxy surveys.

Note: We have treated only matter dominated epoch. In radiation dominated epoch another term is entering the differential equation:

$$\ddot{D} + \frac{2\dot{a}}{a}\dot{D} + (\frac{v_s^2 k^2}{a^2} - 4\pi G\bar{\rho})D = 0$$

with $v_s = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{adiabatic}}$ the speed of sound and $k = \frac{2\pi}{\lambda}$ the wave number.

This leads to oscillating solution for $D(t)$ if

$$\frac{v_s^2 k^2}{a^2} - 4\pi G\bar{\rho} > 0.$$

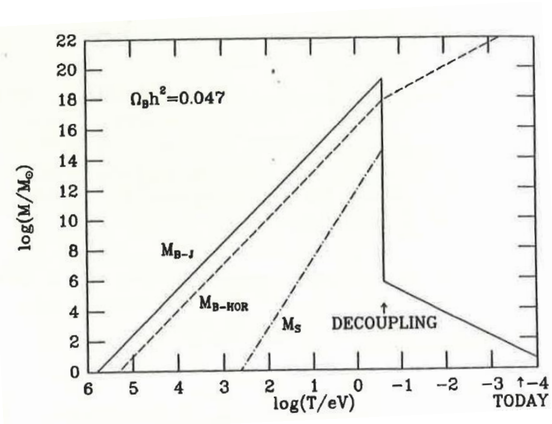
This means that perturbations are propagating as sound waves. This is the mechanism that leads to structure seen in the CMB anisotropy (acoustic peaks).

If above term becomes negative, i.e. for $k_J < \sqrt{\frac{4\pi G\bar{\rho}a^2}{v_s^2}}$ structures become unstable, i.e. there will be a gravitationally collapse. In other words: If the mass inside a sphere with radius $\lambda_J = \frac{2\pi}{k_J}$ is larger than the Jeans mass

$$M_J = \frac{4\pi}{3}\left(\frac{\pi}{k_j}\right)^3 \bar{\rho} = \frac{\pi^{5/2}}{6}\frac{v_s^3}{\bar{\rho}^{1/2}}G^{-3/2} \cong 5 \cdot 10^{18}(\Omega_b h^2)T_{eV}^{-3}M_{\odot}$$

the volume is instable against gravitational collapse. M_J is varying during the evolution of the universe; during radiation domination $M_J > M_{HOR}$, i.e. structures do not collapse gravitationally. During decoupling radiation does cease to effect matter, hence M_J is decreasing below M_{HOR} and structure growth starts.

Intriguing: $M_{B-J} \sim M_{globular\ cluster}$



Jeans Mass as a function of temperature for a baryon dominated universe. (Taken from Kolb& Turner, The Early Universe, Westview Press, 1990)

This behavior is qualitatively valid for any gravitational non relativistic component in the early universe going out of thermal equilibrium: Decoupled non relativistic component can start structure formation much earlier! We have seen that linear growth by baryons only is not enough to explain seen structures

→ <u>Dark Matter component?</u>

Different models lead to different evolution → Observation of large scale structure and comparison to models can give information on cosmological parameters.

Note that for $\delta\rho \cong \rho$ linear perturbation theory can no longer be applied → growth of density fluctuations is no linear anymore!

→ Simulations are needed!

Note that we have not treated any decoupled relativistic component of the universe, i.e. Neutrinos! This component is referred to as a <u>free streaming</u> component. If there was a free streaming component in the epoch of thermal equilibrium of the baryon-photon fluid: structures are damped by gravitational pull of ~ homogeneous density distribution!

- ➔ Information on neutrino mass!

<u>Representation of large scale structure:</u>

Density contrast $\delta(\vec{x},t)$ describes density fluctuations of one unique universe with very strong dependence on initial conditions of fluctuations.

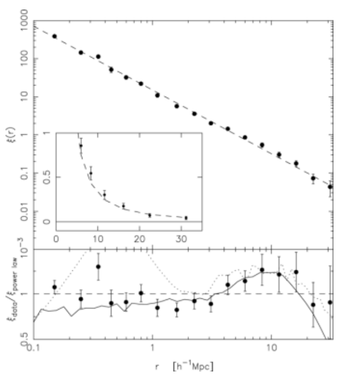
In order to describe models: need to treat statistical properties:

Two simulations of a universe are considered equivalent if all their statistical properties are identical, while their individual maps can be different. For example: Surface of a lake

→ <u>The two point correlation function</u>

$$P = (\bar{n}dV)^2[1 + \xi(\vec{x},\vec{y})]$$

quantifies the probability to find a galaxy in volume element dVat coordinates $\vec{y}$ if there is a galaxy in the volume element dV at coordinates $\vec{x}$, with $\bar{n}$ the mean galaxy density.



Two point correlation function as measured by the 2dF survey. The function is well described by a power law.

→ <u>The power spectrum</u>:

Description of fluctuations as function of scale (see CMB):

$\delta(\vec{x}) = \sum_k a_k \cos(\vec{x} \cdot \vec{k})$

Decomposition of relative density contrast into plane waves with wave vector $\vec{k}$ and amplitude a_k. The power spectrum $P(k)$ describes the mean of the squared amplitudes a_k^2 over all wave vectors with the same length.

The power spectrum and the correlation function are related by Fourier transformation:

$$P(k) = 2\pi \int_0^\infty dr\, r^2 \frac{\sin kr}{kr} \xi(r).$$

→ $\xi(r)$ also depends quadratically on average apmlitudes of fluctuations, i.e. density contrast.

Dependence of $P(k)$ and $\xi(r)$ on $D_+(t)$ in linear regime can be written as

$$P(k,t) = D_+^2(t) P(k,t_0) \text{ and } \xi(k,t) = D_+^2(t)\xi(k,t_0).$$

<u>Observations</u>:

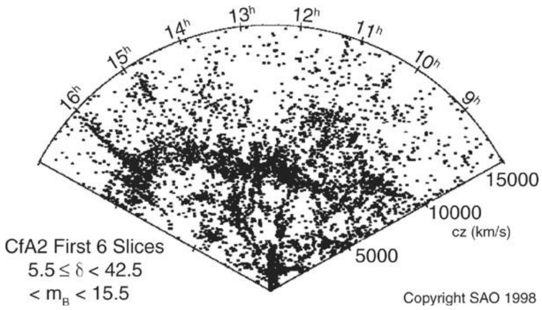
In the 1980's first large scale surveys were started.

1980s: *Center for Astrophysics* (CfA) survey measured redshifts of ~14000 galaxies with distances up to $cz{\sim}15000\,\frac{km}{s}$ ($z{\sim}0.05$): Discovery of great wall, "Fingers of God" due to higher average velocities of galaxies in clusters), first hints for voids.

1990s: *Las Campanas Redshift survey (LCRS)* measured redshifts of ~26.000 galaxies with $cz{\sim}60000\,\frac{km}{s}$ ($z{\sim}0.2$): Clear indication for galaxies lying on filaments, honeycomb (void) structure of galaxy distribution. No structures at scales comparable to the extent of the survey!

2000s: *2dF survey*: Simultaneous measurement of up to 400 spectra with 4m telescope. 230.000 galaxies measured!

Sloan Digital Sky Survey (SDSS): Dedicated 2.5m telescope! Scanned ~quarter of the sky in five photometric bands. ~200.000.000 objects scanned! > 1.000.000 galaxy redshifts, ~ 90.000 Quasar spectra.

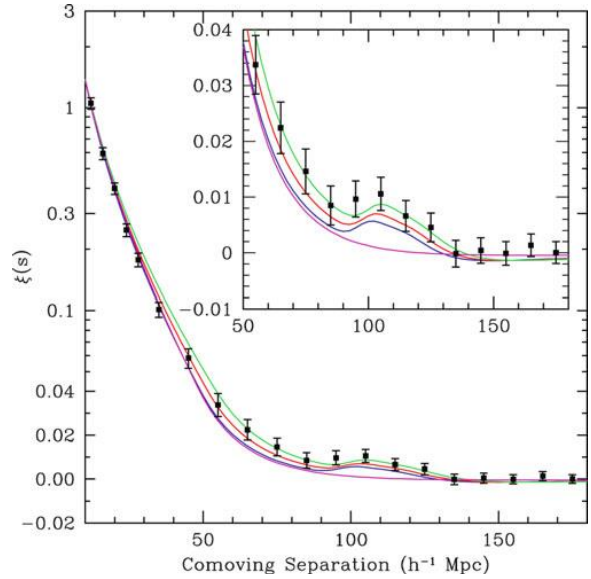


Results from the CfA survey. A "great wall" of galaxies and voids (regions with low galaxy density) can be seen. Some "Fingers of God" are also visible.

Large scale surveys like SDSS and 2dF have made it possible to extract cosmologically relevant information from the galaxy maps.

→ <u>Observation of Baryon Acoustic Oscillations</u> (BAO): Characteristic size of baryon-photon fluid oscillations imprinted on CMB power spectrum is also visible as a characteistic peak in the two point correlation for galaxies at the characteristic separation of $\sim 150 Mpc$!

The strength of the BAO peak is sensitive to the matter conent of the universe.



Two point correlation function for galaxies as measured by the SDSS. The BAO peak appears at commoving separation of about $100\ h^{-1}\ Mpc\ \cong 150\ Mpc$. The curves represent predictions for ΛCDM models with $\Omega_b h^2 = 0.024$ and $\Omega_m h^2 = 0.12, 0.13, 0.14$ as well as the prediction for a universe with $\Omega_b = 0$. Taken from: D. Eisenstein et ApJ 633 (2005) 560

<u>Formation of Strucures</u>:

Structure formation is believed to start well before decoupling of baryons from photons (see above). A Dark Matter component starts forming structure while baryons are rather smoothly distributed until they decouple. Only after decoupling occurred they can fall into the potential wells that already exist through structure formation by dark matter component.

Numerical simulations of Large Scale Structure:

The solution for the differential equation describing growth of density contrast is mostly not analytically calculable. Existing solutions describe only very limited cases.

Gravitational dynamics are *non-linear* hence numerical methods need to be used to calculate structure growth → Simulation

Principle of simulations: It is believed that structure growth is dominated by dark matter. Hence, the evolution of dark matter density is usually simulated. This is enough for many applications. Lately, due rapid evolution of computing power it is also possible to include hydrodynamic processes and radiative transfer → include baryonic/radiative content of universe!

The following steps need to be considered when performing simulations:

- *Chose representative Dark Matter Particles*: It is not possible to treat DM particles individually over the whole universe → simulate "macroscopic DM particles" with mass M assuming that they behave like DM particles in volume $V = \frac{M}{\rho}$

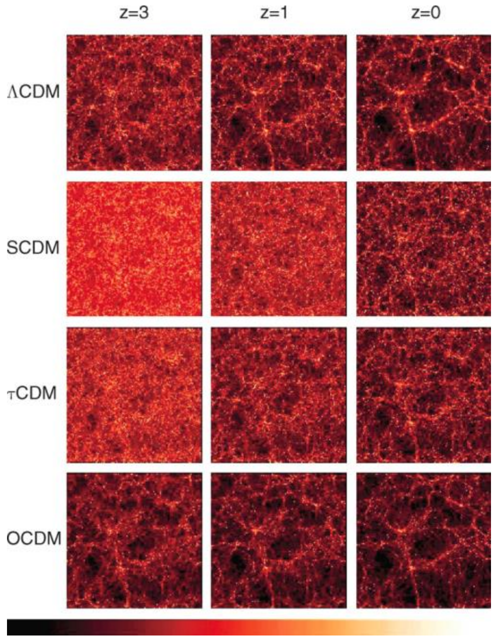
- *Chose representative simulation volume*, i.e. larger cube than largest structures in observable universe with side length $L \cong 200\ h^{-1}Mpc$. This determines (by computational power and time available) resolution in terms of M.

- *Periodic boundary conditions*: As universe is not empty outside the box, force from outside the box has to be taken into account → Chose periodic boundaries, i.e. a particle leaving the box on one side enter the box on the other side

- *Softening length*: Choice of representive DM particles M falsifies interaction with small impact parameter. Scattering for low impact parameter needs to be "softened" → Definition of spatial resolution of simulation.

- *Computation of Force field on each DM particle*: Need approximation of total force $F_i = \sum_{i\neq j} \frac{M^2(r_j - r_i)}{|r_j - r_i|^3}$(sum of forces from all particles in the simulation, including periodic boundary, i.e. ∞): Introduce adaptive grid and shift mass of all particles to closest grid point to calculate field by FFT.

- *Set Initial conditions*: Start at very high redshift with distribution of particles having power spectrum resembling Gaussian random field of the model considered (can be theretically calculated).



Simulated structures (VIRGO consortium) for different model universes: ΛCDM: $\Omega_\Lambda = 0.7, \Omega_m = 0.3$ *; SCDM and τCDM:* $\Omega_\Lambda = 0.0, \Omega_m = 1.0$ *and OCDM:* $\Omega_\Lambda = 0.0, \Omega_m = 0.3$*. SCDM and τCDM differ by initial shape of the power spectrum.* 256^3 *particles were tracked.*

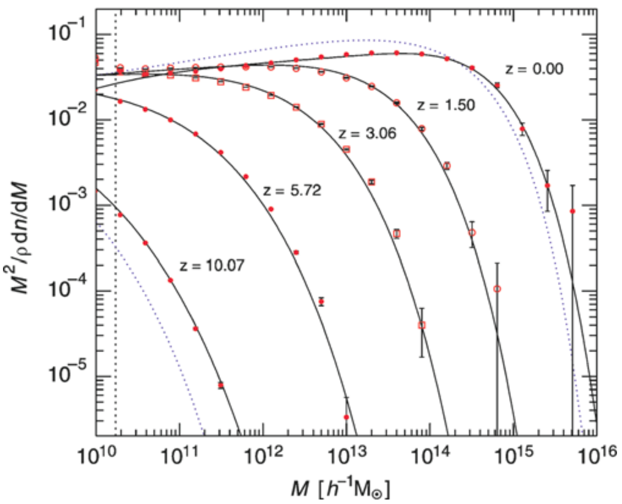
Simulation of the evolution of large scale structures are done for different cosmological parameters. The statistical properties of the simulated structures can be directly compared to the ones obtained from the existing galaxy surveys.

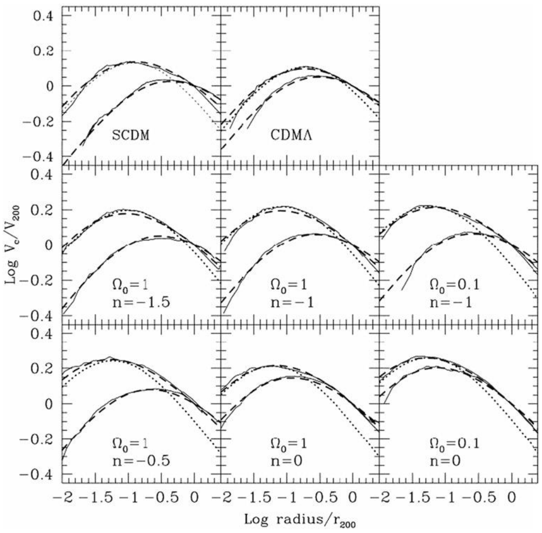
Simulations can resolve structures down to the size of galactic halos (defined as a spherical volume in which the density is ~200 times the critical density at respective redshift). From simulations it appears that there is a **universal** galactic halo density profile halo → The **NFW profile (Navarro-Frenk-White)**:

$$\rho(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2}$$

With ρ_s the amplitude of the profile and r_s a characteristic length.
For $r \ll r_s$: $\rho \propto r^{-1}$, while for $r \gg r_s$: $\rho \propto r^{-3}$ → r_s represents the characteristic length at which the slope of the density distribution changes. No analytical argument has so far been found to explain such a universal profile. In fact latest simulations have obtained slightly different results, in particular in the center regions of the galaxies. (probably resolution issues)

Reconstructions of density profiles of low surface brightness (LSB) galaxies, that are assumed to be dominated by dark matter, do not seem to be in agreement with the NFW profile (See Fig. below).





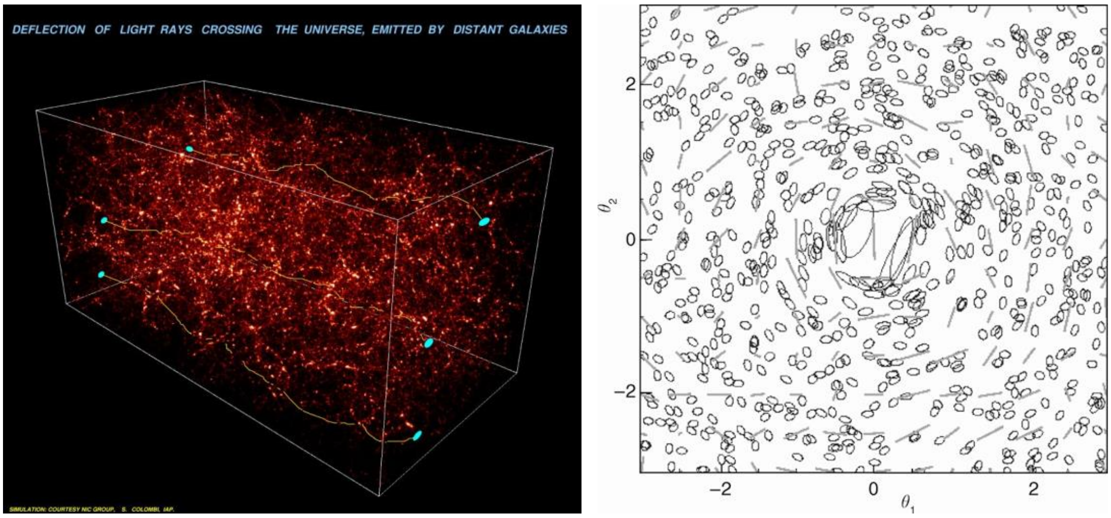
Left: Simulated mass spectrum of dark matter halos for different redshifts as obtained from the millennium simulation. Shown are also results obtained from combination of different simulations (solid lines). Dotted lines represent predictions (Press Schechter model with $P(k) \propto k^n$*). Right: Comparison galactic rotation curves obtained from simulated dark matter halos (solid lines) with those expected from a NFW profile (dashed curves) for different model universes. The dotted curves represent a different halo profile model (Hernquist profile). (both Figures taken from P. Schneider, Extragalactic Astronomy and Cosmology)*

Lyman α-forest:

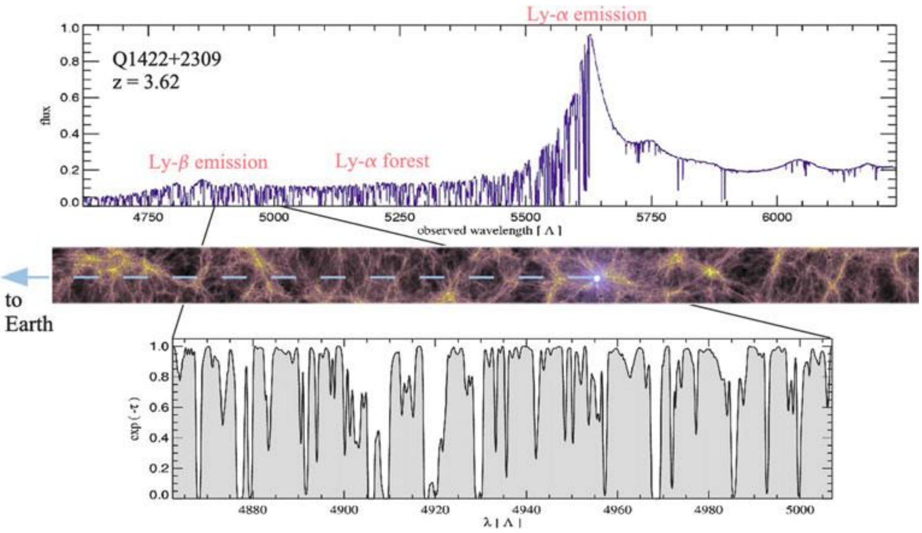
Lyman series of hydrogen: transition from/and to energy level $n = 1$ of hydrogen. Transition between levels $n = 1$ and $n = 2$ → Ly-α at $\lambda_{\mathrm{Ly-\alpha}} = 1216$ Å.

The redshifted Ly-α line is often found in quasars at redshifted wavelength $\lambda_{obs} = (1 + z_{em}\lambda_{\mathrm{Ly-\alpha}})$.

At lower wavelengths often a "forest" of absorption lines can be detected. These result from $Ly - \alpha$ absorption of light emitted from the quasar in intergalactic neutral hydrogen with lower redshift. This allows tracing of the hydrogen density along the line of sight of individual cosmological objects.



The effects of cosmic shear and weak lensing: light from cosmologically distant galaxies travels through the gravitational potential of structures and is thus distorted (left, taken from http://www.cfhat.hawaii.edu/News/Lensing). From the deviation of the average galaxy shapes conclusions can be drawn on the matter density on the line of sight – cosmic shear. If a galaxy cluster is passed, a deformation tangential to the center of mass of the galaxy cluster can be reconstructed – weak lensing(right, taken from P. Schneider, Extragalactic astronomy and Cosmology).



Spectrum of a quasar with $z = 3.62$ (top). The $Ly - \alpha$ forest can be reproduced by simulation of ΛCDM models (center, bottom) with very high fidelity. They are statistically indistinguishable from observed spectra. (Taken from Nature 440(2006)1137)