Instantons in Supersymmetric Gauge Theories and D-brane Models

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Foreword

Very extensive literature:

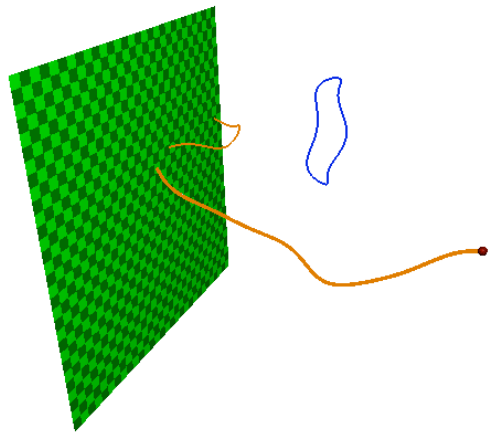
- Earlier work: ADHM, Khoze, Dorey, Mattis, Moore, Nekrasov, Witten, Douglas, Green, Gutperle, ...
- Recent developments: Turin and Rome group, Munich group, Madrid group, Penn, Stanford, ...

For a review see *e.g.*

[R. Blumenhagen, M. Cvetic, S. Kachru, T. Weigand, *arXiv:0902.3251*]

Outline

- Short introduction
  - Multi-instanton calculus
  - Instantons and D-branes (Type IIB, orbifold background)
- Current work
  - Brane model to study instantons on the Higgs branch of $\mathcal{N} = 2$ SYM theory.



- I will focus on theories with $\mathcal{N} = 2$ supersymmetry
  - They are a good theoretical laboratory for testing instanton calculus techniques
  - We can obtain exact results based on the constraints imposed by supersymmetry (*e.g.* Seiberg-Witten).

Multi-instanton Calculus

- Problem: General approach to multi-instanton contributions
- Atiyah-Drinfeld-Hitchin-Manin (ADHM): construction of the solutions of the self-dual equation

$$F = \star F$$

- ADHM can be generalized to study instantons in supersymmetric theories.

Multi-instanton calculus:

1. Construct the ADHM supersymmetric multiplets
2. Perform the semi-classical approximation with the path integral
3. Calculate the instanton effective action $\tilde{S}$
4. Calculate instanton corrections to the correlation functions.

Multi-instanton Calculus

- The result for the partition function is:

$$Z_k \sim e^{2\pi i k \tau} \int d\mathcal{M}_k e^{-\tilde{S}};$$

- Instanton effective action for pure $\mathcal{N} = 2$ theory:

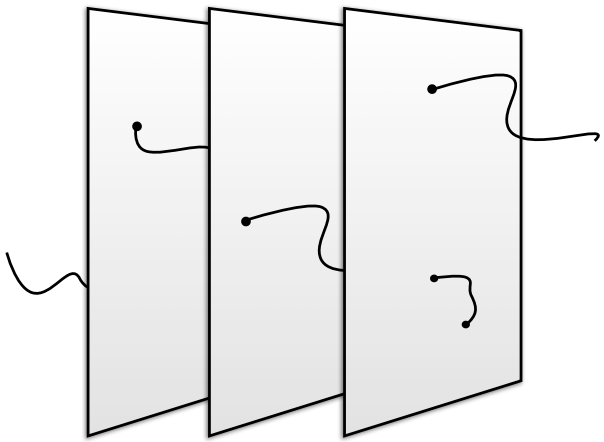
$$\tilde{S} = 4\pi^2 \mathrm{Tr}\left[\bar{\mu}\langle z\rangle\nu - \bar{\nu}\langle z\rangle\mu + \bar{\omega}^{\dot{\alpha}}\langle z\rangle^2\omega_{\dot{\alpha}} - \varphi \mathbf{L}\varphi\right].$$

- These ingredients + powerful integration techniques [N. Nekrasov et al.]: explicit calculation of instanton contributions to the prepotential.

These results have to be compared to the results obtained from the study of the Seiberg-Witten curves. This has been done and there is a perfect agreement.

Dirichlet Branes

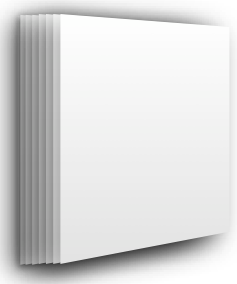
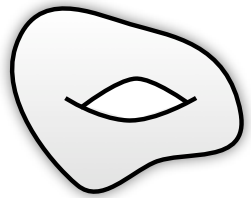
- Fundamental object for Gauge/Gravity: Dirichlet brane
- Dynamic at low energy is described by massless excitations of strings attached to the brane
  - Abelian supersymmetric gauge theory with 16 supercharges on the worldvolume
- Stack of N branes: non-abelian gauge theory with gauge group $U(N)$.



Non-trivial Backgrounds

- Stack of $D3$ branes

  Space-time (10D) = Worldvolume (4D) + Transverse space (6D)

$$\mathcal{M}_{10D} =$$  $$\otimes$$ 

- One simple choice that permits to engineer the $\mathcal{N} = 2$ theory on the $D3$ branes worldvolume is the following orbifold:

$$\mathbb{C} \times \mathbb{C}^2/\mathbb{Z}_2.$$

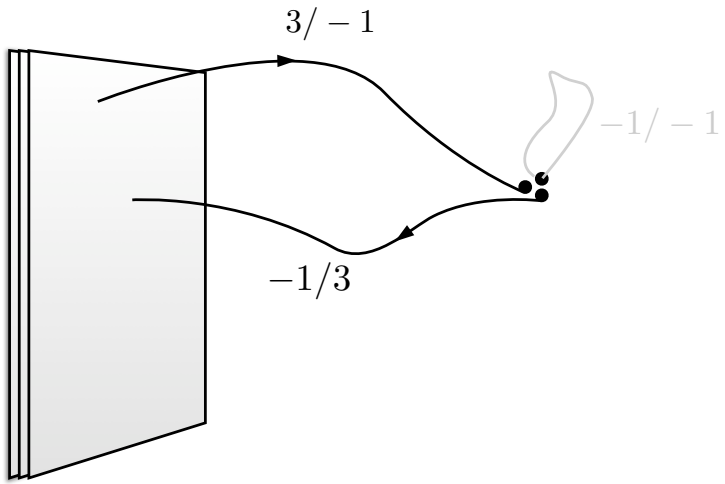
Instantons

- Instantons can be described with brane systems
- $D3/D(-1)$ brane system: the $D(-1)$ brane is an instanton of the gauge theory living on the worldvolume of the $D3$ brane

  [E. Witten, M. Douglas]
- Which is the interpretation of string states suspended between a $D3$ and a $D(-1)$ brane and with both extremities on the $D(-1)$ brane?
- They are in correspondence with the ADHM supersymmetric multiplets of the gauge theory on the $D3$ brane.

D3/D(-1) System on $\mathbb{R}^4 \times \mathbb{C} \times \mathbb{C}^2/\mathbb{Z}_2$

- ADHM moduli for $\mathcal{N} = 2$
- Essential role of $3/(-1)$ strings: twisted states



Twisted NS sector

$\omega_{\dot\alpha}$ $\quad$ $\bar\omega_{\dot\alpha}$

Twisted R sector

μ^A $\quad$ $\bar\mu^A$

NS sector

a'_μ $\quad$ χ $\quad$ D

R sector

$M^{\alpha A}$ $\quad$ $\lambda_{\dot\alpha A}$

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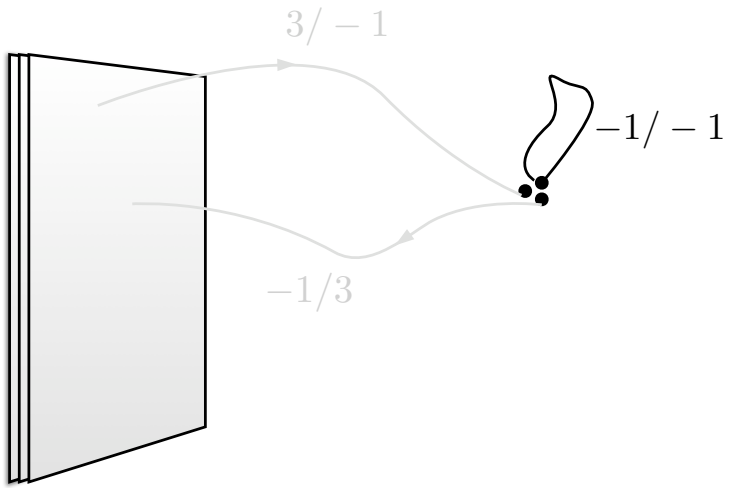

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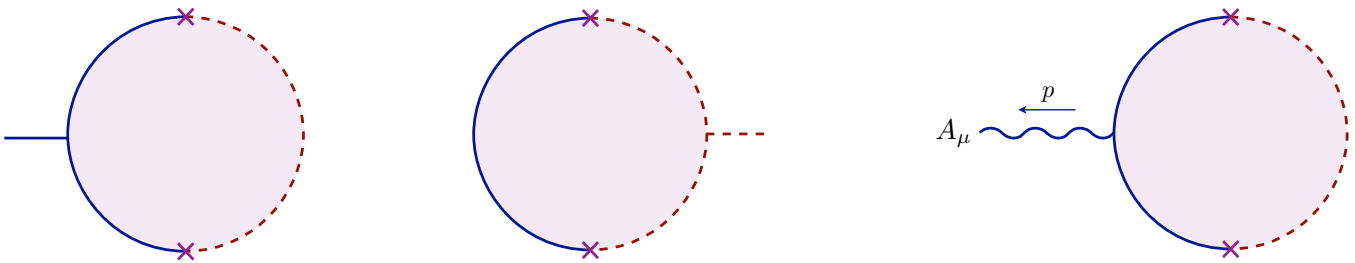
Instanton Calculus and Open Strings

Open strings $\longrightarrow$ multi-instanton calculus:

- Instanton effective action $\tilde{S}$
  - Computation of scattering amplitudes between open string states with mixed boundary conditions: mixed disk diagrams
- Instanton profiles
  - Mixed disks with emission of a gauge field
- Prescription for the integration: $S_{eff}^{(k)} \sim e^{2\pi i k \tau} \int d\mathcal{M}_k e^{-\tilde{S}}$.

[M. Billó, M. Frau, I. Pesando, F. Fucito, A. Lerda, A. Liccardo]



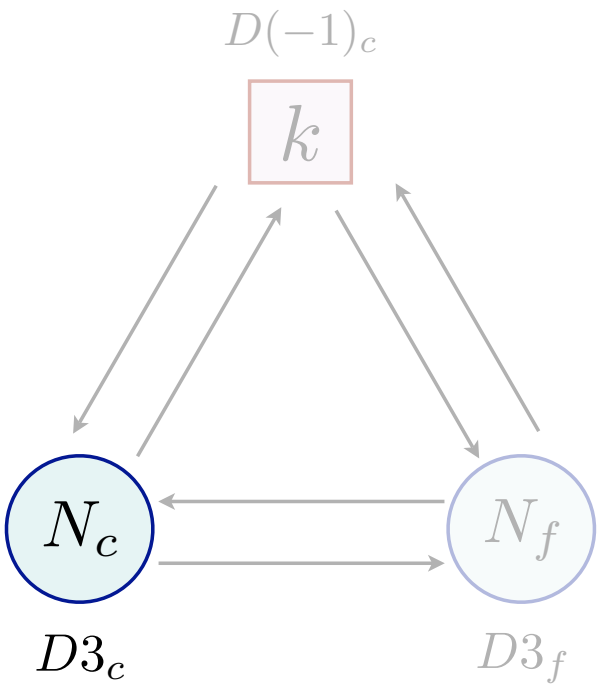
Instantons on the Higgs Branch

As an application of the methods described above I will show how to calculate the instanton effective action for instantons on the Higgs branch of the $\mathcal{N} = 2$ SYM theory with matter.

- It would be interesting to see if there is an instanton contribution in this case
- The instanton effective action calculated with the instanton calculus techniques in field theory coincide with the string theory result (Gauge/Gravity checked).

Brane model analysis

The model: $D3_c/D3_f/D(-1)_c$ on $\mathbb{R}^4 \times \mathbb{C} \times \mathbb{C}^2/\mathbb{Z}_2$



$D3_c/D3_c$	$\longrightarrow$	Chiral vector multiplet Φ
$D3_c/D3_f$	$\longrightarrow$	Hypermultiplet $\mathcal{Q}$
$D(-1)_c/D(-1)_c$		
$D3_c/D(-1)_c$	$\longrightarrow$	ADHM moduli
$D3_f/D(-1)_c$		

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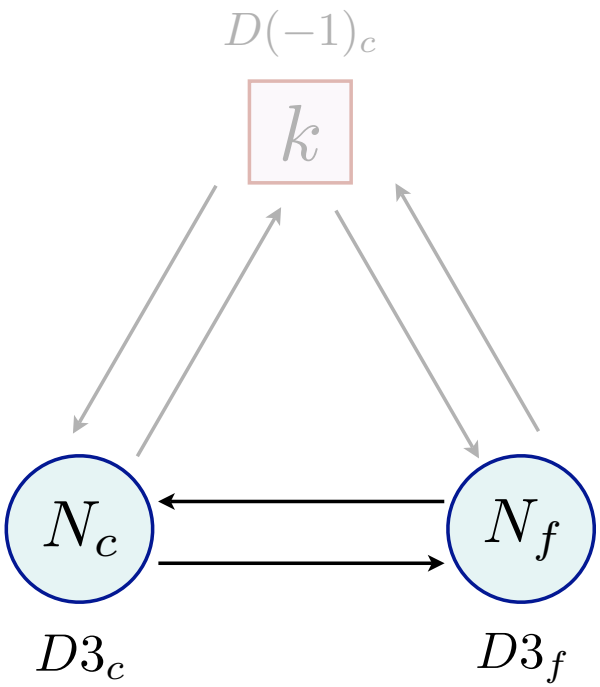

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$D(-1)_c/D(-1)_c$

$D3_c/D(-1)_c \longrightarrow$ ADHM moduli

$D3_f/D(-1)_c$

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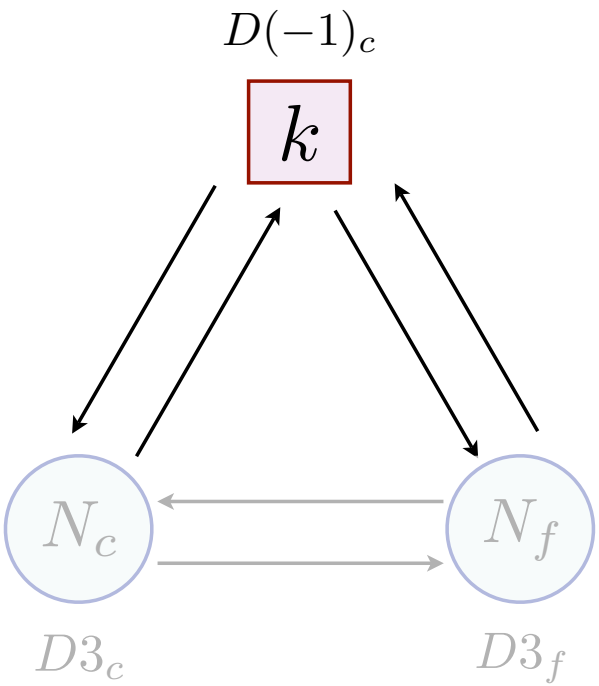

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$D3_c/D3_f$ ⟶ Hypermultiplet $\mathcal{Q}$

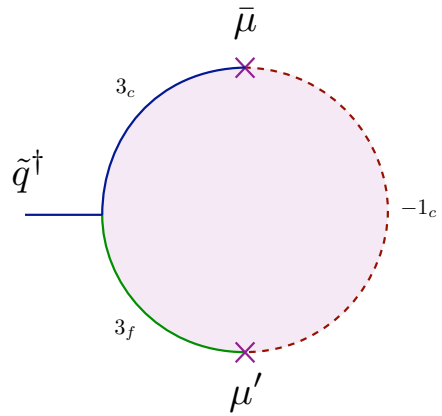
$D(-1)_c/D(-1)_c$

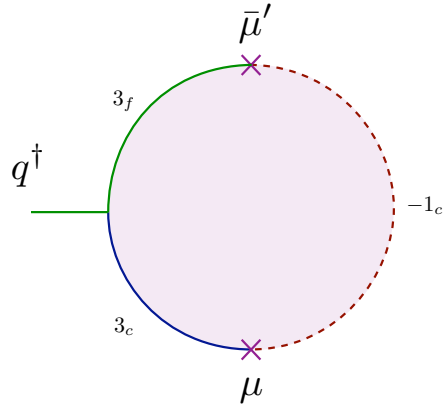
$D3_c/D(-1)_c$ ⟶ ADHM moduli

$D3_f/D(-1)_c$

Disk Amplitudes

- The instanton effective action $\tilde{S}$ is obtained from scattering amplitudes on disks between open strings attached to the branes.
- Let us consider amplitudes with insertions of ADHM moduli and hypermultiplet. Three-point amplitudes:

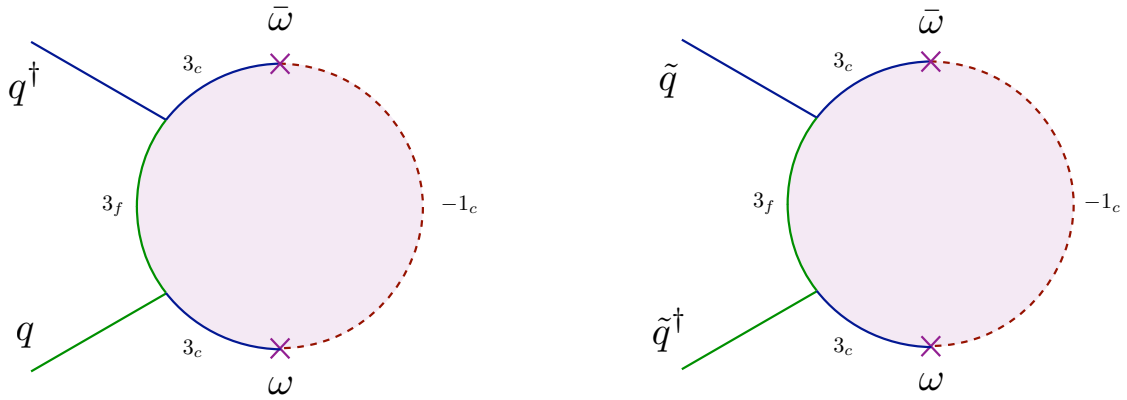




$$\tilde{\mathcal{S}}_3 = -\langle\!\langle\, \mathcal{V}_{\bar{\mu}} \mathcal{V}_{\tilde{q}^\dagger} \mathcal{V}_{\mu'} \,\rangle\!\rangle = -\frac{8\pi^2}{g^2} \int \frac{\prod_i dz_i}{dV_{CKV}} \left\langle \mathcal{V}_{\bar{\mu}}(z_1) \mathcal{V}_{\tilde{q}^\dagger}(z_2) \mathcal{V}_{\mu'}(z_3) \right\rangle$$
$$= \cdots = -i\bar{\mu}(\tilde{q})^\dagger \mu'.$$

Disk Amplitudes

Four-point amplitudes:



$$\tilde{\mathcal{S}}_4 = -\left\langle\!\left\langle\, \mathcal{V}_{\bar{\omega}} \mathcal{V}_{q^\dagger} \mathcal{V}_q \mathcal{V}_\omega \,\right\rangle\!\right\rangle = \cdots = \bar{\omega}^{\dot{\alpha}} q^\dagger q \omega_{\dot{\alpha}}$$

The scalars can be promoted to the whole hypermultiplet by supersymmetry:

- $q \longrightarrow q(x) \longrightarrow Q(x, \theta)$
- $\tilde{q} \longrightarrow \tilde{q}(x) \longrightarrow \tilde{Q}(x, \theta)$

Result

Instanton effective action:

$$\begin{aligned}\tilde{S} = S_{ADHM} + \\ &- 2[\chi^\dagger, a'_\mu][\chi, a'^\mu] + (\omega_{\dot\alpha}\chi - \Phi\omega_{\dot\alpha})(\chi^\dagger\bar\omega^{\dot\alpha} - \bar\omega^{\dot\alpha}\bar\Phi) \\ &+ (\omega_{\dot\alpha}\chi^\dagger - \bar\Phi\omega_{\dot\alpha})(\chi\bar\omega^{\dot\alpha} - \bar\omega^{\dot\alpha}\Phi) \\ &+ i\bar\mu^A\epsilon_{AB}(\mu^B\chi^\dagger - \bar\Phi\mu^B) - iM^{\alpha A}\epsilon_{AB}[\chi^\dagger, M^B_\alpha] \\ &+ \bar\omega^{\dot\alpha}(Q)^\dagger Q\omega_{\dot\alpha} + \bar\omega^{\dot\alpha}\tilde{Q}\tilde{Q}^\dagger\omega_{\dot\alpha} - i\bar\mu\tilde{Q}^\dagger\mu' - i\bar\mu' Q^\dagger\mu.\end{aligned}$$

Summary

- Brane models permit to clarify instanton aspects of supersymmetric gauge theories; stringy instantons lead to new phenomena important for phenomenological models.
- I showed a model that permits to engineer the $\mathcal{N} = 2$ theory with matter and to study instantons on the Higgs branch. The instanton effective action is computed from mixed disk diagrams.
- Current work
  - Perform the integration over instanton moduli space:

$$S^{(k)}_{eff}(\Phi, \mathcal{Q}) \sim e^{2\pi i k \tau} \int d\mathcal{M}_k e^{-\tilde{S}(\mathcal{M}, \Phi, \mathcal{Q})}$$

  - Analyze other solitons in field theory (monopoles, vortices) with type IIB brane models on orbifolds; stringy interpretation of Nahm construction, ...

Summary

Thank you!