Chiral-Scale Perturbation Theory

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O. Catà and C. M., arXiv: 1906.01879

Outline

- Chiral Perturbation Theory
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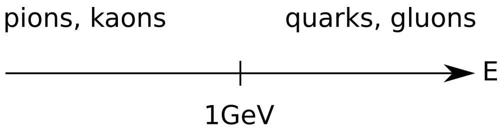
- Comparison at one loop

Chiral Perturbation Theory (χ_3PT)

- QCD becomes non-perturbative at energies $\lesssim 1GeV$
- Change of degrees of freedom



$m_u, m_d, m_s \ll 1GeV \Rightarrow$ neglect light quark masses

$\Longrightarrow$ QCD Lagrangian exhibits $SU(3)_L \times SU(3)_R$ symmetry:

$$\mathcal{L}_{QCD} = \bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R - \frac{1}{2} Tr(G_{\mu\nu} G^{\mu\nu})$$

$$q_L = \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \longrightarrow e^{-i\Theta_L^a T^a} \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} = g_L q_L$$

likewise for q_R

$SU(3)_L \times SU(3)_R$ symmetry predicts the existence of degenerate baryon octets of opposite parity

Observation: There is only one baryon octet in nature!

Spontaneous Symmetry Breaking:
$SU(3)_L \times SU(3)_R \longrightarrow SU(3)_V$

Pseudoscalar mesons (π, K, η) are much lighter than vector mesons or baryons
$\Longrightarrow$ Goldstone bosons

CCWZ formalism $\longrightarrow$ Construct effective field theory for π, K, η:

Chiral Perturbation Theory (Gasser and Leutwyler, 1984)

$$\mathcal{L}_2 = \frac{f_\pi^2}{4} Tr(u_\mu u^\mu + \chi_+)$$

$$u = e^{i\pi^a T^a / f_\pi}, \ u_\mu = iu\partial_\mu (u^\dagger)^2 u$$
$$\chi = 2B_0 \ diag(m_u, m_d, m_s), \ \chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u$$

Applications of $\mathcal{L}_2$:

- Gell-Mann-Oaks-Renner relations ($\hat{m} = m_u = m_d$):

$$m_\pi^2 = 2\hat{m}B_0,\ m_K^2 = (\hat{m} + m_s)B_0,\ m_\eta^2 = \frac{2}{3}(\hat{m} + 2m_s)B_0$$

- Scattering Amplitude $\pi^i\pi^k \longrightarrow \pi^m\pi^n$:

$$\mathcal{A} = -\frac{1}{f_\pi}\left(s\delta_{ik}\delta_{mn} + t\delta_{im}\delta_{kn} + u\delta_{in}\delta_{mk}\right)$$

- Many more...

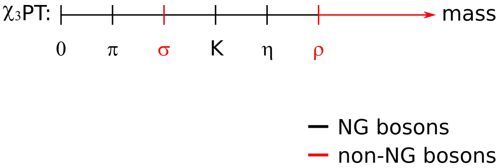
Problems of χ_3PT:

- Less accurate when dealing with K or η scattering
- No explanation of the $\Delta I = \frac{1}{2}$ rule

$\Longrightarrow$ Extend χ_3PT by adding one new degree of freedom

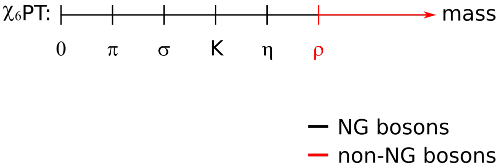
Chiral-Scale Perturbation Theory (χ_σPT)

$\chi_3 PT$: Eight Goldstone bosons (π, K, η)



$\rightarrow$ promote $f(500) = \sigma$ to ninth Goldstone boson

$\chi_\sigma PT$: Nine Goldstone bosons (π, K, η, σ)



Question: Which additional symmetry can be broken?
Answer: Scale symmetry!

Strong trace anomaly:

$$\theta^\mu_\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu} + \Big(1 + \gamma_m(\alpha_s)\Big) \sum_{q=u,d,s} m_q \bar{q} q$$

$$\Rightarrow \beta(\alpha_{IR}) = 0$$

Assumption in χ_σPT:

> There exists an infrared fixed point of the renormalization group equation for the strong coupling constant.

Modify $\mathcal{L}_2$ by introducing scale compensator $\hat{\chi}$:

$$\frac{f_\pi^2}{4} Tr(u_\mu u^\mu) \longrightarrow \frac{f_\pi^2}{4} Tr(u_\mu u^\mu)\hat{\chi}^2$$
$$\frac{f_\pi^2}{4} Tr(\chi_+) \longrightarrow \frac{f_\pi^2}{4} Tr(\chi_+)\hat{\chi}^\gamma$$

Include kinetic and potential terms for $\hat{\chi}$:

$$\frac{f_\sigma^2}{4} \partial_\mu \hat{\chi} \partial^\mu \hat{\chi} + c_1 \hat{\chi}^\delta - c_2 \hat{\chi}^4$$

Fix coefficients:

- Introduce non-linear field $\sigma \Longrightarrow \hat{\chi} = e^{\frac{\sigma}{f_\sigma}}$
- Compute trace of energy momentum tensor and compare with strong trace anomaly $\Longrightarrow \gamma = 3 - \gamma_m$ and $\delta = 4 + \beta'$
- Match m_σ^2 with curvature of potential $\Longrightarrow c = c(m^2, f, \gamma_m, \beta')$

Chiral-Scale Perturbation Theory (Crewther and Tunstall, 2014)

$$\mathcal{L}_2 = \left(\frac{f_\pi^2}{4} Tr(u_\mu u^\mu) + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma \right) e^{\frac{2\sigma}{f_\sigma}} + \\ + \frac{f_\pi^2}{4} Tr(\chi_+) e^{\frac{(3-\gamma_m)\sigma}{f_\sigma}} + c_1 e^{\frac{(4+\beta')\sigma}{f_\sigma}} - c_2 e^{\frac{4\sigma}{f_\sigma}}$$

Comparison at one loop

Apply power counting rules to construct next-to-leading-order Lagrangians

χ_3PT: $$\mathcal{L}_4 = L_1\,Tr(u_\mu u^\mu)^2 + L_2\,Tr(u_\mu u_\nu)^2 + L_3\,Tr\left((u_\mu u^\mu)^2\right) + ...$$

χ_σPT: $$L_i \longrightarrow L_i(\sigma),\;\; G_1(\sigma)(\partial_\mu\sigma\partial^\mu\sigma)^2 + G_2(\sigma)\,Tr(u_\mu u^\mu)\partial_\nu\sigma\partial^\nu\sigma + ...$$

Renormalization via $\widetilde{MS}$ renormalization scheme:

$$L_i = L_i^r + \frac{\Gamma_i}{32\pi^2}\left(\frac{2}{d-4} - \ln(4\pi) + \gamma - 1\right)$$
$$\frac{dL_i^r}{d\mu} = -\frac{\Gamma_i}{16\pi^2\mu}$$

Usual approach via Feynman diagrams is too complicated for $\chi_\sigma PT$

$\rightarrow$ Apply functional methods instead:

- Background field method
- Heat-kernel expansion

Claim: Derive a master formula for one-loop divergences $\longrightarrow$ Γ_i

Background field method:
$\mathcal{L} = \mathcal{L}(\phi, \partial\phi)$
$\phi \to \phi + \varphi$

$$\mathcal{L}(\phi+\varphi) = \mathcal{L}(\phi) + \varphi_i \mathcal{L}'_i(\phi) + \partial_\mu \varphi_i W^{\mu\nu}_{ij} \partial_\nu \varphi_j - \varphi_i 2\Gamma^\mu_{ij} \partial_\mu \varphi_j - \varphi_i M_{ij} \varphi_j + \mathcal{O}(\varphi^3)$$

$\mathcal{L}'_i(\phi) = 0$ (Classical equations of motion)
Only one loop $\to$ drop $\mathcal{O}(\varphi^3)$

Assume $W^{\mu\nu}_{ij} = \delta_{ij} g^{\mu\nu}$, $\Gamma^\mu_{ij} = -\Gamma^\mu_{ji}$ and $M_{ij} = M_{ji}$ (Canonical form)

Advantage: Path integral is Gaussian

Heat-kernel expansion:

$\sigma_{ij} = M_{ij} - (\Gamma_\mu \Gamma^\mu)_{ij}$, $d^\mu_{ij} = \delta_{ij}\partial^\mu + \Gamma^\mu_{ij}$, $\Gamma_{\mu\nu} = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + \Gamma_\mu \Gamma_\nu - \Gamma_\nu \Gamma_\mu$

$$Z = e^{iW} = \int \mathcal{D}\varphi e^{i\int d^4x \varphi_i (d_\mu d^\mu + \sigma)_{ij}\varphi_j}$$

$$W = \frac{i}{2}\int d^4x \ Tr\langle x|\ln(d_\mu d^\mu + \sigma)|x\rangle =$$
$$= \frac{1}{16\pi^2(4-d)}\int d^4x Tr(\frac{1}{2}\sigma^2 + \frac{1}{12}\underbrace{[d_\mu, d_\nu][d^\mu, d^\nu]}_{\Gamma_{\mu\nu}\Gamma^{\mu\nu}} + \frac{1}{6}\underbrace{[d_\mu,[d^\mu,\sigma]]}_{\text{boundary term}}) + ...$$

$\Longrightarrow$ Master Formula ('t Hooft, 1973)

$$\Delta\mathcal{L} = -\mathcal{L}^{div} = -\frac{1}{16\pi^2(4-d)} Tr(\frac{1}{12}\Gamma_{\mu\nu}\Gamma^{\mu\nu} + \frac{1}{2}\sigma^2)$$

Ansatz:

$$\mathcal{L} = \frac{f_\pi^2}{4} Tr(u_\mu u^\mu) f(\sigma) + \frac{f_\pi^2}{4} Tr(\chi_+) g(\sigma) + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma h(\sigma) - V(\sigma)$$

Anomalous Dimensions for χ_3PT (Gasser and Leutwyler, 1984):

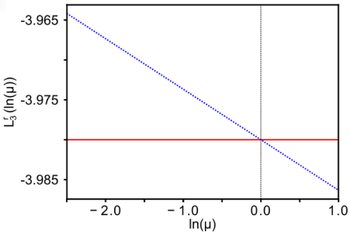
$$\Gamma_1 = \frac{1}{32},\ \Gamma_2 = \frac{3}{16},\ \Gamma_3 = 0$$

Anomalous Dimensions for χ_σPT (Catà and C.M., 2019):

$$\Gamma_1 = \frac{1}{32} - \frac{11}{12}\frac{f_\pi^2}{f_\sigma^2} + \frac{17}{24}\frac{f_\pi^4}{f_\sigma^4},\ \Gamma_2 = \frac{3}{16} - \frac{1}{3}\frac{f_\pi^2}{f_\sigma^2} + \frac{1}{6}\frac{f_\pi^4}{f_\sigma^4},\ \Gamma_3 = 0 + \frac{f_\pi^2}{f_\sigma^2}$$

Comparison between χ_3PT and χ_σPT:

$$L_3^\sigma(\mu) = L_3^\chi(\mu) + \frac{\Gamma_3^\sigma - \Gamma_3^\chi}{16\pi^2} \ln\left(\frac{\mu_0}{\mu}\right) = L_3^\chi(\mu) + \frac{f_\pi^2/f_\sigma^2}{16\pi^2} \ln\left(\frac{\mu_0}{\mu}\right)$$



J. Bijnens, I.Jemos, Nuclear Physics B, Volume 854, 2012

given precision $10^{-4} \longrightarrow$ deviate 2% from reference scale μ_0 for $\frac{f_\pi^2}{f_\sigma^2} \approx 1$

Conclusions

χ_σPT...

- ...enlarges χ_3PT by a ninth degree of freedom
- ...assumes an infrared fixed point of QCD renormalization group equation
- ...can be distinguished from χ_3PT via running of L_i

Thanks for your attention!