A Different Perspective on Strong CP & Axions

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Goal: Present alternative language to understand…

… how axions solve strong CP problem.

… which kind of new physics can spoil the solution.

[Gia Dvali, hep-th/0507215 (2005)]

Maxwell and the θ-Term

Maxwell: $\mathscr{L}_{\text{MW}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Why these terms?

- Gauge invariant under U(1)
- Up to energy dimension 4

$$\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Wait! There is one more: $\mathscr{L}_\theta \sim \theta F_{\mu\nu} \tilde{F}^{\mu\nu}$, $\theta = \text{const}$.

$$\sim \theta \vec{E} \cdot \vec{B}$$

CP violation!

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$\sim \theta\, \vec{E}\cdot\vec{B}$ CP violation!

Wait again! Can write as: $\mathscr{L}_\theta \sim \theta\partial_\mu K^\mu$ Boundary term!

QED is not CP violating

Why QCD makes Life complicated…

QCD: $$\mathscr{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a}$$

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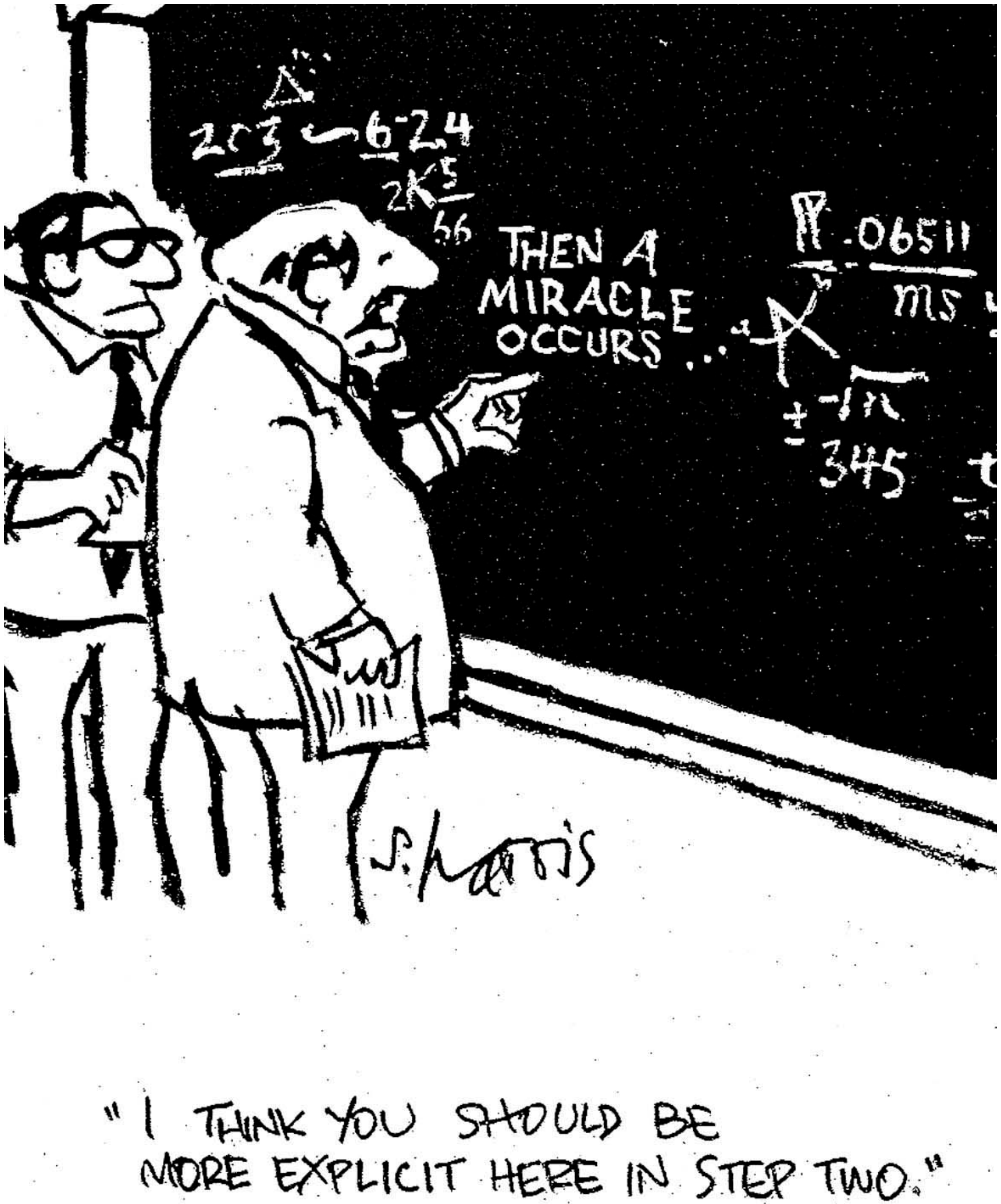
Boundary term!

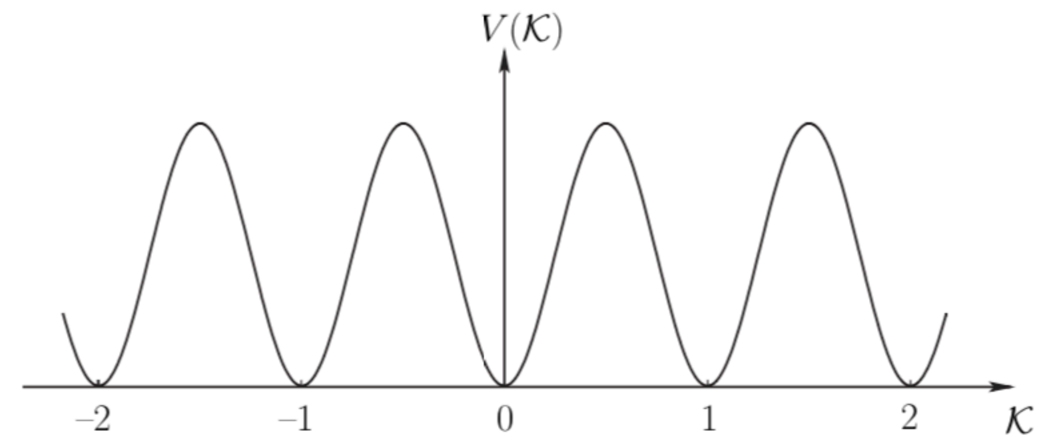
Instantons do not care

QCD is CP violating!

Instantons = Tunneling Processes



- Lead to non-trivial QCD vacuum structure



Main point for now: "Generate correlation to boundary"

How to measure θ?

Contributes to electric dipole moment of e.g. neutron

$\Longrightarrow$ Calculate contribution and compare to experiment to find θ!

We do not measure CP violation! $\Longrightarrow \theta < 10^{-10}$ or so

Strong CP problem: Why is θ so small?

Solution: Axion

lived pseudoscalar boson. [The *SCI*® indicates that this paper has been cited in more than 605 publications.]

The Birth of Axions

Frank Wilczek
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The basic idea of axions occurred to me as a result of my wife's ear infection. This was in the summer of 1975, when we were visiting Fermilab with a small baby daughter in tow. My wife was ill and confined to bed, and I had a rather difficult day trying to cope. Finally, both wife and daughter were safely asleep and it was a beautiful midwestern night with a gorgeous clear sky, and I decided to take a long walk.

Turning with relief from the cares of the day, I decided to think about the Higgs sector. At that time, what is now called the standard model of particle physics, although less than three years old, was already established as far

Solution: Axion

Peccei-Quinn Mechanism:

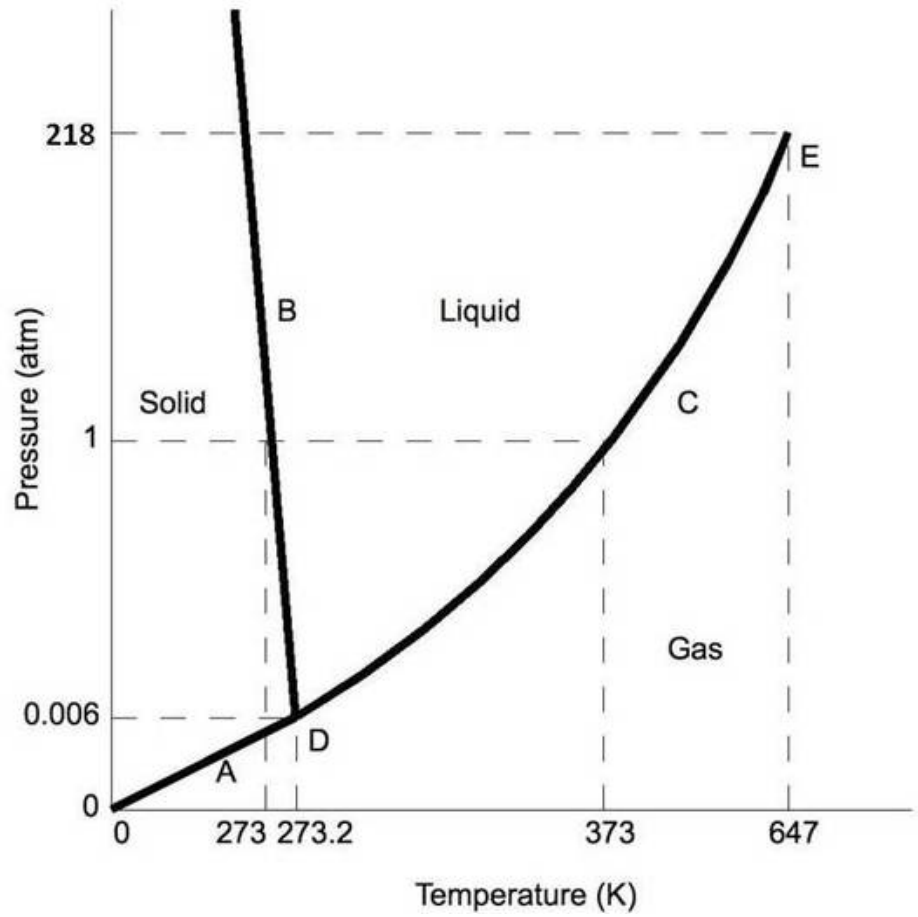
- Propose new global axial symmetry, $U(1)_{PQ}$
- New Higgs particle to spontaneously break $U(1)_{PQ}$
- Axion a is the appearing Goldstone boson

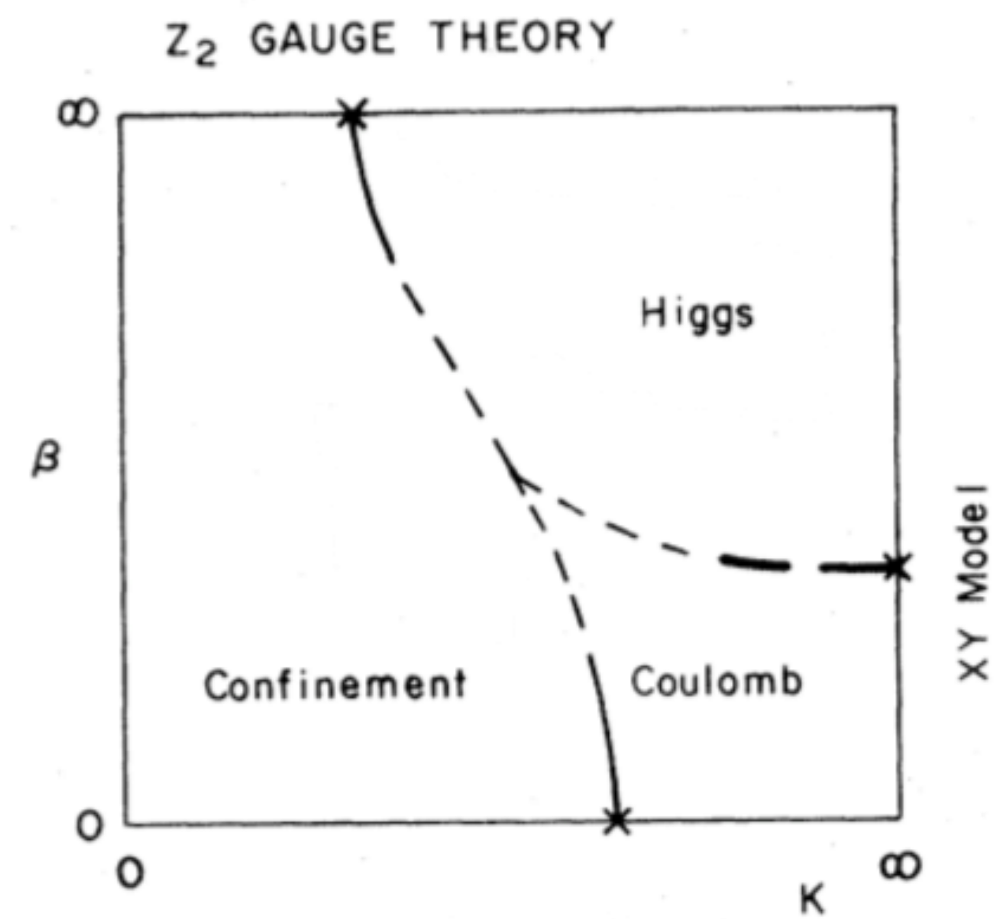
$$\mathscr{L} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + (a+\theta)F^a_{\mu\nu}\tilde{F}^{\mu\nu a}$$

θ now dynamical!

Vafa-Witten: Vacuum where $\theta = 0$

Recap: Phases of Gauge Theories





Coulomb: $V(r) \sim \frac{1}{r}$

Higgs: $V(r) \sim \frac{e^{-mr}}{r}$

Confinement: $V(r) \sim r$

QED in 1+1 Dimensions

Maxwell's Theory: $\mathscr{L}_{\text{MW}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad \mu,\nu = 0,1$

Solution to Maxwells equation: $E(x) = \text{const}.$

- Constant electric field everywhere in space
- Can not be changed

Add θ-term: $\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\theta\,\epsilon_{\mu\nu}F^{\mu\nu}\,, \qquad \theta = \text{const}.$

- Total derivative
- Just redefines the constant value of the electric field

Main Point: θ equivalent to constant electric field in the vacuum

Getting Rid of Global Electric Field

Idea: Bring to Higgs phase

$$\mathscr{L}_{\text{Proca}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}(A_\mu - \partial_\mu B)^2 + \frac{1}{2}\theta\epsilon_{\mu\nu}F^{\mu\nu}$$

Let's Hodge dualize just for fun: $B \longleftrightarrow a$

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu a\partial^\mu a + \frac{1}{2}\left(a+\theta\right)\epsilon_{\mu\nu}F^{\mu\nu}$$

- Constant electric field no longer a solution, now dynamical!
- Unique vacuum: $E = 0,\ a = -\theta$

Axion making θ dynamical
= Putting global electric field in Higgs Phase

Remarks

- Generalizable to arbitrary kinetic term $K(F)$, $F = F_{\mu\nu}\epsilon^{\mu\nu}$ ✓
- Massive fermions give additional contribution to θ, but do not spoil conclusions

Back to QCD

Maxwell in 1+1 d | QCD in 3+1 d

$\mathscr{L}_\theta \sim \theta \epsilon^{\mu\nu} F_{\mu\nu}$ $\longleftrightarrow$ $\mathscr{L}_\theta \sim \theta F^a_{\mu\nu} \tilde{F}^{\mu\nu a} \sim \theta \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta\mu\nu}$

$F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$ $\longleftrightarrow$ $F_{\alpha\beta\mu\nu} = \partial_{[\alpha} C_{\beta\mu\nu]}$

$\mathscr{L} \sim \frac{m^2}{2}(A_\mu - \partial_\mu B)^2$ $\longleftrightarrow$ $\mathscr{L} \sim \frac{m^2}{2}(C_{\beta\mu\nu} - \partial_\beta B_{\mu\nu})^2$

$B \longleftrightarrow a$ $\longleftrightarrow$ $B_{\mu\nu} \longleftrightarrow a$

Wait! QCD in strong coupling regime at low energies

$$\Longrightarrow \mathscr{L} = \frac{1}{2}\theta \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta\mu\nu} + K(F) \qquad \Longrightarrow \checkmark$$

How to spoil axion solution?

- Single axion cannot Higgs more than one 3-form field.

$\Longrightarrow$ New physics must provide independently-massless 3-form field

- Example: Gravity, Chern-Simons spin connection 3-form

$$R\tilde{R} = \epsilon^{\alpha\beta\mu\nu} R^{i}_{j\alpha\beta} R^{j}_{i\mu\nu} = \frac{1}{3}\epsilon^{\alpha\beta\mu\nu}\partial_{[\alpha}G_{\beta\mu\nu]}$$

Conclusion

- Hodge duality is a nice tool
- θ equivalent to constant "electric" field in vacuum
- Axion making θ dynamical
  = Putting global "electric" field in Higgs phase
- New physics must provide independently-massless 3-form field to spoil axion solution

Thank you for your attention!