Obtaining an effective spacetime action from String Field Theory using CFT-Methods

What is String Field Theory and why bother?

- Natural generalization of String Theory:

  String Field $\longrightarrow$ Strings as excitations

  (Gluon Field $\longrightarrow$ Gluons as excitations)

- Two main ways to approaches:

  $\longrightarrow$ Algebraic approach

  $\longrightarrow$ CFT approach

- Advantages: renormalization, non-perturb. effects, gauge sym. manifest

- Complementary point of view to String Theory

Witten's cubic SFT

$$S = -\tfrac{1}{2} \langle \Phi, Q_B \Phi \rangle - \tfrac{g}{3} \langle \Phi, \Phi, \Phi \rangle$$

- classical, bosonic, open String Field Theory
- No higher vertices, all can be constructed from the 3-vertex
- String Field representation in terms of spacetime fields:

$$|\Phi\rangle = \int d^dk \Big(\underbrace{T(k)}_{\text{0th level, } m = -1/\alpha'} + \underbrace{A_\mu(k)\, \alpha^\mu_{-1} + B(k)\, b_{-1} c_0}_{\text{1st level, } m=0} + \dots \Big) \underbrace{|vac\rangle}_{c_1 |k\rangle}$$

- fix gauge : $b_0 |\Phi\rangle = 0 \rightsquigarrow$ all terms with c_0 get set to 0, i.e. $B(k) = 0$

(this corresponds to the Lorentz gauge $k_\mu A^\mu = 0$ for $A_\mu(k)$)

How to evaluate the action?

- Via the Operator-State correspondence relate the String Field state to a CFT-Operator on the 2d-Worldsheet of the String:

$$|\Phi\rangle = \Phi(0)|0\rangle \rightsquigarrow \Phi(0) = \int d^d k \left(T(k) + A_\mu(k)\, c\partial X^\mu + \ldots \right) e^{ik\cdot X}(0)$$

$$\left(|k\rangle = e^{ik\cdot X}|0\rangle \,,\; c_1 \simeq c(0) \,,\; \alpha_{-1}^\mu \simeq \partial X^\mu(0) \right)$$

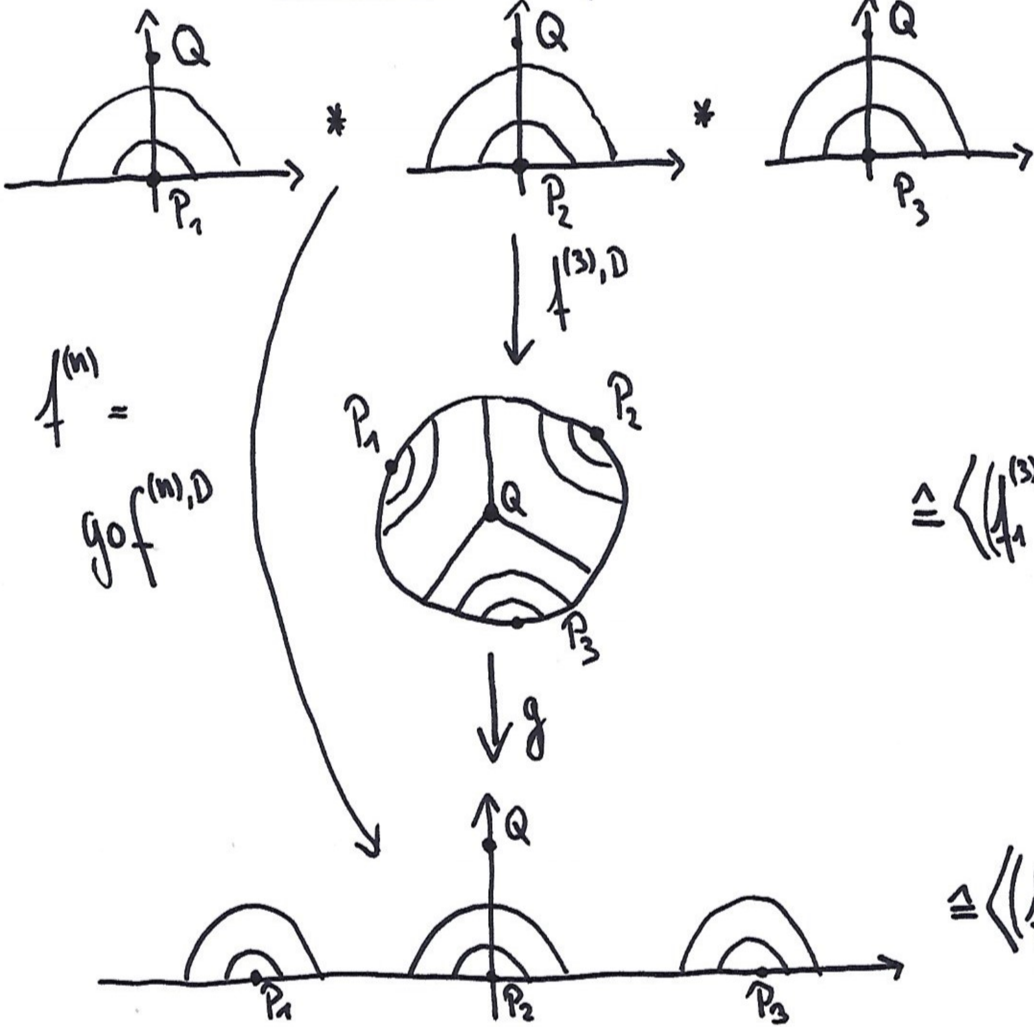
- Map the String Field Products to the UHP where we can use standard CFT-methods to evaluate the correlators (Wick Thm., OPE, ...)

$$\langle \underbrace{\Phi, \ldots, \Phi}_{n \text{ fields}} \rangle \longrightarrow \langle (f_1^{(n)} \circ \Phi)(0) \ldots (f_n^{(n)} \circ \Phi)(0) \rangle_{UHP}$$

$$\hookrightarrow (f \circ \Phi)(0) = (f'(0))^h \, \Phi(f(0))$$

↳ relate the abstract String Field Products to CFT-correlators

The conformal Maps $f_i^{(n)}$



$$\triangleq \left\langle \Phi, \Phi, \Phi \right\rangle$$

$$\triangleq \left\langle \left(f_1^{(3),D} \circ \Phi\right)(0) \left(f_2^{(3),D} \circ \Phi\right)(0) \left(f_3^{(3),D} \circ \Phi\right)(0) \right\rangle_{\text{Disk}}$$

$$\triangleq \left\langle \left(f_1^{(3)} \circ \Phi\right)(0) \left(f_2^{(3)} \circ \Phi\right)(0) \left(f_3^{(3)} \circ \Phi\right)(0) \right\rangle_{\text{UHP}}$$

Deriving the effective action

- effective action = low energy action
  - ↳ only the massless fields can be excited
  - ↳ formally we take $\alpha' \to 0$ ⇝ all massive fields become infinitely heavy ($m \sim 1/\alpha'$), i.e. non-propagating

- String Theory: (on-shell) Amplitudes $\xrightarrow{\text{reconstruct}}$ Lagrangian

  String Field Theory: Lagrangian $\xrightarrow{\text{QFT}}$ Amplitudes

- To obtain a Yang Mills Theory, we need to add Chan-Paton factors to the spacetime fields: $A_\mu \to A^a_\mu$
  - ↳ in the correlators we just trace over the fields

Sketch of the Kinetic term calculation

$$\langle \Phi_A, Q_B \Phi_A \rangle = \langle (f_1^{(2)} \circ \Phi_A)(0) \, (f_2^{(2)} \circ Q_B \Phi_A)(0) \rangle_{UHP} =$$

$$= \int d^dk \, d^dp \, \mathrm{Tr}(A_\mu(k) A_\nu(p)) \langle (f_1^{(2)} \circ c\partial X^\mu e^{ik\cdot X})(0) \, (f_2^{(2)} \circ Q_B c\partial X^\nu e^{ip\cdot X})(0) \rangle_{UHP} =$$

$$= \int d^dk \, d^dp \, \mathrm{Tr}(A_\mu A_\nu) \alpha' p^2 \langle \underbrace{(f_1^{(2)} \circ c\partial X^\mu e^{ik\cdot X})(0)}_{\sim c\partial X^\mu e^{ik\cdot X}} \, \underbrace{(f_2^{(2)} \circ c\partial c \, \partial X^\nu e^{ip\cdot X})(0)}_{\sim c\partial c \partial X^\nu e^{ip\cdot X}} \rangle_{UHP} \sim$$

$$\sim \int d^dk \, d^dp \, \mathrm{Tr}(A_\mu A_\nu) \alpha' p^2 \underbrace{\langle c \, c\partial c \rangle_{ghost}}_{\text{some number}} \underbrace{\langle (\partial X^\mu e^{ikX}) (\partial X^\nu e^{ipX}) \rangle_{matter}}_{\sim \eta^{\mu\nu} \delta^d(k+p)} \sim$$

$$\sim \int d^dk \, \alpha' k^2 \, \mathrm{Tr}(A_\mu(k) A^\mu(-k))$$

$\rightsquigarrow$ Kinetic Yang Mills term in Lorentz gauge