Memory Storage and Thermalization in Multi-Mode Systems

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Outline

Motivation
- System with high entropy
- Specific sub-system

Solution of Sub-System
- Coupling sensitivity
- Stability
- Equilibration
- Thermalization

Outlook: ETH

Summary

System with high entropy

Application to black holes
G. Dvali [Phys. Rev. D 97, 105005 (2018)]

(1) large number of micro-states

$$\Omega \sim e^S$$

$$\hat{H} = \sum_{k=1}^{S} \varepsilon_k \hat{n}_k$$

System with high entropy

Application to black holes
G. Dvali [Phys. Rev. D 97, 105005 (2018)]

(1) large number of micro-states

$$\Omega \sim e^S$$

(2) all S modes $\hat{n}_k$ gapless to fit within infinitesimal energy gap

$$\hat{H} = \left(1 - \frac{\hat{n}_0}{M}\right) \sum_{k=1}^{S} \varepsilon_k \hat{n}_k$$

General system

Individual mass gaps
Interactions among $\hat{a}_j$ modes
Interaction with external $\hat{b}_0$ mode

$$\hat{H} = \sum_j \left(1 - \frac{\hat{n}_0}{M_j}\right) \varepsilon_j \hat{n}_j + \sum_{\substack{j,k \\ j \neq k}} A_{jk} \left(\hat{a}_j^\dagger \hat{a}_k + h.c.\right) + \lambda \left(\hat{a}_0^\dagger \hat{b}_0 + h.c.\right)$$

$j, k = 1, \ldots, D$

Specific sub-system

Common mass gap

$$\hat{H} = \sum_{j} \left(1 - \frac{\hat{n}_0}{M}\right) \varepsilon \hat{n}_j + \sum_{\substack{j,k \\ j \neq k}} A_{jk} \left(\hat{a}_j^\dagger \hat{a}_k + h.c.\right) + \lambda \left(\hat{a}_0^\dagger \hat{b}_0 + h.c.\right)$$

$j, k = 1, \ldots, D$

Solution of sub-system

$D = 2$ solved
G. Dvali [arXiv:1810.02336]

Our generalization: arbitrary D

Observation: $\langle \hat{n}_j \rangle (t)$ independent of (time-varying) common mass gap

Time-evolution

$$|in\rangle = |N_1, \dots, N_D\rangle \; ; \quad N_j \in [0, N] \; , \; \sum_j N_j = N$$

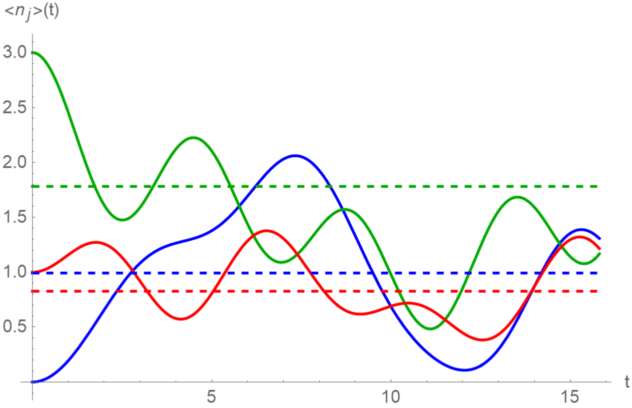

Figure: $\langle \hat{n}_j \rangle (t)$ (solid) and $\overline{\langle \hat{n}_j \rangle (t)}$ (dashed) for $j = 1, 2, 10$ for $D = 10, \; N = 10$.

Coupling sensitivity

$A_{jk} \in [-1, +1]$

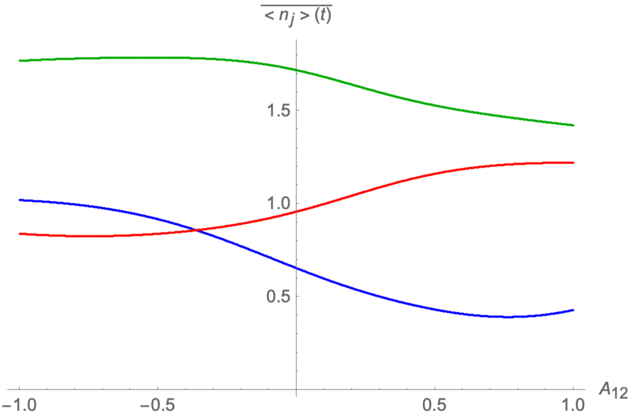

Figure: $\overline{\langle \hat{n}_j \rangle (t)}$ for $j = 1, 2, 10$ vs A_{12} for $D = 10$, $N = 10$.

Stability

$N_1 = 0$, $N_2 = 3$

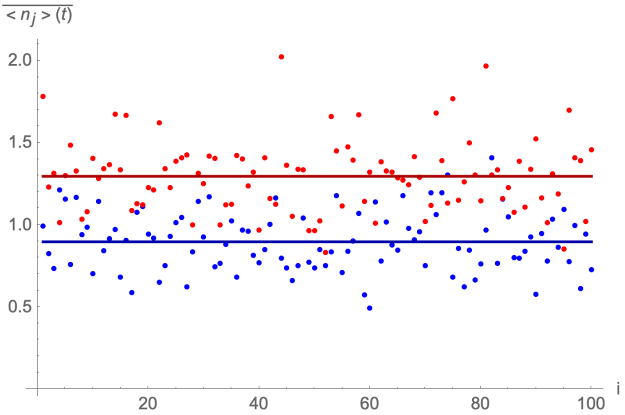

Figure: $\overline{\langle \hat{n}_j \rangle (t)}$ for $j = 1, 2$ for various systems $A_i \in [-1, +1]^{D \times D}$.

Equilibration

$D = 10,\ N = 100\,; \qquad |in\rangle = |N_1, \ldots, N_D\rangle$

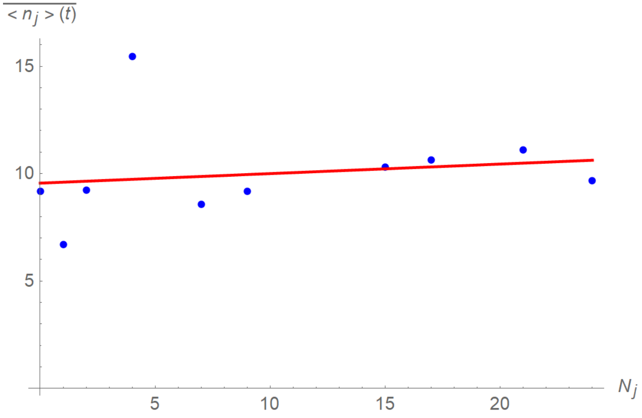

Figure: $\overline{\langle \hat{n}_j \rangle (t)}$ vs N_j with slope k as "memory of $|in\rangle$".

Thermalization

"Memory of $|in\rangle$" k:

- indep. of N
- $\lim_{D\to\infty} k = 0$

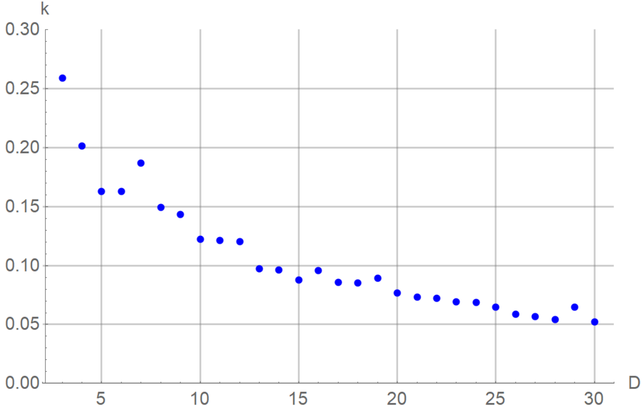

Figure: Mean of k over A_i's and $|in\rangle$'s vs D at $N = 10$.

Thermalization

$$\boxed{\overline{\langle\hat{\mathcal{O}}\rangle(t)} = \langle\hat{\mathcal{O}}\rangle_{mc}}$$

$$\overline{\langle\hat{\mathcal{O}}\rangle(t)} = \lim_{T\to\infty}\frac{1}{T}\int_0^T \langle\hat{\mathcal{O}}\rangle(t)\,\mathrm{d}t$$

$$\langle\hat{\mathcal{O}}\rangle_{mc} = \frac{1}{\mathcal{N}_\varepsilon}\sum_{\substack{\varepsilon \\ |E-\varepsilon|<\delta}} \langle\varepsilon|\,\hat{\mathcal{O}}\,|\varepsilon\rangle$$

Outlook: Eigenstate Thermalization Hypothesis (ETH)

J. M. Deutsch [Phys. Rev. A 43, 2046 (1991)], M. Srednicki [Phys. Rev. E 50, 888 (1994)]

1 of 3 conditions: $\varepsilon_j \simeq \varepsilon_k \Rightarrow \langle\varepsilon_j|\,\hat{\mathcal{O}}\,|\varepsilon_j\rangle \simeq \langle\varepsilon_k|\,\hat{\mathcal{O}}\,|\varepsilon_k\rangle$

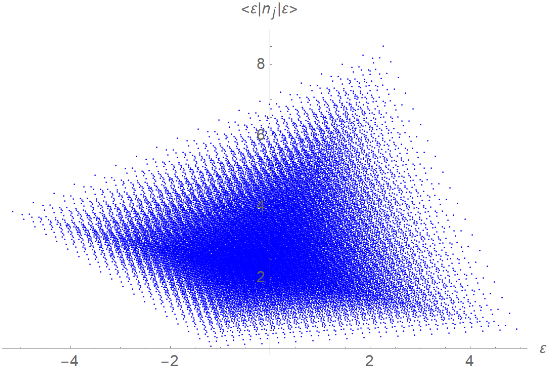

Figure: $\langle\varepsilon|\,\hat{n}_j\,|\varepsilon\rangle$ vs eigenstate energy ε for $D = 6$, $N = 18$.

Summary

Analytic generalization of high S system to arbitrary dim. D

Thermalization for large systems

Indications for thermalization mechanism distinct from that of ETH

Thank You for Your attention!

Any questions?

Appendix

Formulae (solutions)

$$\langle \hat{n}_j \rangle (t) = \sum_{k,l,s} N_s Q_{jl} Q_{jk} Q_{sl} Q_{sk} \cos\left[(\omega_l - \omega_k) t\right]$$

$$\omega_j = \left(Q^{\mathsf{T}} A Q\right)_{jj} = \sum_{i,k} A_{ik} Q_{ij} Q_{kj}$$

$$\overline{\langle \hat{n}_j \rangle (t)} = \sum_{k,s} N_s Q_{jk}^2 Q_{sk}^2$$

$$\langle \hat{n}_{a_0} \rangle (t) = N_{a_0} + \left(\frac{\lambda}{\varphi}\right)^2 (N_{b_0} - N_{a_0}) \sin^2(\varphi t)$$

$$\langle \hat{n}_{b_0} \rangle (t) = N_{b_0} - \left(\frac{\lambda}{\varphi}\right)^2 (N_{b_0} - N_{a_0}) \sin^2(\varphi t)$$

$$\varphi = \frac{1}{2}\sqrt{(2\lambda)^2 + \left(\sum_i \frac{\varepsilon}{M} N_i\right)^2}$$

Formulae (unitarity)

$$\hat{H} \sim A \sum_{i,j} \hat{a}_j^\dagger \hat{a}_i \;, \quad N_i \sim \frac{N}{D}$$

$$\Rightarrow \; \mathcal{M}_{i\to j} = \langle N_j | \, \hat{H} \, | N_i \rangle \sim A \sqrt{N_i} \sqrt{N_j}$$

$$\Rightarrow \; \Gamma_i = \sum_{j=1}^{D} \left| \mathcal{M}_{i\to j} \right|^2 \sim A^2 \frac{N^2}{D} \overset{!}{\sim} const$$

$$\Rightarrow \; A \sim \sqrt{D}/N$$

$$\tau = \frac{\sum_{k,l,s} |N_s Q_{jl} Q_{jk} Q_{sl} Q_{sk}|}{\sum_{k,l,s} |N_s Q_{jl} Q_{jk} Q_{sl} Q_{sk} (\omega_l - \omega_k)|}$$

k vs D at $N = 10$

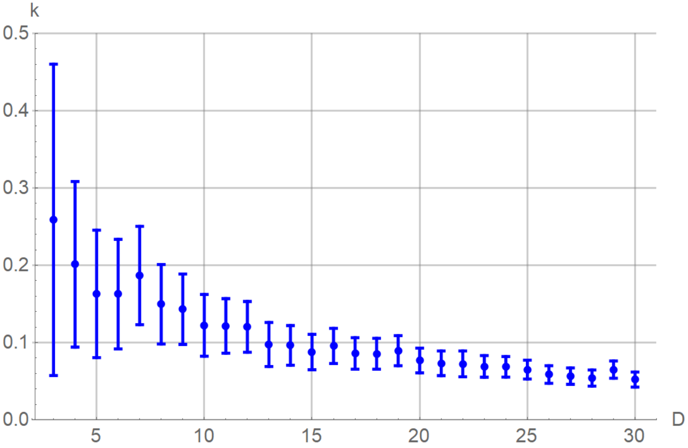

Figure: k vs D at $N = 10$.

k vs N at $D = 10$

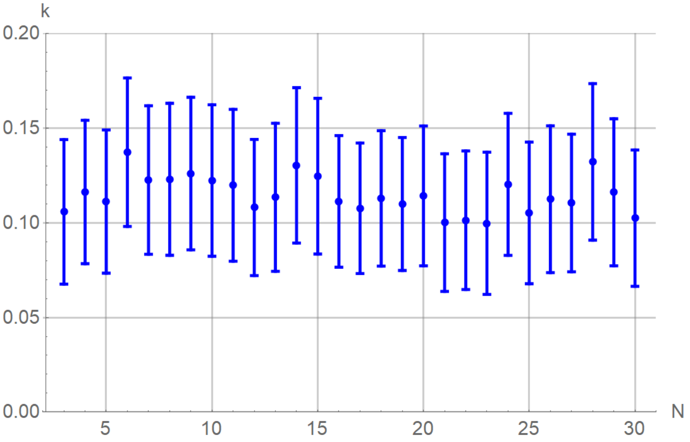

Figure: k vs N at $D = 10$.

k vs D and N

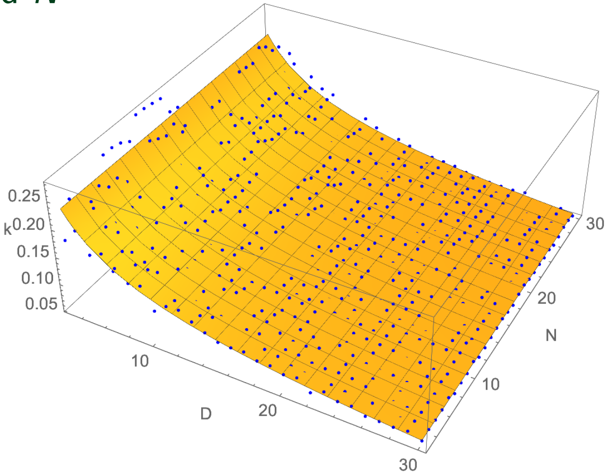

Figure: Mean of k over A_i's and $|in\rangle$'s vs D and N.

τ vs D at $N = 10$

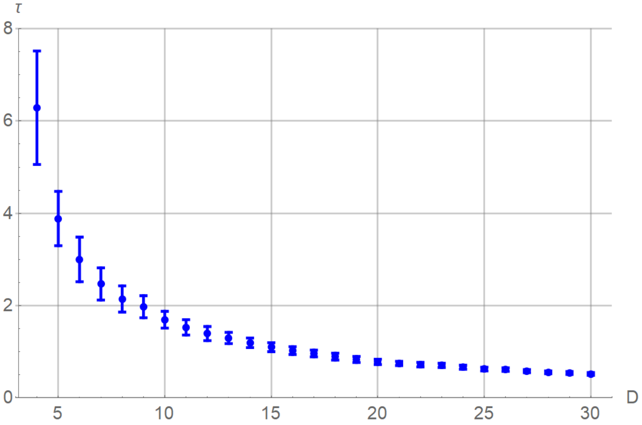

Figure: τ vs D at $N = 10$ for $A_{jk} \in \left[-\sqrt{D}/N, +\sqrt{D}/N\right]$.

τ vs N at $D = 10$

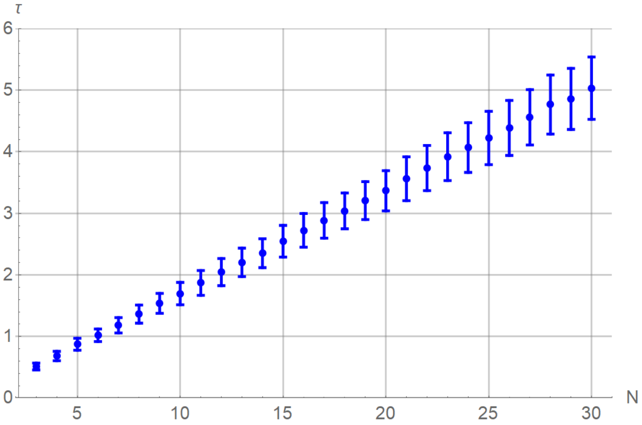

Figure: τ vs N at $D = 10$ for $A_{jk} \in \left[-\sqrt{D}/N, +\sqrt{D}/N\right]$.

Equilibration time scale τ

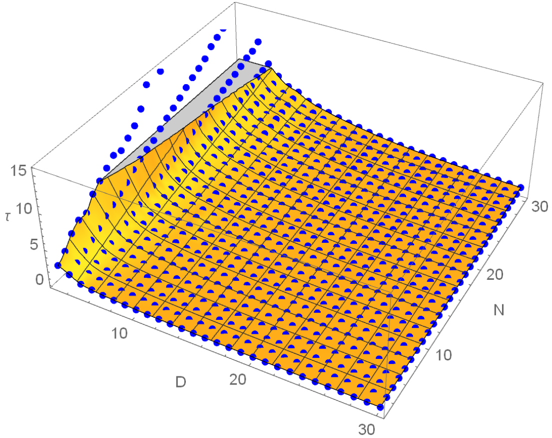

Figure: τ vs D and N for $A_{jk} \in \left[-\sqrt{D}/N, +\sqrt{D}/N\right]$.

ETH: scatter of $\langle \varepsilon | \hat{n}_j | \varepsilon \rangle$

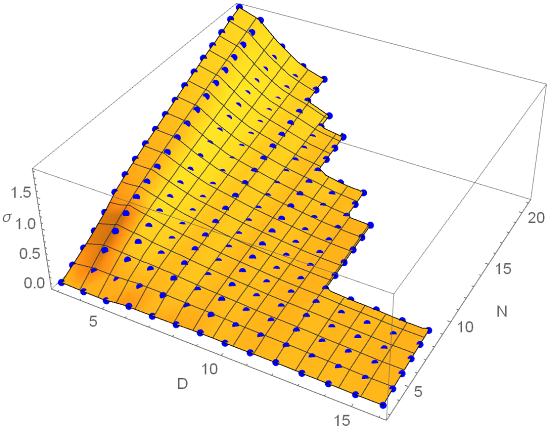

Figure: Scatter σ of $\langle \varepsilon | \hat{n}_j | \varepsilon \rangle$ within appropriate energy bins vs D and N.

Integrability

“A system is *quantum integrable*:
(i) If it exhibits n physically meaningful mutually commuting quantities that are in some sense independent [...] or depend linearly on some parameter of the Hamiltonian. [...]”
[C. Gogolin, J. Eisert, Rep. Prog. Phys. 79, 056001 (2016)]