Soft approximations and possible applications to $g-2$

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MUNICH, 15.07.2019

e μ τ

Anomalous magnetic moment of leptons

Differences between experimental value and theoretical prediction →

New physics??

Expected effects $\sim \frac{m_l^2}{\Lambda^2}$

Anomalous magnetic moment of leptons

μ τ

e

- Known with a precision of sub-part per billions
- Less influenced by new physics effects
- Value of the fine-structure constant α

Giudice, G. F. at al., "Testing new physics with the electron g− 2." *JHEP*, 2012.

Anomalous magnetic moment of leptons

μ τ

e

- Known with a precision of sub-part per billions
- Less influenced by new physics effects
- Value of the fine-structure constant α

→

Now α can be measured very precisely with atomic physics experiments

Giudice, G. F. at al., "Testing new physics with the electron g− 2." *JHEP*, 2012.

Anomalous magnetic moment of leptons

e τ

μ

- Known with a precision of sub part per million
- More influenced by new physics effects
- Nowadays ~ 3,5 σ between experimental value and theoretical predictions

T. Blum et al., "The Muon (g-2) Theory Value: Present and Future", 2013.

Anomalous magnetic moment of leptons

e

τ

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- Known with a precision of sub part per million
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- Nowadays ~ 3,5 σ between experimental value and theoretical predictions

→ New physics??

T. Blum et al., "The Muon (g-2) Theory Value: Present and Future", 2013.

Anomalous magnetic moment of leptons

e μ

τ

- The most influenced by new physics effects
- Very short lifetime: difficult to measure

Eidelman, S., et al., "Theory of the τ lepton anomalous magnetic moment." *Modern Physics Letters A,* 2007.

Anomalous magnetic moment

Exact finite order calculation

Anomalous magnetic moment

~~Exact finite order calculation~~

Soft Approximations

Anomalous magnetic moment

~~Exact finite order calculation~~

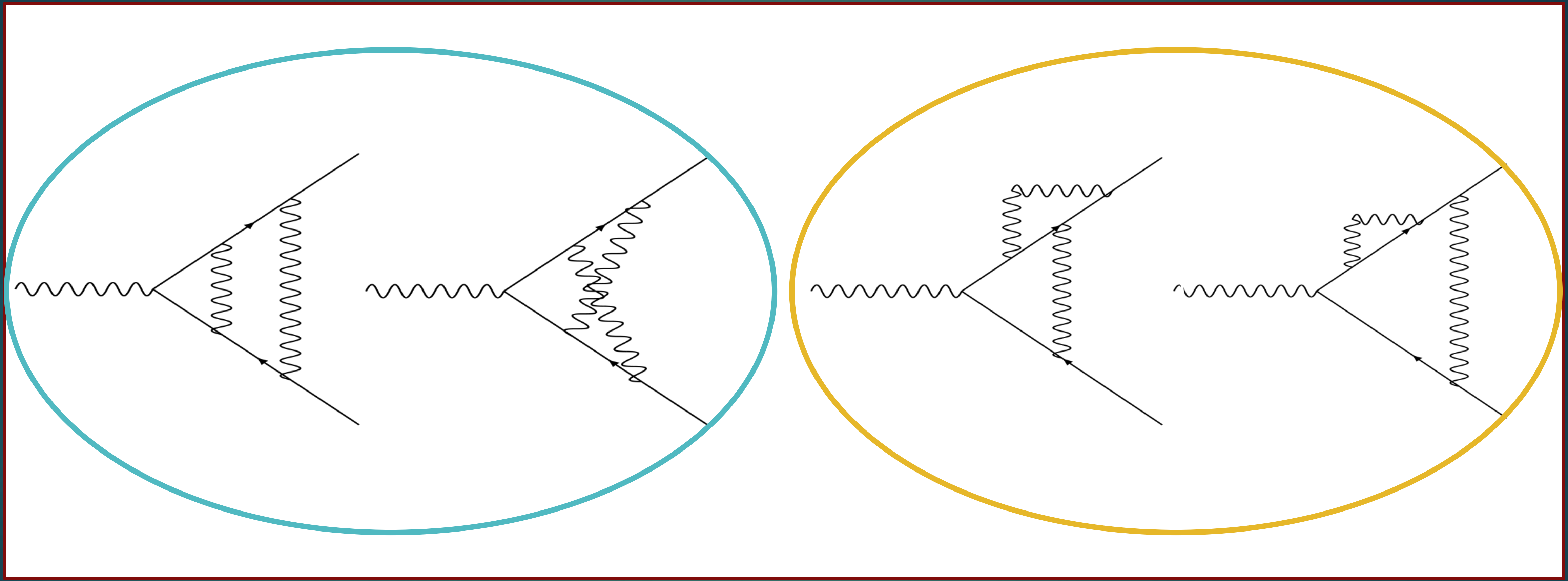
Soft Approximations

Why?

Exponentiation

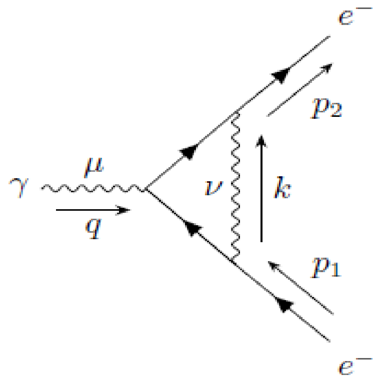
Gauge-set cancellation

Gauge sets



P. Cvitanovic, “QFT and its discontents, a blog," 2018.

Soft limit: how does it work?



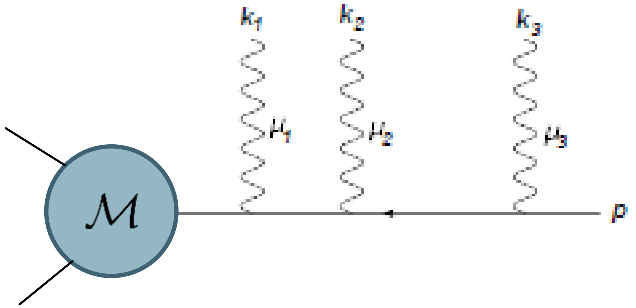
$$= (-\mathrm{ie})\Gamma^{\mu} = -\mathrm{e}^3\mu^{3\epsilon}\int\frac{d^dk}{(2\pi)^d}\frac{\gamma^{\nu}(\not{p}_2-\not{k}+m)\gamma^{\mu}(\not{p}_1-\not{k}+m)\gamma_{\nu}}{[(p_1-k)^2-m^2+i\eta][(p_2-k)^2-m^2+i\eta][k^2+i\eta]}$$

$k \to 0$

$$\Gamma^{\mu} \approx \boxed{\int\frac{d^dk}{(2\pi)^d}\frac{p_1\cdot p_2\gamma^{\mu}}{p_1\cdot k\, p_2\cdot k\, k^2}} + \int\frac{d^dk}{(2\pi)^d}\left[\frac{p_1\cdot p_2\gamma^{\mu}}{2(p_2\cdot k)^2p_1\cdot k}-\frac{\gamma^{\mu}}{p_2\cdot k\, k^2}+\frac{\not{k}\not{p}_1\gamma^{\mu}}{2p_1\cdot k\, p_2\cdot k\, k^2}\right]+$$

$$+\int\frac{d^dk}{(2\pi)^d}\left[\frac{p_1\cdot p_2\gamma^{\mu}}{2(p_1\cdot k)^2p_2\cdot k}-\frac{\gamma^{\mu}}{p_1\cdot k\, k^2}+\frac{\gamma^{\mu}\not{p}_2\not{k}}{2p_1\cdot k\, p_2\cdot k\, k^2}\right]$$

Eikonal approximation



$$= \mathcal{M}^{\mu_1\cdots\mu_n}(p,k_i) = \mathcal{M}_0(p)\frac{\not{p}+\not{K}_1+m}{(p+K_1)^2-m^2}\gamma^{\mu_1}\cdots\frac{\not{p}+\not{K}_n+m}{(p+K_n)^2-m^2}\gamma^{\mu_n}u(p)$$

$$K_i = \sum_{m=i}^{n} k_m$$

Eikonal approximation and Dirac Equation

$$\mathcal{M}^{\mu_1\cdots\mu_n}(p,k_i) = \mathcal{M}_0(p)\frac{p^{\mu_1}\cdots p^{\mu_n}}{(p\cdot K_1)\cdots(p\cdot K_n)}$$

Sterman, George. "Partons, Factorization and Resummation, TASI95." *arXiv preprint hep-ph/9606312* (1996).

- Physical quantity → Sum over permutations of the emitted photons

$$E^{\mu_1\cdots\mu_n}(p,k_i) = \frac{1}{n!}\, p^{\mu_1}\ldots p^{\mu_n} \sum_{\pi} \frac{1}{p\cdot k_{\pi_1}} \frac{1}{p\cdot(k_{\pi_1}+k_{\pi_2})} \cdots \frac{1}{p\cdot(k_{\pi_1}+\ldots+k_{\pi_n})}$$

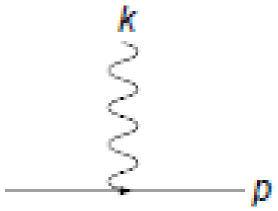
- Eikonal Identity →

$$\sum_{\pi}\left[\frac{1}{p\cdot k_{\pi(1)}}\cdots\frac{1}{p\cdot(\sum_{i=1}^{n} k_{\pi(i)})}\right] = \prod_{i=1}^{n}\frac{1}{p\cdot k_i}$$

$$E^{\mu_1\cdots\mu_n}(p,k_i) = \prod_{i}\frac{p^{\mu_i}}{p\cdot k_i}$$

Factorization

1. Eikonal Feynman rule



$$= \frac{p^\mu}{p\cdot k}$$

2. Define a subset of Feynman diagrams generating the full eikonal amplitude

Exponentiation

$$\mathcal{M} = \mathcal{M}_0 \exp\left[\sum G_c\right]$$

Next-to-Eikonal approximation

- Spin dependent
- Remainder term

Exponentiation

$$\mathcal{M} = \mathcal{M}_0 \exp\left[\mathcal{M}_{\mathrm{Eik}} + \mathcal{M}_{\mathrm{NE}}\right] (1 + \mathcal{M}_r) + O(\mathrm{NNE})$$

Laenen, E., Magnea, L., Stavenga, G., & White, C. D. (2011). "Next-to-eikonal corrections to soft gluon radiation: a diagrammatic approach." *JHEP*, *2011*(1), 141.

Form factors

$$\bar{u}(p_2)\,\Gamma^\mu\,u(p_1) = \bar{u}(p_2)\,\left[F_1(q^2)\gamma^\mu + \mathrm{i}\,\sigma^{\mu\nu}\,\tfrac{q_\nu}{2m}\,F_2(q^2)\right]\,u(p_1)$$

$$\Big\downarrow$$

$$a \equiv \tfrac{g-2}{2} = F_2(0)$$

$$\Big\downarrow$$

$$a\Big|_{\text{one loop}} = \frac{\alpha}{2\,\pi}$$

Peskin, M. E. (2018). *An introduction to quantum field theory*. CRC Press.
Schwinger, J. (1948). On quantum-electrodynamics and the magnetic moment of the electron. *Physical Review*, *73*(4), 416.

Form factors

$$\bar{u}(p_2)\,\Gamma^\mu\,u(p_1) = \bar{u}(p_2)\left[F_1(q^2)\gamma^\mu + \mathrm{i}\,\sigma^{\mu\nu}\,\frac{q_\nu}{2m}F_2(q^2)\right]u(p_1)$$

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Where does it come from?

Peskin, M. E. (2018). *An introduction to quantum field theory*. CRC Press.
Schwinger, J. (1948). On quantum-electrodynamics and the magnetic moment of the electron. *Physical Review*, *73*(4), 416.

Analytical one-loop calculation

$$\Gamma^\mu\big|_{\text{one loop}}$$

$$I = \int \frac{d^dk}{(2\pi)^d} \frac{1}{[(p_1-k)^2-m^2+i\eta][(p_2-k)^2-m^2+i\eta][k^2+i\eta]}$$

$$C^\mu = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu}{[(p_1-k)^2-m^2+i\eta][(p_2-k)^2-m^2+i\eta][k^2+i\eta]}$$

$$C^{\mu\nu} = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu}{[(p_1-k)^2-m^2+i\eta][(p_2-k)^2-m^2+i\eta][k^2+i\eta]}$$

Which of these contribute to $F_2(0)$?

I C^{μ} $C^{\mu\upsilon}$

$\cancel{I}$ $\downarrow$ No contribution

C^{μ}

$C^{\mu\upsilon}$

$\cancel{I}$

$\Big\downarrow$

No contribution

C^{μ}

$\Big\downarrow$

$$\frac{\alpha}{\pi}$$

$C^{\mu v}$

$\Big\downarrow$

$$-\frac{\alpha}{2\pi}$$

$\cancel{I}$

$\downarrow$

No contribution

$$\underbrace{\begin{array}{ccc} C^{\mu} & & C^{\mu\upsilon} \\ \downarrow & & \downarrow \\ \dfrac{\alpha}{\pi} & & -\dfrac{\alpha}{2\pi} \end{array}}_{\dfrac{\alpha}{2\pi}}$$

$$\cancel{I} \;\Big\downarrow$$

No contribution

$$C^{\mu} \;\Big\downarrow\; \frac{\alpha}{\pi} \qquad\qquad C^{\mu\upsilon} \;\Big\downarrow\; -\frac{\alpha}{2\pi}$$

$$\underbrace{\frac{\alpha}{\pi} \qquad\qquad -\frac{\alpha}{2\pi}}_{\frac{\alpha}{2\pi}}$$

Only term with one or two powers of k at numerator give contributions to the anomaly → Eikonal approximation does not suffice to obtain information about $g - 2$

One loop in the soft limit

Massless case → At NNE order I can reconstruct the whole form factor

What happens if I put the masses?

One loop in the soft limit

$$\boxed{\Gamma^\mu\big|_{\text{one loop}} = \Gamma^\mu\big|_{\text{Eik}} + \Gamma^\mu\big|_{\text{NE}} + \Gamma^\mu\big|_{\text{NNE}} + O(\text{N}^3\text{E})}$$

Only terms with at least one power of k^μ at numerator contribute at the anomalous magnetic moment

I compute the contribution from the NE and the NNE terms

Next-to-Eikonal order

$$\Gamma^\mu\Big|_{\rm NE} = (\cdots) + \int \frac{d^d k}{(2\pi)^d}\,\frac{\not{k}\not{p}_1\gamma^\mu}{2\,p_1\!\cdot\! k\,p_2\!\cdot\! k\,k^2} + \int \frac{d^d k}{(2\pi)^d}\,\frac{\gamma^\mu\not{p}_2\not{k}}{2\,p_1\!\cdot\! k\,p_2\!\cdot\! k\,k^2}$$

Next-to-Eikonal order

$$\Gamma^\mu\big|_{\mathrm{NE}} = (\cdots) + \int \frac{d^dk}{(2\pi)^d}\,\frac{\slashed{k}\,\slashed{p}_1\,\gamma^\mu}{2\,p_1\!\cdot\! k\; p_2\!\cdot\! k\; k^2} + \int \frac{d^dk}{(2\pi)^d}\,\frac{\gamma^\mu\,\slashed{p}_2\,\slashed{k}}{2\,p_1\!\cdot\! k\; p_2\!\cdot\! k\; k^2}$$

I have to calculate →

$$I^\mu = \int \frac{d^dk}{(2\pi)^d}\,\frac{k^\mu}{2\,p_1\!\cdot\! k\; p_2\!\cdot\! k\; k^2} = \frac{\mathrm{i}}{(4\pi)^2}\,\frac{p_1^\mu + p_2^\mu}{2\,m^2}$$

Next-to-Eikonal order

$$\Gamma^\mu|_{\mathrm{NE}} = (\cdots) + \int \frac{d^dk}{(2\pi)^d}\,\frac{\not{k}\not{p}_1\gamma^\mu}{2\,p_1\!\cdot\! k\; p_2\!\cdot\! k\; k^2} + \int \frac{d^dk}{(2\pi)^d}\,\frac{\gamma^\mu\not{p}_2\not{k}}{2\,p_1\!\cdot\! k\; p_2\!\cdot\! k\; k^2}$$

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$$\bar{u}(p_2)\Gamma^\mu|_{\mathrm{NE}}u(p_1) = (\cdots) + \bar{u}(p_2)\left[-\mathrm{i}\frac{\alpha}{4\pi}\,\frac{\sigma^{\mu\nu}q_\nu}{m}\right]u(p_1)$$

Next-to-Eikonal order

$$\Gamma^\mu\big|_{\mathrm{NE}} = (\cdots) + \int \frac{d^dk}{(2\pi)^d}\,\frac{\not{k}\,\not{p}_1\,\gamma^\mu}{2\,p_1\!\cdot\! k\; p_2\!\cdot\! k\; k^2} + \int \frac{d^dk}{(2\pi)^d}\,\frac{\gamma^\mu\,\not{p}_2\,\not{k}}{2\,p_1\!\cdot\! k\; p_2\!\cdot\! k\; k^2}$$

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$$\bar{u}(p_2)\Gamma^\mu\big|_{\mathrm{NE}} u(p_1) = (\cdots) + \bar{u}(p_2)\left[-\mathrm{i}\frac{\alpha}{4\pi}\,\frac{\sigma^{\mu\nu}q_\nu}{m}\right] u(p_1)$$

$$F_2(0)\big|_{\mathrm{NE}} = -\frac{\alpha}{2\pi}$$

Next-to-Next-to-Eikonal order

$$\Gamma^{\mu}\big|_{\mathrm{NNE}} = (\cdots) + \int \frac{d^d k}{(2\pi)^d}\, \frac{\not{k}\,\not{p}_1\,\gamma^{\mu}}{4\,(p_1\!\cdot\! k)^2\; p_2\!\cdot\! k} + \int \frac{d^d k}{(2\pi)^d}\, \frac{\not{k}\,\not{p}_1\,\gamma^{\mu}}{4\,p_1\!\cdot\! k\;(p_2\!\cdot\! k)^2} + \\ + \int \frac{d^d k}{(2\pi)^d}\, \frac{\gamma^{\mu}\,\not{p}_2\,\not{k}}{4\,(p_1\!\cdot\! k)^2\; p_2\!\cdot\! k} + \int \frac{d^d k}{(2\pi)^d}\, \frac{\gamma^{\mu}\,\not{p}_2\,\not{k}}{4\,p_1\!\cdot\! k\;(p_2\!\cdot\! k)^2} - (1-\epsilon)\int \frac{d^d k}{(2\pi)^d}\, \frac{\not{k}\,\gamma^{\mu}\,\not{k}}{2\,p_1\!\cdot\! k\; p_2\!\cdot\! k\; k^2}$$

Next-to-Next-to-Eikonal order

$$\Gamma^\mu\big|_{\mathrm{NNE}} = (\cdots) + \int \frac{d^dk}{(2\pi)^d}\,\frac{\not{k}\not{p}_1\gamma^\mu}{4\,(p_1\cdot k)^2\,p_2\cdot k} + \int \frac{d^dk}{(2\pi)^d}\,\frac{\not{k}\not{p}_1\gamma^\mu}{4\,p_1\cdot k\,(p_2\cdot k)^2} + \int \frac{d^dk}{(2\pi)^d}\,\frac{\gamma^\mu\not{p}_2\not{k}}{4\,(p_1\cdot k)^2\,p_2\cdot k} + \int \frac{d^dk}{(2\pi)^d}\,\frac{\gamma^\mu\not{p}_2\not{k}}{4\,p_1\cdot k\,(p_2\cdot k)^2} - (1-\epsilon)\int \frac{d^dk}{(2\pi)^d}\,\frac{\not{k}\gamma^\mu\not{k}}{2\,p_1\cdot k\,p_2\cdot k\,k^2}$$

$$I^{\mu\nu} = \int \frac{d^dk}{(2\pi)^d}\,\frac{k^\mu\,k^\nu}{2\,p_1\cdot k\,p_2\cdot k\,k^2}$$

Next-to-Next-to-Eikonal order

$$\Gamma^\mu|_{\mathrm{NNE}} = (\cdots) + \int \frac{d^d k}{(2\pi)^d} \frac{\slashed{k}\slashed{p}_1\gamma^\mu}{4\,(p_1\cdot k)^2\,p_2\cdot k} + \int \frac{d^d k}{(2\pi)^d} \frac{\slashed{k}\slashed{p}_1\gamma^\mu}{4\,p_1\cdot k\,(p_2\cdot k)^2} +$$

$$+ \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^\mu\slashed{p}_2\slashed{k}}{4\,(p_1\cdot k)^2\,p_2\cdot k} + \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^\mu\slashed{p}_2\slashed{k}}{4\,p_1\cdot k\,(p_2\cdot k)^2} - (1-\epsilon)\int \frac{d^d k}{(2\pi)^d} \frac{\slashed{k}\gamma^\mu\slashed{k}}{2\,p_1\cdot k\,p_2\cdot k\,k^2}$$

$$I^{\mu\nu} = \int \frac{d^d k}{(2\pi)^d} \frac{k^\mu\,k^\nu}{2\,p_1\cdot k\,p_2\cdot k\,k^2}$$

$$I_1^\mu = \int \frac{d^d k}{(2\pi)^d} \frac{k^\mu}{4\,(p_1\cdot k)^2\,p_2\cdot k}$$

$$I_2^\mu = \int \frac{d^d k}{(2\pi)^d} \frac{k^\mu}{4\,p_1\cdot k\,(p_2\cdot k)^2}$$

On going

Beyond one loop

Some questions have to be answered

- What happens inside each gauge set? On going
- Can we see and understand the cancellation?
- Can this method be generalized to all orders?

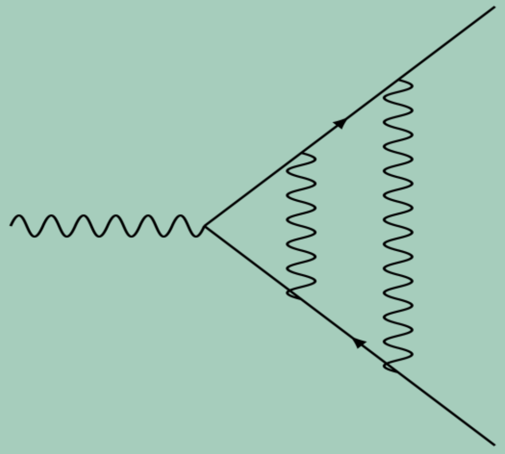
Thank you for your attention!

Thank you for your attention!

Backup slides

Glance beyond one loop

$$\sum_{\text{Perm}} \Bigg[$$



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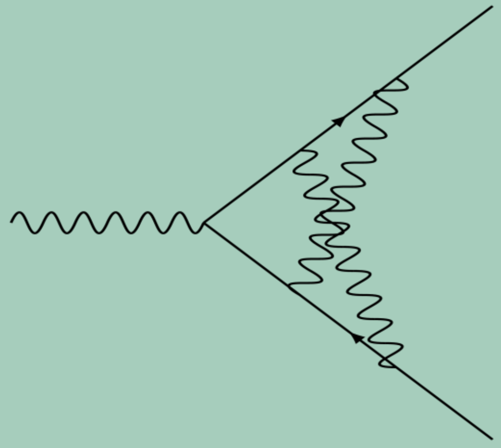
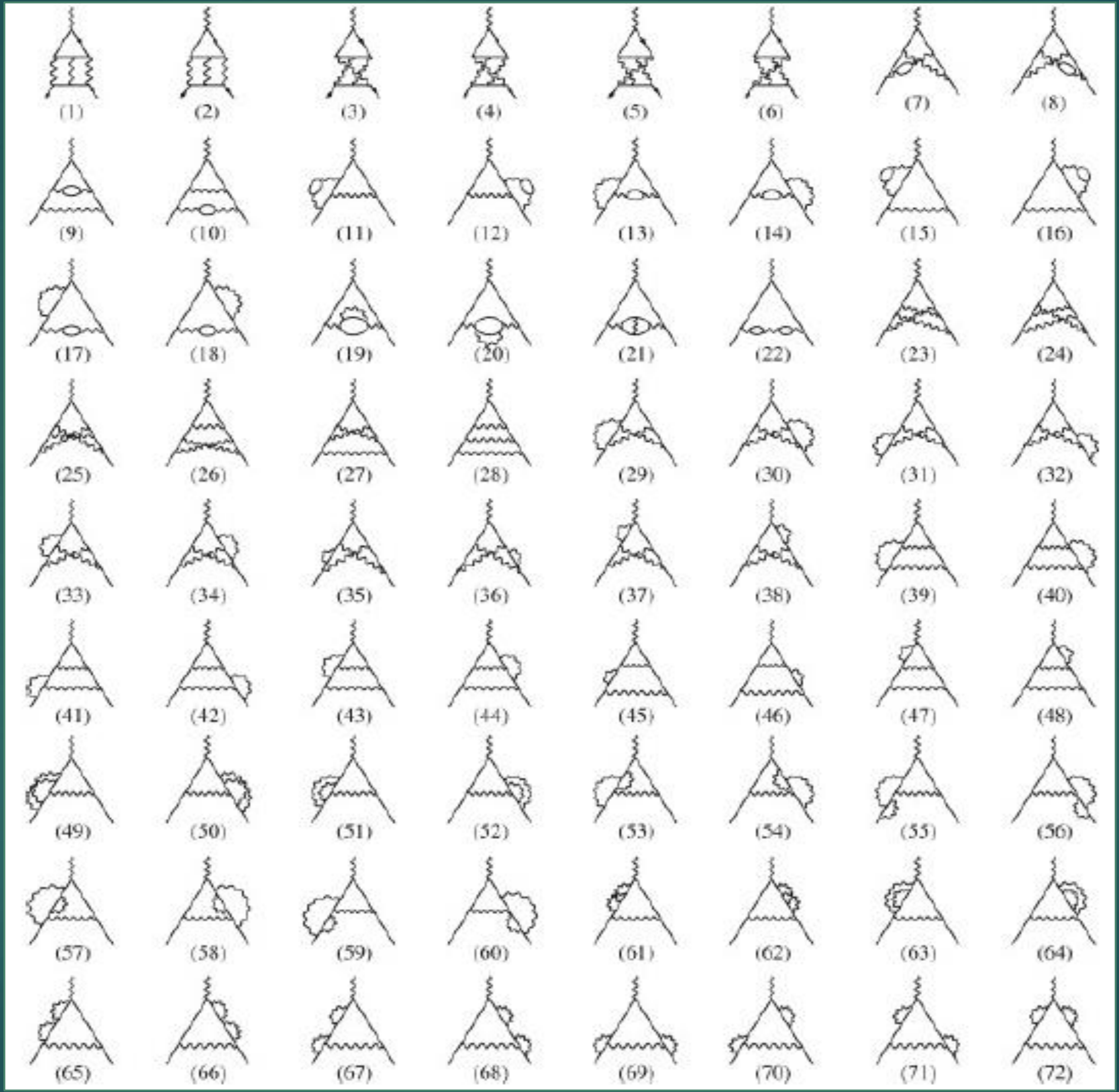


$$\Bigg] =$$

$$= -\int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{m p_1 \cdot p_2 (k_1^\mu + k_2^\mu)}{p_2 \cdot k_1 \, p_2 \cdot k_2 \, p_1 \cdot k_1 \, p_1 \cdot k_2 \, k_1^2 \, k_2^2}$$

$$+ \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{p_1 \cdot p_2 (\not{k}_1 + \not{k}_2)(p_1^\mu + p_2^\mu)}{p_2 \cdot k_1 \, p_2 \cdot k_2 \, p_1 \cdot k_1 \, p_1 \cdot k_2 \, k_1^2 \, k_2^2}$$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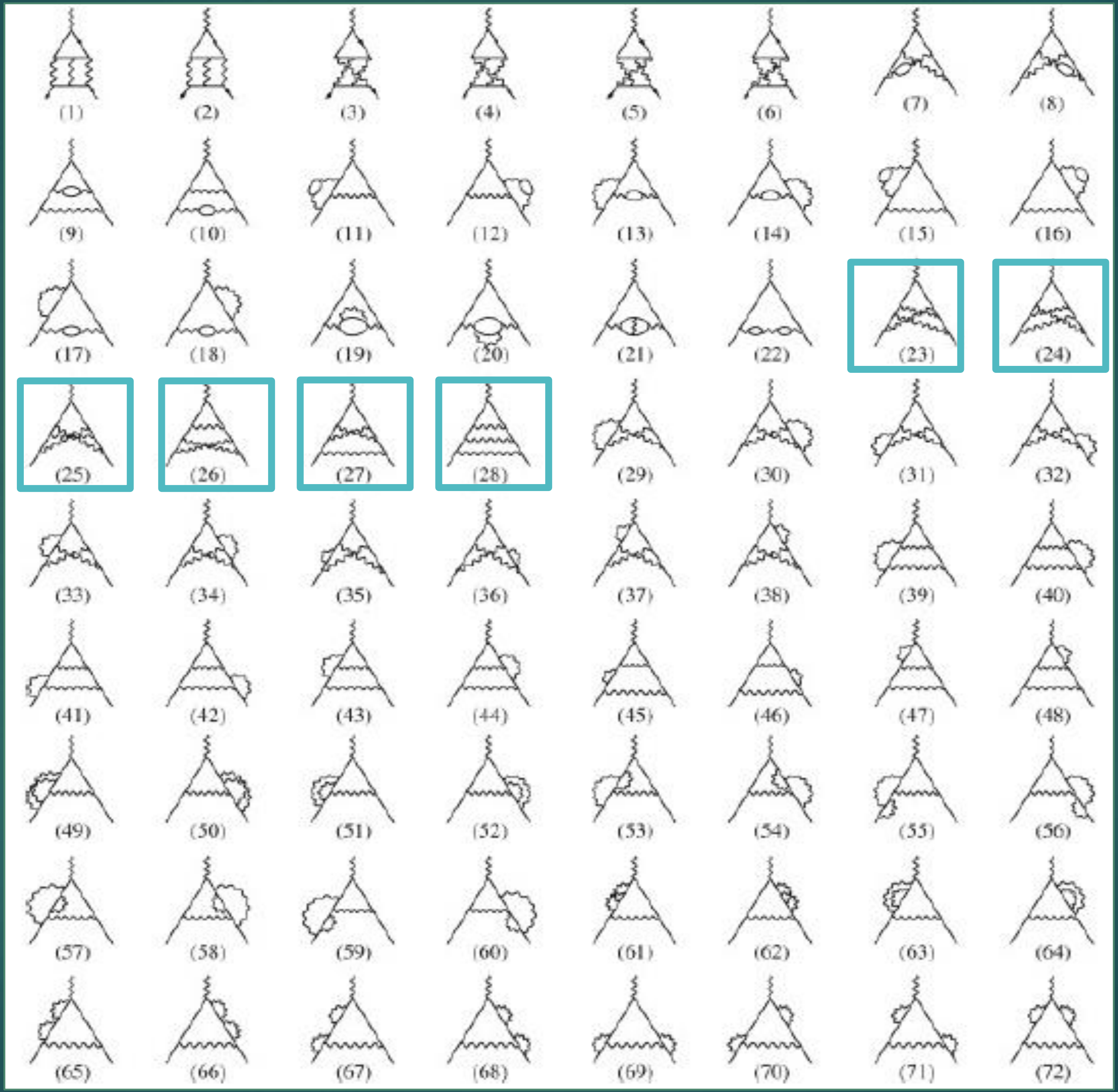
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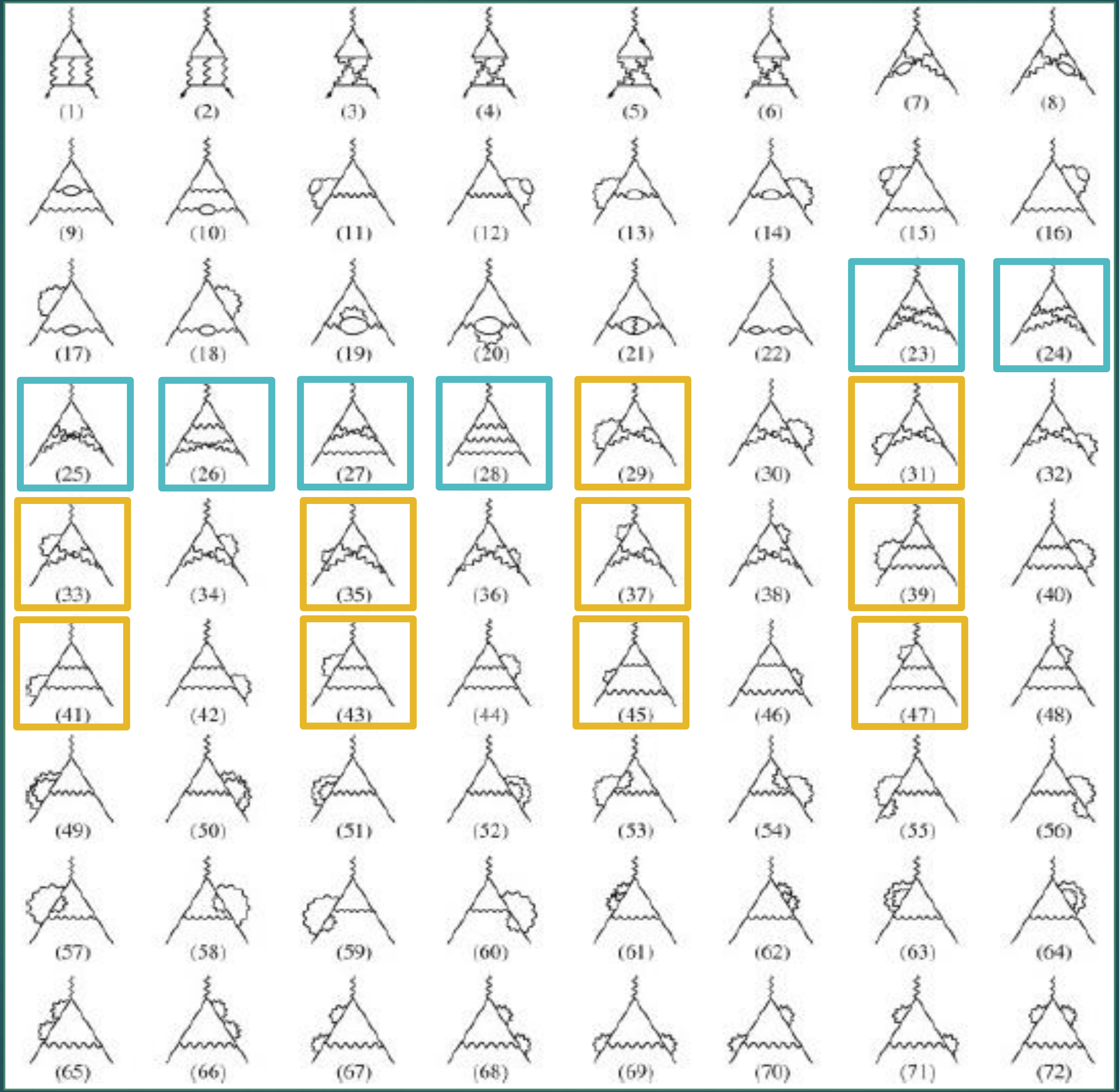
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