Soft approximations and possible applications to g - 2

ANNA RINAUDO MUNICH, 15.07.2019



Anomalous magnetic moment of leptons

Differences between experimental value and theoretical prediction







e

Anomalous magnetic moment of leptons

- Known with a precision of sub-part per billions
 - Less influenced by new physics effects
 - Value of the finestructure constant α

Giudice, G. F. at al., "Testing new physics with the electron g- 2." *JHEP*, 2012.



e

Anomalous magnetic moment of leptons

- Known with a precision of sub-part per billions
 - Less influenced by new physics effects
 - Value of the finestructure constant α

Now α can be measured very precisely with atomic physics experiments

2/19

Giudice, G. F. at al., "Testing new physics with the electron g- 2." JHEP, 2012.

Anomalous magnetic moment of leptons

Known with a precision of sub part per million

• More influenced by new physics effects

• Nowadays ~ 3,5 σ between experimental value and theoretical predictions

T. Blum et al., "The Muon (g-2) Theory Value: Present and Future", 2013.





Anomalous magnetic moment of leptons

 Known with a precision of sub part per million

• More influenced by new physics effects

• Nowadays ~ 3,5 σ between experimental value and theoretical predictions

New physics??

3/19

T. Blum et al., "The Muon (g-2) Theory Value: Present and Future", 2013.

μ



Anomalous magnetic moment of leptons

• The most influenced by new physics effects

• Very short lifetime: difficult to measure

Eidelman, S., et al., "Theory of the T lepton anomalous" magnetic moment." *Modern Physics Letters A*, 2007.

Anomalous magnetic moment

5/19

Exact finite order calculation

Anomalous magnetic moment

Exact finite order calculation

Soft Approximations

Anomalous magnetic moment

Why?

Exact finite order calculation

Soft Approximations

5/19

Exponentiation

Gauge-set cancellation

Gauge sets





P. Cvitanovic, "QFT and its discontents, a blog," 2018.

Soft limit: how does it work?

$$\gamma \underbrace{\mu}_{q} \underbrace{\nu}_{p_{2}}^{\mu} = (-ie)\Gamma^{\mu} = -e^{3}\mu^{3\epsilon} \int \frac{d^{d}k}{(2\pi)^{d}} \frac{\gamma^{\nu}(p_{2}^{\prime} - k + m)\gamma^{\mu}(p_{1}^{\prime} - k + m)\gamma_{\nu}}{[(p_{1} - k)^{2} - m^{2} + i\eta][(p_{2} - k)^{2} - m^{2} + i\eta][k^{2} + i\eta]}$$



$$\Gamma^{\mu} \approx \int \frac{d^{d}k}{(2\pi)^{d}} \frac{p_{1} \cdot p_{2} \gamma^{\mu}}{p_{1} \cdot k \, p_{2} \cdot k \, k^{2}} + \int \frac{d^{d}k}{(2\pi)^{d}} \left[\frac{p_{1} \cdot p_{2} \gamma^{\mu}}{2(p_{2} \cdot k)^{2} p_{1} \cdot k} - \frac{\gamma^{\mu}}{p_{2} \cdot k \, k^{2}} + \frac{k p / 1 \gamma^{\mu}}{2p_{1} \cdot k \, p_{2} \cdot k \, k^{2}} \right] + \\ + \int \frac{d^{d}k}{(2\pi)^{d}} \left[\frac{p_{1} \cdot p_{2} \gamma^{\mu}}{2(p_{1} \cdot k)^{2} p_{2} \cdot k} - \frac{\gamma^{\mu}}{p_{1} \cdot k \, k^{2}} + \frac{\gamma^{\mu} p / 2 k}{2p_{1} \cdot k \, p_{2} \cdot k \, k^{2}} \right]$$

Eikonal approximation







Eikonal approximation and Dirac Equation

Sterman, George. "Partons, Factorization and Resummation, TASI95." arXiv preprint hepph/9606312 (1996).

$$\mathcal{M}^{\mu_1\cdots\mu_n}(p,k_i) = \mathcal{M}_0(p) \frac{p^{\mu_1}\cdots p^{\mu_n}}{(p\cdot K_1)\cdots(p\cdot K_n)}$$

Physical quantity

Sum over permutations ^{9/19} of the emitted photons

$$E^{\mu_1\dots\mu_n}(p,k_i) = \frac{1}{n!} p^{\mu_1}\dots p^{\mu_n} \sum_{\pi} \frac{1}{p \cdot k_{\pi_1}} \frac{1}{p \cdot (k_{\pi_1} + k_{\pi_2})} \dots \frac{1}{p \cdot (k_{\pi_1} + \dots + k_{\pi_n})}$$

Eikonal Identity

$$\sum_{\pi} \left[\frac{1}{p \cdot k_{\pi(1)}} \cdots \frac{1}{p \cdot (\sum_{i=1}^{n} k_{\pi(i)})} \right] = \prod_{i=1}^{n} \frac{1}{p \cdot k_i}$$

$$E^{\mu_1\dots\mu_n}(p,k_i) = \prod_i \frac{p^{\mu_i}}{p \cdot k_i}$$

Factorization

1. Eikonal Feynman rule

$$\underline{\overset{k}{\leq}}_{p} = \frac{p^{\mu}}{p \cdot k}$$

2. Define a subset of Feynman diagrams generating the full eikonal amplitude

10/19

Exponentiation

$$\mathcal{M} = \mathcal{M}_0 \exp\left[\sum G_c\right]$$

Next-to-Eikonal approximation

• Spin dependent

• Remainder term

Exponentiation

11/19

$$\mathcal{M} = \mathcal{M}_0 \exp \left[\mathcal{M}_{\rm Eik} + \mathcal{M}_{\rm NE} \right] \, (1 + \mathcal{M}_r) + O(\rm NNE)$$

Laenen, E., Magnea, L., Stavenga, G., & White, C. D. (2011). "Next-to-eikonal corrections to soft gluon radiation: a diagrammatic approach." JHEP, 2011(1), 141.

Form factors

12/19

$$\bar{u}(p_2) \Gamma^{\mu} u(p_1) = \bar{u}(p_2) \left[F_1(q^2) \gamma^{\mu} + i \sigma^{\mu\nu} \frac{q_{\nu}}{2m} F_2(q^2) \right] u(p_1)$$



Peskin, M. E. (2018). An introduction to quantum field theory. CRC Press. Schwinger, J. (1948). On quantum-electrodynamics and the magnetic moment of the electron. *Physical Review*, 73(4), 416.

Form factors

12/19

$$\bar{u}(p_2) \Gamma^{\mu} u(p_1) = \bar{u}(p_2) \left[F_1(q^2) \gamma^{\mu} + i \sigma^{\mu\nu} \frac{q_{\nu}}{2m} F_2(q^2) \right] u(p_1)$$



Where does it come from?

Peskin, M. E. (2018). An introduction to quantum field theory. CRC Press. Schwinger, J. (1948). On quantum-electrodynamics and the magnetic moment of the electron. *Physical Review*, 73(4), 416.

Analytical one-loop calculation

$$I = \int \frac{d^d k}{(2\pi)^d} \frac{1}{[(p_1 - k)^2 - m^2 + i\eta][(p_2 - k)^2 - m^2 + i\eta][k^2 + i\eta]}$$

13/19

$$C^{\mu} = \int \frac{d^d k}{(2\pi)^d} \frac{k^{\mu}}{[(p_1 - k)^2 - m^2 + i\eta][(p_2 - k)^2 - m^2 + i\eta][k^2 + i\eta]}$$

$$C^{\mu\nu} = \int \frac{d^d k}{(2\pi)^d} \frac{k^{\mu}k^{\nu}}{[(p_1 - k)^2 - m^2 + i\eta][(p_2 - k)^2 - m^2 + i\eta][k^2 + i\eta]}$$

Which of these contribute to $F_2(0)$?

 $\Gamma^{\mu}|_{\text{one loop}}$









14/19

No contribution







Only term with one or two powers of k at numerator give contributions to the anomaly Eikonal approximation does not suffice to obtain information about g-2

One loop in the soft limit





At NNE order I can reconstruct the whole form factor

15/19

What happens if I put the masses?



One loop in the soft limit

$\Gamma^{\mu}|_{\text{one loop}} = \Gamma^{\mu}|_{\text{Eik}} + \Gamma^{\mu}|_{\text{NE}} + \Gamma^{\mu}|_{\text{NNE}} + O(\text{N}^{3}\text{E})$

Only terms with at least one power of k^{μ} at numerator contribute at the anomalous magnetic moment

I compute the contribution from the NE and the NNE terms

$$\Gamma^{\mu}|_{\rm NE} = (\cdots) + \int \frac{d^d k}{(2\pi)^d} \frac{k p_1' \gamma^{\mu}}{2 p_1 \cdot k p_2 \cdot k k^2} + \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^{\mu} p_2' k}{2 p_1 \cdot k p_2 \cdot k k^2}$$

$$\Gamma^{\mu}|_{\rm NE} = (\cdots) + \int \frac{d^d k}{(2\pi)^d} \frac{\not\!\!\!\! k p_1 \gamma^{\mu}}{2 p_1 \cdot k p_2 \cdot k k^2} + \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^{\mu} p_2' \not\!\!\! k}{2 p_1 \cdot k p_2 \cdot k k^2}$$

I have to calculate

$$I^{\mu} = \int \frac{d^d k}{(2\pi)^d} \frac{k^{\mu}}{2 \, p_1 \cdot k \, p_2 \cdot k \, k^2} = \frac{\mathrm{i}}{(4\pi)^2} \frac{p_1^{\mu} + p_2^{\mu}}{2 \, m^2}$$

$$\Gamma^{\mu}|_{\rm NE} = (\cdots) + \int \frac{d^d k}{(2\pi)^d} \frac{k p_1' \gamma^{\mu}}{2 p_1 \cdot k p_2 \cdot k k^2} + \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^{\mu} p_2' k}{2 p_1 \cdot k p_2 \cdot k k^2}$$

I have to calculate

$$I^{\mu} = \int \frac{d^d k}{(2\pi)^d} \frac{k^{\mu}}{2 p_1 \cdot k p_2 \cdot k k^2} = \frac{i}{(4\pi)^2} \frac{p_1^{\mu} + p_2^{\mu}}{2 m^2}$$

$$\bar{u}(p_2)\Gamma^{\mu}|_{\mathrm{NE}}u(p_1) = (\cdots) + \bar{u}(p_2)\left[-\mathrm{i}\frac{\alpha}{4\pi}\frac{\sigma^{\mu\nu}q_{\nu}}{m}\right]u(p_1)$$

$$\Gamma^{\mu}|_{\rm NE} = (\cdots) + \int \frac{d^d k}{(2\pi)^d} \frac{k p_1' \gamma^{\mu}}{2 p_1 \cdot k p_2 \cdot k k^2} + \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^{\mu} p_2' k}{2 p_1 \cdot k p_2 \cdot k k^2}$$

I have to calculate

$$I^{\mu} = \int \frac{d^d k}{(2\pi)^d} \frac{k^{\mu}}{2 \, p_1 \cdot k \, p_2 \cdot k \, k^2} = \frac{\mathrm{i}}{(4\pi)^2} \frac{p_1^{\mu} + p_2^{\mu}}{2 \, m^2}$$

$$\bar{u}(p_2)\Gamma^{\mu}|_{\mathrm{NE}}u(p_1) = (\cdots) + \bar{u}(p_2)\left[-\mathrm{i}\frac{\alpha}{4\pi}\frac{\sigma^{\mu\nu}q_{\nu}}{m}\right]u(p_1)$$

$$F_2(0)|_{\rm NE} = -\frac{\alpha}{2\pi}$$

Next-to-Next-to-Eikonal order

$$\Gamma^{\mu}|_{\text{NNE}} = (\cdots) + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{k p_{1}' \gamma^{\mu}}{4 (p_{1} \cdot k)^{2} p_{2} \cdot k} + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{k p_{1}' \gamma^{\mu}}{4 p_{1} \cdot k (p_{2} \cdot k)^{2}} + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{\gamma^{\mu} p_{2}' k}{4 (p_{1} \cdot k)^{2} p_{2} \cdot k} + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{\gamma^{\mu} p_{2}' k}{4 p_{1} \cdot k (p_{2} \cdot k)^{2}} - (1 - \epsilon) \int \frac{d^{d}k}{(2\pi)^{d}} \frac{k \gamma^{\mu} k}{2 p_{1} \cdot k p_{2} \cdot k k^{2}}$$

Next-to-Next-to-Eikonal order

$$\Gamma^{\mu}|_{\text{NNE}} = (\cdots) + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{\not k \not p_{1}' \gamma^{\mu}}{4(p_{1}\cdot k)^{2} p_{2}\cdot k} + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{\not k \not p_{1}' \gamma^{\mu}}{4p_{1}\cdot k(p_{2}\cdot k)^{2}} + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{\gamma^{\mu} \not p_{2}' \not k}{4(p_{1}\cdot k)^{2} p_{2}\cdot k} + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{\gamma^{\mu} \not p_{2}' \not k}{4p_{1}\cdot k(p_{2}\cdot k)^{2}} - (1-\epsilon) \int \frac{d^{d}k}{(2\pi)^{d}} \frac{\not k \gamma^{\mu} \not k}{2p_{1}\cdot k p_{2}\cdot k k^{2}}$$

$$I^{\mu\nu} = \int \frac{d^{d}k}{(2\pi)^{d}} \frac{k^{\mu} k^{\nu}}{2 \, p_{1} \cdot k \, p_{2} \cdot k \, k^{2}}$$

Next-to-Next-to-Eikonal order

$$\Gamma^{\mu}|_{\text{NNE}} = (\cdots) + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{kp_{1}^{\prime}\gamma^{\mu}}{4(p_{1}\cdot k)^{2}p_{2}\cdot k} + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{kp_{1}^{\prime}\gamma^{\mu}}{4p_{1}\cdot k(p_{2}\cdot k)^{2}} + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{\gamma^{\mu}p_{2}^{\prime}k}{4(p_{1}\cdot k)^{2}p_{2}\cdot k} + \int \frac{d^{d}k}{(2\pi)^{d}} \frac{\gamma^{\mu}p_{2}^{\prime}k}{4p_{1}\cdot k(p_{2}\cdot k)^{2}} - (1-\epsilon) \int \frac{d^{d}k}{(2\pi)^{d}} \frac{k\gamma^{\mu}k}{2p_{1}\cdot kp_{2}\cdot kk^{2}}$$

$$I^{\mu\nu} = \int \frac{d^d k}{(2\pi)^d} \frac{k^{\mu} k^{\nu}}{2 p_1 \cdot k p_2 \cdot k k^2} \qquad I^{\mu}_1 = \int \frac{d^d k}{(2\pi)^d} \frac{k^{\mu}}{4 (p_1 \cdot k)^2 p_2 \cdot k} \qquad I^{\mu}_2 = \int \frac{d^d k}{(2\pi)^d} \frac{k^{\mu}}{4 p_1 \cdot k (p_2 \cdot k)^2}$$
On going

Beyond one loop

Some questions have to be answered

What happens inside solve
 each gauge set?

- Can we see and understand the cancellation?
- Can this method be generalized to all orders?

Thank you for your attention!

Thank you for your attention!

Backup slides

Glance beyond one loop



 $= -\int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{m p_1 \cdot p_2(k_1^{\mu} + k_2^{\mu})}{p_2 \cdot k_1 \, p_2 \cdot k_2 \, p_1 \cdot k_1 \, p_1 \cdot k_2 \, k_1^2 \, k_2^2}$

 $+ \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{p_1 \cdot p_2 (k_1 + k_2) (p_1^{\mu} + p_2^{\mu})}{p_2 \cdot k_1 \, p_2 \cdot k_2 \, p_1 \cdot k_1 \, p_1 \cdot k_2 \, k_1^2 \, k_2^2}$

5 X 团 (2) (7) (4) (6) (8) (3) (1) × (15) 0 (10) 6 gmin 9 Juni hoy that (14) (9) (11) (13) (12) (16) And (19) Anta (17) (18) (21) 8 (22) (23) (24) and Lind april (27) (29) bot Arany pro (25) (26) (30) (31) (28) (32) ş Sac bod 820 Sas bad Jun Spol fund (33) (34) (35) (37) (38) (36) (40) (39) (45) frend human (41) (42) \$ (43) (44) § (46) (47) (48) ş thend Ç. Frans Frund Juni (56) min (51) (49) (50) (53) (55) (52) (54) £ Anna (And F han (57) (63) (58) (61) (62) (64) (59) (60) 0 fund mint fund Frind and smind fund mo (66) (71) (72) (65) (68) (69) (70) (67)

5 团 (2) 343 (7) (6) (4) (8) (3) (1) Z (15) 0 6 (10) gam σ hoy that (14) (9) (11) (13) (12) (16) (19) frend And (21) (17) how 20 (22) (23) (18) (24) Lind gha find (29) and (range 1202 (27) (30) (31) (26) (32) (25) (28) South bod grad. Zand And fund (33) (35) (37) (38) (34) (40) (36) (39) A (45) fund ----human (41) (42) \$ (43) (44) (46) (47) (48) Ç. France time Frund (56) Juni inni (51) (49) (50) (53) (52) (54) (55) Ø. £ Anna (Am F (57) (63) (61) (58) (62) (64) (59) (60) 0 front hund Frind mint and fund mo smin (65) (71) (66) (69) (70) (72) (67) (68)

5 X (2) 10 (7) (4) (3) (6) (8)(1) × (15) 0 6 (10) σ how how (9) (11) (14) (13) (12) (16) (19) And Assa (21) (17) 80 (22) ~~~~ (23) (18) (24) - Char phot and (soil) ports and (27) (30) (31) (32) (25) (26) (28) (29) São bod bad 22a (mag (Zinda) Soc fund (35) (38) (33) (34) (36) (37) (40) (39) A 2 Z line hand Z..... Anna (46) han (43) (41) (42) (44) (47) (48) ş Jun Frank Frand find (56) Juni form (49) (50) (51) (52) (53) (54) (55) Ø Z Ann (And F -(57) (61) (63) (58) (64) (59) (62) (60) Ũ fund Frink min and front fund mo smin (65) (66) (69) (70)(71) (72) (67) (68)

(5) X (2) 10 (7) (4) (3) (6) (8)(1) (15) 6 (10) 102 how (14) (9) (11) (13) (12) (16) (19) Assa And (21) (17) mon 20) Lono (23) (18) (22) (24) Land - ghad and And and 300 march (27) (30) (25) (26) (28) (29) (31) (32) 600 Sad bod (Zm) (And yood humi (35) (33) (38) (34) (37) (39) (36) (40)A Zand Land hund -----Sund in (42) (43) (46) (47) (41) (44) (45) (48) \$ fund Ş. Frund Frank (56) Frend (49) (50) (51) (52) (53) (54) (55) £ Ann An 5 in (57) (61) (63) (64) (58) (59) (62) (60) Ũ Frink and front Xms fund mos smin fring (65) (66) (69) (70)(71) (72) (67) (68)