GUTs from F-Theory

(Particle Physics meets Algebraic Geometry)

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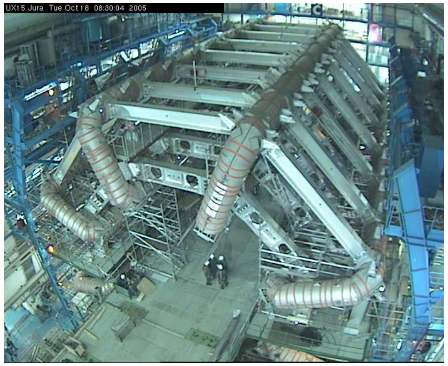
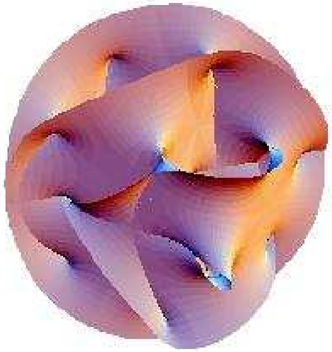
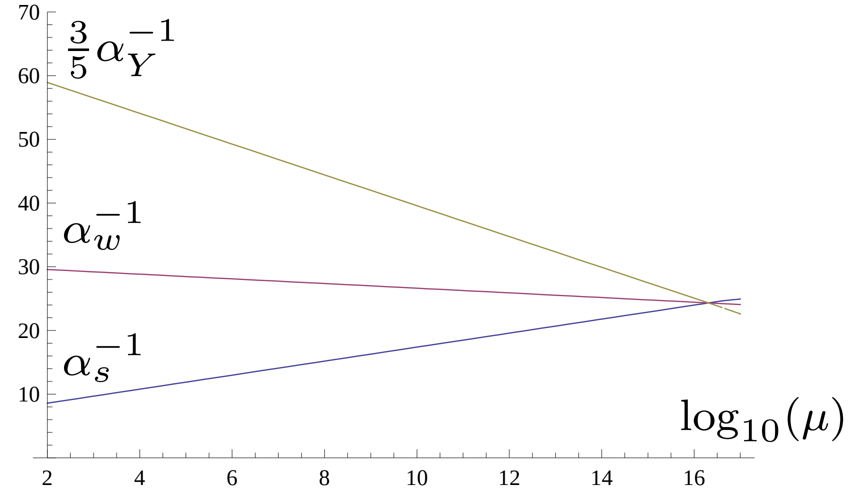




Grand Unification

Grand Unification

- One-loop running of the three Standard Model gauge couplings with MSSM matter spectrum above the TeV scale, (Ellis, Kelley, Nanopoulos), (Amaldi, de Boer, Fürstenau), (Langacker, Luo)



- Evidence for a supersymmetric Grand Unification at $M_X = 2.1 \cdot 10^{16}$ GeV, as for instance:
  - Gauge group: $SU(5)$
  - chiral matter in $\mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$
  - Higgs field: $\mathbf{5}_H + \overline{\mathbf{5}}_H$
  - Yukawa couplings: $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$, $\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}_H$, $\overline{\mathbf{5}}\,\mathbf{1}\,\mathbf{5}_H$

Beyond CFT: F-Theory

Beyond CFT: F-Theory

Consider Type IIB: $SL(2,\mathbb{Z})$ symmetry acting as

$$\tau' = \frac{a\tau + b}{c\tau + d}, \qquad \begin{pmatrix} F_3' \\ H_3' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} F_3 \\ H_3 \end{pmatrix}$$

- magnetically charged (p,q) 7-branes
- fundamental string can end on a D7-brane $\rightarrow$ (p,q) string can end on a (p,q) 7-brane
- backreaction of D7-brane $\tau = \frac{1}{2\pi i}\log z$ induce an $SL(2,\mathbb{Z})$ monodromy $M_{\mathrm{D7}} = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$
- general (p,q) 7-brane induces a monodromy $M_{(p,q)} = \left(\begin{smallmatrix} 1-pq & p^2 \\ -q^2 & 1+pq \end{smallmatrix}\right)$

F-Theory

F-Theory

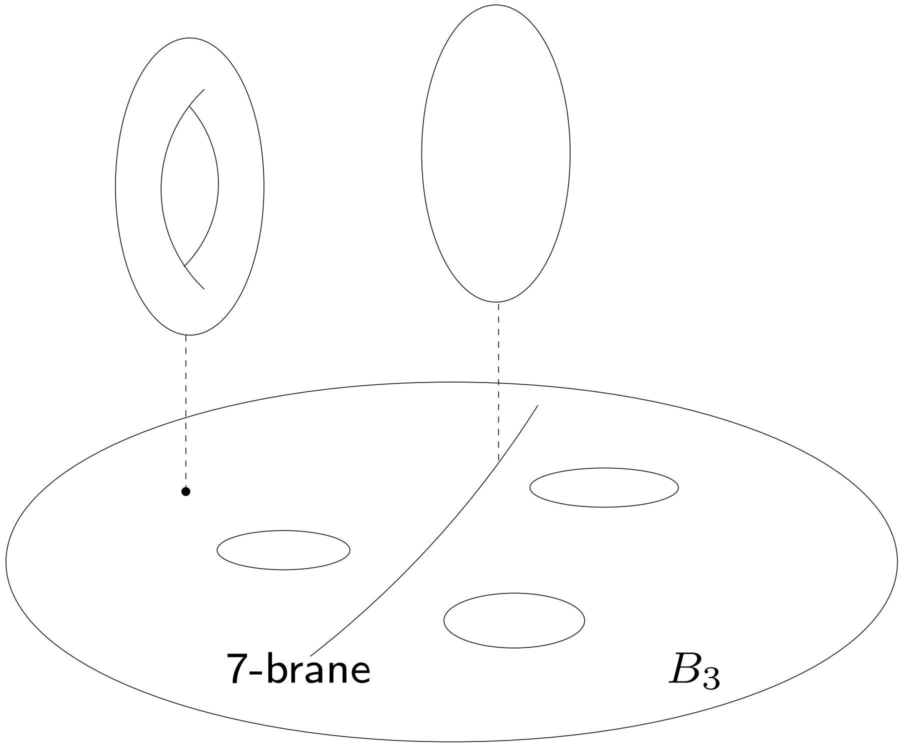
F-theory is a way of book-keeping of the positions of more general (p,q)-7-branes in Type IIB $\mathcal{N}=1$ compactifications

$$\text{elliptic fibration :} \qquad Y \to B_3$$

Susy $\to Y$ Calabi-Yau 4-fold. Elliptic curve $y^2 = x^3 + f(u)\,x + g(u)$ with complex structure:

$$\tau = C_0 + i\,e^{-\varphi}$$

with $j(\tau) = \frac{4(24)\,f^3}{4f^3+27\,g^2}$



(Vafa,Nucl.Phys.B469:403,1996), (Beasley, Heckman, Vafa, arXiv:0802.3391+0806.0102), (Donagi, Wijnholt. arXiv:0802.2969)

F-Theory

F-Theory

- Consistency condition: Degeneration loci can be described by a compact Calabi-Yau fourfold Y
- D3-tadpole:

$$N_{\rm D3} + \frac{1}{2}\int_Y G_4 \wedge G_4 = \frac{\chi(Y)}{24}$$

- Gauge symmetry on co-dimension one loci in B : Degeneration of elliptic curve, ADE Kodaira classification
- Matter localized on further enhancements over co-dimension two loci. Example: $SO(10)$ enhances to E_6: $\mathbf{16}$ or to $SO(12)$: $\mathbf{10}$
- Yukawa couplings localized over triple intersection on co-dimension 3 loci
- 4D chirality only for $G_4 \neq 0$

F-Theory

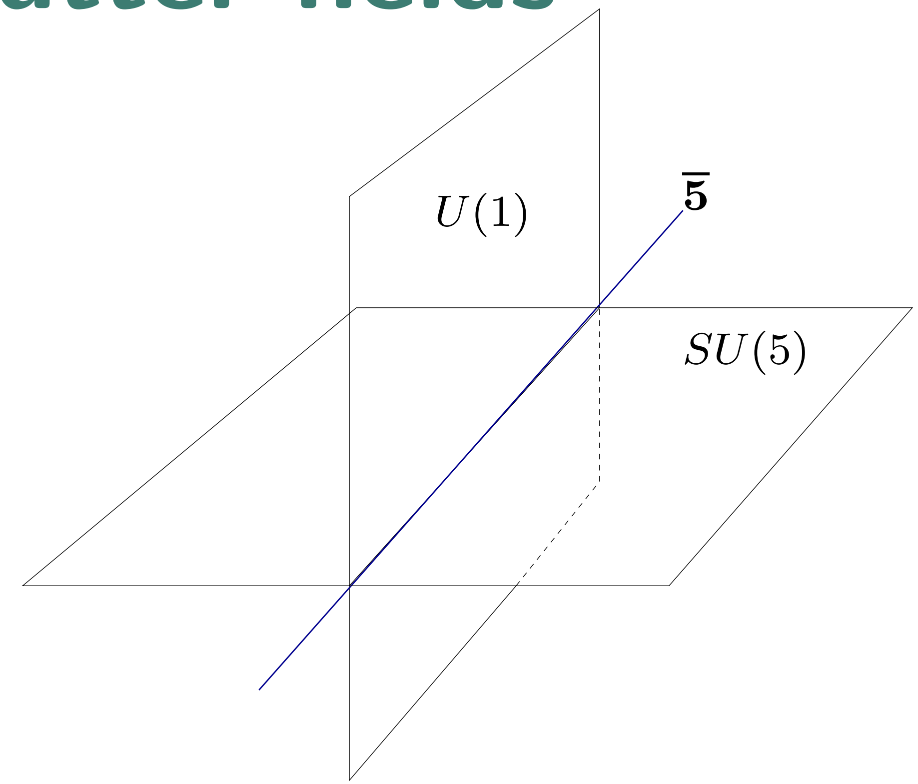
F-Theory

$ord(f)$	$ord(g)$	$ord(\Delta)$	fiber	singularity	comp.	monod.
≥ 0	≥ 0	0	I_0	smooth	1	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
0	0	1	I_1	dbl. point	1	$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
0	0	n	I_n	A_{n-1}	n	$\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$
2	≥ 3	$n+6$	I_n^*	D_{n+4}	$n+5$	$\begin{pmatrix} -1 & -n \\ 0 & -1 \end{pmatrix}$
≥ 2	3					
≥ 3	4	8	IV^*	E_6	7	$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$
3	≥ 5	9	III^*	E_7	8	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
≥ 4	5	10	II^*	E_8	9	$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$

Matter fields

Matter fields

Matter fields are generally localised on curves: $C = D_a \cap D_b$



F-theory: Enhancement of the singularity over the intersection: $SU(5) \times U(1) \to SU(6)$

$$\mathbf{35} = \mathbf{24}_0 + \mathbf{1}_0 + \mathbf{5}_1 + \overline{\mathbf{5}}_{-1}$$

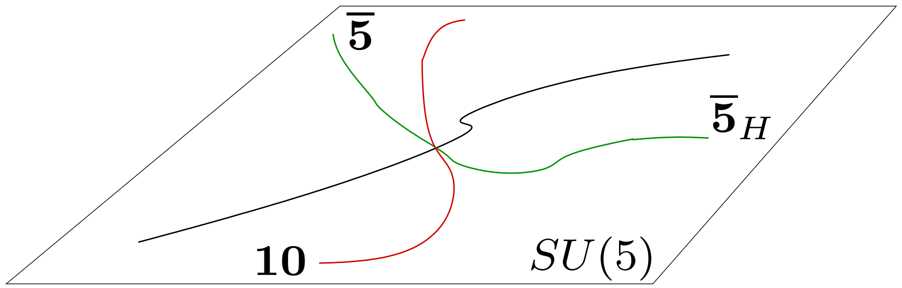
resp. $SU(5) \times U(1) \to SO(10)$

$$\mathbf{45} = \mathbf{24}_0 + \mathbf{1}_0 + \mathbf{10}_2 + \overline{\mathbf{10}}_{-2}$$

Yukawa couplings

Yukawa couplings

Yukawa couplings:



The Yukawa couplings which give masses to the MSSM fields after GUT and electroweak symmetry breaking are

$$\mathbf{10}^{(2,0)}\,\mathbf{10}^{(2,0)}\,\mathbf{5_H}^{(1,-1)}, \quad \mathbf{10}^{(2,0)}\,\overline{\mathbf{5}}^{(-1,-1)}\,\overline{\mathbf{5}}_\mathbf{H}^{(-1,1)},$$

$$\mathbf{1}_N^{(0,2)}\,\overline{\mathbf{5}}^{(-1,-1)}\,\mathbf{5_H}^{(1,-1)}$$

F-theory: Enhancement to $SU(5) \times U(1) \times U(1) \rightarrow E_6$

$$\mathbf{78} \rightarrow \mathbf{adj}. + \mathbf{10}_{(-1,3)} + \mathbf{10}_{(4,0)} + \mathbf{5}_{(-3,-3)} + \mathbf{1}_{(5,3)} + c.c.$$

Yukawa: $\mathbf{10}\mathbf{10}\mathbf{5}_H$ of O(1) possible in F-theory!

GUT symmetry breaking

GUT symmetry breaking

Symmetry breaking via gauge flux F_Y: $c_1(L_Y) \in H^2(D)$ has to be trivial in $H^2(X)$, i.e. $\iota_*(L_Y) = 0$.

$$\mathbf{24} \longrightarrow (\mathbf{8},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{3},\mathbf{2})_5 + (\overline{\mathbf{3}},\mathbf{2})_{-5}$$

To avoid exotic states, need to change embedding to

$$\begin{aligned} F \;=\; & \sum_{a=1}^{8} F^a_{SU(3)} \begin{pmatrix} \lambda_a/2 & 0 \\ 0 & 0 \end{pmatrix} + \sum_{i=1}^{3} F^i_{SU(2)} \begin{pmatrix} 0 & 0 \\ 0 & \sigma_i/2 \end{pmatrix} + \\ & \tfrac{1}{6}\, F_Y \begin{pmatrix} -2_{3\times 3} & 0 \\ 0 & 3_{2\times 2} \end{pmatrix} + \\ & \left(\overline{f}_a + \tfrac{2}{5}\,\overline{f}_Y\right) \begin{pmatrix} 1_{3\times 3} & 0 \\ 0 & 1_{2\times 2} \end{pmatrix} + \tfrac{1}{5}\,\overline{f}_Y \begin{pmatrix} -2_{3\times 3} & 0 \\ 0 & 3_{2\times 2} \end{pmatrix}, \end{aligned}$$

Compact models

Compact models

Problem: Realisations of all these local features in genuine compact F-theory

First step: Type IIB orientifolds compactified on Calabi-Yau 3-folds containing appropriate del-Pezzo surfaces (Bhg, Braun, Grimm, Weigand)

- del-Pezzo transitions of the Quintic $\mathbb{P}^4[5]$: sing. $x_5^2\, P_3(x_1, x_2, x_3, x_4) = 0 \rightarrow dP_6$
- uplift to F-theory (Bhg, Grimm, Jurke, Weigand)
- study degenerations of the elliptic fiber with $SU(5)$ enhancement and appropriate matter curves and Yukawa couplings
- choice of G_4-form flux resp. gauge flux with 3 generations, spectral cover construction (Bhg, Grimm, Jurke, Weigand)

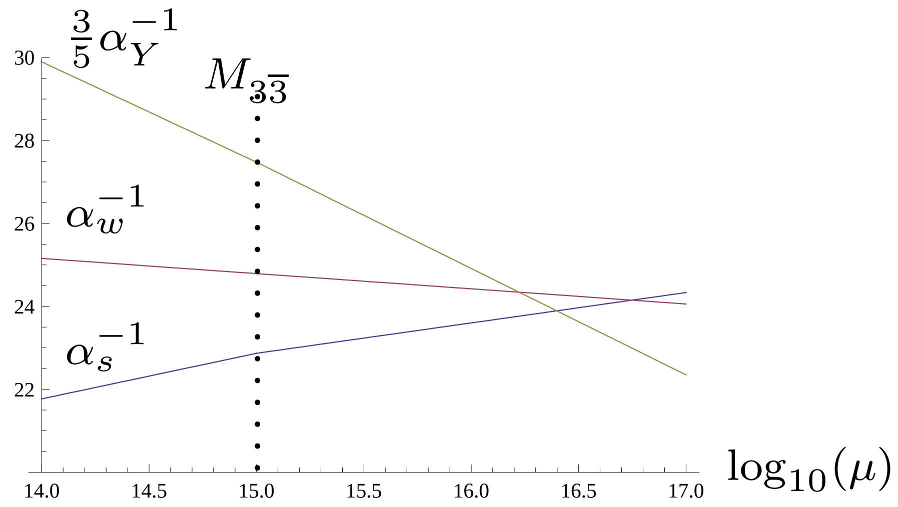
Gauge coupling unification

Gauge coupling unification

The gauge couplings at the string/GUT scale are changed due to the $U(1)_Y$ flux and satisfy the relation (Bhg, arXiv:0812.0248)

$$\frac{1}{\alpha_Y(M_s)} = \frac{1}{\alpha_w(M_s)} + \frac{2}{3\,\alpha_s(M_s)}\ .$$

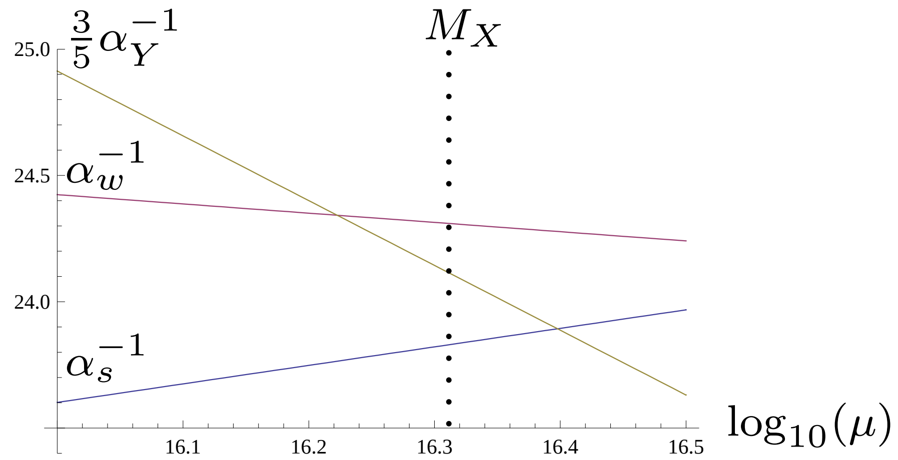
Include the Higgs triplet above a scale $1\mathrm{TeV} < M_{3\bar{3}} < M_X$ in the running:



Gauge coupling unification

Gauge coupling unification

zooming further in



The three MSSM gauge couplings satisfy the F-theory GUT relation at

$$M_X = 2.1 \cdot 10^{16}\,\mathrm{GeV}$$

independent of the triplet mass scale $M_{3\overline{3}}$.

Work in progress

Work in progress

F-theory/IIB orientifold compactifications provide new and very promising string theory realisations of $SU(5)$ GUTs:

- Better understanding of CY 4-folds and its degenerations
- Instanton effects in F-theory, in particular with charged zero modes.
- Include moduli stabilisation in the construction, susy breaking (improved LARGE volume scenario, gauge mediated), soft terms, collider signatures (Bhg, Conlon, Krippendorf, Moster, Quevedo)
- Implications for the so-called landscape of string vacua $\longrightarrow$ landscape deconstruction?

Workshop: *GUTs and Strings*, MPI, February, 2010

Structure of the group at MPI

Structure of the group at MPI

Research topics in the string theory group in 2009:

- Low string scale signatures at LHC: D. Lüst, S. Stieberger + D. Härtl, O. Schlotterer
- Flux compactifications and generalized geometry: D. Lüst + J. Held
- F-theory GUTs: R. Blumenhagen + B. Jurke, T. Rahn, H. Roschny
- AdS-CFT at finite T and Quark-Gluon plasma : J. Erdmenger + M. Ammon, V. Graß , C. Greubel, S. Halter, P. Kerner, H. Ngo Thanh
- M. Zagermann (Emmy Noether research group leader) moved to University of Hannover
- PhDs: C. Caviezel, S. Körs, R. Meyer, E. Plauschinn, F. Rust, M. Schmidt-Sommerfeld

Structure of the group at MPI

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Book projects

- *String Cosmology*, Erdmenger (ed.), Wiley VCH
- *Introduction to Conformal Field Theory*, Blumenhagen, Plauschinn, Lecture Notes in Physics, Springer
- *Basic Concepts of String Theory*, Blumenhagen, Lüst, Theisen, in preparation, Springer