The mass of the lightest Higgs boson in the MSSM

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Outline

1. **the MSSM and its Higgs bosons**
2. **the lightest Higgs-boson mass M_{h^0}**
3. **precise prediction of M_{h^0}**

Minimal Supersymmetric Standard Model (MSSM)

Construction:

- start out with the Standard Model (SM)
  - **fermions:** leptons and quarks
  - **bosons:** gauge bosons (W-, Z-bosons, photon, gluons)
    Higgs boson

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- make it supersymmetric:

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  Supersymmetry?:

  fermionic degrees of freedom

  $\updownarrow$

  bosonic degrees of freedom

Minimal Supersymmetric Standard Model (MSSM)

Construction:

- start out with the Standard Model (SM)

- add superpartners:

  to each fermion $\longrightarrow$ one boson

  to each boson $\longrightarrow$ one fermion

Note: Particle and corresponding SUSY-partner:

same quantum numbers except for
spin quantum number

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  (replace fields by **superfields** (bos./ferm. comp.))

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⇝ a supersymmetric model

It is not that easy...

In a supersymmetric theory:

mass of particles $\equiv$ mass of their superpartners

Problem: no SUSY-particles have been observed

if realised in nature $\rightsquigarrow$ supersymmetry must be broken

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Solution for the MSSM:

Add soft supersymmetry breaking terms ,

explicit symmetry breaking (many new parameters!)

(soft: relations between dimensionless couplings remain unchanged, no higher than logarithmic divergences)

A difference concerning the Higgs sector:

Standard Model (SM):

Recall: Wherefore a Higgs boson?

theory:

mass terms for gauge bosons are explicitly forbidden by gauge symmetry

experiment:

Massive gauge bosons were observed (W, Z)

$\longrightarrow$ contradiction (?) $\longleftarrow$

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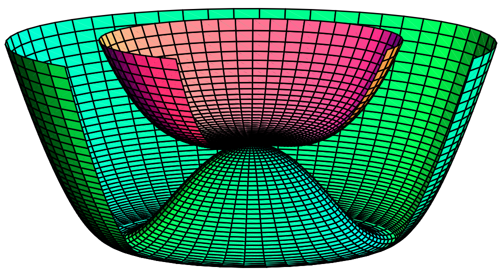
possible **realisation**:

scalar field ϕ (*Higgs field*) with a finite vacuum expectation value exists



$V = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$

$\Rightarrow$ Generation of gauge boson and fermion masses (in the SM)

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Standard Model (SM):

quark masses generated by terms proportional to:

- the Higgs doublet H for the down-type quarks $H = \begin{pmatrix} G^+ \\ v + \frac{1}{\sqrt{2}}(h + iG^0) \end{pmatrix}$
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MSSM:

Problem: term generating up-type quark masses

Start with: term proportional to H_c

then: fields $\longrightarrow$ superfields (bos./ferm. comp.)

$\Rightarrow$ new term: not supersymmetric

Solution: instead of H_c: Second Higgs doublet!

Minimal Supersymmetric StandardModel (MSSM)

Construction:

- start out with the Standard Model (SM)
- add a second Higgs doublet
- add superpartners

  (replace fields by **superfields** (bos./ferm. comp.))
- add soft supersymmetry breaking terms

Higgs bosons in the MSSM

physical mass eigenstates:

- 5 Higgs-bosons: 3 neutral H^0, h^0, A^0
  2 charged $H^\pm$

masses of the Higgs-bosons:

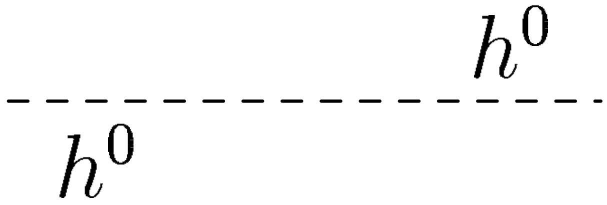
- not all independent:
  common: A^0-boson mass M_A as free parameter
- lightest Higgs-boson: h^0

Upper theoretical **Born** mass limit: $M_{h^0} \leq M_Z$

with quantum corrections of **higher orders**: $M_{h^0} \lesssim 135$ GeV

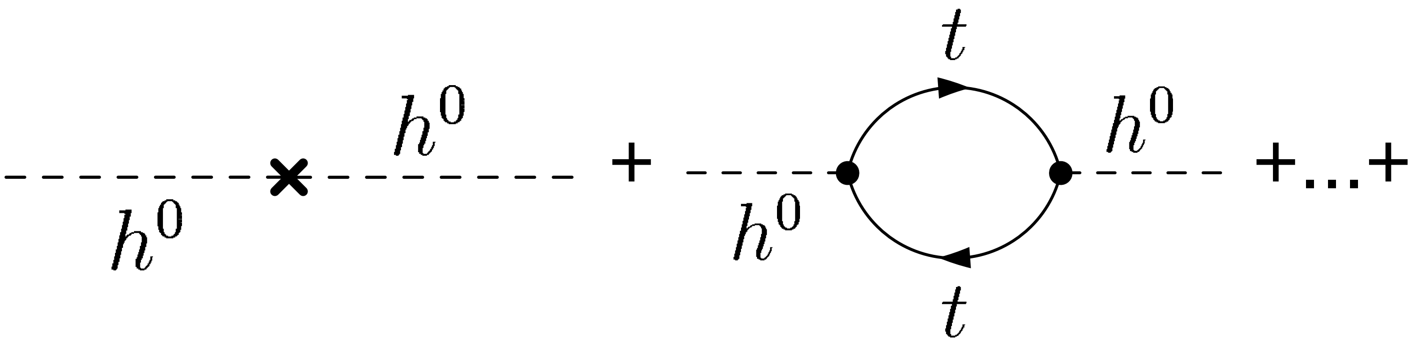
Higgs-propagator

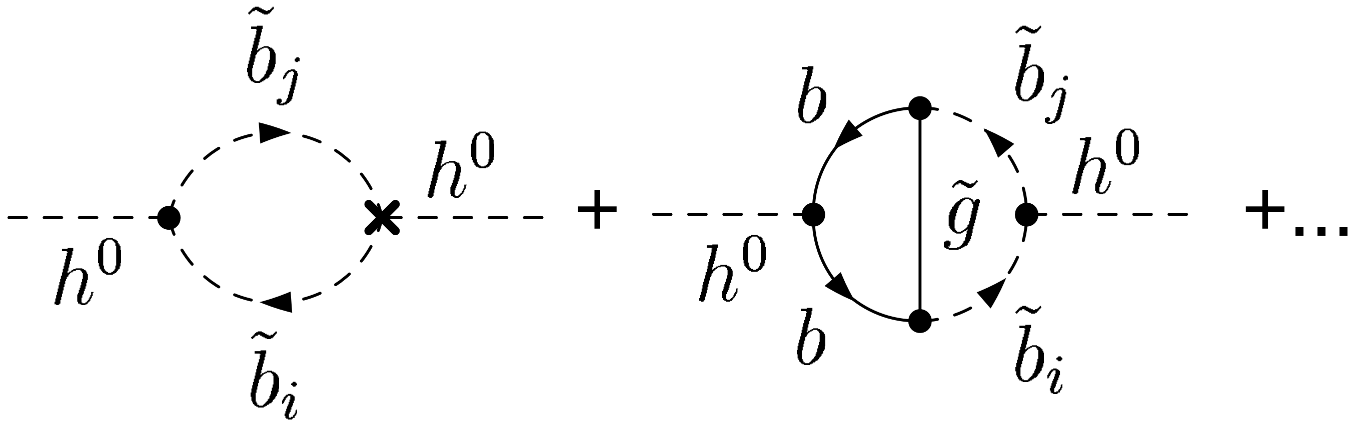
on **Born** level:



$$\hat{=} \quad \frac{i}{k^2 - M_{h^0}^2}$$

with **quantum** corrections:





$$\Rightarrow \frac{i}{k^2 - (M_{h^0} + \Delta M_{h^0})^2}$$

ΔM_{h^0} depends on the MSSM-parameters

M_{h^0} as precision observable

- **Discovery** of the Higgs-boson:

  accurate measurement & precise prediction
  of the mass:

  $\Rightarrow$ strong bounds on the MSSM-parameters,
  e.g. on A_t (A_t: SUSY-breaking parameter of the top squark sector)

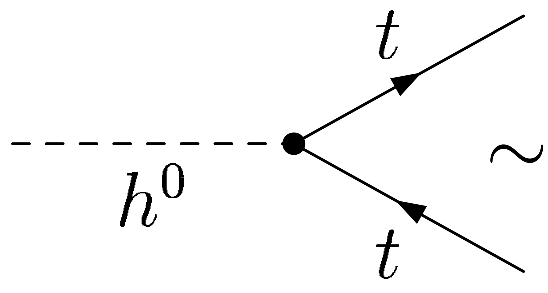
  at the LHC: $\Delta M_{h^0}^{\text{exp}} = 0.2$ GeV (LC: $\Delta M_{h^0}^{\text{exp}} = 0.05$ GeV)

  $\Rightarrow$ small theoretical uncertainty necessary
  (truncation of perturbation series $\Rightarrow$ theoretical uncertainty)

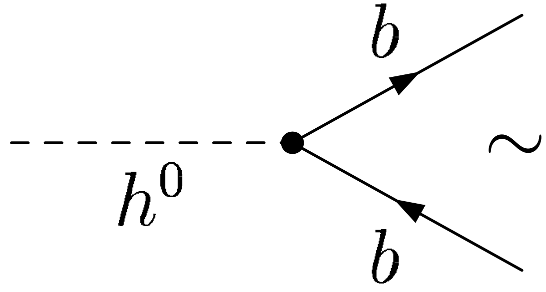
- **Before** the discovery:

  Exclusion of parts of the parameter space possible

ΔM_{h^0}: main contributions



$\sim \quad \lambda_t \sim \frac{m_t}{\sin\beta}$



$\sim \quad \lambda_b \sim \frac{m_b}{\cos\beta}$

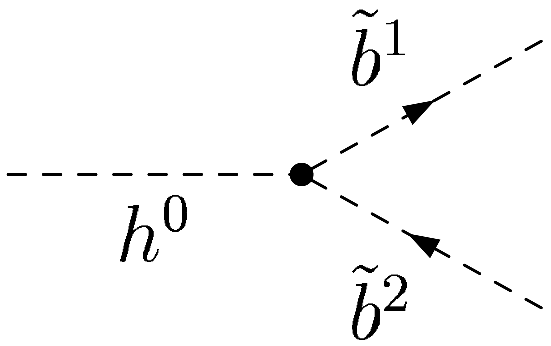
with: $$\frac{\lambda_b}{\lambda_t} = \frac{m_b}{m_t}\tan\beta$$

(λ: Yukawa coupling, $\alpha_t \sim \lambda_t^2$, $\alpha_b \sim \lambda_b^2$)

($\tan\beta = \frac{v_2}{v_1}$; v_1, v_2: Higgs vac. exp. values)

Large contribution: – from the top sector

– from the bottom sector for large $\tan\beta$



- Yukawa part:

α: mixing angle of h^0, H^0

A_b: SUSY-breaking parameter

μ: Higgsino mass term

$$\sim \lambda_b(A_b^* \sin\alpha + \mu\cos\alpha)$$

$\Rightarrow$ bottom-contribution **large** for μ and $\tan\beta$ large

Scheme dependence

Within the 2-loop calculation $\mathcal{O}(\alpha_s \alpha_{\{t,b\}})$:

- **parameters of the top/bottom sector** are defined at **one-loop**:

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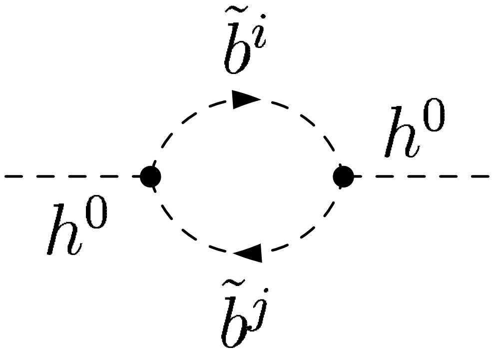
  Problem: Loop integrals are **UV-divergent**:

  Example:



$$\sim \int d^4k \frac{1}{((p+k)^2 - m^2_{\tilde{b}^i})(k^2 - m^2_{\tilde{b}^j})}$$

$\longrightarrow$ need a **regularisation scheme**, e.g. use a Cut-off parameter

New problem:

free parameters of the theory $\overset{\text{relation}}{\longleftrightarrow}$ observables

$\underbrace{\hspace{6em}}$ dependent on unphysical parameters

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  New problem:

  $$\underbrace{\text{free parameters of the theory} \overset{\text{relation}}{\longleftrightarrow} \text{observables}}_{\text{dependent on unphysical parameters}}$$

  $\longrightarrow$ need **renormalisation**

  Replace:

  $$\mathcal{M} \to Z_{\mathcal{M}} \mathcal{M} = \underbrace{\mathcal{M}}_{\text{finite}} + \underbrace{\delta\mathcal{M}^{(1)}}_{\substack{\text{dependence on} \\ \text{unphysical parameter:} \\ \textbf{one-loop order}}} + \underbrace{\delta\mathcal{M}^{(2)}}_{\substack{\text{dependence on} \\ \text{unphysical parameter:} \\ \textbf{two-loop order}}} + \ldots$$

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Counterterms ($\delta \ldots$) of input parameters:

- absorption of the dependence on the unphysical parameter ($\hat{=}$ divergences)
- freedom in the choice of finite part (but for perturbative calculations: finite part should be small **!**)

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  $\Rightarrow$ **information** about **size** of **missing higher order** contributions

  $\Rightarrow$ theoretical error estimate

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Here: top sector: only one scheme (masses/mixing angle on-shell)

bottom sector: 4 different schemes

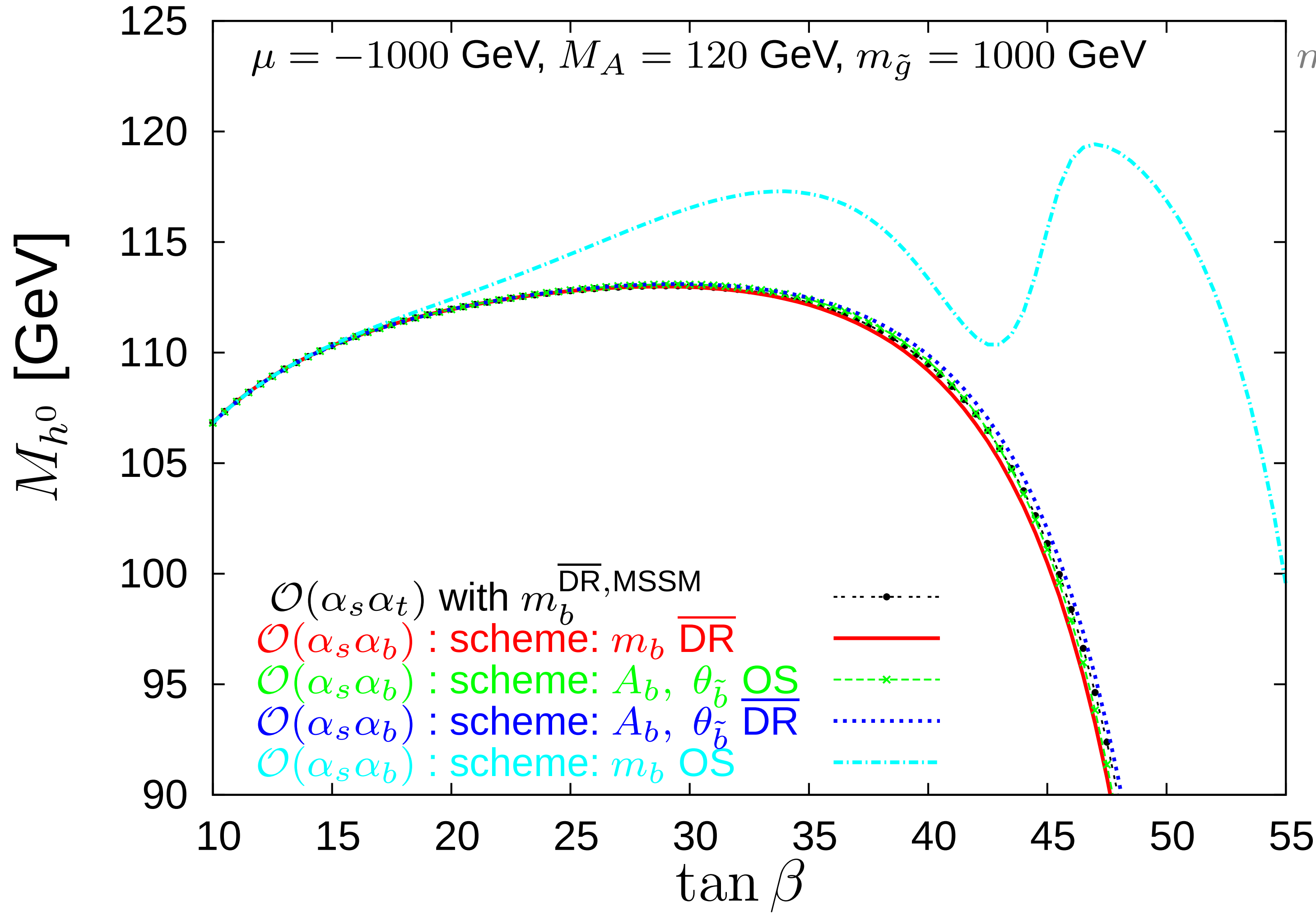
Different schemes

Bottom sector:

scheme	b-mass m_b	A_b	mixing angle $\theta_{\tilde{b}}$
m_b $\overline{\text{DR}}$	running ($\overline{\text{DR}}$)	running ($\overline{\text{DR}}$)	dep.
A_b, $\theta_{\tilde{b}}$ OS	dep.	on-shell	on-shell
A_b, $\theta_{\tilde{b}}$ $\overline{\text{DR}}$	dep.	running ($\overline{\text{DR}}$)	running ($\overline{\text{DR}}$)
m_b OS	on-shell	dep.	on-shell

analog. top sector

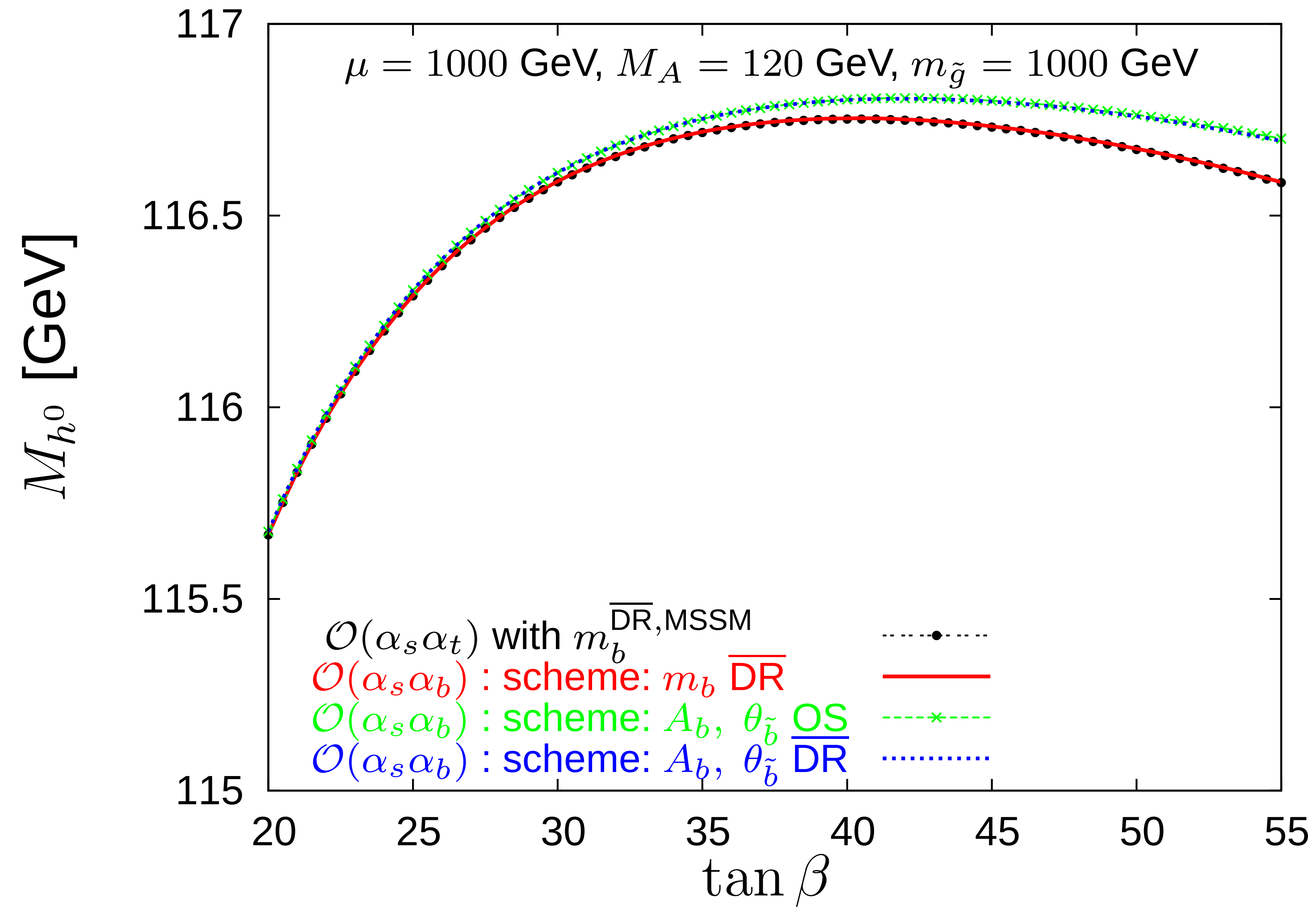
Results: $\tan\beta$-dependence (μ negative)



$m_{\tilde{g}}$: gluino mass

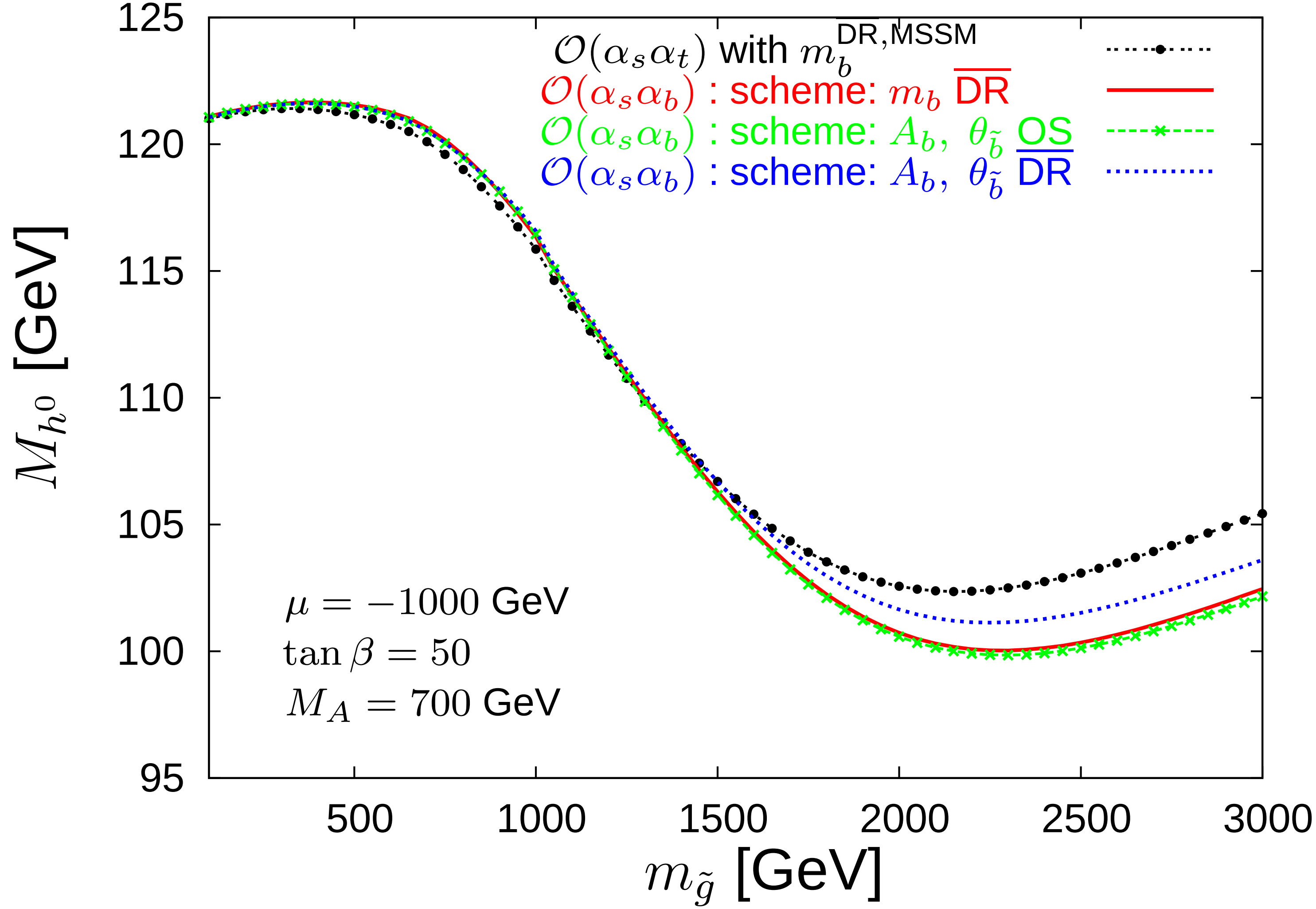
- scheme m_b OS: very large corrections, unpractical scheme
- other schemes: sizeable differences, up to $\mathcal{O}(1$ GeV$)$, for large $\tan\beta$

Results: $\tan\beta$-dependence (μ positive)



- tiny differences between schemes, max. $\mathcal{O}(0.1 \text{ GeV})$

Results: $m_{\tilde{g}}$-dependence



- subleading corrections up to $\mathcal{O}(3\ \text{GeV})$
- scheme differences of the order of $\mathcal{O}(2\ \text{GeV})$

Summary

- mass of the lightest MSSM-Higgs-boson M_{h^0}
  = interesting **precision observable**

- The **knowledge** of quantum corrections is **necessary** for **precise** theoretical predictions of M_{h^0}.

- bottom-quark/squark-corrections:
  - ⋆ relevant for **large** $\boldsymbol{\mu}$ and $\boldsymbol{\tan\beta}$
  - ⋆ subleading two-loop contributions of $\mathcal{O}(\alpha_s\alpha_b)$ can yield shifts up to 3 GeV.

- **first comparison** between different schemes for $\mathcal{O}(\alpha_s\alpha_b)$:
  - ⋆ for positive μ: corrections are under control
  - ⋆ for negative μ: differences between schemes are of the order of $\mathcal{O}(\pm 2$ GeV$)$