# Compactification of String/M-theory

## - How to get rid of 6 and 7 dimensions?

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- M-theory: $11 = 4 + 7$
- String Theory: $10 = 4 + 6$
  (see Talk of Florian)
- Outlook

# Motivation

`String/M-theory` has several features:

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- Strings/membranes are one/two dimensional objects
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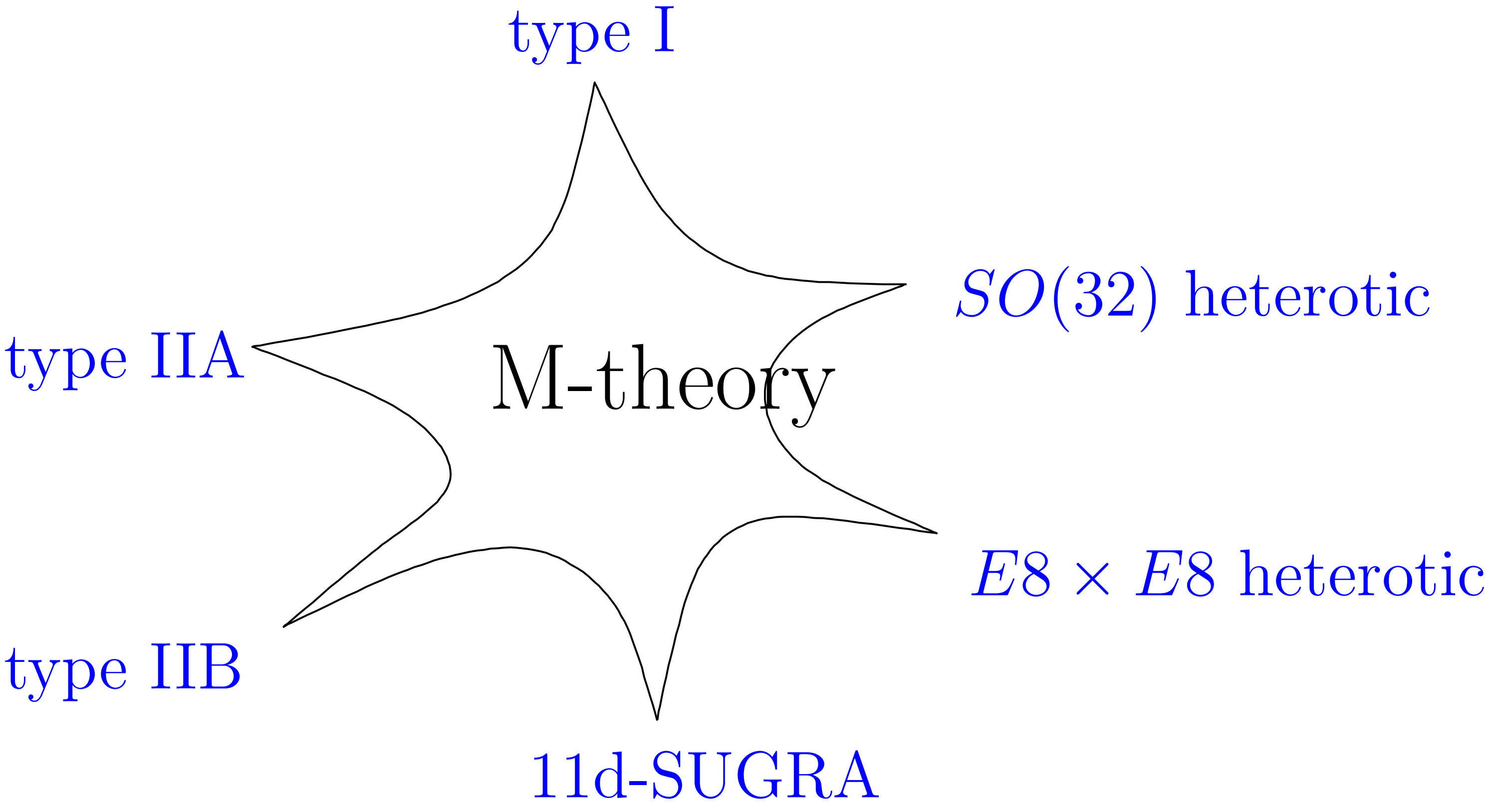
- Quantization leads to spectrum
- Usual particles appear as zero-modes
- Good news: graviton is included
- Bad news: must live in ten/eleven dimensions

# Limits of an unknown theory



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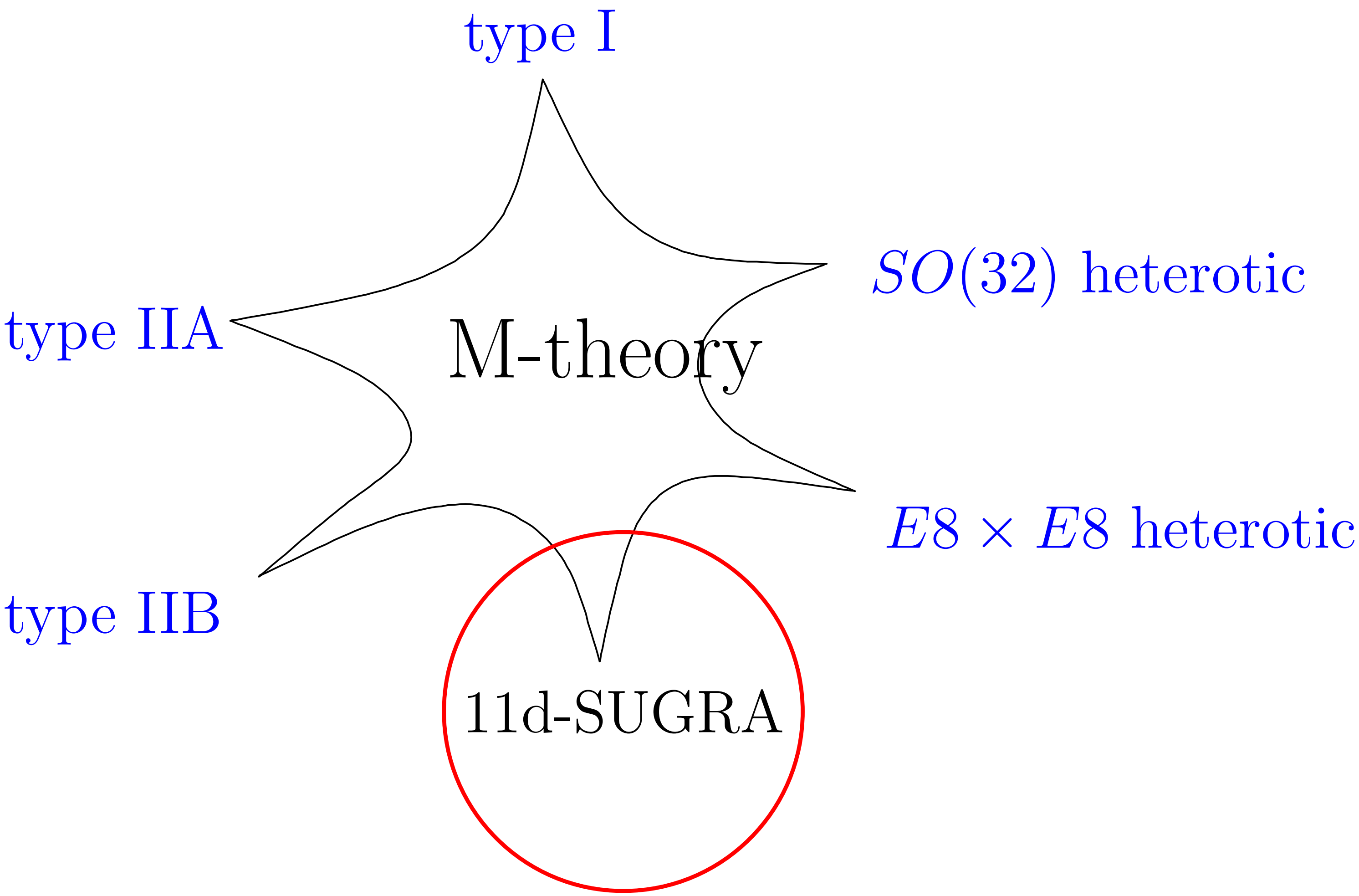

# Back to our good old four dimensions

Way out:

- Split dimensions: direct-sum
  - M-theory: $11 = 4 + 7$
  - String Theory: $10 = 4 + 6$

- Choose tiny, compact internal space

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- Vice versa: Can internal space dictate 4-dim physics?

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$\rightarrow$solve eqns $\equiv$ find vacuum manifold

# M-theory: $11 = 4 + 7$

if smallest length scale $\gg$ Planck length then

M-theory $\rightarrow d = 11$ SUGRA

Action for bosonic part:

$$S = \int \sqrt{g} R \cdot *1 - \frac{1}{2} F \wedge *F - \frac{1}{6} C \wedge F \wedge F$$

$g$ is metric, $R$ is scalar curvature

$C$ is 3-form potential, where locally $F = dC$

# SUSY-variations

fields: metric $g$, 3-form $C$, gravitino $\Psi_X$

$\longrightarrow$ calculate SUSY-variations

$\longrightarrow$ only $\delta_\varepsilon \Psi_X$ is of interest (vacuum)

$$\delta_\varepsilon \Psi_X = \nabla^S_X \cdot \varepsilon = 0$$

$$\nabla^S_X \cdot \varepsilon \stackrel{!}{=} \nabla^{LC}_X \cdot \varepsilon + \frac{1}{144}\Big( X \lrcorner F - 8\, X \wedge F \Big) \cdot \varepsilon$$

$\varepsilon$: SUSY-parameter (spinor in 11d)

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Bad news: no unique solution

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- Make Ansatz: $\mathbb{R}^{1,3} \times M^7$
  and $F$ is only defined on $M^7$

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- $F$ is only defined on $M^7$
- SUSY (vacuum) variation is zero

$$0 = \nabla_X^{LC} \cdot \varepsilon + \frac{1}{144}\Big( X \lrcorner F - 8\, X \wedge F \Big) \cdot \varepsilon$$

# Easy going with $F = 0$

- Via the splitting $\varepsilon = \chi \otimes \xi$ we obtain

$$0 = \nabla_g \varepsilon \; = \; \chi \; \otimes \; \nabla_{g(M^7)} \xi \, ,$$

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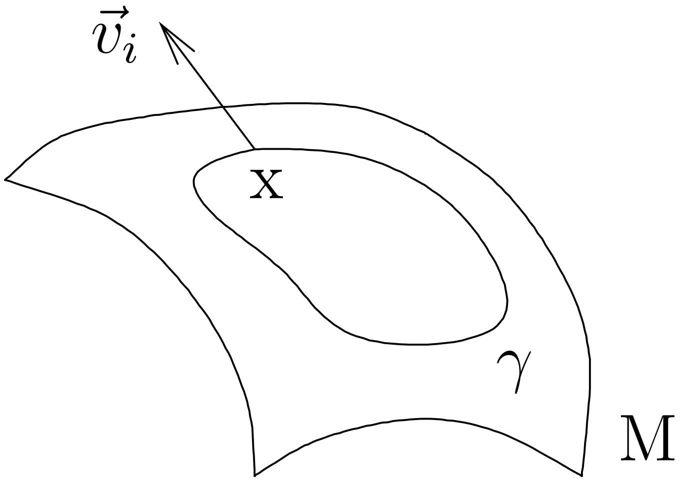
Equivalently: precisely one internal covariant constant spinor

$\longrightarrow$ Study holonomy theory

# Idea of holonomy

Let $(M, g)$ be a Riemannian manifold with metric $g$
$\rightarrow$ Levi-Civitá connection $\nabla^{LC}$ exists

Parallel transport of a vector via $\nabla^{LC}$:

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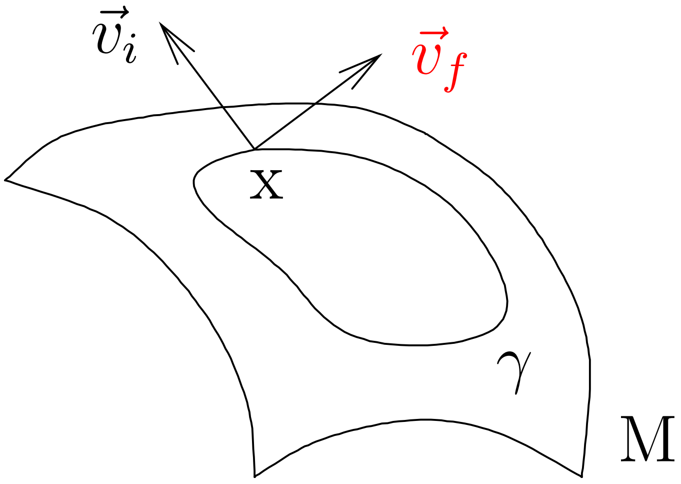


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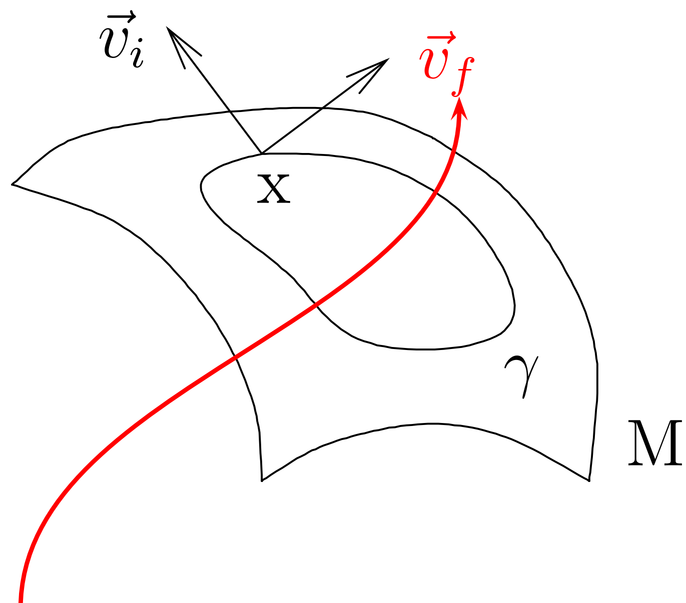


holonomy group $Hol$ measures difference
$\rightarrow$ group $Hol$ characterize manifold

Example: take flat space $\rightarrow$ $Hol$=identity

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A: group theory tells us $Hol = G_2$ does the job
since we have: $\Delta \rightarrow \mathbf{8} = \mathbf{1} + \mathbf{7}$

Precisely $\mathcal{N} = 1$ in $d = 4$ if $M^{11} = \mathbb{R}^{1,3} \times M^7$
and $M^7$ has $G_2$-holonomy

# More General: $F \neq 0$

- idea: interpret additional terms, e.g. $F$, by torsion $T$

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Solution: weak $G_2$-manifold
$A = const, \qquad F^4 = 0 \qquad T = m \in X_1 \qquad X_{2,3,4} = 0$

# Outlook

- Topology of $M^7$ tells us e.g. $\#$ of generations
- include singularities $\rightarrow$ YM-bundles, chiral fermions
- apply same procedure for string theories
- investigate duality transformation, e.g. mirror symmetry
- Topological sigma model (see Talk of Florian)

present work: string theory and generalized $G \times G$-structure