Four-fermion production at the $\gamma\gamma$ Collider

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Contents

Lowest-order predictions:

Calculation of $\gamma\gamma \to 4f$ und $\gamma\gamma \to 4f\gamma$ in lowest order

Construction of a Monte Carlo generator

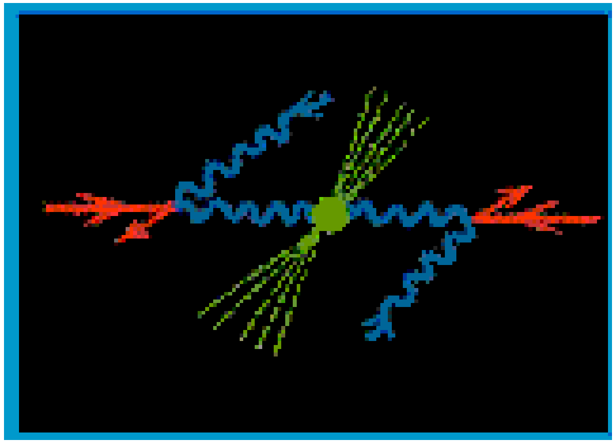
Anomalous couplings, Effective Higgs coupling,...

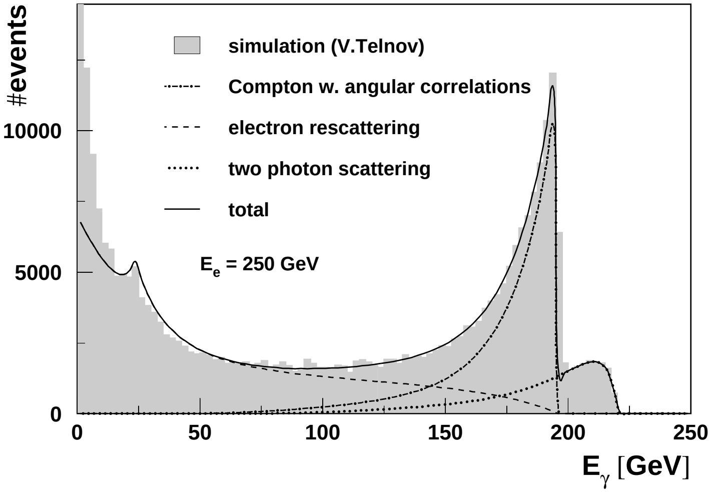
Radiative Corrections:

Methods for treating real corrections

$\gamma\gamma$ collider

- Option at the linear collider
- Photons are produced via Compton backscattering off electrons
- Energies and luminosities $\sim$ 80% of corresponding $\mathrm{e}^+\mathrm{e}^-$ collision
- however: γ energy distribution



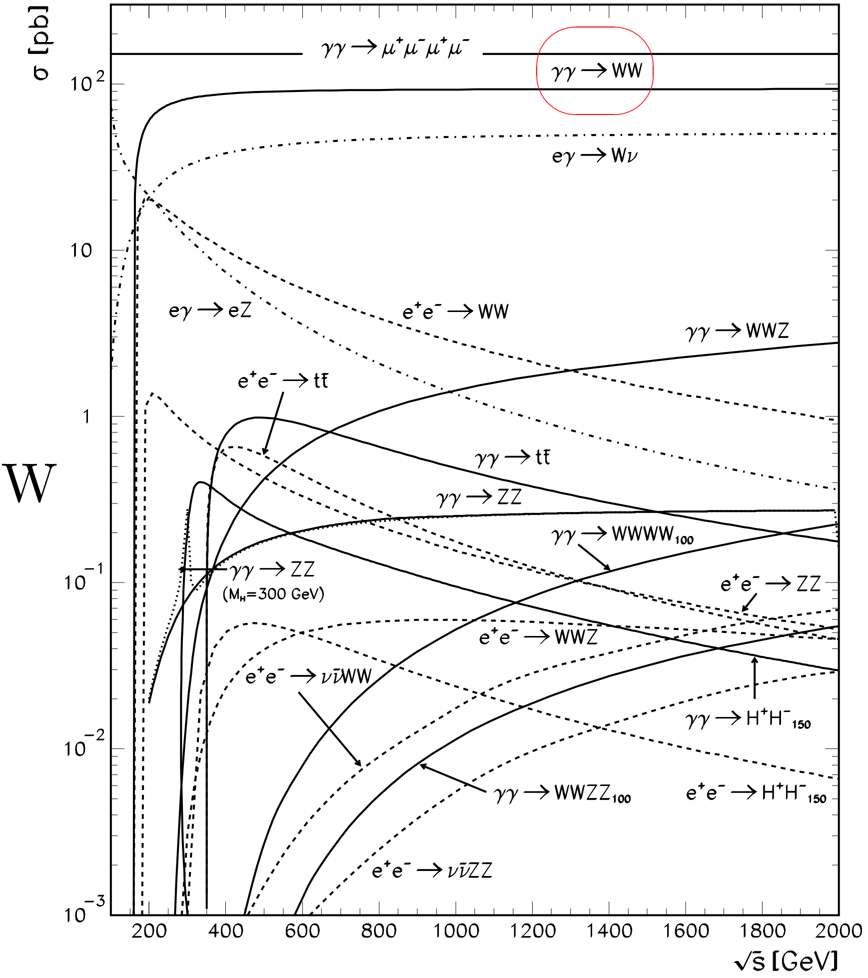


Motivation

$\gamma\gamma \to \mathrm{WW}$

- one of the largest cross sections
- contains gauge-boson couplings $\gamma\mathrm{WW}$ and $\gamma\gamma\mathrm{WW}$, insight into EW sector of SM, limits on anomalous couplings
- if $M_{\mathrm{H}} \gtrsim 160\,\mathrm{GeV} \Rightarrow \gamma\gamma \to \mathrm{H} \to \mathrm{WW}$ possible through loops of charged massive particles
- sensitive to extra dimensions

Baillargeon, et al. '95



Motivation

W bosons are unstable $\Rightarrow$ $\gamma\gamma \to \mathrm{WW} \to 4f$ (“W-pair signal diagrams”)

Experimental precision requires

- inclusion of single and non-resonant diagrams (“background diagrams”) in lowest order $\mathcal{O}(\Gamma_\mathrm{W}/M_\mathrm{W})$, $\mathcal{O}(\Gamma_\mathrm{W}/M_\mathrm{W})^2$
  $\Rightarrow$ $\gamma\gamma \to 4f$
- inclusion of radiative corrections $\mathcal{O}(\alpha) \sim \mathcal{O}(\Gamma_\mathrm{W}/M_\mathrm{W})$

$\gamma\gamma \to 4f\gamma$:

Building block for real corrections to $\gamma\gamma \to \mathrm{WW} \to 4f$

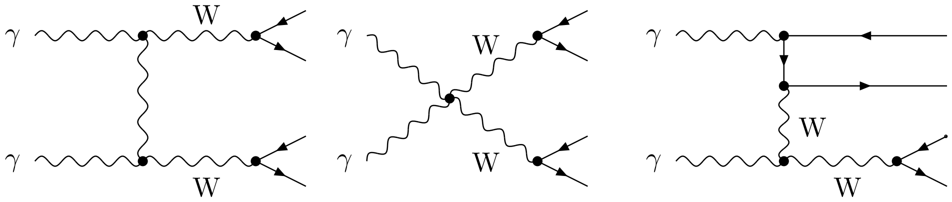
Existing studies:

$\gamma\gamma \to 4f$: Moretti ’96; Baillargeon et al. ’97; Boos, Ohl ’97

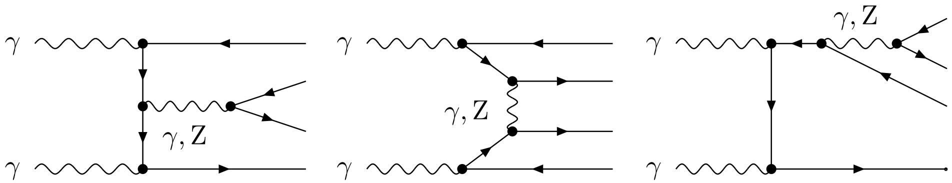
Amplitudes

- Helicity amplitudes
- Weyl–van-der-Waerden formalism
- fermion masses neglected

charged current (CC) (dominates)



neutral current (NC)

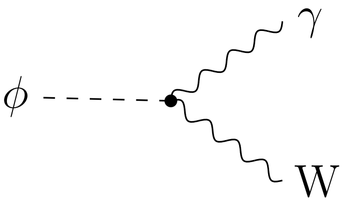


gluon-exchange diagrams

representative final states

final state	reaction type	$\gamma\gamma \to$
leptonic	CC	$\mathrm{e}^-\bar{\nu}_\mathrm{e}\nu_\mu\mu^+$
	NC(a)	$\mathrm{e}^-\mathrm{e}^+\nu_\mu\bar{\nu}_\mu$
		$\mathrm{e}^-\mathrm{e}^+\mu^-\mu^+$
	NC(b)	$\mathrm{e}^-\mathrm{e}^+\mathrm{e}^-\mathrm{e}^+$
	CC/NC	$\mathrm{e}^-\mathrm{e}^+\nu_\mathrm{e}\bar{\nu}_\mathrm{e}$
semi-leptonic	CC(c)	$\mathrm{e}^-\bar{\nu}_\mathrm{e}\mathrm{u}\bar{\mathrm{d}}$
	NC(a)	$\nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{u}\bar{\mathrm{u}}$
		$\nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{d}\bar{\mathrm{d}}$
		$\mathrm{e}^-\mathrm{e}^+\mathrm{u}\bar{\mathrm{u}}$
		$\mathrm{e}^-\mathrm{e}^+\mathrm{d}\bar{\mathrm{d}}$
hadronic	CC	$\mathrm{u}\bar{\mathrm{d}}\mathrm{s}\bar{\mathrm{c}}$
	NC(a)	$\mathrm{u}\bar{\mathrm{u}}\mathrm{c}\bar{\mathrm{c}}$
	NC(a)	$\mathrm{u}\bar{\mathrm{u}}\mathrm{s}\bar{\mathrm{s}}$
	NC(a)	$\mathrm{d}\bar{\mathrm{d}}\mathrm{s}\bar{\mathrm{s}}$
	NC(b)	$\mathrm{u}\bar{\mathrm{u}}\mathrm{u}\bar{\mathrm{u}}$
	NC(b)	$\mathrm{d}\bar{\mathrm{d}}\mathrm{d}\bar{\mathrm{d}}$
	CC/NC	$\mathrm{u}\bar{\mathrm{u}}\mathrm{d}\bar{\mathrm{d}}$

Amplitudes

non-linear gauge: 

vanishes

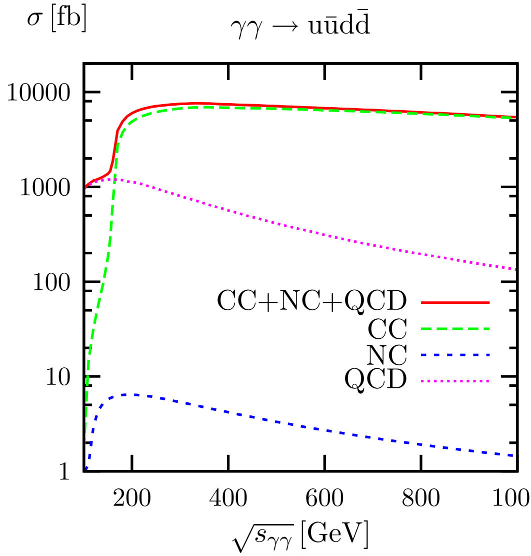
diagrams: 6 ($e^-e^+\nu_\mu\bar{\nu}_\mu$) to 588 ($u\bar{u}d\bar{d}+\gamma$)

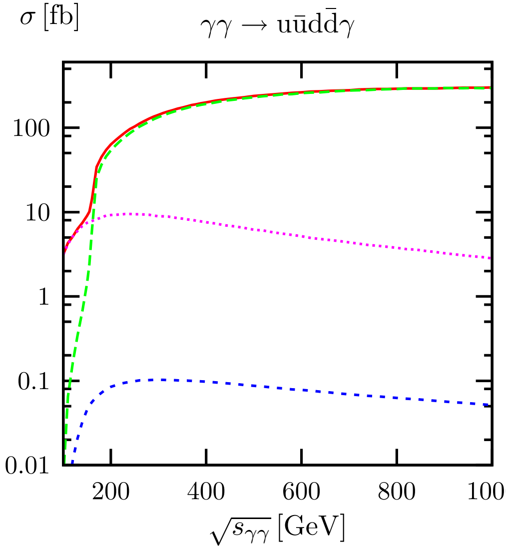
$\gamma\gamma \to 4f$: calculation with general
gauge spinor of γ, drops out in the end

use of discrete symmetries
$\to$ only 2 independent helicity ampl. for $\gamma\gamma \to 4f$

gluon diagrams similar to NC diagrams,
add colour structure

check against Madgraph (Stelzer, Long '94)





Phase-space integration

Problem: rich peaking structure of integrand

RacoonWW
Denner,Dittmaier,Roth,Wackeroth '01

"importance sampling" : more points near peaks

$$\int \underbrace{dx}_{\substack{\downarrow \\ \text{random numbers}}} f(x) = \int dx\, g(x) \frac{f(x)}{g(x)} = \int dy\, \underbrace{\frac{f(x(y))}{g(x(y))}}_{\text{"weight"} \sim \mathrm{const}}$$

$$\int_0^x d\bar{x}\, g(\bar{x}) = y(x)$$: "mapping" → integrand flattened

many Feynman diagrams/propagators → "multi-channel"

one phase-space generator per diagram with appropriate "mapping"

Photon spectrum (CompAZ (Zarnecki '02; Telnov '95; Chen et al. '95)):

$$\mathrm{d}\sigma = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, f(x_1) f(x_2) \mathrm{d}\sigma(x_1 P_1, x_2 P_2)$$

"stratified sampling" + adaptive optimization

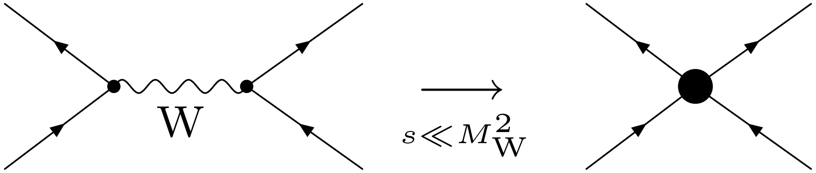
Comparison with Whizard&Madgraph (Kilian '01; Stelzer, Long '94) → good agreement

Anomalous Couplings

General approach in search for new physics:

Effective Lagrangian: low energy limit of theory beyond SM, deviation of SM Lagrangian in terms of new operators

cf. Fermi theory



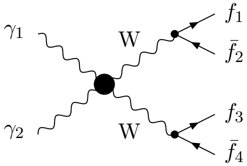
Example: triple couplings induced via dimension-6 operators

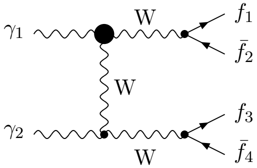
assumption: symmetries of SM are respected

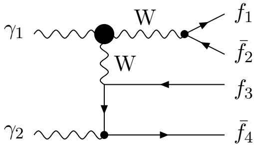
$$\mathcal{L}^{\rm ATGC}_{\rm CC} = {\rm i}g_1 \frac{\alpha_{{\rm B}\phi}}{M_{\rm W}^2} (D_\mu\Phi)^\dagger B^{\mu\nu}(D_\nu\Phi) - {\rm i}g_2 \frac{\alpha_{{\rm W}\phi}}{M_{\rm W}^2} (D_\mu\Phi)^\dagger \boldsymbol{\sigma}\cdot\mathbf{W}^{\mu\nu}(D_\nu\Phi)$$
$$-g_2 \frac{\alpha_{\rm W}}{6M_{\rm W}^2} \mathbf{W}^{\mu}{}_{\nu}\cdot(\mathbf{W}^{\nu}{}_{\rho}\times\mathbf{W}^{\rho}{}_{\mu}),$$

→ γWW (and related $\gamma\gamma$WW)

coefficients are related to $\Delta\kappa_\gamma$ and λ_γ (LEP2)



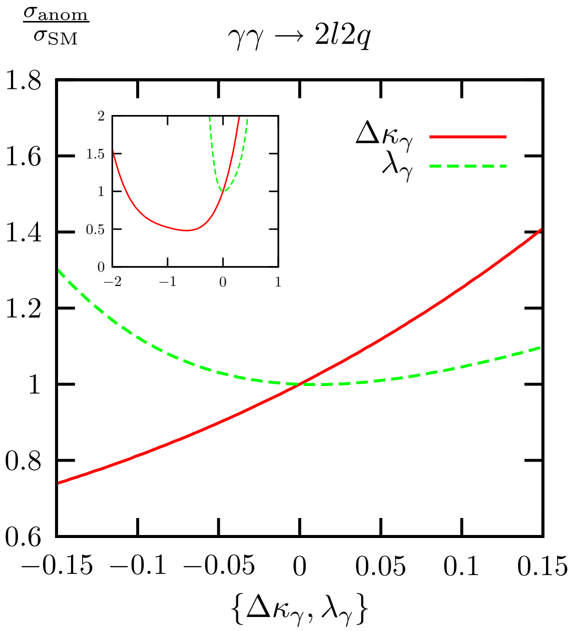


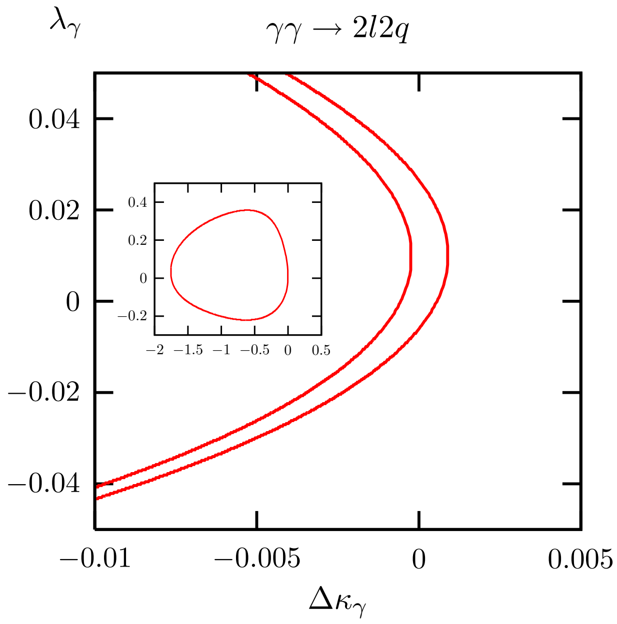


Anomalous triple couplings

$\gamma\gamma \to 4f$ all semi-leptonic final states photon spectrum included

$\sqrt{s_{\mathrm{ee}}} = 500\,\mathrm{GeV}$ $\int L dt = 100\,\mathrm{fb}^{-1}$ $\chi^2 = 1$ $\chi^2 \equiv \frac{(N(a_i) - N_{\mathrm{SM}})^2}{N_{\mathrm{SM}}}$





$\longrightarrow$ large interference with SM amplitude

expected limits comparable to $\mathrm{e^+e^-}$-mode (see also Baillargeon et al. '97; Bozovic-Jelisavcic et al. '02)

full study requires consideration of distributions

Anomalous quartic couplings

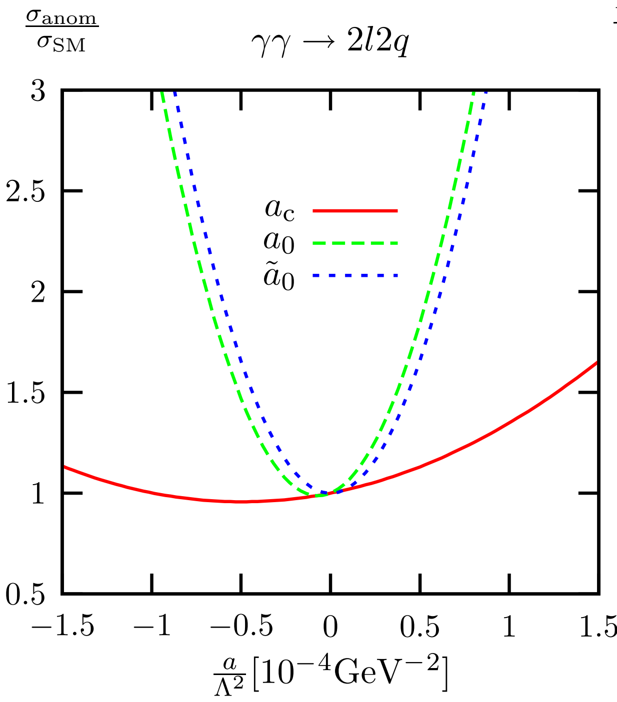
Assumption: CP, $\mathrm{U}(1)_{\mathrm{em}}$, $\mathrm{SU}(2)_{\mathrm{cust}}$

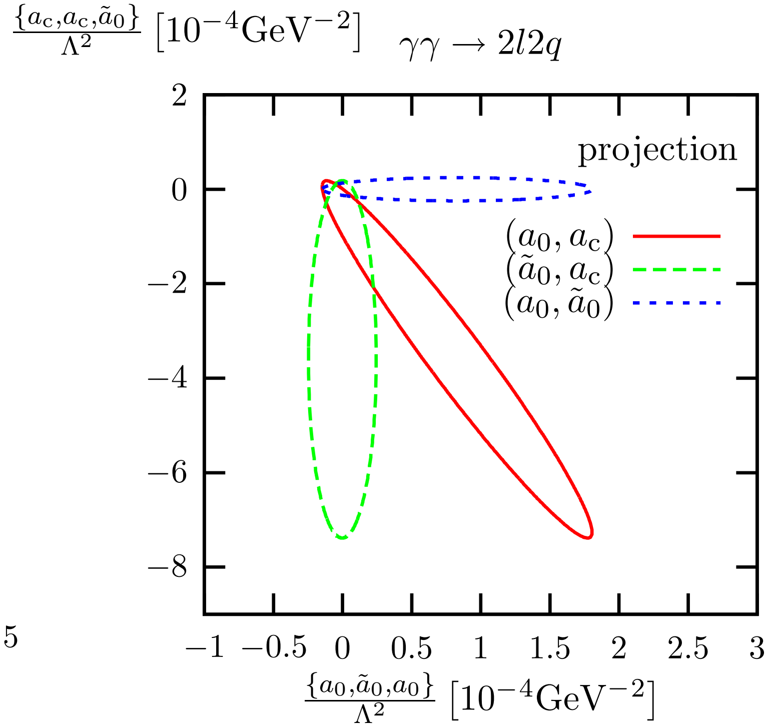
Bélanger,Boudjema '92; Abu Leil,Stirling '95;
Stirling,Werthenbach '00; Denner,Dittmaier,Roth,Wackeroth '01

$$\mathcal{L}_{\mathrm{anom}} = -\frac{e^2}{16\Lambda^2}\left(a_0\, F^{\mu\nu} F_{\mu\nu} \overline{\mathbf{W}}_\alpha \overline{\mathbf{W}}^\alpha + a_c\, F^{\mu\alpha} F_{\mu\beta} \overline{\mathbf{W}}^\beta \overline{\mathbf{W}}_\alpha + \tilde{a}_0\, F^{\mu\nu} \tilde{F}_{\mu\nu} \overline{\mathbf{W}}_\alpha \overline{\mathbf{W}}^\alpha \right)$$

$$\overline{\mathbf{W}}_\mu = \left(\overline{W}^1_\mu, \overline{W}^2_\mu, \overline{W}^3_\mu\right) = \left(\frac{1}{\sqrt{2}}(W^+ + W^-)_\mu, \frac{\mathrm{i}}{\sqrt{2}}(W^+ - W^-)_\mu, \frac{1}{c_{\mathrm{W}}} Z_\mu\right)$$

$\rightarrow$ $\gamma\gamma\mathrm{WW}$ and $\gamma\gamma\mathrm{ZZ}$



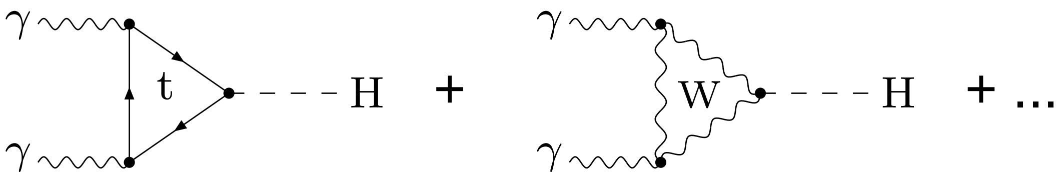


better than in

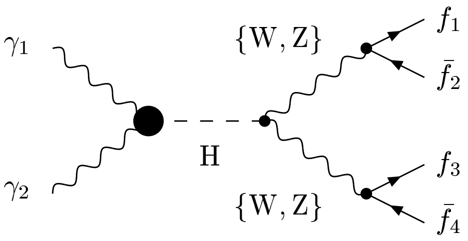
$\mathrm{e}^+\mathrm{e}^- \to 4f\gamma$

Effective Higgs coupling

In the SM through radiative corrections



$$\mathcal{L}_{\gamma\gamma\mathrm{H}} = -\frac{g_{\gamma\gamma\mathrm{H}}}{4} F^{\mu\nu} F_{\mu\nu} \frac{H}{v}$$

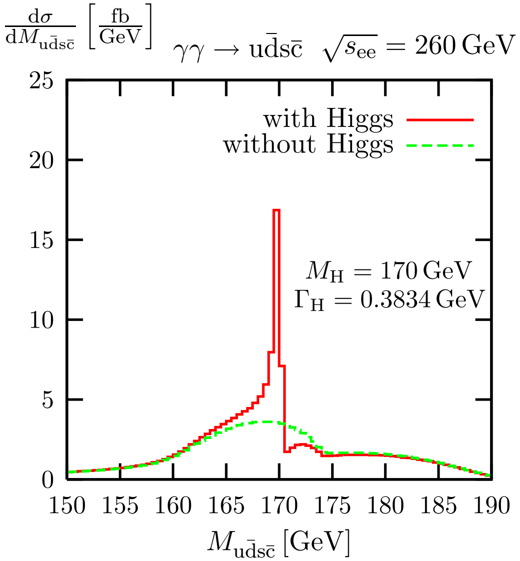


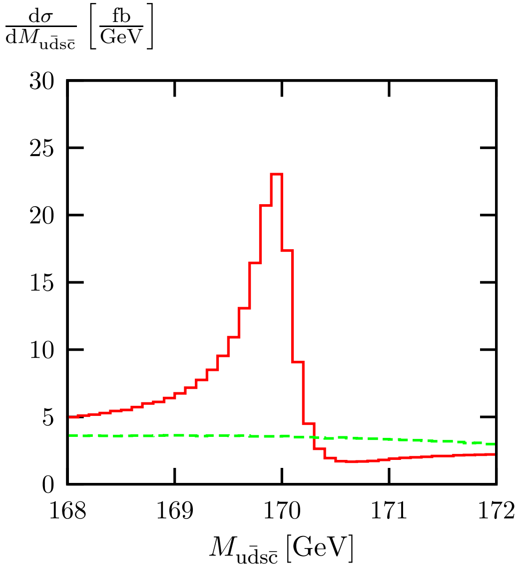
Calculate $g_{\gamma\gamma\mathrm{H}}$ by matching to

SM radiative corrections at $s = M_\mathrm{H}^2$

Motivation: enhanced at $s \sim M_\mathrm{H}^2$,

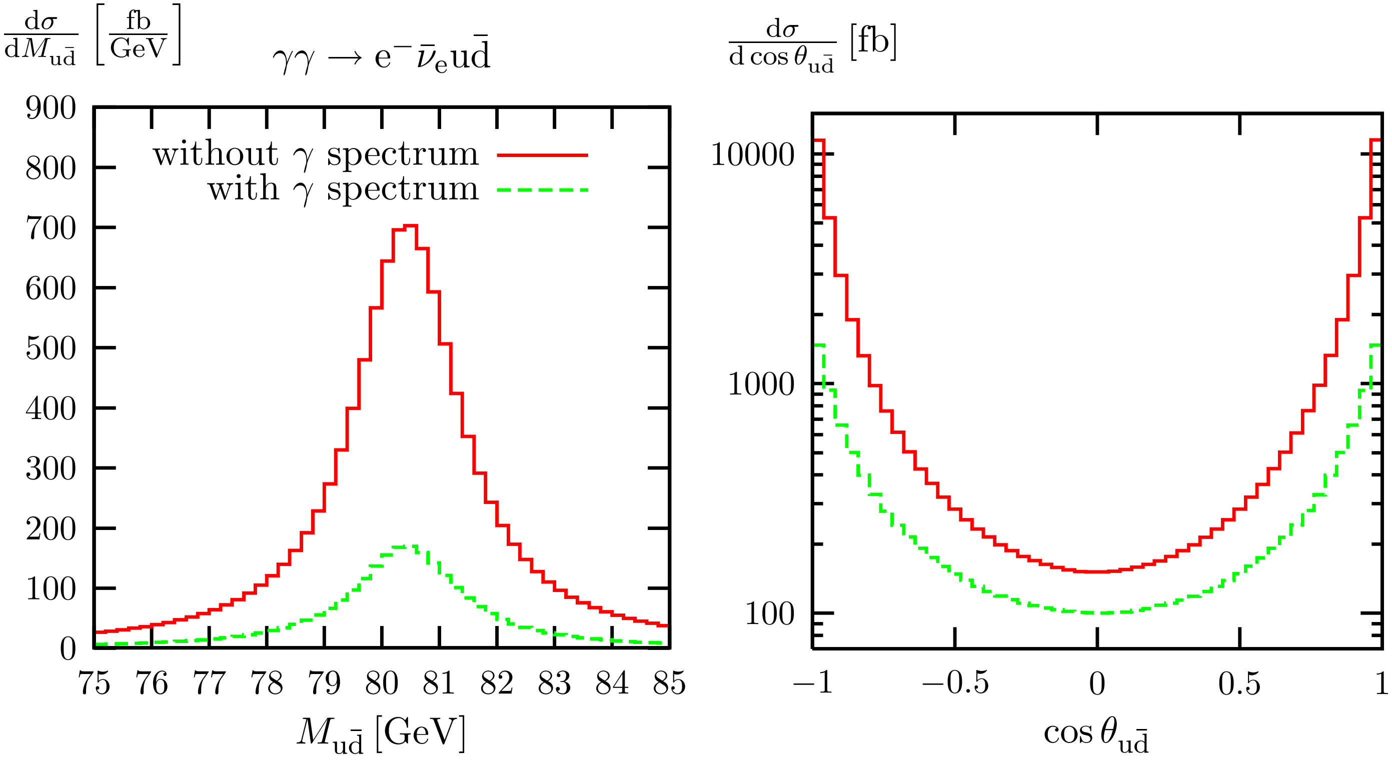
result gauge invariant





Distributions

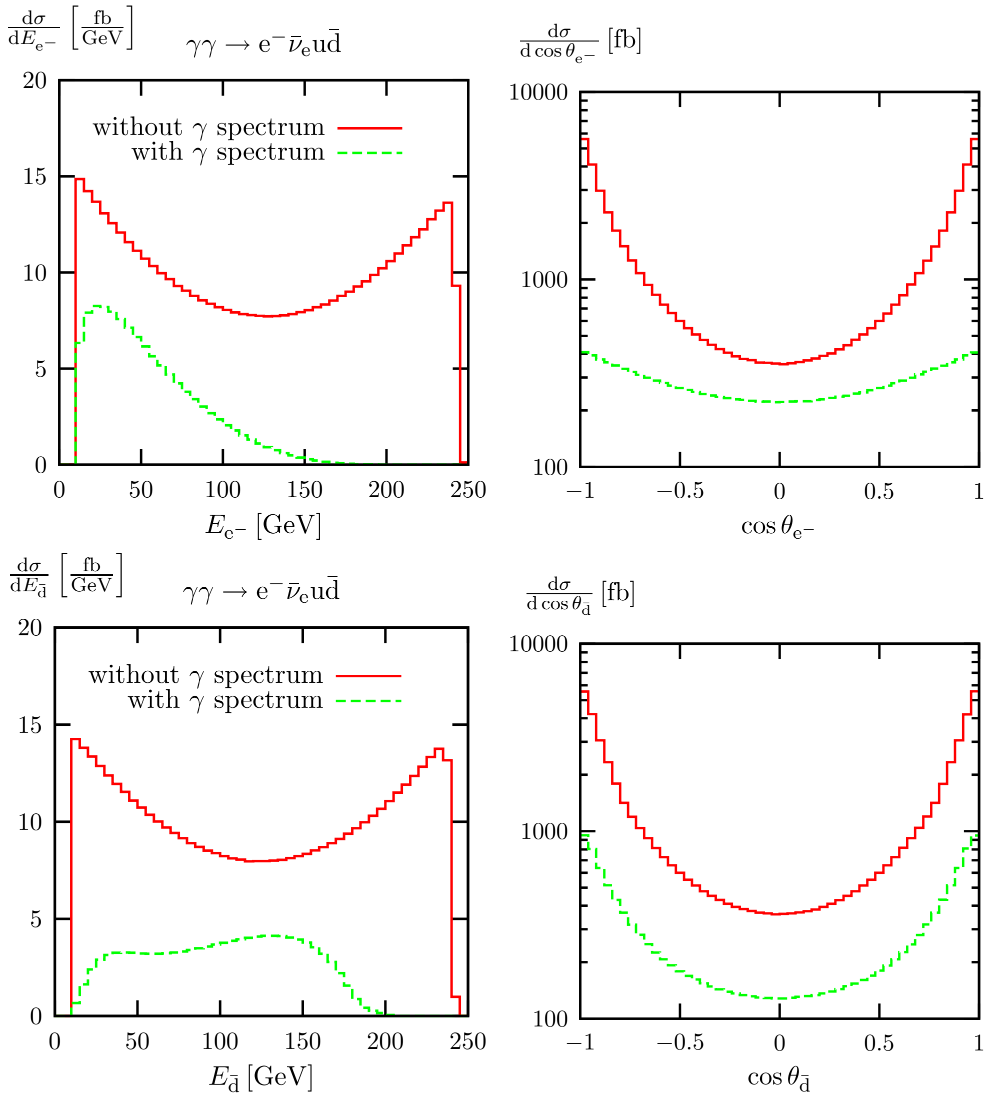
Invariant mass and production angle of the W^+ boson in $\gamma\gamma \to \mathrm{e}^-\bar{\nu}_\mathrm{e}\mathrm{u}\bar{\mathrm{d}}$



Distributions

Energy and production angle of e^- and $\bar{\mathrm{d}}$ in $\gamma\gamma \to \mathrm{e}^-\bar{\nu}_\mathrm{e}\mathrm{u}\bar{\mathrm{d}}$

Convolution over photon spectrum changes energy and angular distributions due to the effective polarisation of the $\gamma\gamma$ system



Finite gauge-boson width

fixed width: $P_V(p^2) = \frac{1}{p^2 - M^2 + iM\Gamma}$ $U(1), \cancel{SU(2)}$

step width: $P_V(p^2) = \frac{1}{p^2 - M^2 + iM\Gamma\theta(p^2)}$ $\cancel{U(1)}, \cancel{SU(2)}$

running width: $P_V(p^2) = \frac{1}{p^2 - M^2 + iM\Gamma\frac{p^2}{M^2}\theta(p^2)}$ $\cancel{U(1)}, \cancel{SU(2)}$

complex-mass scheme: $M^2 \to M^2 - iM\Gamma$ (e.g. in $\cos\theta_W = \frac{M_W}{M_Z}$) gauge invariant

Denner,Dittmaier,Roth,Wackeroth et al. ’99

For $\gamma\gamma \to 4f(\gamma)$ (massless fermions and non-linear gauge)

fixed width equivalent to complex-mass scheme

$\sigma(\gamma\gamma \to \mathrm{e}^-\bar{\nu}_\mathrm{e}\nu_\mu\mu^+\gamma)$					
$\sqrt{s_{\gamma\gamma}}$ [GeV]	500	800	1000	2000	10000
fixed width	39.230(45)	47.740(73)	49.781(91)	43.98(18)	4.32(23)
step width	39.253(45)	47.781(73)	49.881(96)	44.01(18)	4.31(24)
running width	39.251(49)	47.781(74)	49.898(95)	44.48(22)	10.83(28)
complex mass	39.221(45)	47.730(73)	49.770(91)	43.97(18)	4.31(23)

Double-pole approximation

Naive W-pair signal:

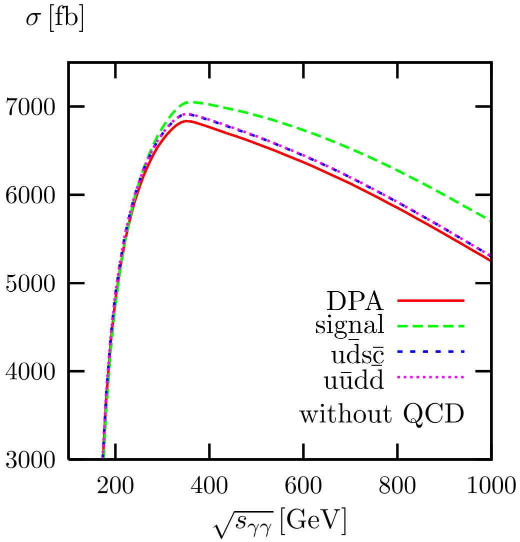
only diagrams with two resonant
W propagators (not gauge invariant)
not sufficient

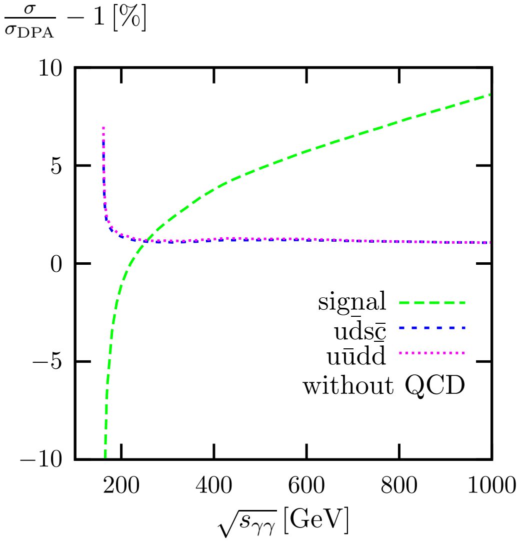
DPA = signal + “on-shell projection”

gauge invariant
uncertainty of $\mathcal{O}(\Gamma_\mathrm{W}/M_\mathrm{W}) \sim 1 - 3\%$
breakdown at WW threshold

→ promising approach:

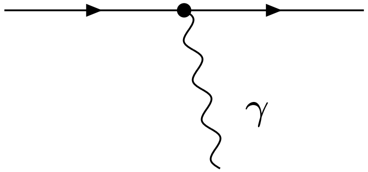
radiative corrections in DPA
uncertainty of $\mathcal{O}(\alpha/\pi \times \Gamma_\mathrm{W}/M_\mathrm{W}) \sim 0.1 - 0.5\%$



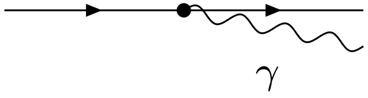


Real corrections

IR Divergences: soft collinear



$\sim \log \frac{E}{E_{\gamma,min}}$



$\sim \log \frac{E}{m_f}$

Remember: $m_f \to 0$

Hard photon: detector cuts (minimal energy and minimal angle)

real corrections: Bloch-Nordsieck theorem: cancellation together with virtual corrections

However: Problem in numerical integration, regularization needed, large cancellations

$$\sigma \quad = \quad \int \mathrm{d}\phi_{4f}\, |\mathcal{M}_{\mathrm{virt}}|^2 \quad + \quad \int \mathrm{d}\phi_{4f\gamma}\, |\mathcal{M}_{\mathrm{real}}|^2$$

Solution: Treat singular regions analytically

Two approaches: Phase-space slicing and subtraction

Phase-space slicing

Procedure:

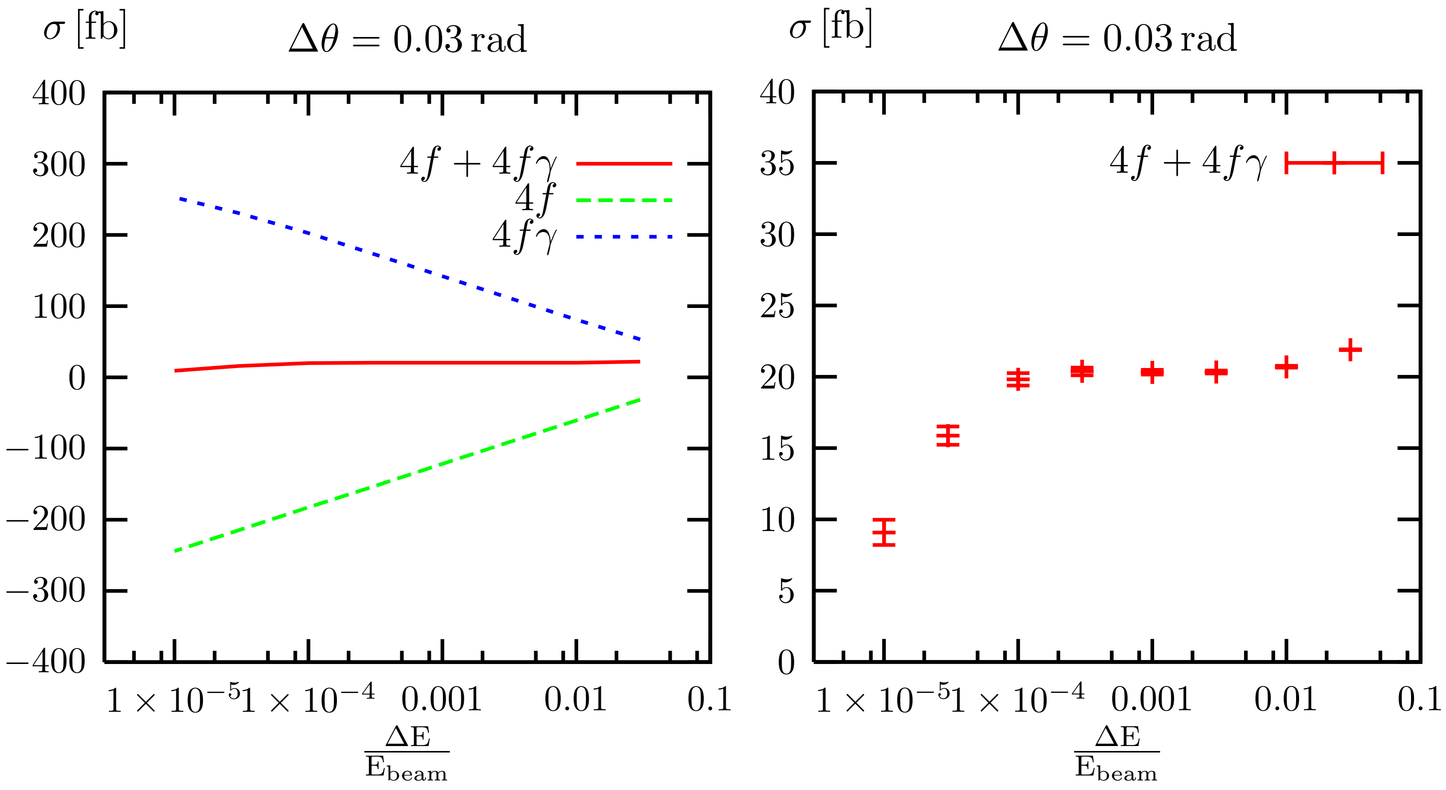
- Decompose phase space ($\phi_{4f\gamma}$) into singular and non-singular regions, singular region: $\mathrm{E}_\gamma < \Delta\mathrm{E}$ or $\theta(\gamma, f_i) < \Delta\theta$ dependence on cut-off parameters $\Delta\mathrm{E}, \Delta\theta$
$$\mathrm{d}\phi_{4f\gamma}\,|\mathcal{M}_{\mathrm{real}}|^2 \quad = \quad \mathrm{d}\phi_{4f\gamma}^{\mathrm{sing}}\,|\mathcal{M}_{\mathrm{real,sing}}|^2 + \mathrm{d}\phi_{4f\gamma}^{\mathrm{finite}}\,|\mathcal{M}_{\mathrm{real,finite}}|^2$$
- In soft and collinear limit factorization:
$$\mathrm{d}\phi_{4f\gamma}^{\mathrm{sing}} = \mathrm{d}\phi_{4f}\mathrm{d}\phi_\gamma, \qquad |\mathcal{M}_{\mathrm{real,sing}}|^2 = |\mathcal{M}_{\mathrm{Born}}^{4f}|^2 f(k),$$
integrate $\mathrm{d}\phi_\gamma f(k)$ over photon momentum k analytically,

$4f$ part with no or invisible photon: $\int \mathrm{d}\phi_{4f}\,\left(|\mathcal{M}_{\mathrm{virt}}|^2 + \int \mathrm{d}\phi_\gamma |\mathcal{M}_{\mathrm{real,sing}}|^2\right)$

$4f\gamma$ part with visible photon: $\int \mathrm{d}\phi_{4f\gamma}\,|\mathcal{M}_{\mathrm{real,finite}}|^2$

Slicing

$$\gamma\gamma \to \nu_e e^+ \mu^- \bar{\nu}_\mu, \quad E_{\text{beam}} = 250\,\text{GeV}$$



similar for the angular cut $\Delta\theta$

Subtraction

Basic idea: add and subtract the same quantity

$$\mathrm{d}\phi_{4f}|\mathcal{M}_{\mathrm{virt}}|^2 + \mathrm{d}\phi_{4f\gamma}|\mathcal{M}_{\mathrm{sub}}|^2 + \mathrm{d}\phi_{4f\gamma}\,|\mathcal{M}_{\mathrm{real}}|^2 - \mathrm{d}\phi_{4f\gamma}|\mathcal{M}_{\mathrm{sub}}|^2$$

$|\mathcal{M}_{\mathrm{sub}}|^2 \sim |\mathcal{M}_{\mathrm{real}}|^2$ for $k \to 0$ or $q_i k \to 0$

$\to \int \mathrm{d}\phi_{4f\gamma}\left(|\mathcal{M}_{\mathrm{real}}|^2 - |\mathcal{M}_{\mathrm{sub}}|^2\right)$ is finite

define mapping $\tilde{\phi}_{4f} \to \phi_{4f\gamma}$ such that

$$q_i \underset{k\to 0}{\widetilde{\qquad}} \tilde{q}_i, \qquad q_i + k \underset{kq_i\to 0}{\widetilde{\qquad}} \tilde{q}_i, \qquad q_j \underset{kq_i\to 0}{\widetilde{\qquad}} \tilde{q}_i$$

evaluate process independent function $\int \mathrm{d}\phi_\gamma|\mathcal{M}_{\mathrm{sub}}|^2$

$\to \int \mathrm{d}\phi_{4f}\left(|\mathcal{M}_{\mathrm{virt}}|^2 + \int \mathrm{d}\phi_\gamma|\mathcal{M}_{\mathrm{sub}}|^2\right)$ is finite

In general: statistical uncertainty smaller than with slicing,
construction more involved, but can be done once and for all

Explicit algorithm/method: dipole subtraction Catani,Seymour ’96; Dittmaier ’99; Roth ’99

Summary

- Relevance of $\gamma\gamma \to \mathrm{WW}$ due to its high cross section
- Monte Carlo generator with multi-channel Monte Carlo integration
  - Calculation of Born amplitudes for $\gamma\gamma \to 4f$ and $\gamma\gamma \to 4f\gamma$
  - Inclusion of a realistic photon spectrum
  - Anomalous couplings, Higgs resonance
- Work in progress
  - Double-pole approximation is a promising approach for radiative corrections (cf. RacoonWW)
  - Slicing / Subtraction: two approaches for dealing with soft and collinear divergencies