The Statistics of Supersymmetric D-brane Models

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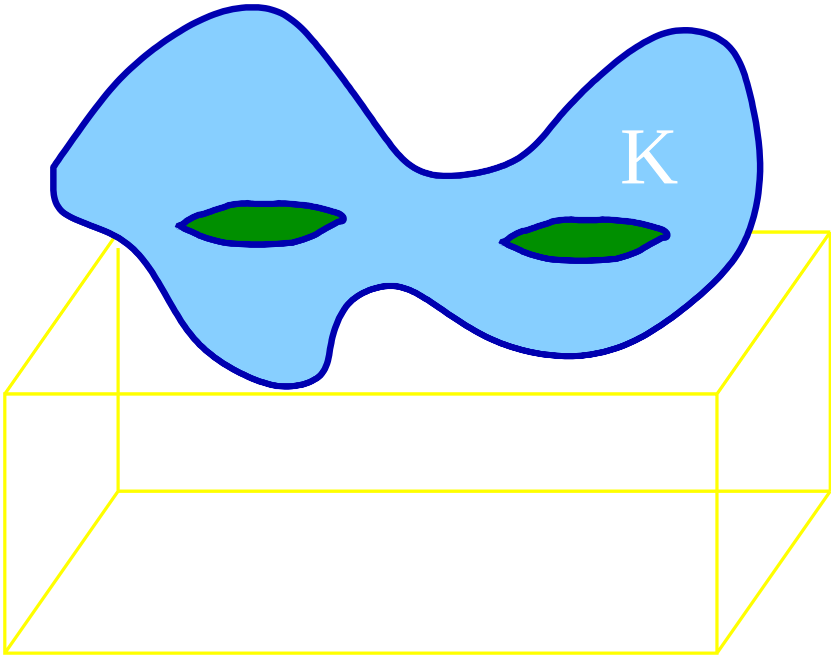
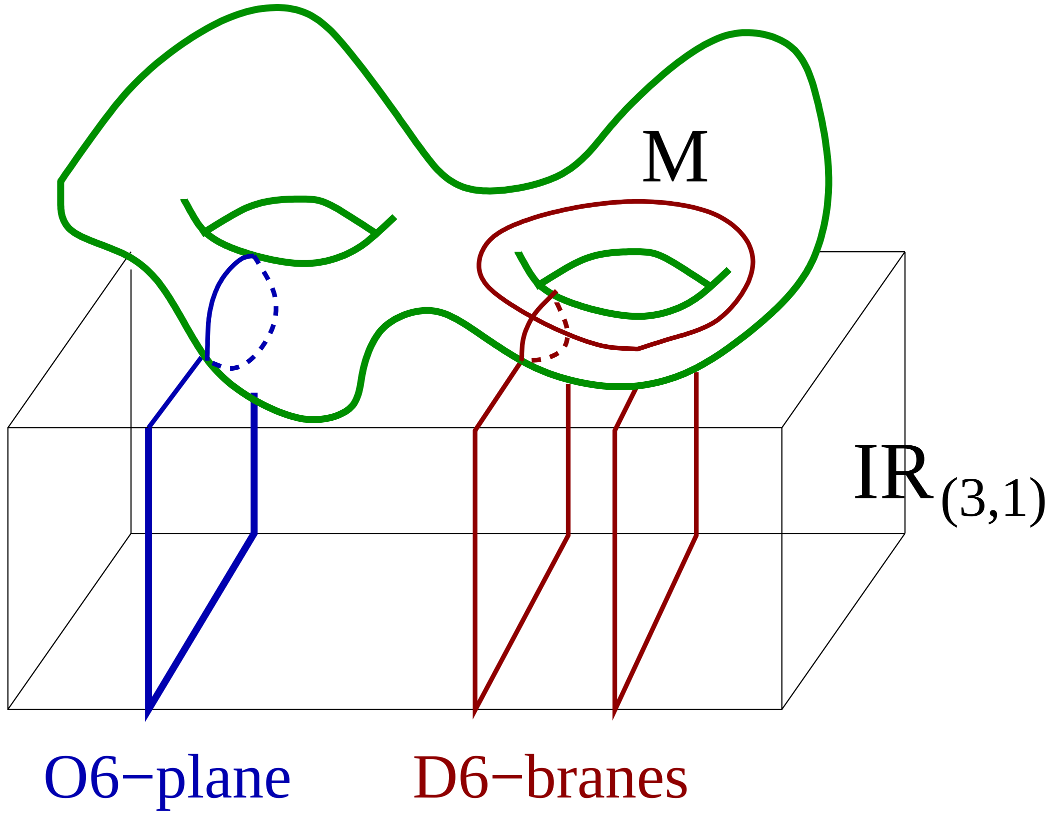
I. Introduction

- String theory is a 10 dimensional finite quantum theory of gravity
- To make contact with low energy physics one has to compactify string theory to 4 dimensions



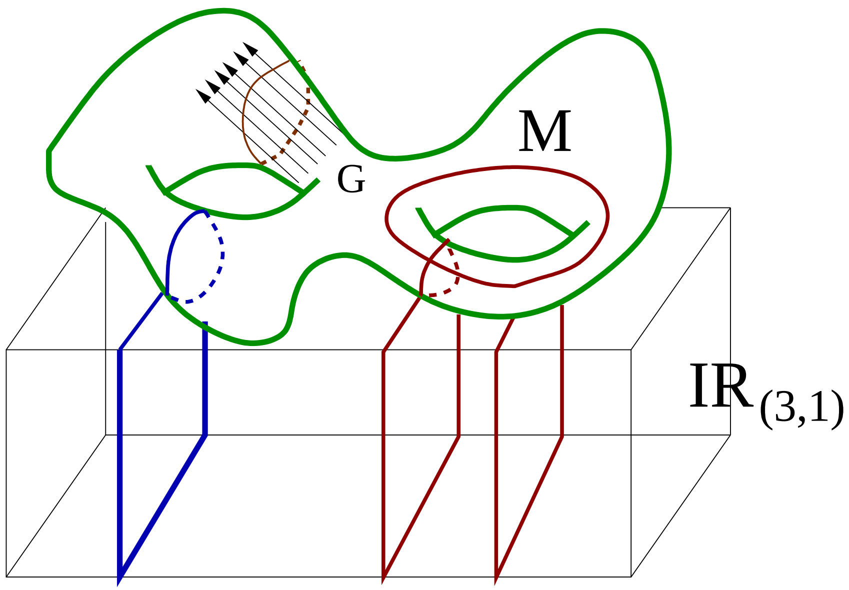
Intersecting D-brane models

- Chiral gauge theories arise on (intersecting) D-branes



Flux compactifications

- In addition one can turn on fluxes



- Freezing of moduli by generation of superpotential

$$W = \int \Omega \wedge G$$

Statistics of String Vacua

- Huge number of flux vacua $\simeq 10^{500}$
- We have to be very lucky to find the realistic string vacuum.
- A statistical approach was proposed by M. Douglas (Douglas, hep-th/0303194), which might allow for:
  - estimates for number of Standard-like models
  - statistical "solutions" of fine tuning problems: Λ, M_H, $\rightarrow$ phenomenological models of split supersymmetry (Arkani-Hamed, Dimopoulos, hep-th/0405159),
  - prospect of falsifying string theory
  - statistical correlations providing evidence for string theory.

Statistics of String Vacua

- Up to now: only flux sector analysed
- Phenomenologically important gauge sector should also be considered
- Hope to find statistical correlations between observables
- How does the statistics depend on coupling to fluxes

II. Stringy consistency conditions

Type IIA string theory orientifolds $/\Omega\overline{\sigma}$ with D6-branes at angles

- D-branes wrap sLag 3-cycles
- Symplectic basis: (α_I, β_I) of $H_3(M, \mathbb{Z})$, where $\alpha_I \in H_3^+(M)$ and $\beta_I \in H_3^-(M)$
- O6-planes

$$\pi_{O6} = \frac{1}{2} \sum_{I=1}^{b_3/2} L_I \, \alpha_I$$

- D6-branes:

$$\begin{aligned} \pi_a &= \textstyle\sum_{I=1}^{b_3/2} (X_{a,I}\, \alpha_I + Y_{a,I}\, \beta_I), \\ \pi'_a &= \textstyle\sum_{I=1}^{b_3/2} (X_{a,I}\, \alpha_I - Y_{a,I}\, \beta_I) \end{aligned}$$

Tadpole cancellation

- Tadpole cancellation: $b_3/2 = 1 + h_{21}$ conditions

$$\sum_{a=1}^{k} N_a \, X_{a,I} = L_I - L_{I,flux}$$

- I_{ab} chiral multiplets in the bifundamental $U(N_a) \times U(N_b)$ representation

$$I_{ab} = \sum_I X_{a,I} \, Y_{b,I} - Y_{a,I} \, X_{b,I}$$

The saddle point approximation

- Brute force computer classification
- Use saddle point method to determine asymptotic expansion of integrals

$$P(L) = \frac{1}{2\pi i}\oint dq \frac{g(q)}{q^{L+1}} = \frac{1}{2\pi i}\oint dq\, e^{f(q,L)}$$

with $f(q,L) = \log g(q) - (L+1)\log q$.

- In the next to leading order SAP it can be approximated as ($f'(q_0) = 0$)

$$\mathcal{N}^{(2)}(L) = \frac{1}{\sqrt{2\pi}} \frac{e^{f(q_0)}}{\sqrt{\frac{\partial^2 f}{\partial q^2}|_{q_0}}}$$

III. Applications

Total number of 8D models:

- Tadpole condition in 8D

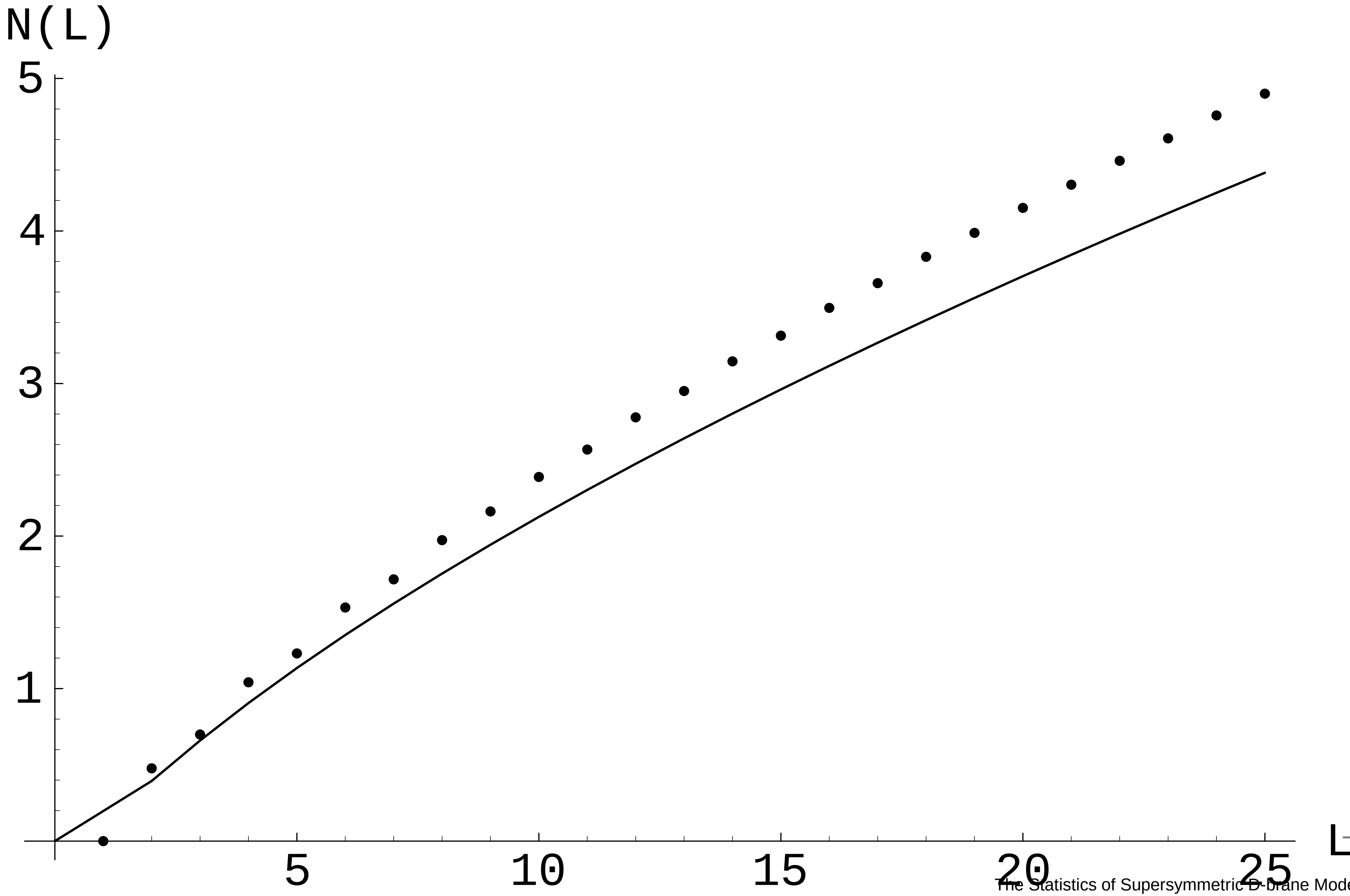
$$\sum_{a=1}^{k} N_a \, X_a = L$$

- Saddle point method gives

$$\mathcal{N}(L) \;=\; \frac{1}{2\pi i} \oint dq \frac{1}{q^{L+1}} \, \exp\left(\sum_{X=1}^{L} \frac{q^X}{1-q^X} \right)$$

- Scaling $\qquad \mathcal{N}(L) \simeq e^{2\sqrt{L \log L}}$

8D Results - N(L)

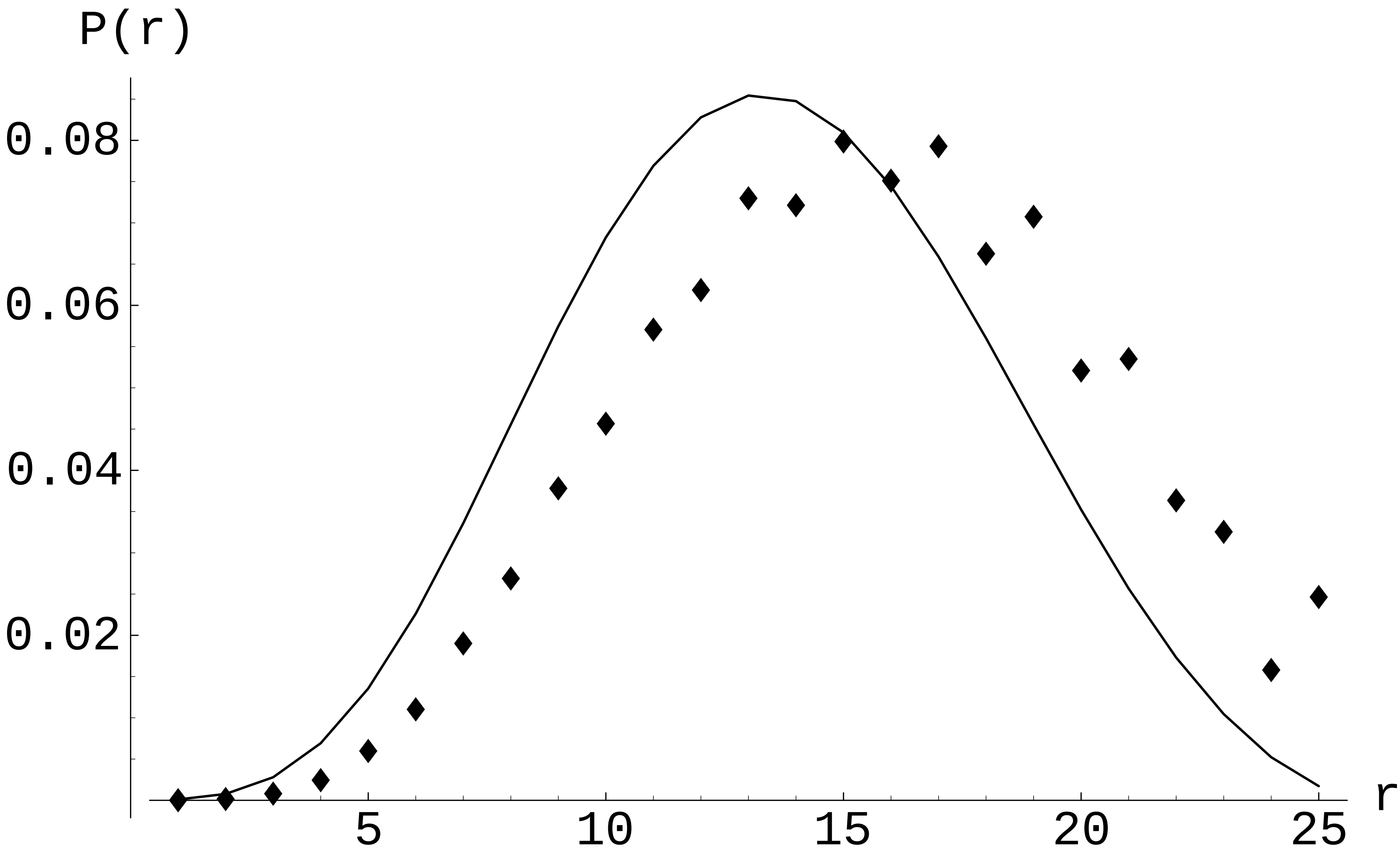


8D Results - rank distribution

- Rank distribution, i.e. probability to get a gauge group of rank r

$$P(r) \simeq \frac{1}{2\pi i \mathcal{N}(L)} \oint dq \frac{1}{q^{L+1}} \oint dz \frac{1}{z^{r+1}} \exp\left(\sum_{X=1}^{L} \frac{z\, q^X}{1 - z\, q^X} \right)$$

8D Results - rank distribution



$L = 25$, Dots: Exact results, Line: SPA

Results

- Similar results in 6D and 4D
- Probability for $SU(N)$ gauge factor

$$P(N) \simeq e^{-\sqrt{\frac{\log L}{L}}\,N}$$

- Probability for number of families

$$P(\chi) \simeq e^{-\kappa\sqrt{\chi}}$$

- Correlation between rank and chirality

4D Results - inclusion of fluxes

Example: rank distribution:

$$\overline{P}(r) = \frac{1}{N_{norm}} \sum_{N_{flux}=0}^{N_{flux}^{max}} (N_{flux}+1)^K \, \mathcal{N}(r; L_0 - N_{flux}, L_1, L_2, L_3)$$

4D results- inclusion of fluxes