Structure Functions at small x

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HERA: F_2 increases as $x \to 0$

DGLAP $\to$ evolution for large $-q^2$ at fixed x

breaks down at small x

BFKL $\to$ $F_2 \sim x^{-\nu(-q^2)}\,, \quad x \to 0$

seems to violate Froissart bound

Saturation models (e.g. "color glass"): $F_2 \sim \ln^p x$

Need non-perturbative input

but NOT amenable lattice simulations

HONEST TITLE:

Structure Functions in 2d asymptotically free models

organizers worried audience would go out for tea!

patience, main result concerns small x

WHY CONSIDER $(1+1)D$?

Test general methods (ideas)

Historical source of "structural inspiration"

* non-Euclidean geometry (Lobachevsky 1829)

* solitons, integrability,...

* duality

* SUSY, ...

* "STRING THEORY"

Beware! Many results special to 2D

* no (usual) spin

* no spontaneous symmetry breaking
  but see Seiler, Niedermaier, Duncan

* non-trivial S-matrices with no particle production

NON-LINEAR O(n) σ-MODEL IN 2d

Fields: $\sigma^a(x),\ a = 1, ..., n,\ \sigma^2 = 1$

Action: $S = \frac{1}{2g_0^2} \int \mathrm{d}^2 x (\partial_\mu \sigma^a)^2$

asymptotically free

Noether current: $j_\mu^{ab} = g_0^{-2} \sigma^a \partial_\mu \sigma^b - (a \leftrightarrow b)$

OPE: $j(x)j(0) \sim_{x \to 0} \sum_j C_j(x) \mathcal{O}_j$

NON-PERTURBATIVE METHODS

$1/n$–expansion

Lattice

$\longrightarrow$ spectrum: one massive O(n) vector multiplet

Form Factor Bootstrap ("on-shell to off-shell")

Integrability $\longrightarrow$ purely elastic S–matrix

$\longrightarrow$ form factors: $\langle\Omega|j_\mu^{ab}(0)|p_1,a_1;\ldots;p_r,a_r\rangle^{\rm in}$

$\longrightarrow$ correlation functions

STRUCTURE FUCTIONS

$$W_{\mu\nu}^{ab;cdef}(p,q) = \frac{1}{4\pi}\int \mathrm{d}^2x \mathrm{e}^{iqx}\langle a,p|\left[j_\mu^{cd}(x), j_\nu^{ef}(0)\right]|b,p\rangle$$

relevant domain: $q^2 < 0, (p+q)^2 \geq M^2$:

$$W_{\mu\nu}^{ab;CD}(p,q) = \pi\langle a,p|j_\mu^C(0)\sum_r |r\rangle\langle r|j_\nu^D(0)|b,p\rangle\delta(p+q-P_r)$$

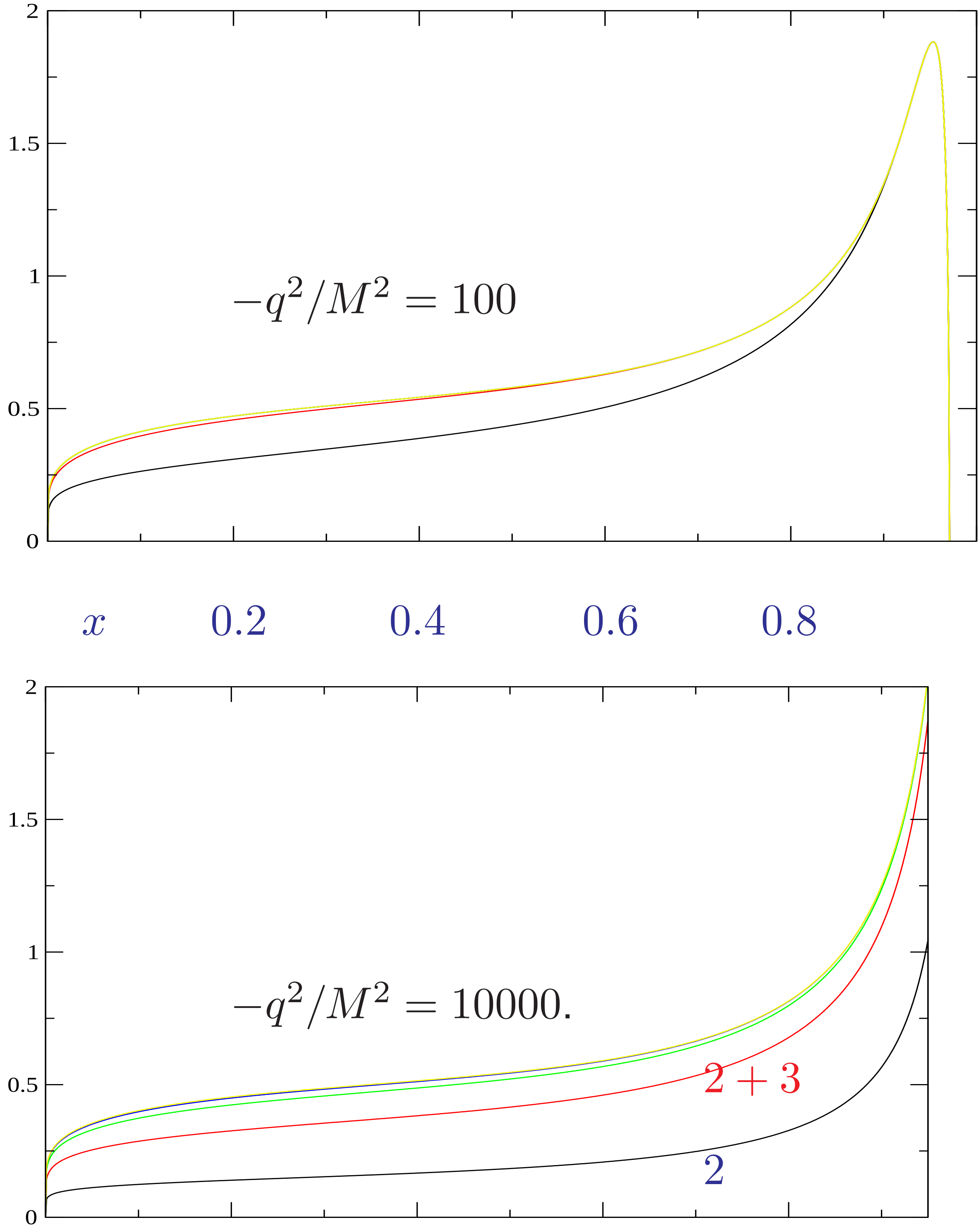
$$= \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\sum_{l=0}^{2} w_l(q^2,x) R_l^{ab;CD},$$

Similarly for the field operator: $\widetilde{w}_l(q^2,x)$

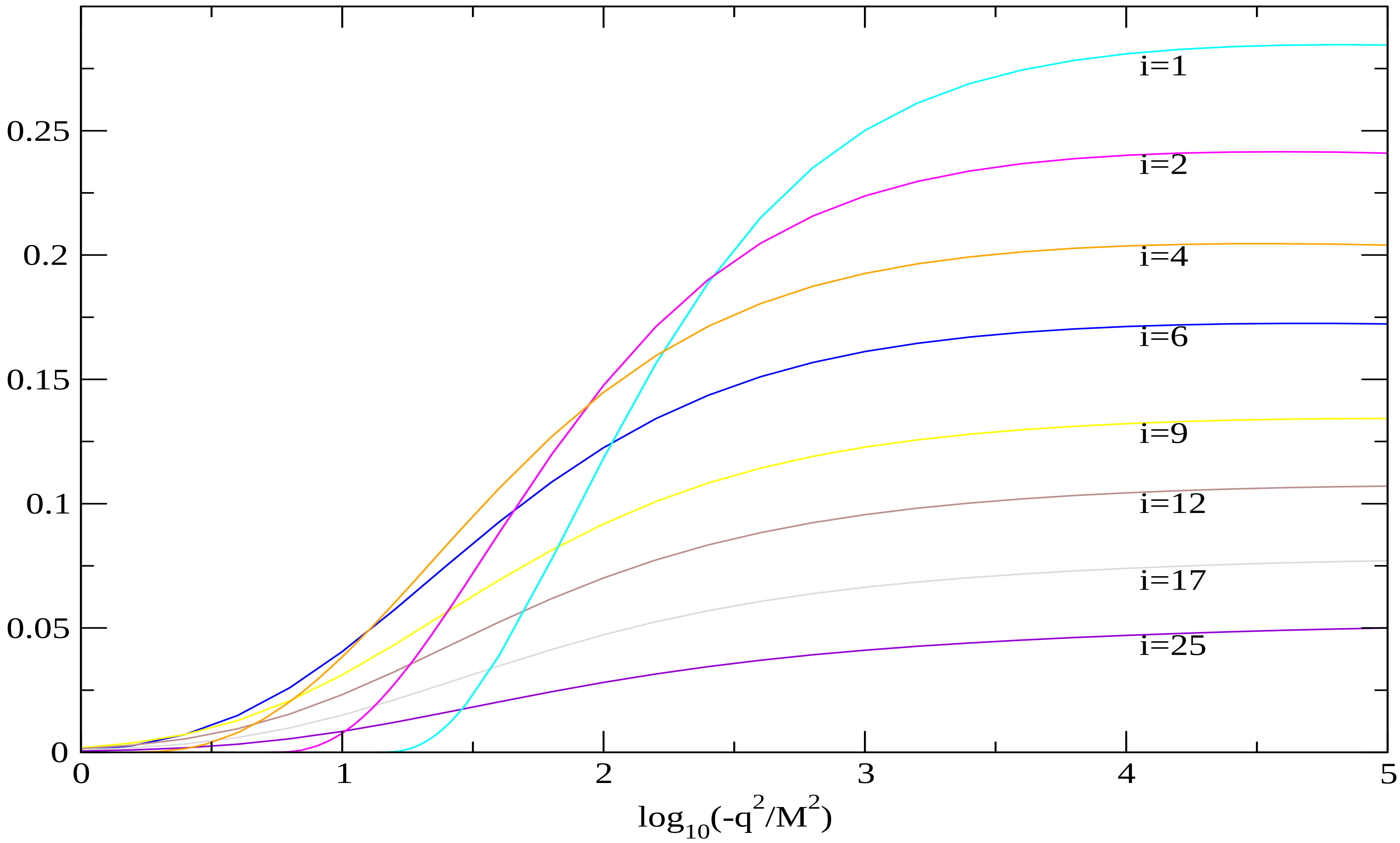
Bjorken $x = -q^2/(2pq)$, $0 \leq x \leq 1$

EXACT NON-PERTURBATIVE STRUCTURE FUNCTIONS!

$$x(w_0 + \widetilde{w}_0)$$



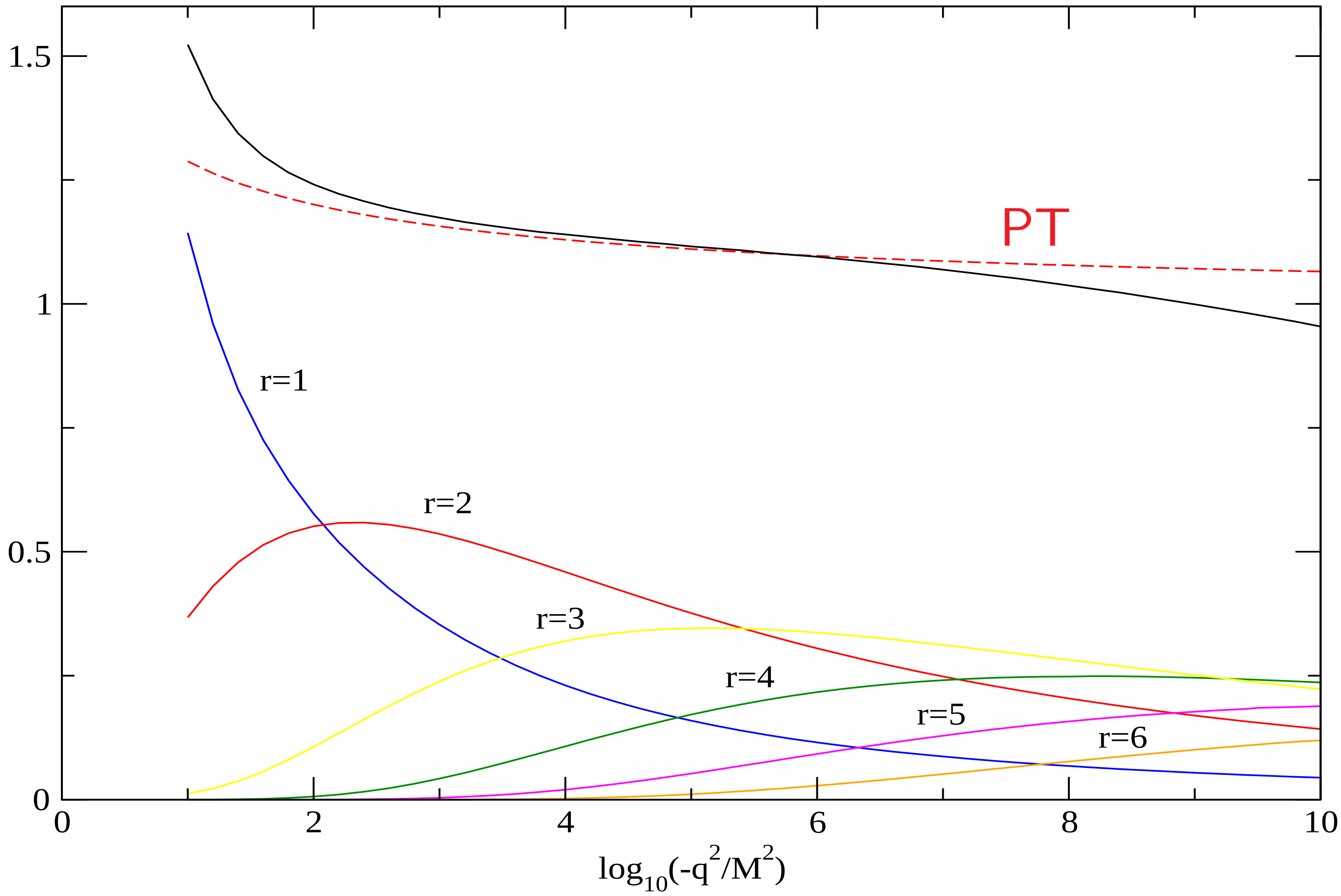
“HERA PLOT”: xw_0 for various values of $x = 10^{-i/5}$.



Bjorken scaling!

Comparison with Perturbation theory

MOMENTS: e.g. $M_{0;2}(q^2) = \int_0^1 \mathrm{d}x x [w_0 + \widetilde{w}_0](q^2, x)$



cf Momentum sum rule in QCD

Exact asymptotic small x behavior

$$xw_l(q^2, x) \sim C(l,n)\frac{1}{\ln^2 x} \times A_J(-q^2)$$

"Adler function": $A_J(z) = -z^2\frac{\partial}{\partial z}I(z)$

$$\int \mathrm{d}^2x \mathrm{e}^{iqx}\langle\Omega|T\left(j_\mu^C(x) j_\nu^D(0)\right)|\Omega\rangle = i\delta^{CD} I(-q^2)(\eta_{\mu\nu}q^2 - q_\mu q_\nu)$$

UNIVERSAL STRUCTURE!

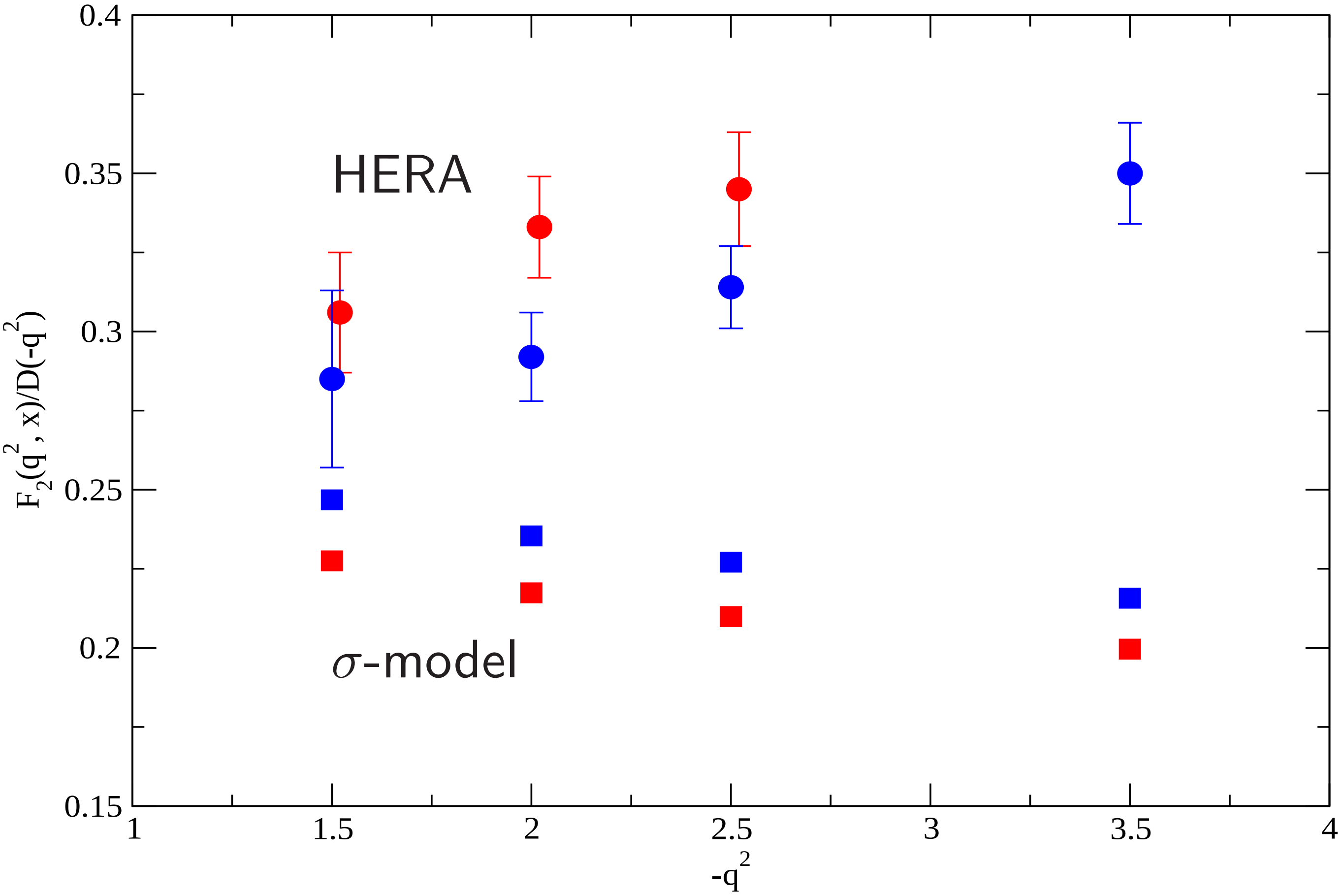
Similar structural aspects in 4d hadronic pysics??

in some HE processes may neglect $p_{\mathrm{transverse}}$

Partons in sigma model?

$n = 3: \quad \sigma^a = \bar{q}\tau^a q\,, \quad \bar{q}q = 1\,, \quad q$ confined

Ratio of the structure function to the Adler function



$x = 8 \times 10^{-5}$, $\quad x = 5 \times 10^{-5}$

At what x does the characteristic behavior “set in”?