

Simulating Neutrino Oscillation Experiments with GLoBES

Part 1: Alexander Merle (amerle@ph.tum.de)

- Theory of Neutrino Oscillations
- State of the Art in Neutrino Physics

Part 2: Joachim Kopp (jkopp@ph.tum.de)

- The GLoBES Software Package
- Simulation of Future Experiments

TU München, Lehrstuhl T30d für Theoretische Elementarteilchenphysik (Prof. Lindner)

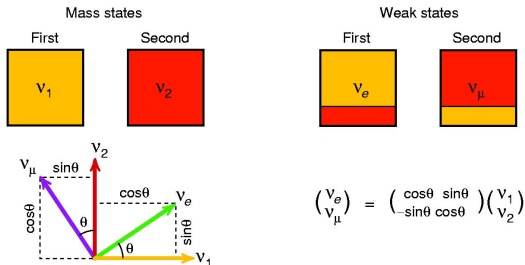
Young Scientists Workshop on Hot Topics in Particle and
Astroparticle Physics
18.07. - 22.07.2005, Ringberg Castle

- 1 The Neutrino Oscillation Framework
 - 2-Flavour Oscillations in Vacuum
 - Matter Effects
 - 3-Flavour Oscillations
- 2 The State of the Art in Neutrino Physics
 - The Oscillation Parameters
 - Mass Hierarchies
 - The Past and the Future
- 3 GLoBES - The General Long Baseline Experiment Simulator
 - GLoBES Basics
 - χ^2 Analysis
- 4 Simulation of Future Oscillation Experiments
 - The R2D2 Setup
 - Superbeams and Neutrino Factories
 - Beta Beams

- 1 The Neutrino Oscillation Framework
 - 2-Flavour Oscillations in Vacuum
 - Matter Effects
 - 3-Flavour Oscillations
- 2 The State of the Art in Neutrino Physics
 - The Oscillation Parameters
 - Mass Hierarchies
 - The Past and the Future
- 3 GLoBES - The General Long Baseline Experiment Simulator
 - GLoBES Basics
 - χ^2 Analysis
- 4 Simulation of Future Oscillation Experiments
 - The R2D2 Setup
 - Superbeams and Neutrino Factories
 - Beta Beams

Mass and flavour eigenstates

- For neutrinos, the eigenstates of the mass operator are not equal to the ones of the weak interaction:



- Massless neutrinos: the mass eigenstates can be arbitrarily chosen \Rightarrow can be defined as identical to the flavour eigenstates \Rightarrow no oscillations! \Rightarrow oscillations are the evidence for non-zero neutrino mass!

The formalism

- Let $(|\nu_e\rangle, |\nu_\mu\rangle)$ be the flavour and $(|\nu_1\rangle, |\nu_2\rangle)$ the mass eigenstates \Rightarrow every flavour-state can be written as a linear combination of mass eigenstates (since it is a rotation one can use a rotation matrix):

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

- Time evolution:

$$|\nu_e(t)\rangle = \cos \theta e^{-iE_1 t} |\nu_1(t=0)\rangle + \sin \theta e^{-iE_2 t} |\nu_2(t=0)\rangle$$

- Energy momentum relation:

$$E_i^2 = p_i^2 + m_i^2 \Rightarrow E_i = \sqrt{p_i^2 + m_i^2} \approx p_i + \frac{m_i^2}{2p_i}$$

Diasappearance Probability

- Assumption: both of the mass eigenstates have the same momentum p (in fact, this does not have to be true, but the longer calculation gives the same result) \Rightarrow with $p = E_\nu$: $E_2 = E_1 - \frac{\Delta m^2}{2E_\nu}$ where $\Delta m^2 = m_1^2 - m_2^2$
- E. g. for the muon-neutrino we get:

$$|\nu_\mu\rangle = e^{-i(p + \frac{m_1^2}{2p})t} \cdot \left[-\sin\theta |\nu_1(0)\rangle + \cos\theta e^{\frac{i\Delta m^2}{2E_\nu}t} |\nu_2(0)\rangle \right]$$

- With a simple calculation (trigonometric identities, etc.) one can construct the probability for detecting a ν_μ as a ν_e at a time t :

$$\begin{aligned} P_{\nu_\mu \rightarrow \nu_e}(t) &= |\langle \nu_e(0) | \nu_\mu(t) \rangle|^2 = |(\cos\theta \langle \nu_1(0) | + \sin\theta \langle \nu_2(0) |) \cdot |\nu_\mu(t)\rangle|^2 \\ &= \sin^2(2\theta) \cdot \sin^2\left(\frac{t\Delta m^2}{4E_\nu}\right) \Rightarrow \text{oscillates with time!} \end{aligned}$$

- Probability to see the ν_μ again as a ν_μ :

$$P_{\nu_\mu \rightarrow \nu_\mu}(t) = 1 - P_{\nu_\mu \rightarrow \nu_e}(t) = 1 - \sin^2(2\theta) \cdot \sin^2\left(\frac{t\Delta m^2}{4E_\nu}\right)$$

- Neutrinos travel (nearly) with the speed of light \Rightarrow one can set $t=x$, where x is the distance to the place where the neutrino was born
- Oscillation length: distance after which the ν_μ is again a 100 % ν_μ

$$\sin^2\left(\frac{x\Delta m^2}{4E_\nu}\right) \equiv \sin^2\left(\frac{\pi x}{L_0}\right) \Rightarrow L_0 = \frac{4\pi E_\nu}{\Delta m^2} \propto \frac{E_\nu}{\Delta m^2}$$

\rightarrow important quantity for experiments

- Notice, that only the squared mass difference Δm^2 can be determined by oscillation experiments!

- The Hamiltonian in the basis of the mass eigenstates is diagonal
⇒ Schrödinger's equation (one can skip the first p from $E = p + \frac{m^2}{2p}$ since it just gives a global phase):

$$i \frac{d}{dt} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = H_M \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \frac{1}{2p} \cdot \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

- To go into the flavour basis, one has to rotate the states as well as the Hamiltonian by a unitary matrix U , which leads to the Hamiltonian H_F in the flavour basis:

$$\begin{aligned} H_F &= \frac{1}{2p} \cdot \begin{pmatrix} m_{ee}^2 & m_{e\mu}^2 \\ m_{e\mu}^2 & m_{\mu\mu}^2 \end{pmatrix} = U H_M U^\dagger = \\ &= \frac{1}{2p} \cdot \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \cdot \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = \end{aligned}$$

$$= \frac{1}{4p} \left[\Sigma \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + D \begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix} \right]$$

$$\text{where } \Sigma = m_1^2 + m_2^2 \text{ and } D = m_2^2 - m_1^2$$

- Since it is diagonal, the first term just gives a common phase \Rightarrow has no physical meaning
- Coherent forward scattering in matter: ν_e and $\bar{\nu}_e$ can - different from all other flavours - interact with the electrons via CC-reactions \Rightarrow change of the oscillation probabilities!
- The Hamiltonian in the flavour basis has to be changed:

$$H_F \rightarrow H_{F,mat} = H_F + \frac{1}{2p} \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}$$

where $A = 2\sqrt{2}G_F N_e p$ (G_F : Fermi's constant, N_e : electron density)

- Back to the mass eigenstates \Rightarrow after a lengthy calculation we get:

$$H_{M,mat} = \frac{1}{2p} \cdot \begin{pmatrix} m_1^2 + A \cos^2(\theta) & A \sin(\theta) \cos(\theta) \\ A \sin(\theta) \cos(\theta) & m_2^2 + A \sin^2(\theta) \end{pmatrix}$$

\Rightarrow this Hamiltonian is not diagonal anymore \Rightarrow the mass eigenstates of the vacuum are not eigenstates in matter

- So one has to diagonalize again to get the new mass eigenstates $|\nu_{1m}\rangle$ and $|\nu_{2m}\rangle$ in matter, which gives

$$m_{1,2m}^2 = \frac{1}{2} \cdot \left[(\Sigma + A) \pm \sqrt{(A - D \cos(2\theta))^2 + D^2 \sin^2(2\theta)} \right]$$

and so the squared mass difference in matter is (remember that $D = m_2^2 - m_1^2$)

$$D_m = m_{2m}^2 - m_{1m}^2 = D \cdot \sqrt{\left(\frac{A}{D} - \cos(2\theta)\right)^2 + \sin^2(2\theta)}$$

- So we also have in matter a new mixing angle θ_{mat} and so a new transition probability

$$P_{mat, \nu_e \rightarrow \nu_\mu}(\mathbf{x}) = \sin^2(2\theta_{mat}) \sin^2\left(\frac{x D_m}{4p}\right)$$

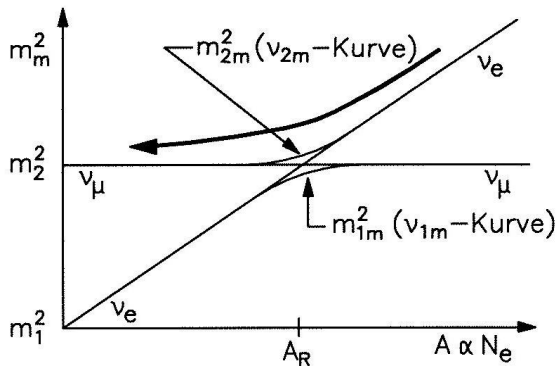
- The oscillation amplitude is

$$\sin^2(2\theta_{mat}) = \frac{\sin^2(2\theta)}{\left(\frac{A}{D} - \cos(2\theta)\right)^2 + \sin^2(2\theta)} = f(A/D) = f(E_\nu, N_e)$$

\Rightarrow the ratio $\frac{A}{D}$ determines the oscillation probabilities in matter \Rightarrow for $\frac{A}{D} \approx \cos(2\theta)$ we have a resonance effect \Rightarrow **a muon-neutrino can at certain distances be a 100% electron-neutrino!!!**

Flavour-Flipping/The MSW-Effect

(Mikheyev-Smirnov-Wolfenstein)



e.g. the first mass eigenstate (ν_{1m} -curve) is most probably a ν_e below and most probably a ν_μ above the resonant matter density A_R

3-Flavour Oscillations

- For 3 neutrino-flavours the mixing looks like:

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U \cdot \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

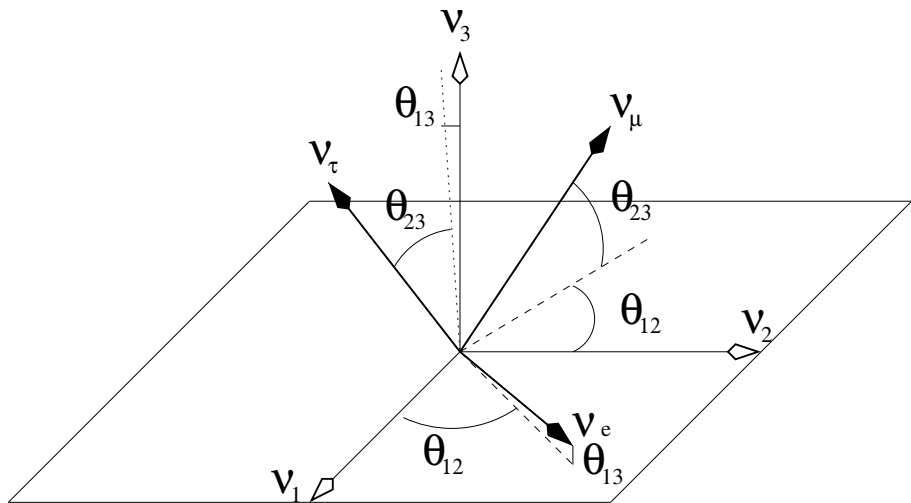
with $U^\dagger = U^{-1}$ and

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$

- Parameters: 3 mixing angles (θ_{12} , θ_{23} and θ_{13}), 1 CP-violating phase (δ) and 2 squared mass differences ($(\Delta m^2)_{sol} = (\Delta m^2)_{21}$ and $(\Delta m^2)_{atm} = (\Delta m^2)_{31}$)

3-Neutrino-Mixing



- 1 The Neutrino Oscillation Framework
 - 2-Flavour Oscillations in Vacuum
 - Matter Effects
 - 3-Flavour Oscillations
- 2 The State of the Art in Neutrino Physics
 - The Oscillation Parameters
 - Mass Hierarchies
 - The Past and the Future
- 3 GLoBES - The General Long Baseline Experiment Simulator
 - GLoBES Basics
 - χ^2 Analysis
- 4 Simulation of Future Oscillation Experiments
 - The R2D2 Setup
 - Superbeams and Neutrino Factories
 - Beta Beams

The Oscillation Parameters

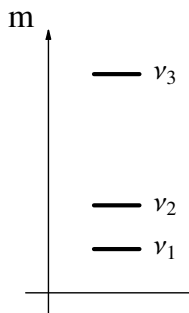
| Parameter | Best fit value and $\pm 3\sigma$ range | Experiments |
|----------------------------------|--|-------------------------|
| $\theta_{12} \approx 32.6^\circ$ | $\sin^2(\theta_{12}) = 0.29_{-0.06}^{+0.08}$ | solar/reactor neutrinos |
| $\theta_{23} \approx 45.0^\circ$ | $\sin^2(\theta_{23}) = 0.50 \pm 0.16$ | atmospheric/accelerator |
| $\theta_{13} \lesssim 9.6^\circ$ | $\sin^2(\theta_{13}) = 0.000 + 0.047$ | CHOOZ-experiment |
| δ | ????? | several proposals |
| $(\Delta m^2)_{21}$ | $(8.1_{-0.8}^{+1.0}) \cdot 10^{-5} \text{ eV}^2$ | solar/reactor |
| $(\Delta m^2)_{31}$ | $(2.2_{-0.8}^{+1.1}) \cdot 10^{-3} \text{ eV}^2$ | atmospheric/accelerator |

taken from Maltoni, Schwetz et al., New J. Phys. 6 (2004)
see also: Bahcall et al., JHEP 08 (2004) or Bandyopadhyay et al., Phys. Lett. B608 (2005)

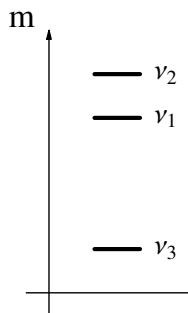
- Two large mixing angles and one small mixing angle (\neq CKM!).
- No information about CP violation.
- Precise measurement of neutrino mixing parameters is essential for a deeper theoretical understanding of flavour physics and physics beyond the Standard Model.

Mass Hierarchies

Problem: since oscillation only measures squared mass differences, and we do not know the sign of $(\Delta m^2)_{atm}$ yet, there could be different hierarchies



Normal

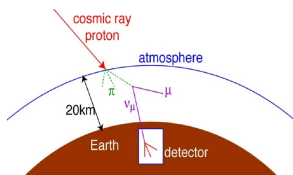


Invertiert

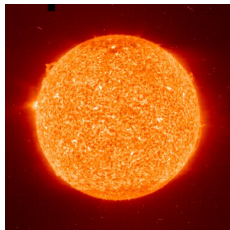
⇒ from the oscillations we get only the information, that neutrinos do have mass, but nothing about the absolute values

Where do we know all that from?

- Super-Kamiokande (atmospheric ν 's), K2K,... $\Rightarrow \theta_{23}, (\Delta m^2)_{31}$



- Solar neutrinos, KamLAND, ... $\Rightarrow \theta_{12}, (\Delta m^2)_{21}$



- CHOOZ $\Rightarrow \theta_{13}$

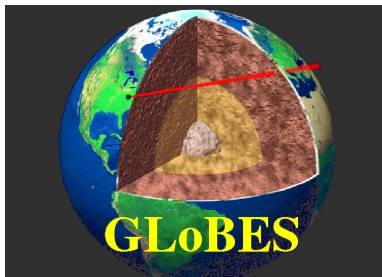
The next steps

- Achieving a better precision for the known mixing parameters
- Measurement of the small mixing angle θ_{13}
- Measurement of the CP-phase δ

AND (what we do): Making Simulations with GLoBES

- 1 The Neutrino Oscillation Framework
 - 2-Flavour Oscillations in Vacuum
 - Matter Effects
 - 3-Flavour Oscillations
- 2 The State of the Art in Neutrino Physics
 - The Oscillation Parameters
 - Mass Hierarchies
 - The Past and the Future
- 3 **GLOBES - The General Long Baseline Experiment Simulator**
 - **GLOBES Basics**
 - χ^2 Analysis
- 4 Simulation of Future Oscillation Experiments
 - The R2D2 Setup
 - Superbeams and Neutrino Factories
 - Beta Beams

The GLoBES software package



- C library for simulating neutrino oscillation experiments
- Provides sophisticated functions for high level and low level access to the simulation
- AEDL — Abstract Experiment Definition Language for defining experimental setups
- Developed at TUM by P. Huber, M. Lindner and W. Winter

Comput. Phys. Commun. 167:195, 2005, hep-ph/0407333

Components of a GLOBES simulation

Neutrino Sources

- Energy and flavour spectrum
- Signal and Background

Oscillations

- Fundamental parameters:
 $\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m_{21}^2, \Delta m_{31}^2$.
- Baseline and matter density profile

Detector

- Interaction cross sections
- Energy resolution
- Systematical errors

Analysis

- Number and width of energy bins
- Fitting parameters to the simulated data
- χ^2 analysis

Calculation of event rates

$$\begin{aligned}
 \frac{dn_f^{\text{IT}}}{dE'} &\propto \sum_{f_i} \int \int dE d\hat{E} \underbrace{\Phi_{f_i}(E)}_{\text{Production}} \times \\
 &\quad \underbrace{\frac{1}{L^2} P_{(f_i \rightarrow f)}(E, L, \rho; \theta_{23}, \theta_{12}, \theta_{13}, \Delta m_{31}^2, \Delta m_{21}^2, \delta)}_{\text{Propagation}} \times \\
 &\quad \underbrace{\sigma_f^{\text{IT}}(E) k_f^{\text{IT}}(E - \hat{E})}_{\text{Interaction}} \times \underbrace{T_f(\hat{E}) V_f(\hat{E} - E')}_{\text{Detection}}
 \end{aligned}$$

P. Huber, M. Lindner, W. Winter, Nucl. Phys. B645:3-48, 2002, hep-ph/0204352

E = Neutrino energy

\hat{E} = Charged Lepton energy

E' = Reconstructed energy

$\Phi_{f_i}(E)$ = Source flux

L = Baseline

$P_{(f_i \rightarrow f)}(\dots)$ = Oscillation probability

$\sigma_f^{\text{IT}}(E)$ = Cross sections

$k_f^{\text{IT}}(E - \hat{E})$ = Charged lepton distribution

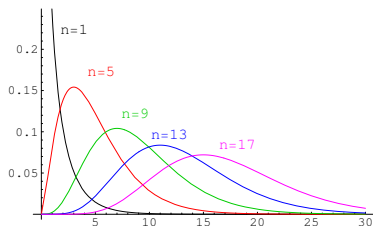
$T_f(\hat{E})$ = Threshold function

$V_f(\hat{E} - E')$ = Energy resolution function

Sensitivity of a reactor experiment to $\sin^2(2\theta_{13})$

- 1 Define what you want to measure: What 90 % C.L. limit can the experiment set on $\sin^2(2\theta_{13})$, assuming the true value is 0?
- 2 Define the setup using AEDL (Abstract Experiment Definition Language)
- 3 Calculate the “true” event rates.
- 4 Now calculate the expected event rates for nonzero values of $\sin^2(2\theta_{13})$.
- 5 Perform a χ^2 analysis to find out which values produce event rates that are still compatible with $\sin^2(2\theta_{13}) = 0$ at the chosen C.L.

- If X_i are normally distributed with $\langle X_i \rangle = 0$ and $\sigma = 1$, then $\chi^2 = \sum_{i=1}^n X_i^2$ is distributed according to the χ^2 distribution with n degrees of freedom.



- For n energy bins with event rates following the Poisson distribution

$$\chi^2 = \sum_{i=1}^n 2(N_i(\vec{\Lambda}) - N_i(\vec{\Lambda}_0)) + 2N_i(\vec{\Lambda}) \log \frac{N_i(\vec{\Lambda})}{N_i(\vec{\Lambda}_0)}$$

follows a χ^2 distribution with $\dim \vec{\Lambda}$ degrees of freedom. Here:

$$\begin{aligned} N_i(\vec{\Lambda}_0) &= \text{Event rates for true parameter values } \vec{\Lambda}_0. \\ N_i(\vec{\Lambda}) &= \text{Event rates for hypothesised parameters } \vec{\Lambda}. \end{aligned}$$

Inclusion of systematics and external input

$$\chi^2 = \underbrace{\sum_{i=1}^n 2(N_i(\vec{\Lambda}, \vec{x}) - N_i(\vec{\Lambda}_0, \vec{x}_0)) + 2N_i(\vec{\Lambda}, \vec{x}) \log \frac{N_i(\vec{\Lambda}, \vec{x})}{N_i(\vec{\Lambda}_0, \vec{x}_0)}}_{\text{Statistical part}} + \underbrace{\sum_{j=1}^{\dim \vec{\Lambda}} \frac{(\vec{\Lambda}_j - \vec{\Lambda}_{0j})^2}{\sigma_{\vec{\Lambda}_j}^2}}_{\text{External input}} + \underbrace{\sum_{k=1}^{\dim \vec{x}} \frac{(\vec{x}_k - \vec{x}_{0k})^2}{\sigma_{\vec{x}_k}^2}}_{\text{Systematics}}$$

$\vec{\Lambda}, \vec{\Lambda}_0$ = True and hypothesised oscillation parameters

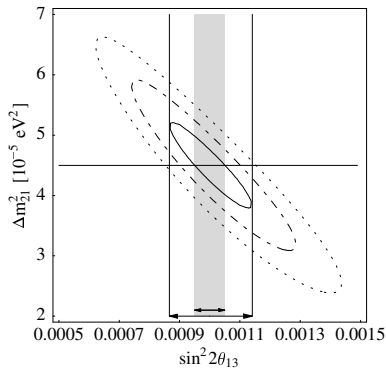
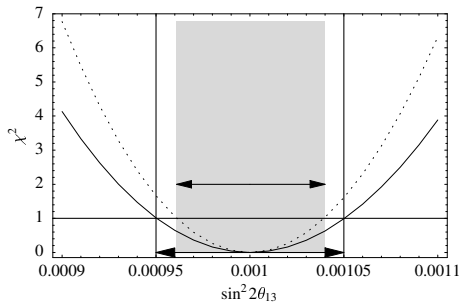
\vec{x}, \vec{x}_0 = Systematics parameters

$N_i(\vec{\Lambda}, \vec{x})$ = Expected event rates

$\sigma_{\vec{\Lambda}_j}$ = Standard errors of external parameter input

$\sigma_{\vec{x}_k}$ = Standard errors of systematics parameters

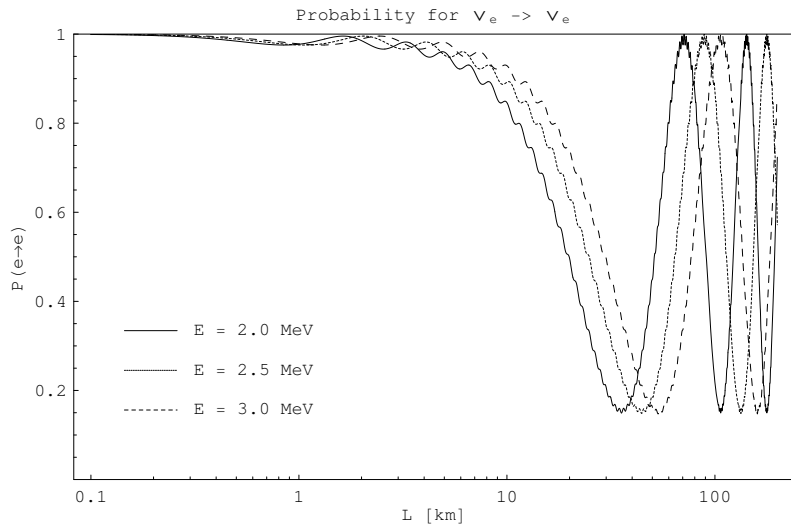
χ^2 Analysis in GLOBES



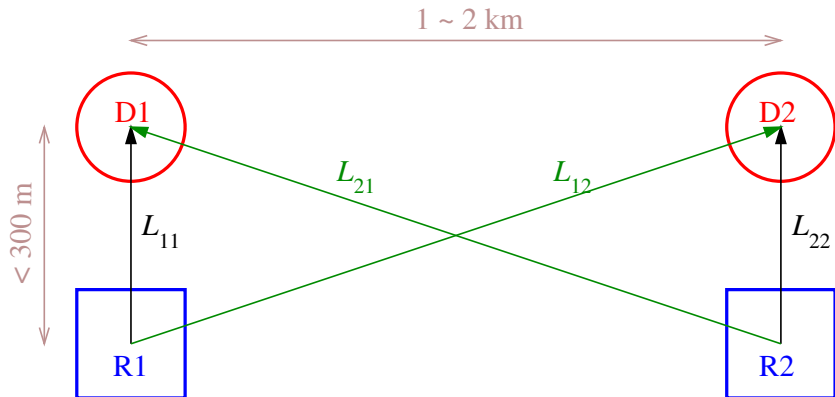
P. Huber, M. Lindner, W. Winter, Nucl. Phys. B645:3-48, 2002, hep-ph/0204352

- 1 The Neutrino Oscillation Framework
 - 2-Flavour Oscillations in Vacuum
 - Matter Effects
 - 3-Flavour Oscillations
- 2 The State of the Art in Neutrino Physics
 - The Oscillation Parameters
 - Mass Hierarchies
 - The Past and the Future
- 3 GLoBES - The General Long Baseline Experiment Simulator
 - GLoBES Basics
 - χ^2 Analysis
- 4 **Simulation of Future Oscillation Experiments**
 - **The R2D2 Setup**
 - **Superbeams and Neutrino Factories**
 - **Beta Beams**

The $\bar{\nu}_e$ disappearance channel



The R2D2 Setup

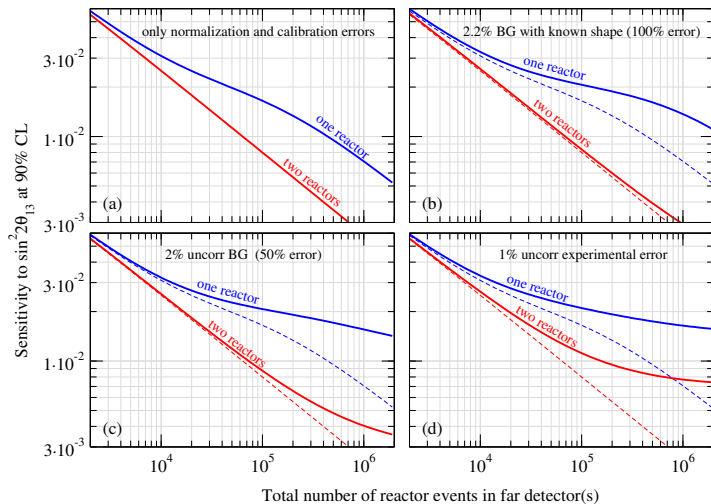


P. Huber, M. Lindner, T. Schwetz, JHEP 0502:029, 2005, hep-ph/0411166

- Near and far detectors \rightarrow Elimination of flux normalization error
- Two reactors \rightarrow Elimination of detector systematics

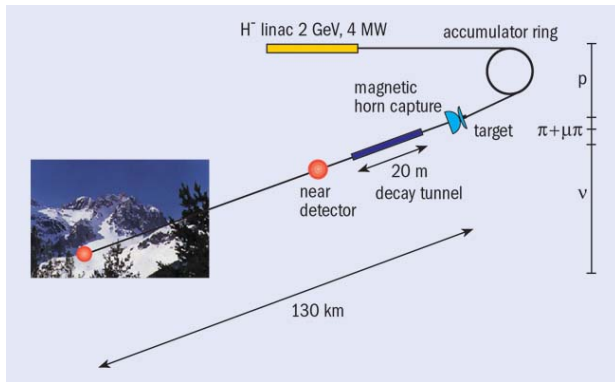
The R2D2 Setup

Two detectors and one (or two) reactor(s), $L_N = 300$ m, $L_F = 1300$ m



P. Huber, M. Lindner, T. Schwetz, JHEP 0502:029, 2005, hep-ph/0411166

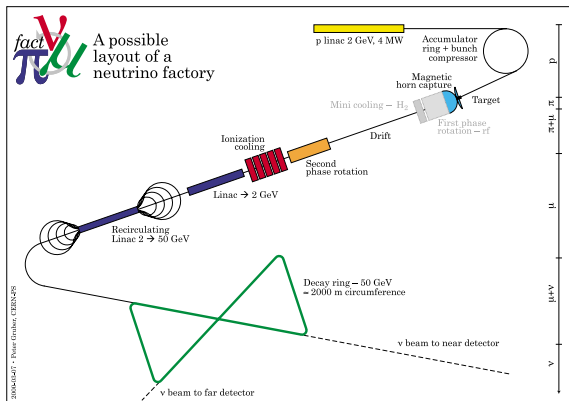
Superbeams



J. Bouchez, CERN Courier 42, No. 5

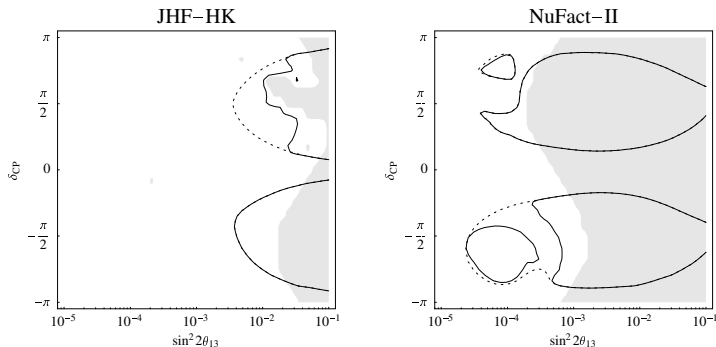
- High neutrino energy → Easy to detect
- ν_{μ} beam with ν_e , $\bar{\nu}_e$ and $\bar{\nu}_{\mu}$ contamination → limited physics potential

Neutrino Factories



- High neutrino energy → Easy to detect
- Long Baselines → Exploiting matter effects
- Beam contains only ν_{μ} and $\bar{\nu}_e$ → “Wrong sign muon” signature: Detection of μ^+ indicates oscillation

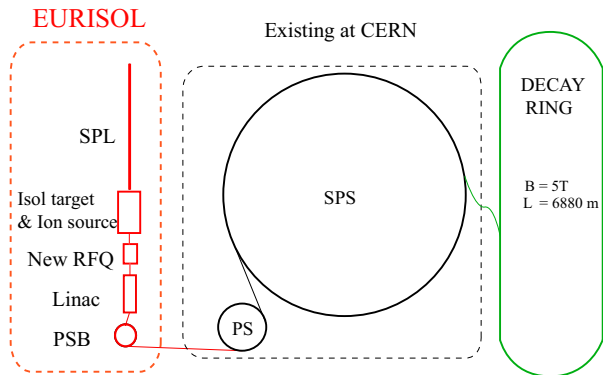
CP Violation in Superbeams and Neutrino Factories



P. Huber, M. Lindner, W. Winter, Nucl. Phys. B645:3-48, 2002, hep-ph/0204352

- JHF-HK: $E_p = 50$ GeV, $L = 295$ km, Target Power 4 MW, 1 Mt Water Čerenkov detector
- Nu-Fact II: $E_\mu = 50$ GeV, $L = 3000$ km, Target Power: 4 MW, $5 \cdot 10^{20} \frac{\mu}{\text{year}}$, 50 kt magnetized iron calorimeter

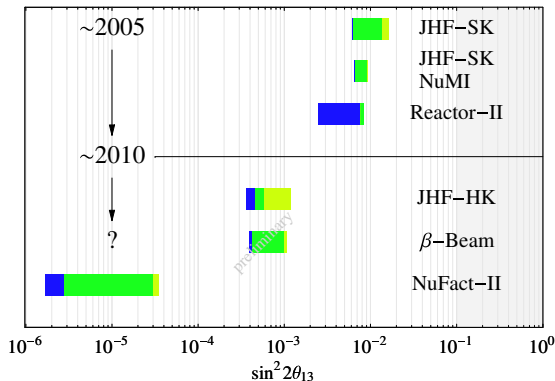
Beta Beams



J. Bouchez, M. Lindroos, M. Mezzetto, AIP Conf. Proc. 721:37-47, 2004, hep-ex/0310059

- High neutrino energy \longrightarrow Easy to detect
- Long Baselines \longrightarrow Exploiting matter effects
- Pure ν_e or $\bar{\nu}_e$ beam

Limitations of Neutrino Oscillation Experiments



C. Albright et al., physics/0411123

- Correlations: Sensitivity only to combination of parameters
- Degeneracies: Distinct regions in parameter space can explain the experiment

- Neutrino oscillations are theoretically well understood and experimentally clearly verified.
- A precise determination of the oscillation parameters is desirable to achieve a deeper understanding of particle physics (flavour structure, ...).
- GLoBES is a tool for simulating future neutrino oscillation experiments and determine their physics potential.
- Nuclear reactors, Superbeams, Neutrino Factories and Beta Beams are excellent tools for solving many open questions.