2-Loop Electroweak Corrections to the Effective Leptonic Weak Mixing Angle

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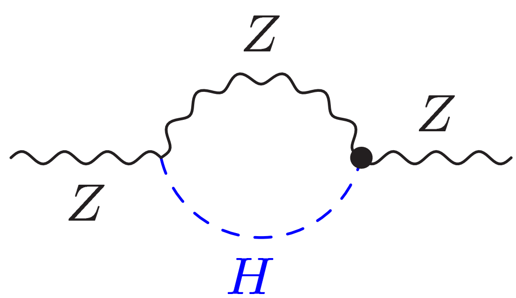
Topics

- tests of the SM
- calculation of radiative corrections
- the effective leptonic weak mixing angle

testing the SM

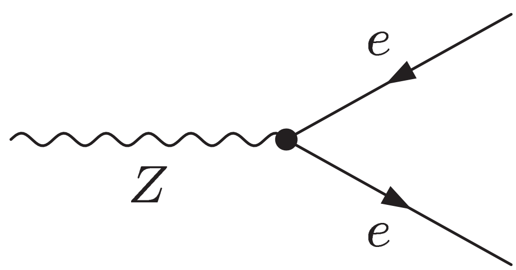
- precise measurements
- precise theoretical predictions
  (calculation of radiative corrections)
- access to all sectors of the SM,
  e.g. Higgs sector



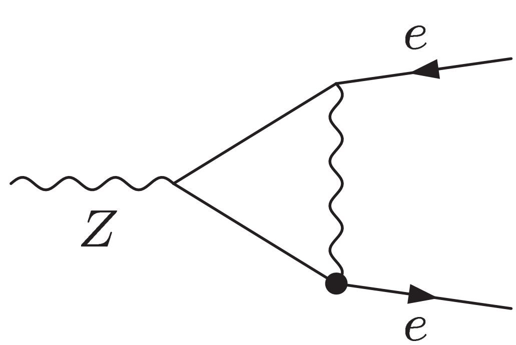
- SM: treat Higgs mass as a free parameter
  - bounds on the Higgs mass
  - comparison with experimental results
    from direct Higgs search

radiative corrections

- 0.order (tree level):
  no closed loops



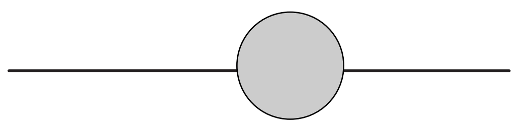
- 1. order : one closed loop



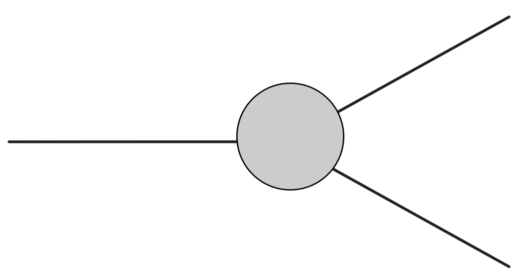
- ...

N-point functions

- processes with two external particles:
  2-point functions (self-energies)



- processes with three external particles:
  3-point functions (vertices)



- ...

regularization/renormalization

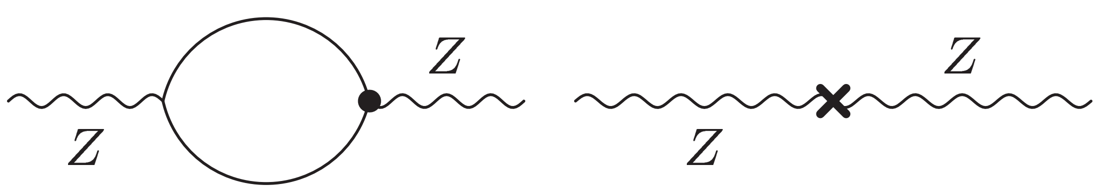
- integration over loop momenta
  ⇒divergences

- regularization : introduce a cut-off

- relations
  free parameters ↔ observables
  cut-off dependent

- renormalization: replace bare parameters/fields:
  $$M_0^2 = M^2 + \delta M^2_{(1)} + \delta M^2_{(2)} + \cdots$$

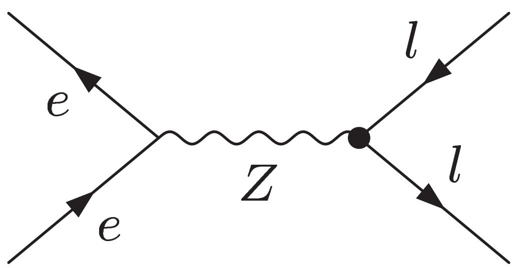
- ⇒ ren. Lagrangian + counter term part



Z resonance

- LEP1: many events

$$e^+e^- \to l^+l^- \text{ at } s \sim M_Z^2$$



$$\sim \frac{1}{s - M_Z^2 + iM_Z\Gamma_Z}$$

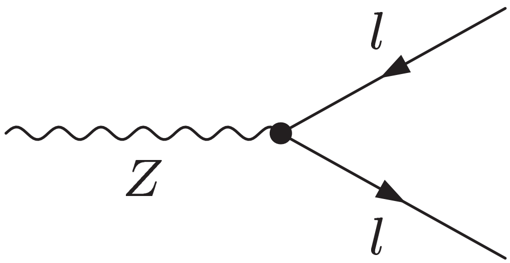
- theory: Z pole approximation
  - neglect non-resonating contributions
  - set $s = M_Z^2$

$\Rightarrow$ radiative corrections can be absorbed into the couplings:

$\Rightarrow$ **effective couplings**

Zll-**vertex**

tree-level:

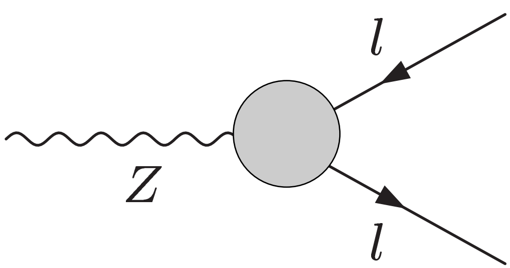


matrix element:

$$\mathcal{M}_{born} \;=\; \bar{u}\gamma_\alpha \left[g_v - g_a\gamma_5\right] v\epsilon_Z^\alpha$$

$g_{v/a}$: vector/axial born coupling

loop-order:



$$\mathcal{M}_{eff} \;=\; \bar{u}\gamma_\alpha \left[\mathcal{G}_V^{eff} - \mathcal{G}_A^{eff}\gamma_5\right] v\epsilon_Z^\alpha$$

$G_{V/A}^{eff}$: vector/axial effective coupling

effective leptonic mixing angle

- tree level: $\sin^2\Theta_W = \frac{1}{4}\left(1 - \frac{g_v}{g_a}\right)$

- def.: $\sin^2\Theta^{lept}_{eff} := \frac{1}{4}\left(1 - \Re\left\{\frac{\mathcal{G}^{eff}_V}{\mathcal{G}^{eff}_A}\right\}\right)$

- very sensitive to M_H

- exp.: asymmetries of the Z resonance
  $\sin^2\Theta^{lept}_{eff} = 0.23150 \pm 0.00016$

- linear collider: expected precision 1.3×10^{-5}!

$\Rightarrow$ precise theoretical prediction needed

existing calculations (summer 2004)

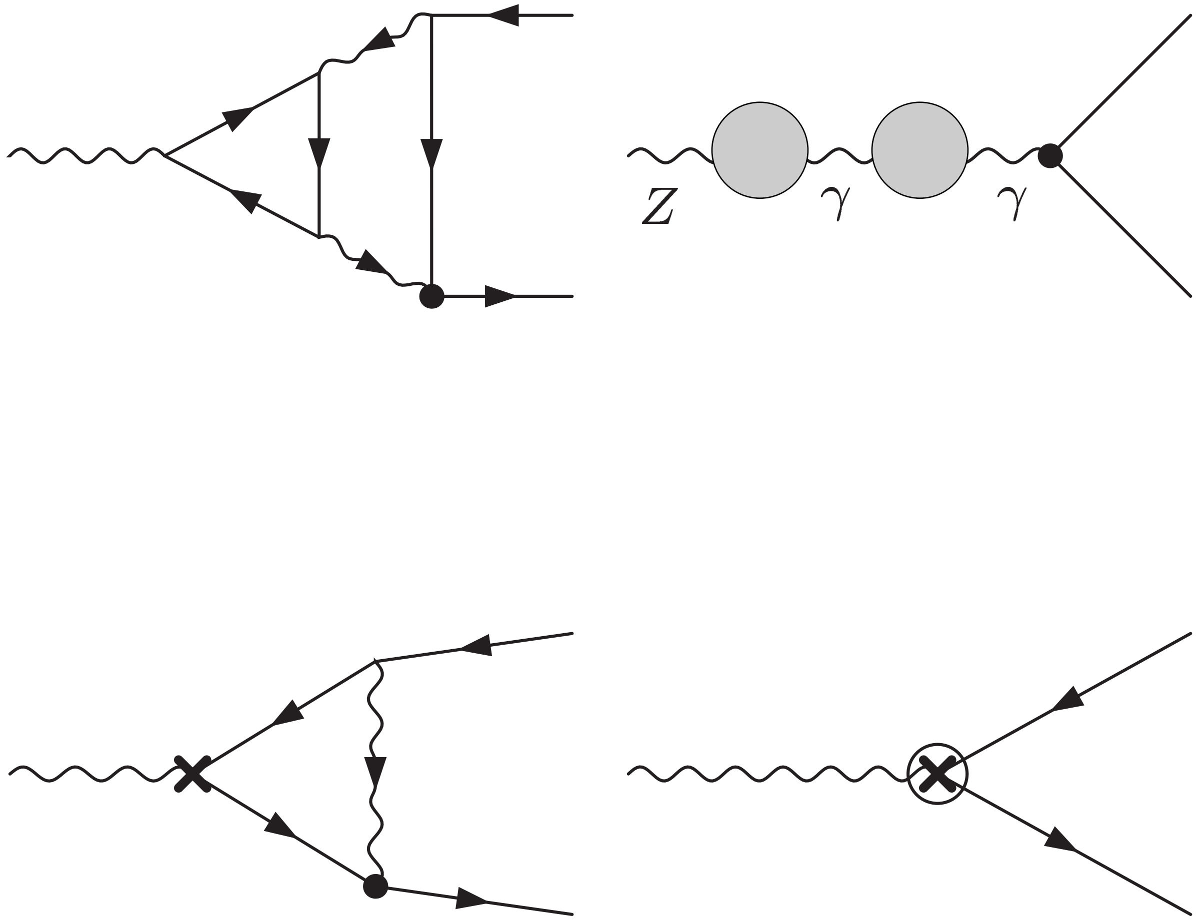
- One loop corrections $\mathcal{O}(\alpha)$
  Marciano, Sirlin '80

- QCD corrections $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}\left(\alpha\alpha_s^2\right)$
  Djouadi '88
  Chetyrkin, Kühn, Steinhauser '95

- electroweak 2 loop corrections $\mathcal{O}\left(\alpha^2\right)$
  - 2 gauge-invariant subsets:
    fermionic/bosonic corrections
  - only leading terms in a top-mass expansion $\mathcal{O}\left(\alpha^2 M_t^4\right)$ and $\mathcal{O}\left(\alpha^2 M_t^2\right)$
    Degrassi, Gambino, Sirlin '97

$\Rightarrow$ complete $\mathcal{O}\left(\alpha^2\right)$ corrections necessary

typical diagrams



Calculation of two-loop corrections

- problems:
  - 2-loop renormalization
  - 2-loop vertices

- strategy: split into two UV-finite parts

$$\hat{\Gamma}^{Z\bar{l}l}_{(2)}\left(M_Z^2\right) = \Gamma^{Z\bar{l}l}_{(2)}\left(M_Z^2\right) + \delta Z^{Z\bar{l}l}_{(2)} =$$

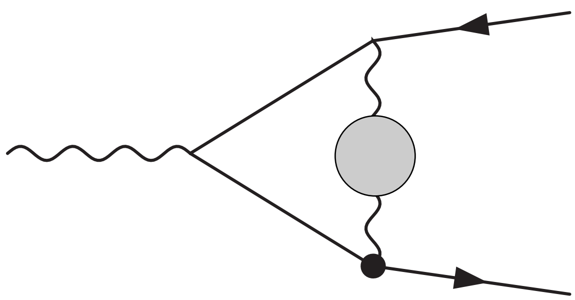
$$\underbrace{\left(\Gamma^{Z\bar{l}l}_{(2)}(0) + \delta Z^{Z\bar{l}l}_{(2)}\right)}_{\text{finite}} + \underbrace{\left(\Gamma^{Z\bar{l}l}_{(2)}\left(M_Z^2\right) - \Gamma^{Z\bar{l}l}_{(2)}(0)\right)}_{\text{finite}}$$

  - first term : complete 2-loop renormalization but no 2-loop vertices
  - second term : all 2-loop vertices but simple divergence-structure

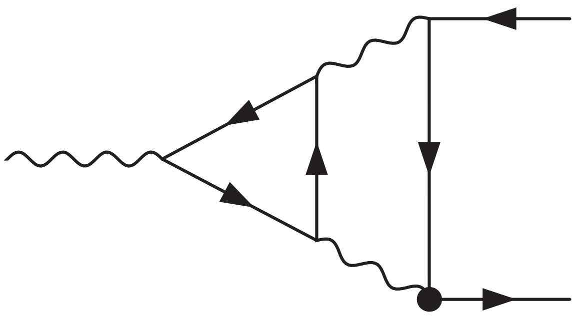
Calculation of two-loop corrections

- main problem: 2-loop vertices
- different approaches to calculate them
- fermionic contributions calculated
  Awramik, Czakon, Freitas, Weiglein '04
- new: independent calculation
  with different methods

typical diagrams



dispersion relation
$\rightarrow$ one-dimensional integral representation



Feynman-parameters, analytical manipulations
Ferroglia, Passera, Passarino, Uccirati
$\rightarrow$ up to 4-dimensional integration

results for $\Delta\kappa$ $\left([\Delta\kappa] = 10^{-4}\right)$

$\sin^2\theta_{eff}^{lept} =: (1 - M_W^2/M_Z^2)\,(1 + \Delta\kappa).$

$M_H\,[GeV]$	$\mathcal{O}(\alpha)$	$\mathcal{O}\left(\alpha^2\right)$	prev. calc.
100	438.937	-0.633(1)	-0.63
200	419.599	-2.161(1)	-2.16
600	379.560	-5.008(1)	-5.01
1000	358.619	-4.733(1)	-4.73

$M_H\,[GeV]$	2 ferm. loops	1 ferm. loop
100	13.758	-14.391(1)
200	13.758	-15.919(1)
600	13.758	-18.766(1)
1000	13.758	-18.491(1)

$M_W = 80.426$ GeV, $M_Z = 91.1876$ GeV,
$\Gamma_Z = 2.4952$ GeV, $m_t = 178.0$ GeV,
$\Delta\alpha\left(M_Z^2\right) = 0.05907$, $\alpha_s\left(M_Z^2\right) = 0.117$,
$G_\mu = 1.16637 \times 10^{-5}$.

Conclusions and outlook

- effective leptonic weak mixing angle
  is an important precision observable

- electroweak 2-loop corrections
  are needed

- we performed an independent calculation
  of the fermionic corrections in agreement
  with Awramik, Czakon, Freitas, Weiglein

- bosonic corrections will be approached
  with the same methods