vNRQCD

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- Theory to Calculate $\sigma_{\rm tot}(e^+e^- \to t\,\bar{t}\,)$ at Threshold
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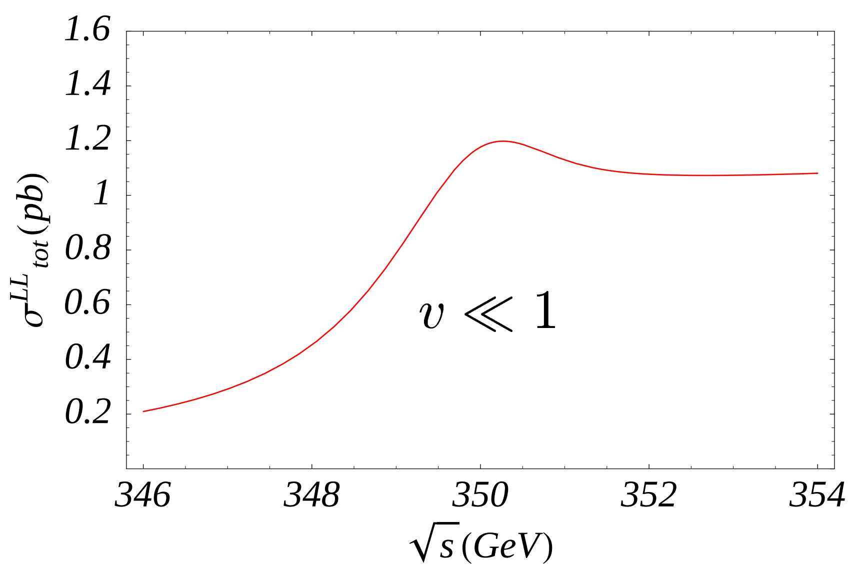
Motivation for Top Quark Threshold Physics

Top Physics at the ILC:

Main focus: $t\,\bar{t}$ production at threshold, e.g. $e^+e^- \to t\,\bar{t}$

$\Gamma_t \approx 1.5\,GeV \gg \Lambda_{QCD} \quad \Rightarrow$ non-perturbative effects suppressed, no resonance peaks expected



Aims:

$m_t \quad \leftarrow$ position of rise

$\delta m_t \sim 100 MeV$ ✓

$y_t, \alpha_s, \Gamma_t \quad \leftarrow$ shape, normalization

$\frac{\delta\sigma^{theo}_{\rm tot}}{\sigma_{\rm tot}} \lesssim 2-3\,\%$ required !

Theoretical Problems at Threshold

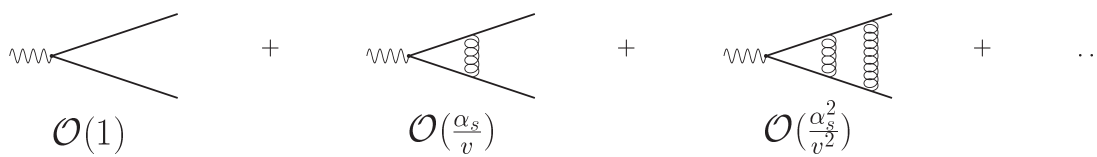
3 Scales: $m_t \gg |\vec{p}| \sim m_t v \gg E_{kin} \sim m_t v^2 \quad (\sim \Gamma_t \gg \Lambda_{QCD})$

(soft) (ultrasoft)

$\Rightarrow$ Two (main) problems :

1. Problem: **Coulomb singularities**

threshold $\Rightarrow v \sim \alpha_s \sim 0.1 \Rightarrow$ breakdown of perturbation theory:



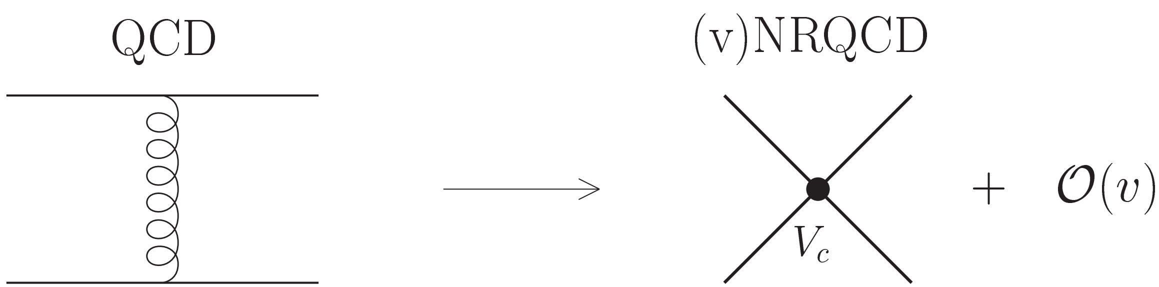
Solution: non-rel. field theory (NRQCD) $\rightarrow$ summation of $\left(\frac{\alpha_s}{v}\right)^n$- terms in a Schrödinger formalism

Theoretical Problems at Threshold

Solution to the problem with Coulomb singularities

- Construction of a non-rel. theory (NRQCD):

  “integrate out” non-resonant degrees of freedom, e.g.:



“potential” gluon integrated out

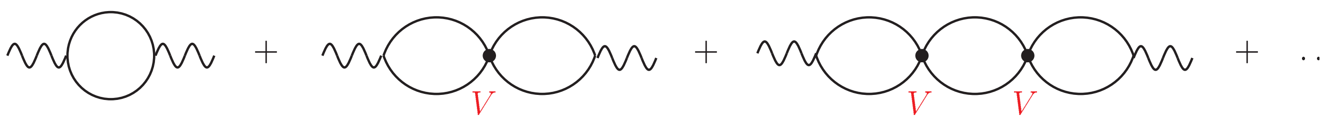
$$\Rightarrow \quad \mathcal{L}_{\rm non-rel.} = \mathcal{L}_{\rm kin} + \left[V_c^{(0)} + V^{(1)} + V^{(2)} + \ldots \right] \psi^\dagger \psi \, \chi^\dagger \chi + \ldots$$

- Separate center-of-mass motion

  $\Rightarrow$ Schrödinger Eq.: $\quad E\,\Psi = \left[\frac{\mathbf{p}^2}{m} + V_c^{(0)} + V^{(1)} + \ldots \right] \Psi$

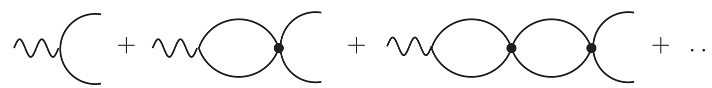
Theoretical Problems at Threshold

- Green function: $\left[-\frac{\nabla_{\vec{r}}^2}{m} + V - E \right] G(\vec{r}, \vec{r'}, E) \;=\; \delta^{(3)}(\vec{r} - \vec{r'})$

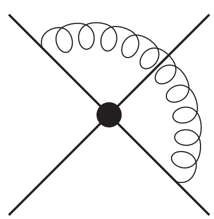


$\sim G(0,0,E)$

- Optical theorem $\Rightarrow$

$\sigma_{\text{tot}} \sim \int d\text{PS} \,\Big|$  $\Big|^2 \sim \text{Im}\left[G(0,0,E)\right]$

- Running of potentials, e.g.:



$$\Rightarrow\; V \;=\; V(\mu) \quad \Rightarrow \quad G(0,0,E) \;=\; G(0,0,E,\mu)$$

Theoretical Problems at Threshold

3 Scales: $m_t \gg |\vec{p}| \sim m_t v \gg E_{kin} \sim m_t v^2 \quad (\sim \Gamma_t \gg \Lambda_{QCD})$

(soft) (ultrasoft)

2. Problem: large logarithms

log's at dyn. scales: $\mathrm{Ln}\left(\frac{m^2}{E^2}\right)$, $\mathrm{Ln}\left(\frac{m^2}{p^2}\right)$, $\mathrm{Ln}\left(\frac{p^2}{E^2}\right)$ z.B $\alpha_s \mathrm{Ln}\left(\frac{m^2}{E^2}\right) \sim 1$

Solution: vNRQCD (effective QFT)

Status of the Theory

Effective Theory:

Production current (cm. frame): $j_{\mu}(x) \;\rightarrow\; c_1(\mu)\cdot \vec{j}_1^{\,\text{eff}}(x) \;+\; \ldots$

$$\sigma_{\text{tot}} \;\sim\; |c_1(\mu)|^2 \cdot \text{Im}\Big[-i\int d^4x\; e^{i\hat{q}x}\,\langle 0|\,T\,\vec{j}_1^{\,\text{eff}}(x)\,\vec{j}_1^{\,\text{eff}}(0)\,|0\rangle\Big]$$

$$\sim\; |c_1(\mu)|^2 \cdot \text{Im}\big[G(0,0,E,\mu)\big]$$

$G(0,0,E,\mu)$:

LL ✓ $\leftarrow \left(\frac{\alpha_s}{v}\right)^n,\ \left(\alpha_s \text{Ln}(v)\right)^m$

NLL ✓ $\leftarrow \alpha_s\left(\frac{\alpha_s}{v}\right)^n,\ \alpha_s\left(\alpha_s \text{Ln}(v)\right)^m$

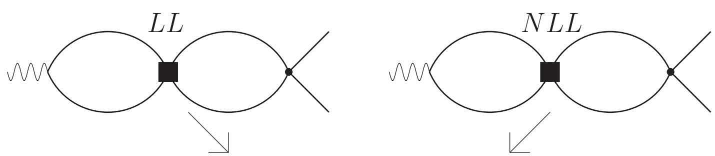
NNLL ✓ $\leftarrow \alpha_s^2\left(\frac{\alpha_s}{v}\right)^n,\ \alpha_s^2\left(\alpha_s \text{Ln}(v)\right)^m$

[Hoang, Manohar, Stewart, Teubner, Penin et al., Beneke et al.]

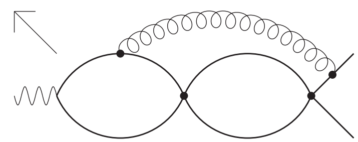
Status of the Theory

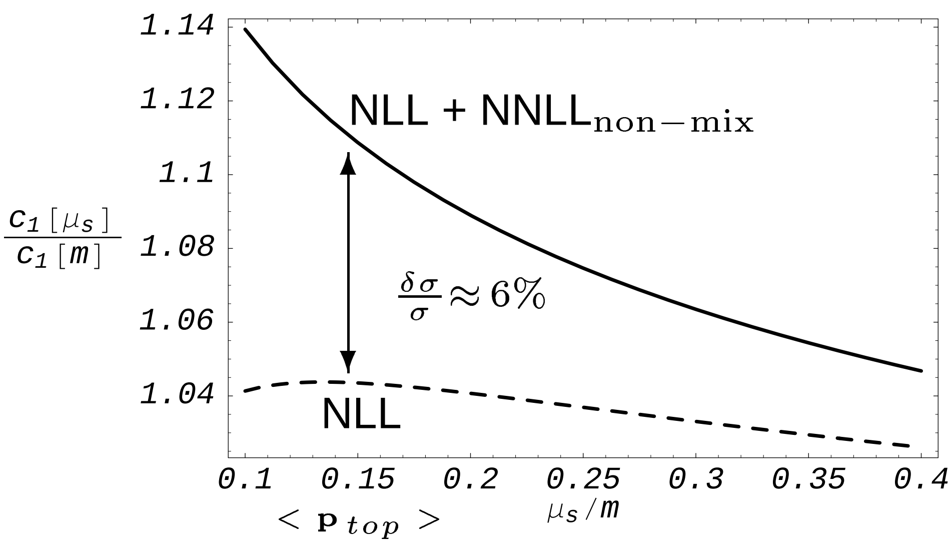
vNRQCD-Calculation of $c_1(\mu)$: (LL) ✓, NLL ✓, NNLL incomplete!

[Hoang, Luke, Manohar, Rothstein]



$$\mathrm{Ln}\left[\frac{c_1(\mu)}{c_1(1)}\right] = \xi^{\mathrm{NLL}} + \xi^{\mathrm{NNLL}}_{\mathrm{mix}} + \xi^{\mathrm{NNLL}}_{\mathrm{non-mix}}$$





large NNLL correction ?

First step:

ultrasoft part of $\xi^{\mathrm{NNLL}}_{\mathrm{mix}} \rightarrow V^{NLL}_{\mathcal{O}(v^2)}$

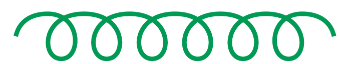
this talk: light quark corrections

vNRQCD

- Resonant degrees of freedom:

  non-rel. quark: $\psi_{\mathbf{p}}(x)$

  soft gluon: $(k^\mu \sim mv)$ $A_q(x)$ 

  ultrasoft gluon: $(k^\mu \sim mv^2)$ $A(x)$ 

- Non-resonant effects have been integrated out!

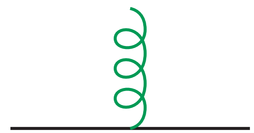
- Systematic expansion in v $\rightarrow$ consistent power counting

- Two renormalization scales: $\mu_s \sim mv\,,\ \mu_u \sim mv^2$

  $\rightarrow$ correlation: $\mu_u = \frac{\mu_s^2}{m}$ $\Rightarrow$ "v"NRQCD

vNRQCD

$$\mathcal{L}_{\rm vNRQCD} = \mathcal{L}_{\rm us} + \mathcal{L}_{\rm pot} + \mathcal{L}_{\rm soft} + \ldots$$

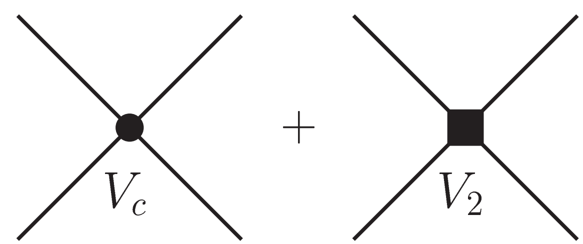


- $\mathcal{L}_{\rm us} \sim \psi^\dagger_{\mathbf{p}}(x) \left\{ iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m} + \ldots \right\} \psi_{\mathbf{p}}(x) - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \ldots$

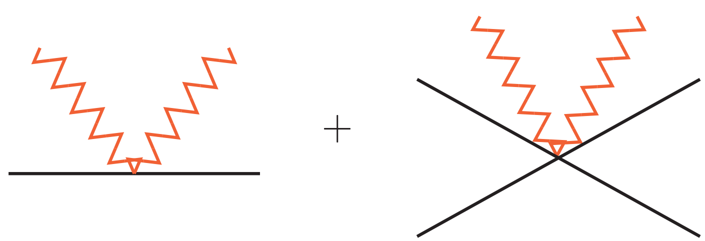
  with $D^\mu = \partial^\mu + igA^\mu(x)$

- $\mathcal{L}_{\rm pot} \sim -V\, \psi^\dagger_{\mathbf{p}'} \psi_{\mathbf{p}} \chi^\dagger_{-\mathbf{p}'} \chi_{-\mathbf{p}} + \ldots$

  $$\boxed{V \sim \frac{\mathcal{V}_c}{(\mathbf{p} - \mathbf{p}')^2} + \frac{\mathcal{V}_2}{m^2} + \ldots}$$ $\to$



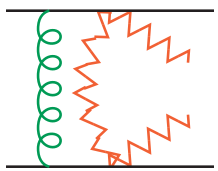
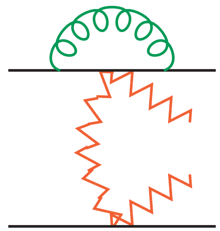
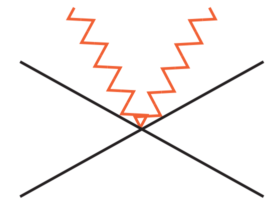
- $\mathcal{L}_{\rm soft} \to$

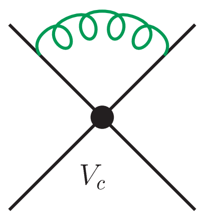


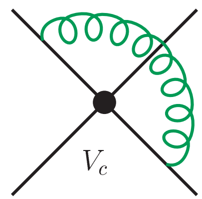
Diagrammatic Sample Calculation

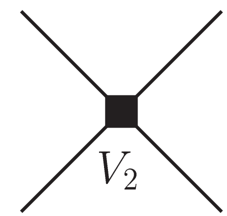
Anomalous dimension of $\mathcal{O}(\frac{1}{m^2})$-potential $\mathcal{V}_2(\mu)$:

One loop (LL):

- (A)  +  $+ \ldots \Rightarrow \delta\mathcal{C}_{2i} \xrightarrow{RGE} \mathcal{C}_{2i}^{LL}(\mu)$ 

- (B)  + 

+ 

$+ \ldots \Rightarrow \delta\mathcal{V}_2 \xrightarrow{RGE} \mathcal{V}_2^{LL}(\mu)$ 

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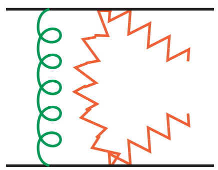
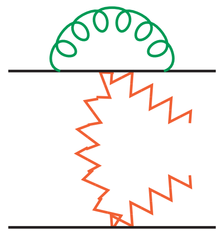
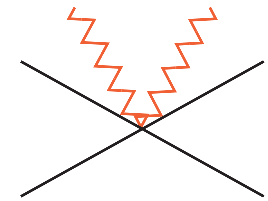
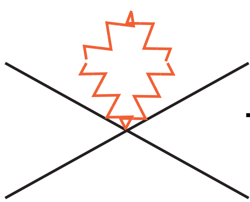
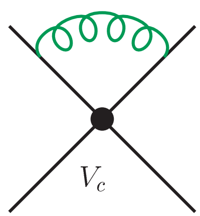
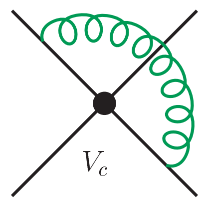
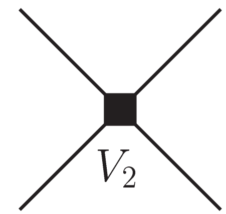
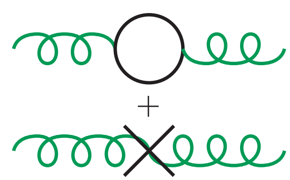
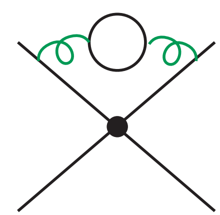
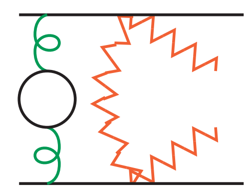
- (A)  +  + $\ldots \Rightarrow \delta\mathcal{C}_{2i} \xrightarrow{RGE} \mathcal{C}_{2i}^{LL}(\mu)$ 

- (B)  + 

+ 

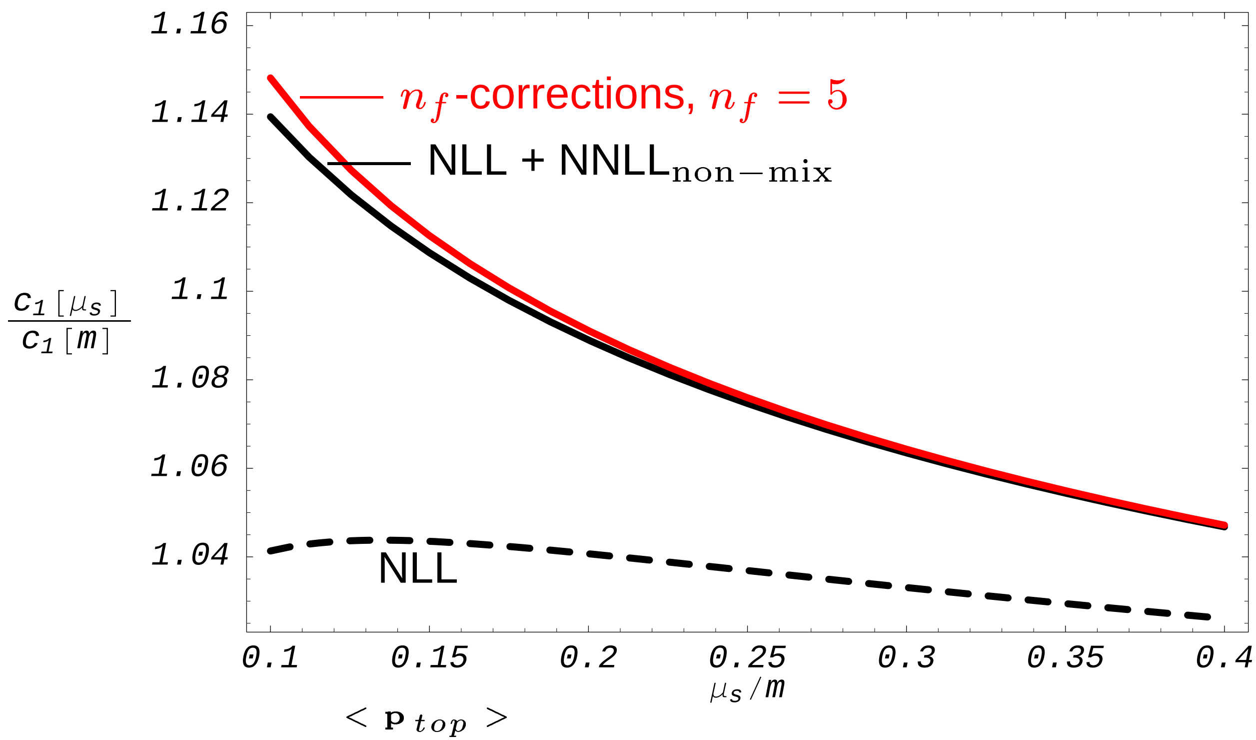
+ $\ldots \Rightarrow \delta\mathcal{V}_2 \xrightarrow{RGE} \mathcal{V}_2^{LL}(\mu)$ 

<u>Two loop (NLL):</u>

- Ultrasoft n_f-corrections:  $\rightarrow$  e.g.:  , 
  $\Rightarrow \mathcal{V}_2^{NLL}(\mu)$, $\ldots$

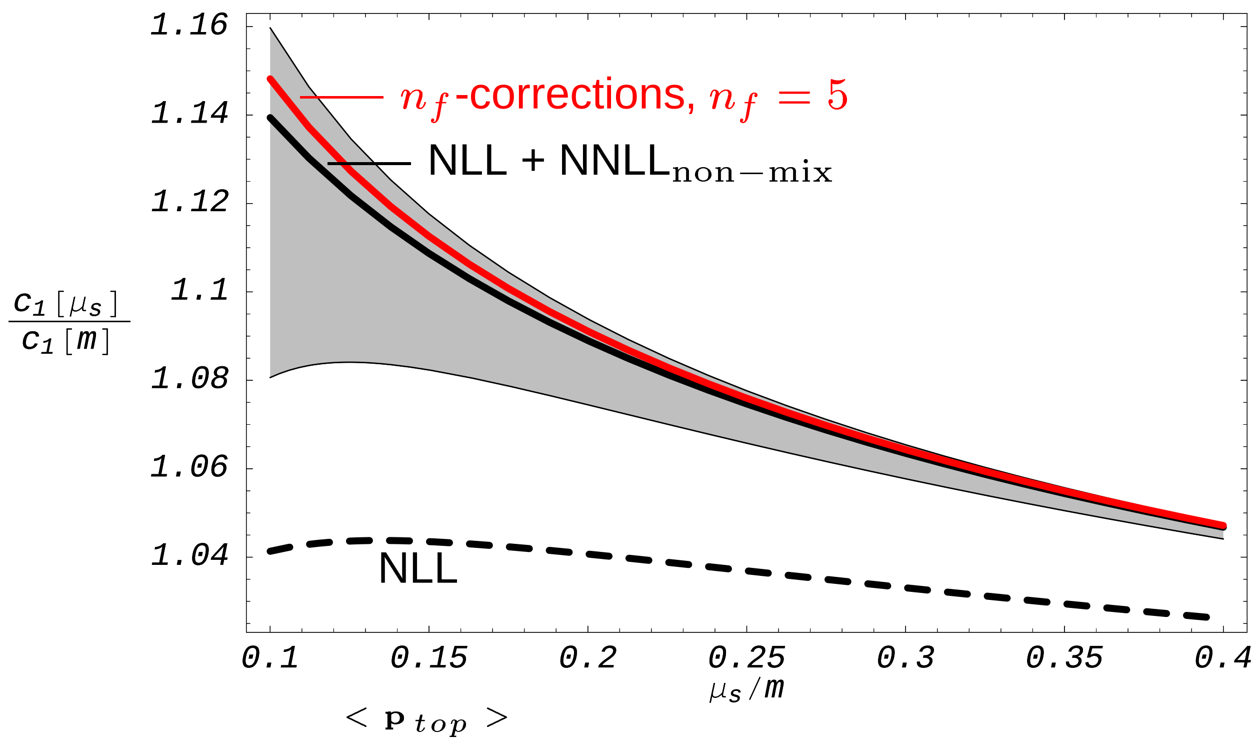
$\Rightarrow$ Contribution to $\xi_{\text{mix}}^{\text{NNLL}}$ (NNLL-running of production current)

Numerical Result



- Estimation of full us-contributions via “naive non-Abelianization”
- Us-n_f-corrections behave “normal”
- Next step: calculation of full us-contributions

Numerical Result



non-fermionic contributions:

$$-\tfrac{2}{3}n_f \to \beta_0(1 \pm 2)$$

$(\beta_0 = 11 - \tfrac{2}{3}n_f)$

- Estimation of full us-contributions via “naive non-Abelianization”
- Us-n_f-corrections behave “normal”
- Next step: calculation of full us-contributions

Summary

- Top pair production: $\frac{\delta\sigma}{\sigma} \lesssim 2 - 3\,\%$ required
  $\rightarrow\ y_t,\ \alpha_s,\ \Gamma_t$

- $\sigma \sim |c_1(\mu)|^2 \cdot \mathrm{Im}\left[G(0,0,E)\right]$, G: NNLL ✓

- $c_1(\mu)$: us-n_f-part of $\xi_{\rm mix}^{\rm NNLL}$ computed ✓ (first step)

- Next step: calculate us-non-fermionic-part of $\xi_{\rm mix}^{\rm NNLL}$ . . .