Top Instability and Electroweak Effects at the $t\bar{t}$ Production Threshold

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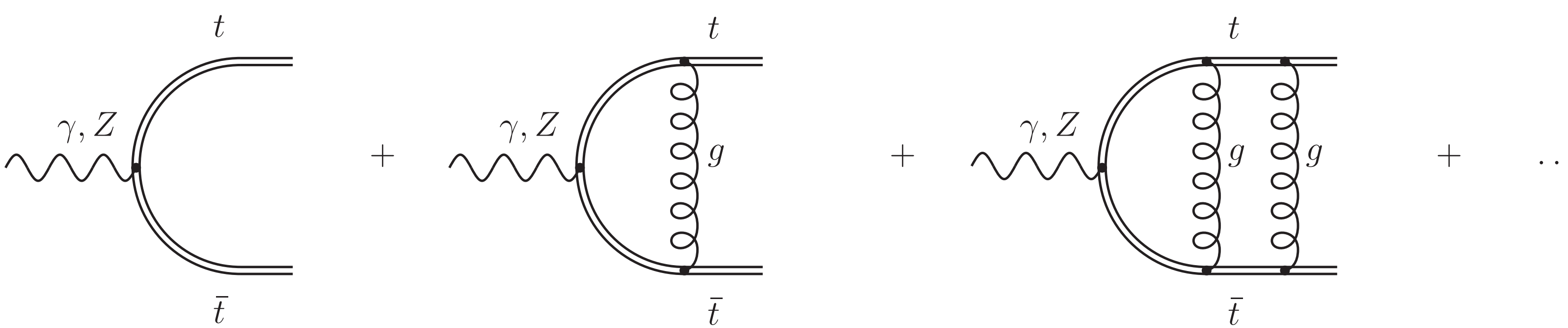
Outline

- Motivation
- Status for $\sigma_{\mathrm{tot}}(\mathrm{e^+e^-} \to \mathrm{t\bar{t}})$ at threshold
- Non-relativistic QCD (vNRQCD)
- Effects of the top instability
- (Usual) electroweak effects
- Numerical analysis
- Summary and Outlook

$t\bar{t}$ Production Threshold

<u>e^+e^- collisions:</u> c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are non-relativistic $\boxed{v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1}$



$\sim 1 \qquad \sim \frac{\alpha_s}{v} \qquad \sim \left(\frac{\alpha_s}{v}\right)^2$

$\Rightarrow$ Perturbation theory in α_s breaks down $\boxed{v \sim \alpha_s}$

$\Rightarrow$ Non-relativistic QCD $\simeq$ Schrödinger theory at LO

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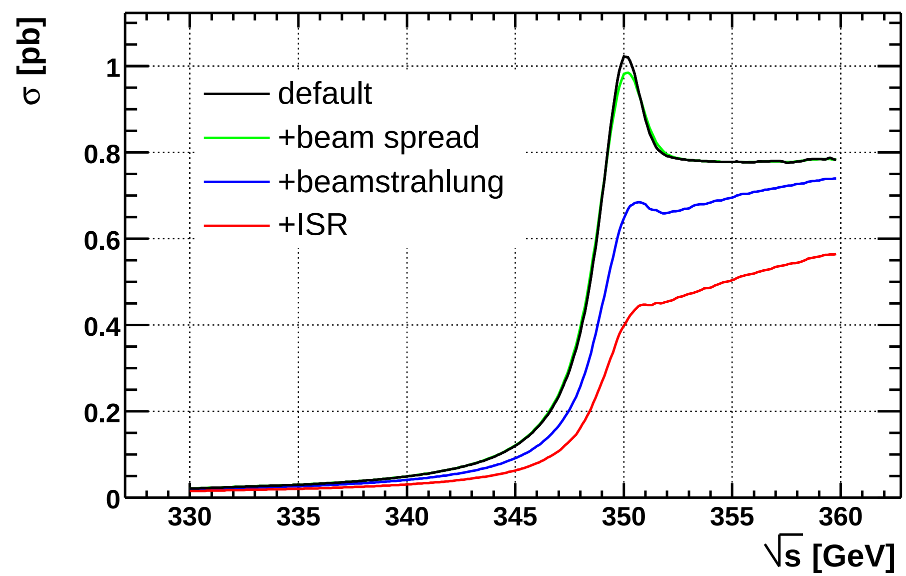
- Top quarks decay fast $\boxed{\Gamma_t \approx 1.5\ \text{GeV} \gg \Lambda_{QCD}}$

$\Rightarrow$ No bound states

$\Rightarrow$ Smooth line-shape

$\Rightarrow$ Non-perturbative effects suppressed

[Fadin,Khoze]



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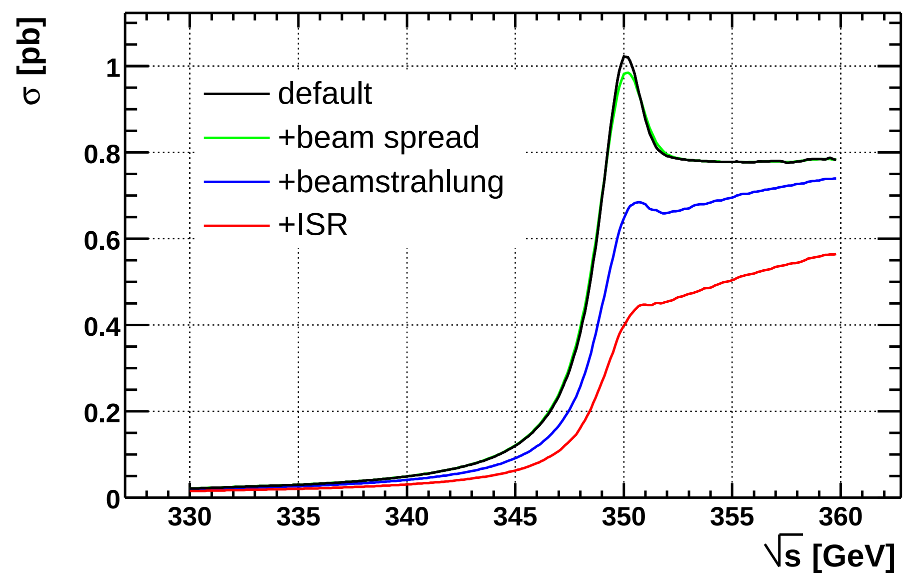
- Top quarks decay fast $\boxed{\Gamma_t \approx 1.5\ \text{GeV} \gg \Lambda_{\text{QCD}}}$
- Measured cross section $\boxed{\sigma^{\text{obs}}(s) = \int_0^1 dx\, \mathcal{L}(x) \sigma^{\text{theo}}(x^2 s)}$ contains
  - beam spread
  - beamstrahlung
  - ISR



Measurements

Simulations of Threshold Scan ($\mathcal{L} \sim 300\,\mathrm{fb}^{-1}$):
[Martinez, Miquel]

- Top quark mass

$$(\delta m_t)^{\exp} \sim 50 \text{ MeV}$$

- Top Yukawa coupling

$$(\delta y_t / y_t)^{\exp} \sim 0.35 \text{ (light Higgs)}$$

- Top decay width

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$\Rightarrow$ Theory needs

$$\boxed{(\delta\sigma_{\mathrm{tot}}/\sigma_{\mathrm{tot}}) \leq 3\,\%}$$

Theory Status

RGE methods to sum large logs $(\alpha_s \ln v)^m$

- QCD effects

LL $\checkmark$	LL $\sim \left(\frac{\alpha_s}{v}\right)^n$
NLL $\checkmark$	NLL $\sim \alpha_s \left(\frac{\alpha_s}{v}\right)^n$
NNLL (nearly) $\rightarrow \left(\frac{\delta\sigma_{\rm tot}}{\sigma_{\rm tot}}\right)^{\rm NNLL} \sim \pm 6\,\%$	NNLL $\sim \alpha_s^2 \left(\frac{\alpha_s}{v}\right)^n$

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- Electroweak effects

  LL ✓

  NLL (partly)

  NNLL ?

Theory Status

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- Electroweak effects

  LL ✓

  NLL (partly) $\rightarrow$ New parametric NLL corrections

  NNLL ? $\rightarrow$ NNLL top decay corrections

vNRQCD (stable quarks)

[Hoang, Luke, Manohar, Rothstein, Stewart]

Relevant scales

$$m_t \text{ (hard)} \quad \gg \quad \mathbf{p} \sim m_t v \text{ (soft)} \quad \gg \quad E \sim m_t v^2 \text{ (ultrasoft)}$$

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Lagrangian (LL) $\qquad \mathcal{L} = \mathcal{L}_{\text{bilinear}} + \mathcal{L}_{\text{potential}}$

$$\mathcal{L}_{\text{bilinear}} = \psi_{\mathbf{p}}^{\dagger}(\mathsf{x}) \left\{ \mathsf{i}D^0 - \frac{\mathbf{p}^2}{2\mathsf{m_t}} - \delta \mathsf{m_t} \right\} \psi_{\mathbf{p}}(\mathsf{x}) + \ldots$$

$$\mathcal{L}_{\text{potential}} = -\frac{\mathcal{V}_{\mathsf{c}}(\mu)}{(\mathbf{p}-\mathbf{p}')^2} \psi_{\mathbf{p}'}^{\dagger} \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^{\dagger} \chi_{-\mathbf{p}} + \ldots$$

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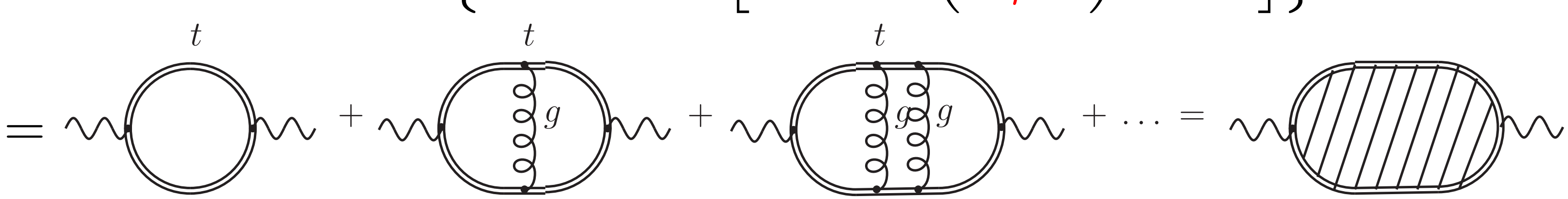
$\Rightarrow$ Schrödinger equation with Coulomb potential

$$\mathsf{G}^{\mathrm{LL}}(0, 0, \sqrt{\mathsf{s}}) = \frac{\mathsf{m_t^2}}{4\pi} \left\{ \mathsf{i\,v} - \mathsf{C_F}\alpha_{\mathsf{s}} \left[\frac{1}{4\epsilon} + \ln\left(\frac{-\mathsf{i\,m_t v}}{\mu} \right) + \ldots \right] \right\}$$



vNRQCD (stable quarks)

Currents for $\mathrm{t\bar{t}}$ production & annihilation

$$\mathbf{O_p} = \mathsf{C}(\mu) \cdot (\psi^\dagger_{\mathbf{p}}\, \boldsymbol{\sigma}\, \tilde{\chi}^*_{-\mathbf{p}}) \qquad ({}^3\mathsf{S}_1)$$

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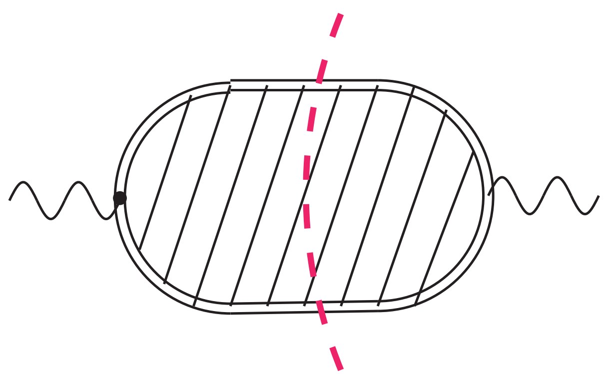
Total cross section from $e^+e^- \to e^+e^-$ using the Optical Theorem [Strassler, Peskin]

$$\sigma_{\rm tot} \quad \propto \quad \mathrm{Im}\left[\int d^4x \, e^{-i\hat{q}\cdot x} \left\langle 0 \,\middle|\, T\, \mathbf{O}^\dagger_{\mathbf{p}}(0)\, \mathbf{O}_{\mathbf{p}'}(x) \,\middle|\, 0 \right\rangle\right]$$

$$\propto \quad \mathrm{Im}\left[\mathsf{C}(\mu)^2 \, \mathsf{G}_(0, 0, \sqrt{s})\right]$$



vNRQCD (unstable quarks)

Optical Theory of absorptive medium

- Complex refractive indices in the Maxwell equations
- Can be derived systematically from microscopic processes

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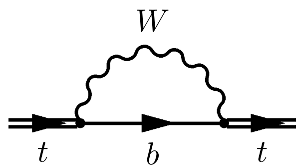
vNRQCD accounting for $t \to Wb$

- Wilson coefficients become complex
- Obtained by matching effective theory to SM
- Effective Lagrangian non-Hermitian
  $\rightarrow$ Total cross section through the optical theorem

vNRQCD (unstable quarks)

Matching conditions accounting for $t \to Wb$

Bilinear quark operators



$$= i\,\Sigma_t\,, \qquad\qquad \mathrm{Im}\,\Sigma_t = \frac{1}{2}\Gamma_t$$

$$\Rightarrow \quad \delta\mathcal{L} = \psi_{\mathbf{p}}^{\dagger}\left[i\frac{\Gamma_t}{2}\left(1 - \frac{\mathbf{p}^2}{2m_t^2}\right)\right]\psi_{\mathbf{p}}$$

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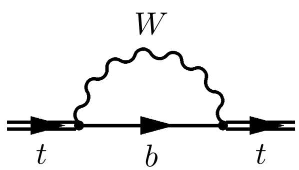

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LL NNLL

Power counting for ew. effects: $\Gamma_t \sim m_t g^2 \sim m_t \alpha_s^2 \Rightarrow \boxed{g \sim g' \sim v \sim \alpha_s}$

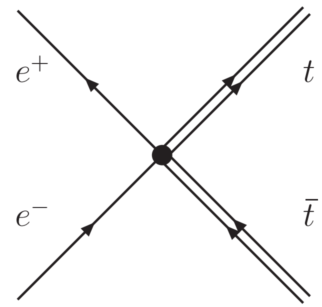
- $E \to E + i\Gamma_t$ replacement to include finite lifetime at LL [Fadin, Khoze] (not sufficient beyond LL order)
- Time dilatation is NNLL

vNRQCD (unstable quarks)

Matching conditions accounting for $\mathrm{t} \to \mathrm{Wb}$

<u>Currents</u> for $\mathrm{t\bar{t}}$ production & annihilation

$$\mathbf{O_p} = \Big[\, \mathrm{C}^{\mathrm{LL}} + \mathrm{C}^{\mathrm{NLL}} + \mathrm{C}^{\mathrm{NNLL}} + \mathrm{i}\,\mathrm{C}^{\mathrm{NNLL}}_{\mathrm{abs}} + \dots \Big] \cdot \Bigg(\quad \Bigg) + \dots$$



vNRQCD (unstable quarks)

Matching conditions accounting for $\mathsf{t} \to \mathsf{Wb}$

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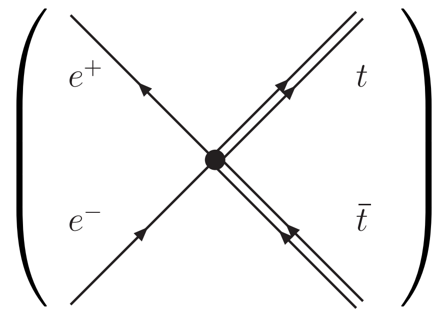

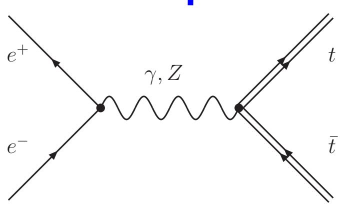

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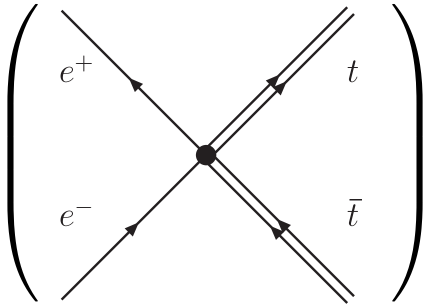
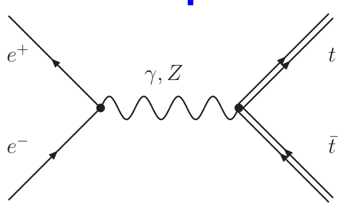
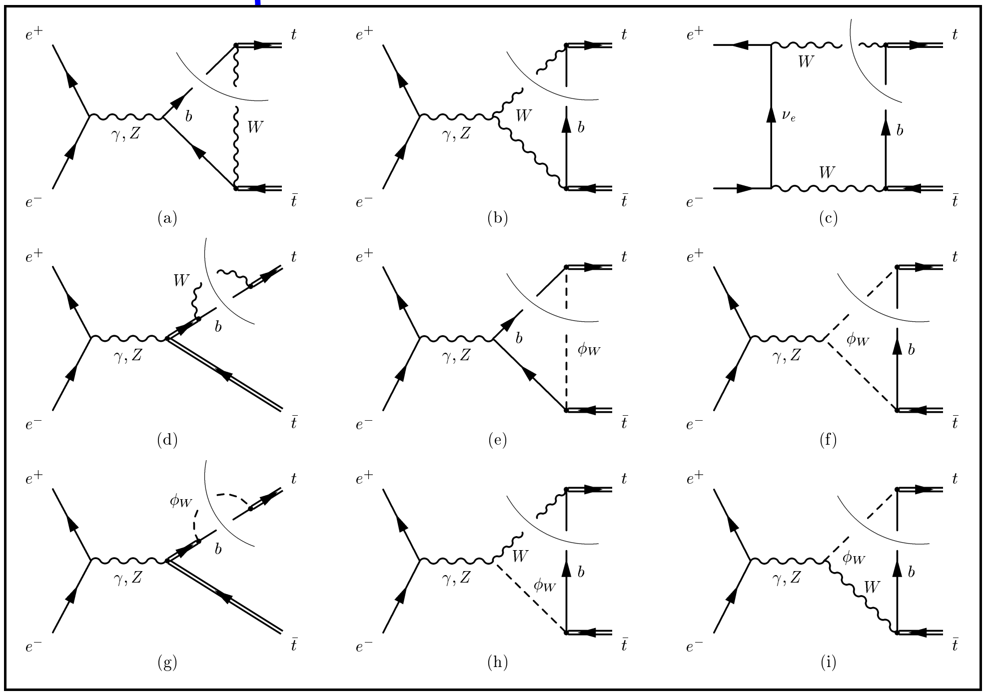


$$+ \ldots$$



> Wb-cuts account for Wb final states

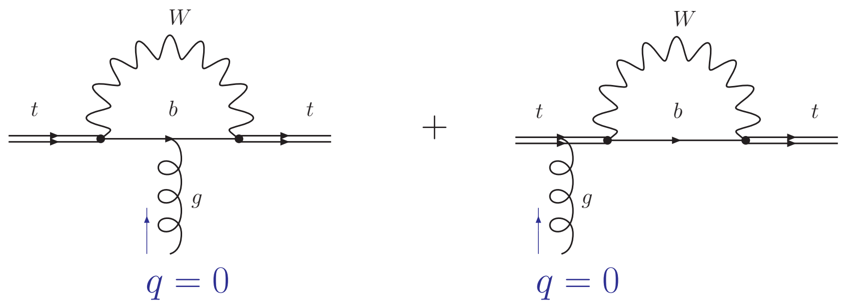
> Wb-cuts are gauge invariant



vNRQCD (unstable quarks)

Interactions in vNRQCD constrained by symmetries

Ward Identity



$= 0$

vNRQCD (unstable quarks)

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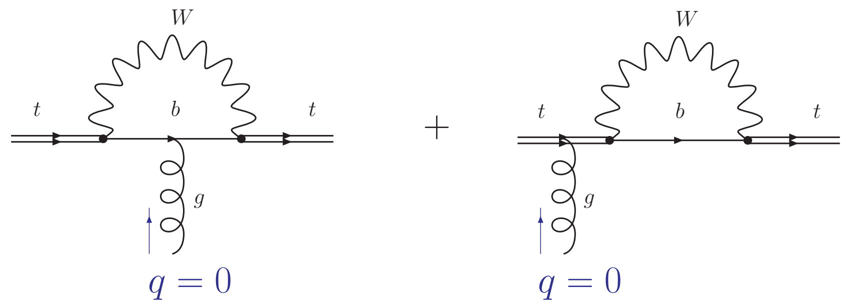
Ward Identity  $= 0$

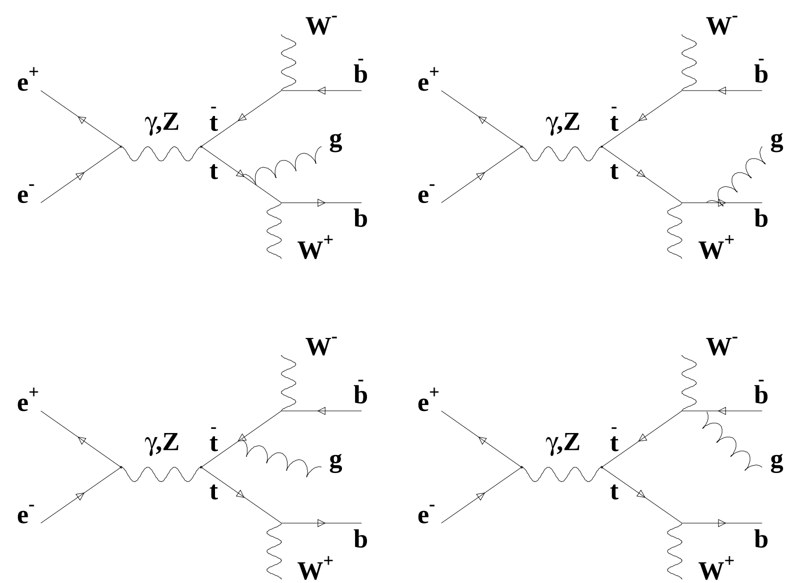
Potentials:

$\longrightarrow$ no NNLL electroweak corrections

Soft, ultrasoft interactions:

$\longrightarrow$ no NNLL electroweak corrections

$\longrightarrow$ also: no non-factorizable effects from ultrasoft gluons [Melnikov, Yakovlev]



Total Cross Section

Optical Theorem $\Rightarrow$ $\boxed{\sigma_{\text{tot}} = 2\,\mathsf{N_c}\,\mathrm{Im}\left[\,\mathsf{C}(\mu)^2\,\mathsf{G}(0,0,\sqrt{\mathsf{s}})\,\right]}$

- NNLL finite lifetime correction:

$$\Delta^{\Gamma,1}\sigma_{\text{tot}} = 2\,\mathsf{N_c}\,\mathrm{Im}\left[2\,\mathsf{C}^{\mathrm{LL}}\,\mathsf{i}\mathsf{C}^{\mathrm{NNLL}}_{\mathrm{abs}}\,\mathsf{G}^{\mathrm{LL}}(0,0,\sqrt{\mathsf{s}}) + \left(\mathsf{C}^{\mathrm{LL}}\right)^2\delta\mathsf{G}^{\mathrm{NNLL}}(0,0,\sqrt{\mathsf{s}})\right]$$

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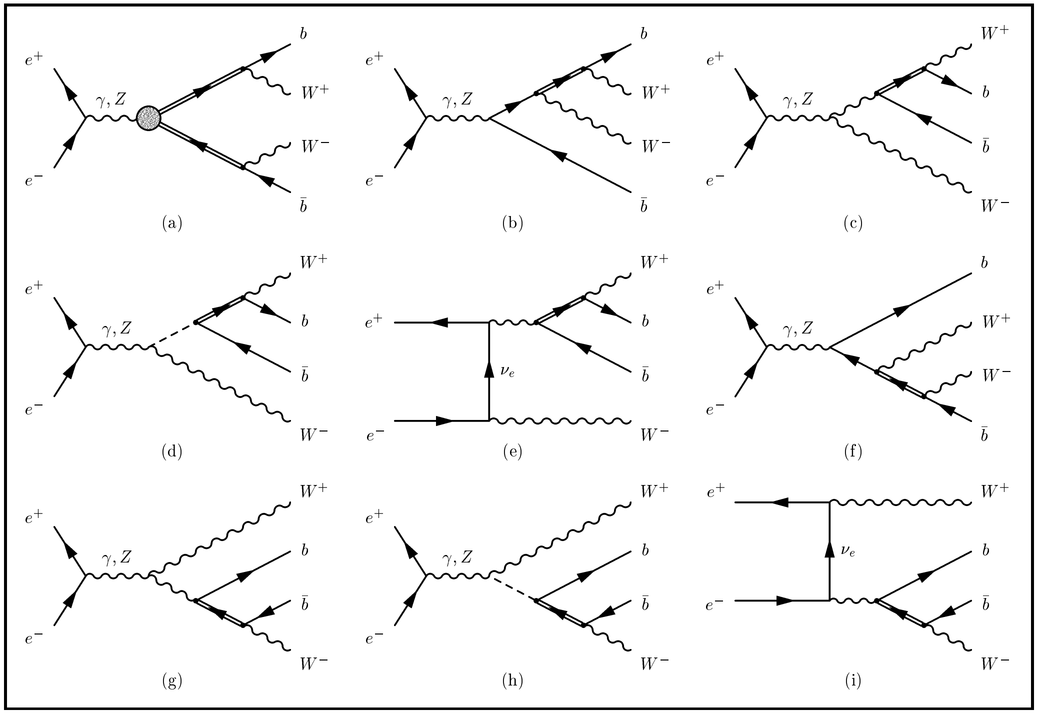

dilatation of the lifetime

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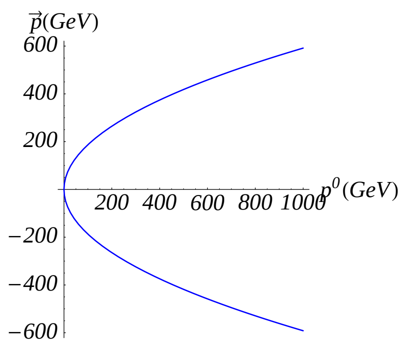
- NLL mixing effect: $\Delta^{\Gamma,1}\sigma_{\rm tot}$ contains

$\boxed{\Gamma_t\,\alpha_s\,\frac{1}{\epsilon}}$ (logarithmic UV phase space divergence due to finite lifetime)

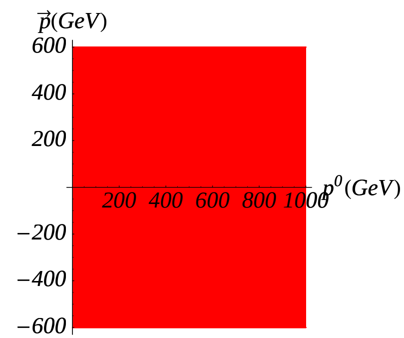
Phase space:

stable tops | unstable tops

$$\frac{1}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\epsilon} \longrightarrow$$



$$\frac{1}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}} \longrightarrow$$



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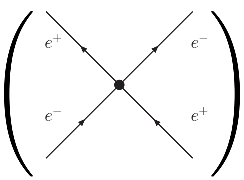
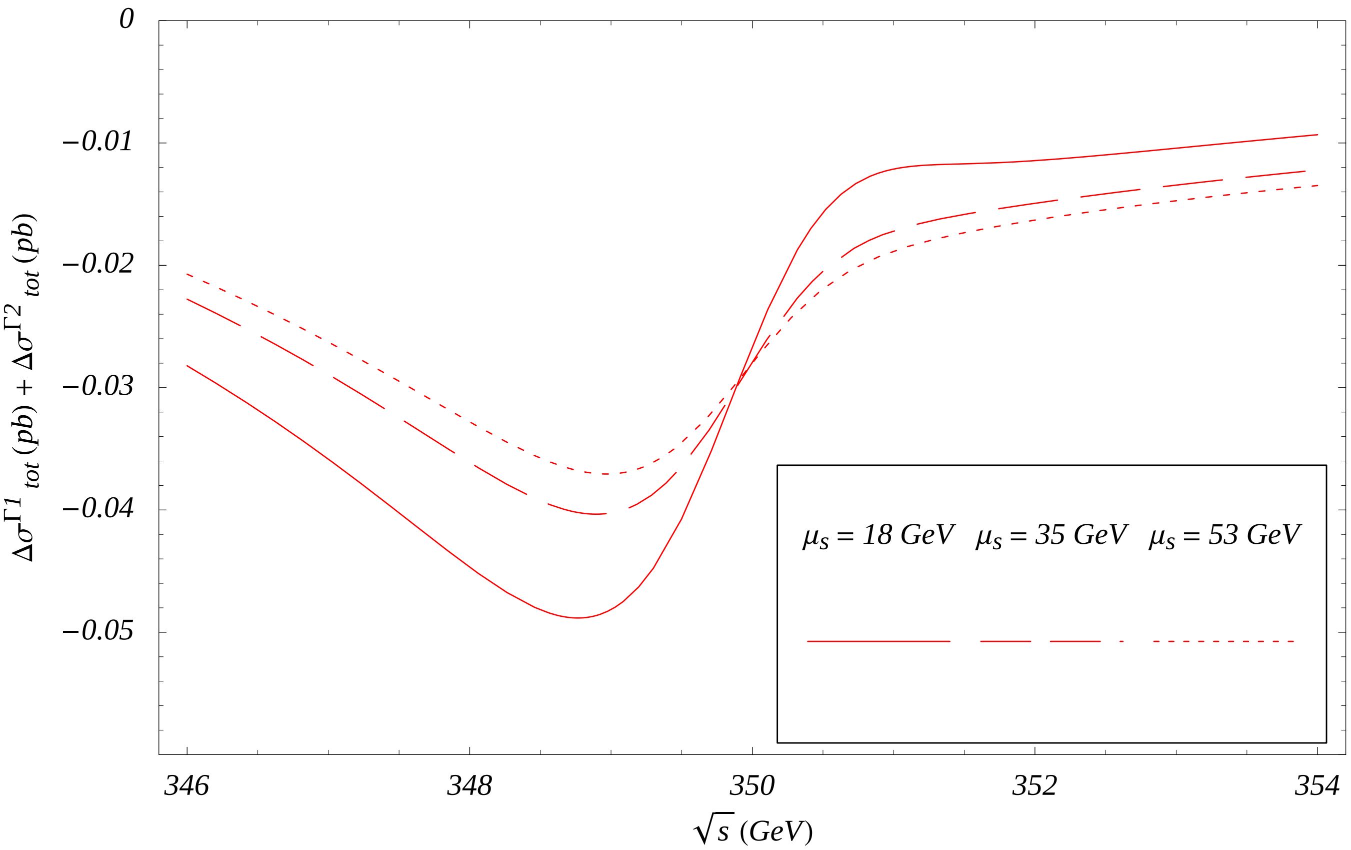
$\Rightarrow$ Anomalous dimension for $(\mathrm{e}^+\mathrm{e}^-)(\mathrm{e}^+\mathrm{e}^-)$ operator:

$\mathrm{i}\,\tilde{\mathrm{C}}^{\rm eeee}(\mu)\cdot$ 

$\Rightarrow$ Correction $\Delta^{\Gamma,2}\sigma_{\rm tot}$
- $\sqrt{\mathrm{s}}$-independent
- scale-dependent

* Matching coefficient $\tilde{\mathrm{C}}^{\rm eeee}(\mu = \mathrm{m_t})$ not yet determined $\rightarrow$ w.i.p.

Absorptive Parts



$\Rightarrow$ Comparable to NNLL QCD corrections

$\Rightarrow$ LL peak position shifted by $30 - 50$ MeV

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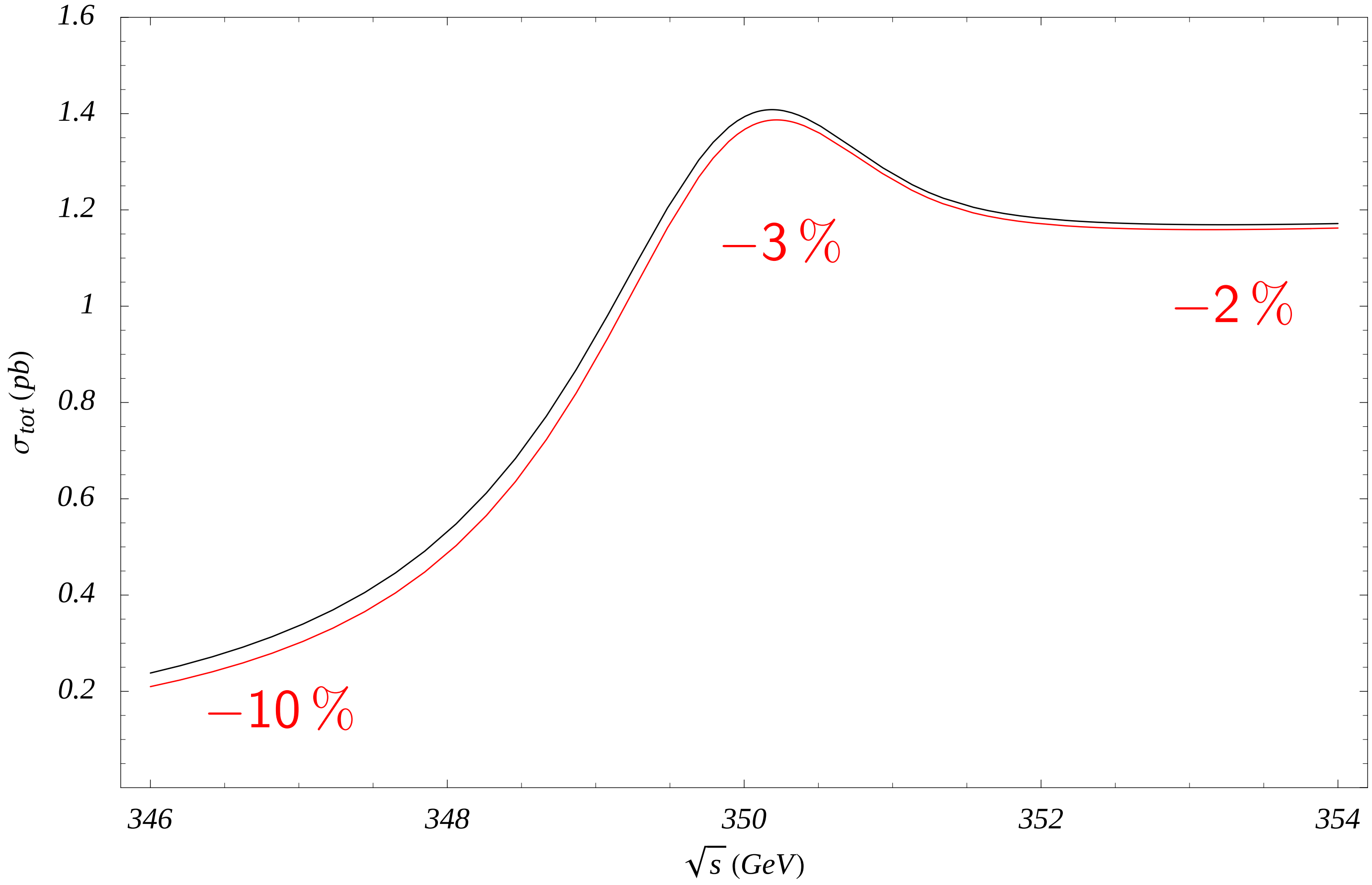

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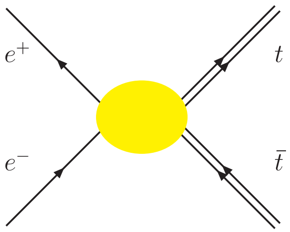
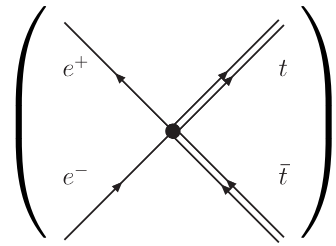
Electroweak Corrections (preliminary)

Electroweak short distance corrections: Real parts $\mathrm{C}^{\mathrm{NNLL}}$



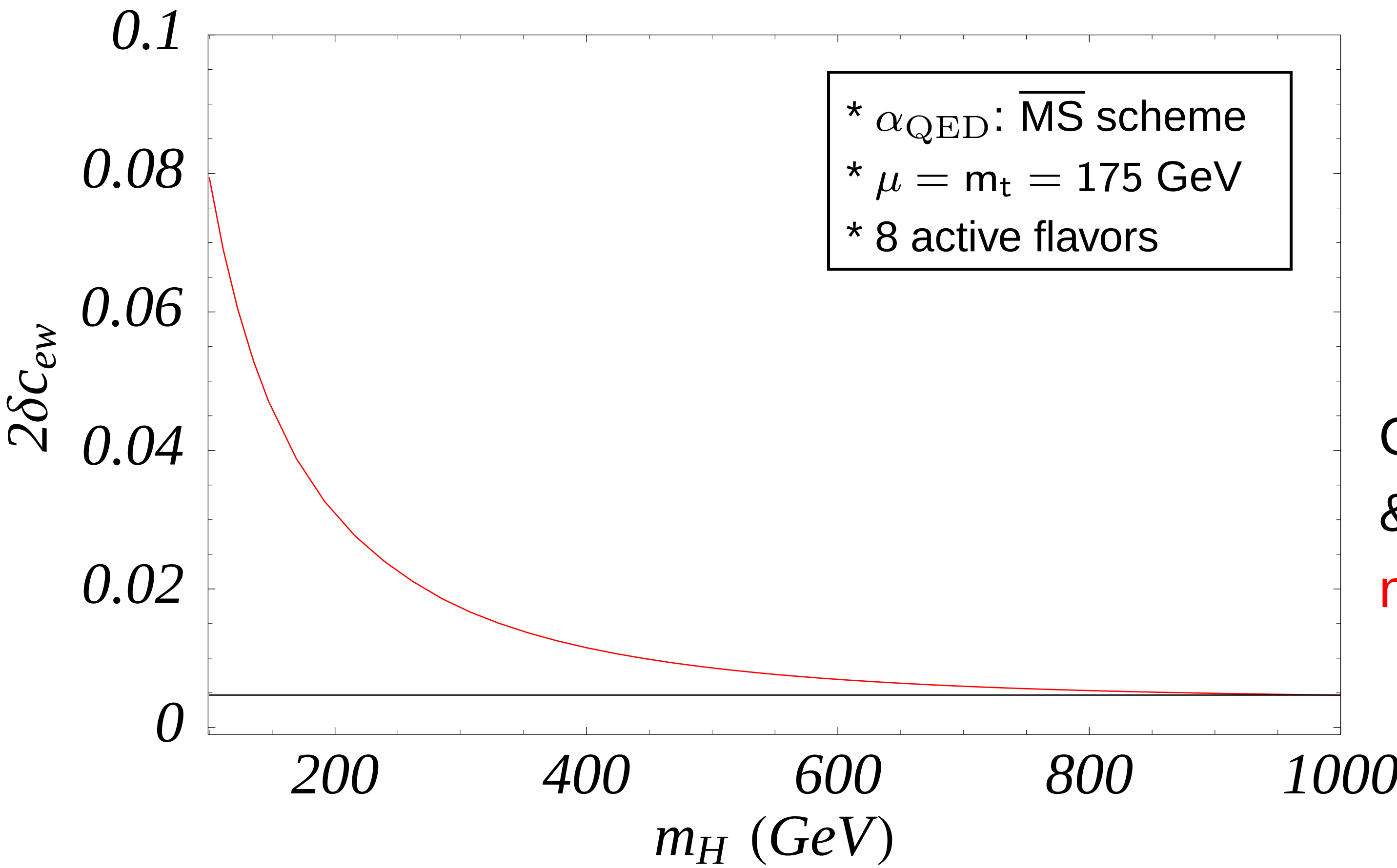
$$= \left[\mathrm{C}^{\mathrm{LL}} + \mathrm{C}^{\mathrm{NLL}} + \mathrm{C}^{\mathrm{NNLL}} + \mathrm{i}\,\mathrm{C}^{\mathrm{NNLL}}_{\mathrm{abs}} + \ldots \right] \cdot$$



- Full electroweak theory one-loop diagrams $\mathrm{O}(\alpha_{\mathrm{em}}) \rightarrow$ NNLL
- Sizable shift of total cross section normalization
- Pure QED & ISR diagrams require separate treatment (SCET) $\rightarrow$ w.i.p.

Electroweak Corrections (preliminary)

Correction to the total cross section normalization: $(1 + 2\delta c_{\rm ew})\,\sigma_{\rm tot}$



QED corrections
& real radiation
not included

Summary and Outlook

Summary

- Threshold scan allows for precise m_t, y_t, Γ_t determination
- Effective theory approach (unstable particles)
  - UV divergencies from instablility
  - Interference of resonant and non-resonant diagrams
  - Symmetries restrict interactions
  - Short distance electroweak corrections
- Corrections comparable to QCD

Outlook

- Phase space matching $\tilde{C}^{eeee}(\mu = m_t)$
- NNLL running of $\tilde{C}^{eeee}(\mu)$
- QED contributions: ISR, beamstrahlung, Coulomb singularities