# FACTORIZATION OF HEAVY-TO-LIGHT FORM FACTORS FOR NON-RELATIVISTIC BOUND STATES

[ G. Bell, Th. Feldmann - hep-ph/0509347, work in progress ]

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LMU

# OUTLINE

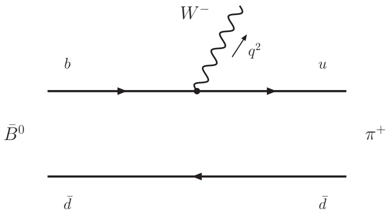
HEAVY-TO-LIGHT FORM FACTORS

NON-RELATIVISTIC APPROXIMATION

FORM FACTORS IN NR LIMIT

CONCLUSIONS

# Heavy-to-light Form Factors



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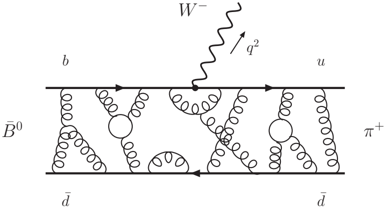


$$\langle \pi(p') | \bar{u} \gamma^\mu b | \bar{B}(p) \rangle = F_+(q^2)\,(p^\mu + p'^\mu) + F_-(q^2)\,q^\mu \qquad (q = p - p')$$

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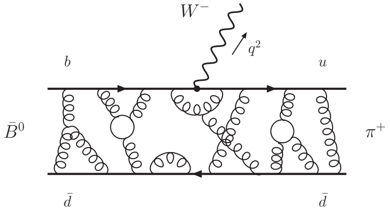


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Knowledge: crucial

- encode strong interaction effects in $B \to \pi \ell \nu, \ \ldots$
  $\rightarrow$ exclusive measurement of $|V_{ub}|$
- appear as non-perturbative input in QCD Factorization $B \to \pi\pi,\ B \to \pi K,\ \ldots$
  $\rightarrow$ important for CKM analysis [Beneke / Buchalla / Neubert / Sachrajda 99, 00]

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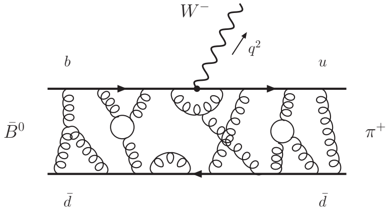


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Understanding: demanding

- contain long-distance effects $\rightarrow$ cannot be calculated in perturbation theory
- $q^2 \gtrsim 16$ GeV$^2$ Lattice QCD $\rightarrow$ energetic pions ?
- $q^2 \lesssim 16$ GeV$^2$ QCD light-cone sum rules $\rightarrow$ model dependence ?
- Alternatives ?

# Factorization

Advantage: perturbative scale $m_b \gg \Lambda_{\text{had}}$

- Form Factors contain long-distance ($\sim \Lambda$) and short-distance ($\sim m_b$) effects
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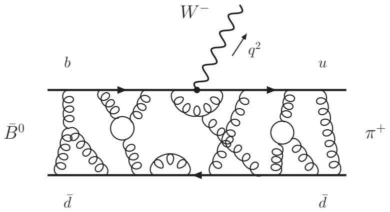
Realization: Effective Field Theories (HQET, SCET, NRQCD, ...)

- systematic power-expansion, can estimate uncertainties
- model-independent
- low energy symmetries become transparent
- resum logarithms using RG-techniques

# Physical Picture

[Bauer / Fleming / Pirjol / Stewart 00,01]
[Beneke / Chapovsky / Diehl / Feldmann 02]




?

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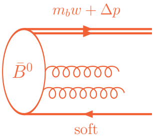


long-distance modes

- heavy quark: $\Delta p^\mu \sim \Lambda$
- soft spectators: $p_s^\mu \sim \Lambda$

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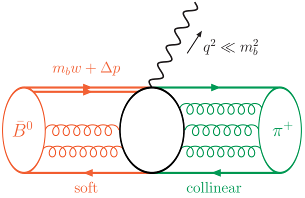


$E_\pi \sim m_b$

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- collinear quarks and gluons:
  $E_c \sim m_b, \quad p_c^2 \sim \Lambda^2$

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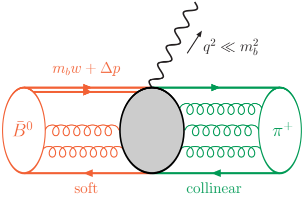


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## short-distance modes

- hard modes:
  $(\text{heavy quark} + \text{coll.})^2 \sim m_b^2$
- hard-collinear modes:
  $(\text{soft} + \text{coll.})^2 \sim m_b\Lambda$

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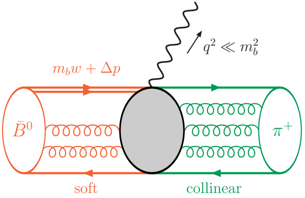


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short-distance modes

- hard modes: (heavy quark + coll.)$^2 \sim m_b^2$
- hard-collinear modes: (soft + coll.)$^2 \sim m_b \Lambda$

scales: $m_b \gg \sqrt{m_b \Lambda} \gg \Lambda$

QCD — $SCET_I$ — $HQET / SCET_{II}$

# Factorization Formula

[Beneke / Feldmann 00,03]
[Bauer / Pirjol / Stewart 02]
[Lange / Neubert 03]

Form factors $F_i(q^2) \propto \langle \pi(p') | \bar{u} \Gamma_i b | \bar{B}(p) \rangle$ for $q^2 \ll m_b^2$

$$F_i(q^2) = T_i^I(q^2)\, \xi_\pi(q^2) \;+\; \phi_B \otimes T_i^{II}(q^2) \otimes \phi_\pi \;+\; \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$$

- $T_i^I(q^2)$ perturbative hard-coefficient function
- $\xi_\pi(q^2)$ universal form factor [$\rightarrow$ symmetry relations]

  [Charles / Le Yaouanc / Oliver / Pène / Raynal 98]
- $T_i^{II}(q^2)$ perturbative hard-scattering kernel
- $\phi_B, \phi_\pi$ process-independent light-cone wave functions

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$$\left. \begin{array}{l} T_i^I(q^2) = 1 + \mathcal{O}(\alpha_s) \\ T_i^{II}(q^2) = \mathcal{O}(\alpha_s) \end{array} \right\}$$

approx. symmetry relations for $F_i(q^2)$
sym. breaking correct. perturbative (and $1/m_b$)

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QCD light-cone sum rules $\rightarrow$ $F_i(q^2)$ [Ball / Zwicky 01,04]

SCET light-cone sum rules $\rightarrow$ $\xi_\pi(q^2)$ [De Fazio / Feldmann / Hurth 05]

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non-factorizable factorizable

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$\xi_\pi(q^2)$ does not factorize into soft and collinear regions

(Feynman mechanism, endpoint singularities, Sudakov Logarithms)

# Non-relativistic Approximation

Consider NR $q\bar{q}$ bound state

- expansion in relative velocity $v = |\vec{v}| \ll 1$ in CMS

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Leading contribution in NR expansion

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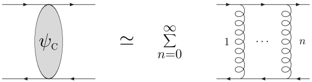


potential gluons

$$q^0 = \mathcal{O}(m_r v^2)$$
$$\vec{q} = \mathcal{O}(m_r v)$$

- properties: $M \simeq m_q + m_{\bar{q}} + E_C$

  $\psi_C(\vec{p}) \propto \frac{1}{\vec{p}^4}$ for $|\vec{p}| \gg m_r v$

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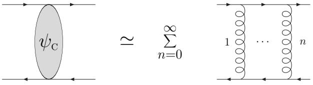


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New ingredients:

| | |
|---|---|
| NR gluons | $q \cdot w = \mathcal{O}(m_r v^2), \quad q_{\perp w} = \mathcal{O}(m_r v)$ |
| NR scale | $\mu_{NR} \sim m_r v$ |

# Heavy-to-light Form Factors in NR picture

Consider $Q\bar{q} \longrightarrow q\bar{q}$ with quark masses $M \gg m \gg \Lambda_{QCD}$

- "Heavy Meson" $M_h \simeq M + m \simeq M$ $p = M_h w$
- "Light Meson" $M_l \simeq 2m$ $p' = M_l w'$

Relevant scales:

- $\mu_h \sim M \quad \gg \quad \mu_{hc} \sim \sqrt{Mm} \quad \gg \quad \mu_{s/c} \sim m \quad \gg \quad \mu_{NR} \sim mv$

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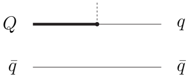
Motivation:

- Form factors explicitly calculable in combined NR + HQ expansion (in particular $\xi_\pi(q^2), \phi_B, \phi_\pi$)
- "toy model" for $B \to \pi$ structure of Fact. Formula, $\xi_\pi(q^2)$
- more realistic case $B_c \to \eta_c$ for $q^2 \ll m_b^2$, $m_c \ll m_b$

# Tree-Level

Direct overlap of NR wave functions




dominant at small recoil
suppressed at large recoil, $\psi_C(\vec{p}) \propto \frac{1}{\vec{p}^4}$

$\longrightarrow$ not sensitive to explicit shape $\psi_C(\vec{p})$
FF only depends on normalization $\longrightarrow$ decay constant

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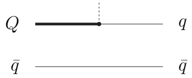


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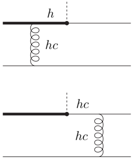
Exchange of one relativistic gluon



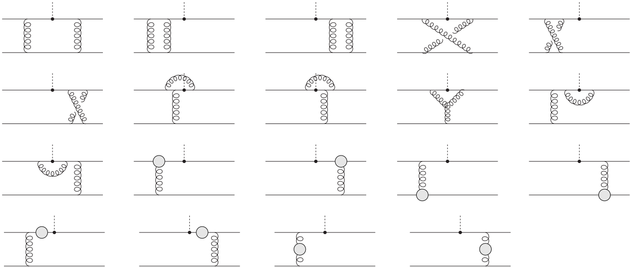

static approximation for external quarks

$$F_+^{\mathrm{LO}}(0) = \frac{4\pi\alpha_s(\mu)C_F}{N_c}\,\frac{3f_h f_l}{mM} \qquad \left(E_\pi = \frac{M}{2}\right)$$

no endpoint singularities

# NLO-Calculation

- various diagrams for $B \longrightarrow \pi$



- expansion in $m/M \quad \longrightarrow \quad$ method of regions within DR [Beneke, Smirnov 97]
- extract non-factorizable and factorizable contributions
- additional flavour-singlet diagrams for $B_c \longrightarrow \eta_c$

# NLO-Result

- endpoint-singularities only appear in non-factorizable contribution $\surd$
- (preliminary) NLO-Result [resummation of Logs to be included!]

$$
\begin{aligned}
F_+^{\mathrm{NLO}}(0) = \frac{4\pi\alpha_s(\mu)C_F}{N_c}\,\frac{3f_hf_l}{mM}\Bigg\{1 + \frac{\alpha_s(\mu)}{4\pi}\Bigg[b_0 \ln\frac{2\mu^2}{mM} - \frac{10}{9}N_F + \frac{1}{3}\ln\frac{M}{4m} \\
+ C_F\left(\frac{1}{2}\ln^2\frac{M}{m} + \frac{35-20\ln 2}{6}\ln\frac{M}{m} + \frac{2}{3}\ln^2 2 + 3\ln 2 + \frac{7}{9}\pi^2 - \frac{103}{6}\right) \\
+ C_A\left(-\frac{1}{6}\ln^2\frac{M}{m} + \frac{1+\ln 2}{3}\ln\frac{M}{m} + \frac{1}{3}\ln^2 2 - \frac{4}{3}\ln 2 - \frac{5}{36}\pi^2 + \frac{73}{9}\right)\Bigg]\Bigg\}
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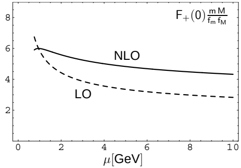
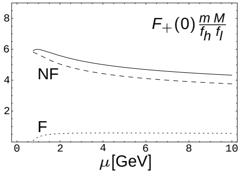
\end{aligned}
$$

- Numerical analysis: $B_c \longrightarrow \eta_c$ scenario $\quad M = 5$ GeV, $m = 1.5$ GeV $\longrightarrow \mu_{\rm hc} \simeq 2.7$ GeV



# Summary

NR-approximation allows for

- explicit analysis of general Factorization Formula
- ”toy model” for $B \to \pi$ transition

Application: $B_c \to \eta_c \ell \nu \quad (q^2 \ll m_b^2)$

- appropriate framework to include relativistic effects
- to be supplemented with higher-order terms from
  non-relativistic expansion $\sim \mathcal{O}(v^2)$
  heavy-quark expansion $\sim \mathcal{O}(m_c/m_b)$

Acknowledgements:
M. Beneke (RWTH Aachen)
Th. Feldmann (University of Siegen)

# Appendix

# Sudakov Logarithms and Endpoint Singularities

- e.g. renormalization of weak decay vertex


$[h] \sim \ln^2 \frac{M}{\mu} + \ldots$

$[hc] \sim \ln^2 \frac{\sqrt{Mm}}{\mu} + \ldots$ $\quad\rightarrow$

$[s] \sim \ln^2 \frac{m}{\mu} + \ldots$

$[h] + [hc] + [s] \sim \ln^2 \frac{M}{m} + \ldots$

Sudakov Logs may be resummed

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# Sudakov Logarithms and Endpoint Singularities

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$$[hc] \sim \ln^2 \frac{\sqrt{Mm}}{\mu} + \ldots \quad \rightarrow$$ Sudakov Logs may be resummed

$$[s] \sim \ln^2 \frac{m}{\mu} + \ldots \qquad QCD \rightarrow SCET_I \rightarrow SCET_{II}$$

- e.g. pentagon diagram


$[c]$, $[s]$ region not separately well-defined in DR, but $[c] + [s]$ is !

$$[hc] \sim \ln^2 \frac{\sqrt{Mm}}{\mu} + \ldots \qquad [hc] + [c] + [s] \sim \ln^2 \frac{M}{m} + \ldots$$

$$[c] + [s] \sim \ln \frac{M}{\mu} \ln \frac{m}{\mu} + \ln^2 \frac{m}{\mu} \ldots \quad \rightarrow$$

Large Logs related to endpoint singularities

$[c]+[s]$ matrix element still depends on high scale $(w \cdot w' \sim \frac{M}{m})$