The Decays $K \to \pi\bar{\nu}\nu$ in the Littlest Higgs Model

Selma Uhlig
Technical University of Munich

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Andrzej J. Buras, Anton Poschenrieder and S.U.

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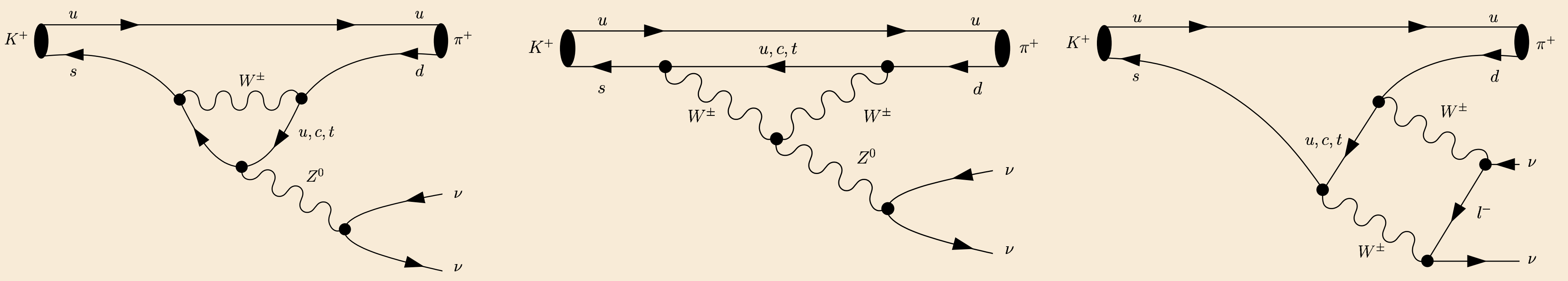
Overview

- **Why calculate $K \to \pi\bar{\nu}\nu$ in NP Models?**
- **Introduction to the Littlest Higgs Model**
- **Issues of Calculations in the Unitary Gauge**
- **Size of the Corrections due to the Littlest Higgs Model Particles**

Why calculate $K \to \pi\bar{\nu}\nu$ in NP Models?

1. General Properties of $K \to \pi\bar{\nu}\nu$



Contributions to $K^+ \to \pi^+\bar{\nu}\nu$ in the SM, similar: $K_L \to \pi^0\bar{\nu}\nu$.

- **$K^+ \to \pi^+\bar{\nu}\nu$ and $K_L \to \pi^0\bar{\nu}\nu$ are:**
  - FCNC (loop-induced within the SM)
  - sensitive to heavy virtual particles and underlying flavour dynamics
  - depend on CKM elements (in particular on V_{td})
- **$K_L \to \pi^0\bar{\nu}\nu$ provides information on CP violation**

- **The branching ratio of $K^+ \to \pi^+ \bar{\nu}\nu$ in the SM**

$$Br(K^+ \to \pi^+ \bar{\nu}\nu) = \frac{G_{\rm F}^2}{2} \frac{\alpha^2}{4\pi^2 \sin^4 \theta_{\rm w}} \sum_{l=e,\mu,\tau} \left| \left(\lambda_c X_{\rm NL}^l + \lambda_t X(x_t)\right) \right|^2 \times |\langle \pi^+ | (\bar{s}d)_{V-A} | K^+ \rangle|^2 \cdot |(\bar{\nu}_l \nu_l)_{V-A}|^2$$

$\lambda_c = V_{cs}^* V_{cd}$, $\lambda_t = V_{ts}^* V_{td}$, $G_{\rm F}$: Fermi constant

- **The branching ratio of $K_{\rm L} \to \pi^0 \bar{\nu}\nu$ in the SM**

$$Br(K_{\rm L} \to \pi^0 \bar{\nu}\nu) = \frac{G_{\rm F}^2}{2} \frac{\alpha^2}{4\pi^2 \sin^4 \theta_{\rm w}} \left({\rm Im}\lambda_t X(x_t)\right) \times \sum_{l=e,\mu,\tau} |\langle \pi^0 | (\bar{s}d)_{V-A} | K^0 \rangle|^2 \cdot |(\bar{\nu}_l \nu_l)_{V-A}|^2$$

2. Theoretical Cleanness of $K^+ \to \pi^+\bar{\nu}\nu$ and $K_L \to \pi^0\bar{\nu}\nu$

- **Essentially no hadronic Uncertainties:**

Hadronic matrix elements can be extracted from the tree-level decay $K^+ \to \pi^0 e^+ \nu_e$ using isospin symmetry:

$$Br(K^+ \to \pi^0 e^+ \nu_e) \propto |\langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle|^2$$

$$\langle \pi^+ | (\bar{s}d)_{V-A} | K^+ \rangle = \sqrt{2}\, \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$$
$$\langle \pi^0 | (\bar{s}d)_{V-A} | K^0 \rangle = \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$$

Isospin breaking corrections:

$Br(K^+) : 10\%$, $Br(K_L) : 6\%$ (Marciano, Parsa, 1996)

- **Long Distance Effects**
  - $Br(K^+) : +6\%$ ▶ small (Isidori, Mescia, Smith, 2005)
  - $Br(K_L) : \leq 1\%$ ▶ negligible (Buchalla, Isidori, 1998)

- **QCD-Corrections**

  LO (Dib, Dunietz, Gilman, 1991)

  NLO (Buchalla, Buras, 1994)

  NNLO (Buras, Gorbahn, Haisch, Nierste, 2005)

▶ **Calculable to a high Degree of Precision:**

Numerical value within the SM

(K^+: Buras, Gorbahn, Haisch, Nierste; K_L: Buras, Schwab, S.U.)

$$Br(K^+ \to \pi^+ \bar{\nu}\nu)_{\rm SM} = (8.0 \pm 1.1) \cdot 10^{-11}$$

$$Br(K_{\rm L} \to \pi^0 \bar{\nu}\nu)_{\rm SM} = (2.8 \pm 0.6) \cdot 10^{-11}$$

3. Experimental Situation

$$Br(K^+ \to \pi^+ \bar{\nu}\nu) = (14.7^{+13.0}_{-8.9}) \cdot 10^{-11} \quad (90\% \text{ C.L.})(E949)$$

$$Br(K_{\rm L} \to \pi^0 \bar{\nu}\nu) < 2.9 \cdot 10^{-7} \quad (90\% \text{ C.L.})(E391a)$$

Model independent bound (Grossman, Nir, 1997)

$$Br(K_{\rm L} \to \pi^0 \bar{\nu}\nu) \leq 4.4 \, Br(K^+ \to \pi^+ \bar{\nu}\nu)$$

$$Br(K_{\rm L} \to \pi^0 \bar{\nu}\nu) \leq 1.4 \cdot 10^{-9} \quad (90\% \text{ C.L})$$

► **A lot of room for new physics** (in particular for $K_{\rm L} \to \pi^0 \bar{\nu}\nu$)

- Large effects in models with new complex phases
- Here: a model of the MFV class, no new phases

The Littlest Higgs (LH) Model

(Arkani-Hamed, Cohen, Katz, Nelson, 2002)

- **Alternative Solution to the Little Hierachy Problem**
  Quadratic divergences are cancelled by particles of the same statistics
- **Symmetries:** global $SU(5)$ and local subgroup
  $[SU(2)_1 \otimes U(1)_1] \otimes [SU(2)_2 \otimes U(1)_2]$
- **Symmetry breaking** at a high scale f:
  global: $SU(5) \rightarrow SO(5)$
  local: $([SU(2) \otimes U(1)])^2 \rightarrow SU(2)_L \otimes U(1)_Y$

- **Particle content of the LH model**
  - SM particles: fermions, gauge bosons ($W_L^{\pm}$, Z_L^0, A_L^0), Higgs boson
  - New heavy gauge bosons: $W_H^{\pm}$, Z_H^0, A_H^0 (heavy "photon")(Masses > 500 GeV)
  - New scalars: H, Φ^0, Φ^P, $\Phi^{\pm}$, $\Phi^{\pm\pm}$
  - A new heavy top quark: T
- **Parameters of the LH model**
  - v/f: v is the SM vev, f the scale of the global symmetry breaking
  - s, s': sines of the mixing angles of the $SU(2)_{1,2}$ and $U(1)_{1,2}$ gauge bosons
  - λ_1, λ_2: Yukawa couplings of t and T, important for phenomenology: $x_L = \frac{\lambda_1^2}{\sqrt{\lambda_1^2+\lambda_2^2}}$

Violation of the CKM Unitarity

- The CKM unitarity relation valid in the SM

$$\lambda_u + \lambda_c + \lambda_t = 0, \qquad \lambda_i = V_{is}^* V_{id}$$

has to be modified, e.g. the V_{ij} have to be generalized.

- Relation between SM CKM elements V_{ij} and LH CKM elements $\hat{V}_{ij}$:

$$\hat{V}_{ij} = V_{ij} \;\text{ for } i,j = u,c, \quad \hat{V}_{tj} = V_{tj}\left(1 - \frac{1}{2}x_L^2 \frac{v^2}{f^2}\right), \quad \hat{V}_{Tj} = V_{tj}\frac{v}{f}x_L$$

- Generalized unitarity relation, valid up to $\mathcal{O}(v^2/f^2)$ is crucial for GIM mechanism to work

$$\hat{\lambda}_u + \hat{\lambda}_c + \hat{\lambda}_t + \hat{\lambda}_T = 0, \qquad \hat{\lambda}_i = \hat{V}_{is}^* \hat{V}_{id}$$

Calculations in the Unitary Gauge

Unitary Gauge in the SM:

- Structure of $\mathcal{H}_{\text{eff}}$ of the Z^0 penguin and the box contribution to $K \to \pi\bar{\nu}\nu$ in the SM (top contribution)

$$\mathcal{H}_{\text{eff}} = \lambda_t \frac{g^4}{64\pi^2} \frac{1}{M_W^2} X(x_t)(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

$$X(x_t) = C(x_t) - 4\,B(x_t), \quad x_t = m_t^2/M_W^2, \quad \lambda_t = V_{ts}^* V_{td}$$

- Propagator of W boson, $\frac{-i}{k^2-M_W^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right)$, implies more divergent integrals than in the Feynman gauge

- The $C(x)$ and $B(x)$ functions are divergent in Unitary Gauge

- The $C(x)$ and the $B(x)$ function in the SM in the Unitary Gauge:

$$C_{0,ug}(x) = \frac{-x}{16\,\varepsilon} + \frac{-7x + x^2}{32(-1+x)} + \frac{(4x - 2x^2 + x^3)\log x}{16(-1+x)^2}$$

$$B_{0,ug}(x) = \frac{-x}{64\,\varepsilon} - \frac{3(5x + x^2)}{128(-1+x)} + \frac{(16x - 8x^2 + x^3)\log x}{64(-1+x)^2}$$

$$X_0(x) = C_{0,ug}(x) - 4\,B_{0,ug}(x) = \frac{x}{8}\left(\frac{x+2}{(-1+x)} + \frac{(3x-6)\log x}{(-1+x)^2}\right)$$

- Divergences from penguins and boxes have to cancel each other, they are not cancelled by GIM mechanism!

Unitary Gauge in NP Models:

- Advantage for models with extended gauge groups: only diagrams with physical particles have to be considered (no GB!)
- Structure of the result in LH model (v: ew scale):

$$\mathcal{H}_{\rm eff} = \lambda_t \frac{g^4}{64\pi^2} \frac{1}{M_{W_L}^2} X_{\rm LH} (\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A}$$

$$C_{\rm LH} = C_{\rm SM}(x_t) + \frac{v^2}{f^2} \Delta C$$
$$B_{\rm LH} = B_{\rm SM}(x_t) + \frac{v^2}{f^2} \Delta B$$
$$X_{\rm LH} = X_{\rm SM}(x_t) + \frac{v^2}{f^2} \Delta X$$

- Mass relations and corrections to SM parameters due to the NP contributions can become important for the cancellation of divergences

- E.G.: Custodial $SU(2)$ symmetry broken in the LH model at $\mathcal{O}(v^2/f^2)$ (tree-level):

$$\frac{M^2_{W^\pm_L}}{M^2_{Z_L}} = \cos^2\theta_{\mathrm{W}}\left(1 + \frac{v^2}{f^2}\frac{5}{4}(c'^2 - s'^2)^2\right)$$

implies the additional divergent contribution

$$\frac{v^2}{f^2}\frac{5}{4}(c'^2 - s'^2)^2\, C_{0,ug}(x_t)$$

▶ **Unitary Gauge in NP models:**

- Less diagrams have to be considered
- Cancellation of additional divergences offer a possibility to test the calculation

$K \to \pi\bar{\nu}\nu$ in the Littlest Higgs Model

• Calculation performed to $\mathcal{O}(v^2/f^2)$

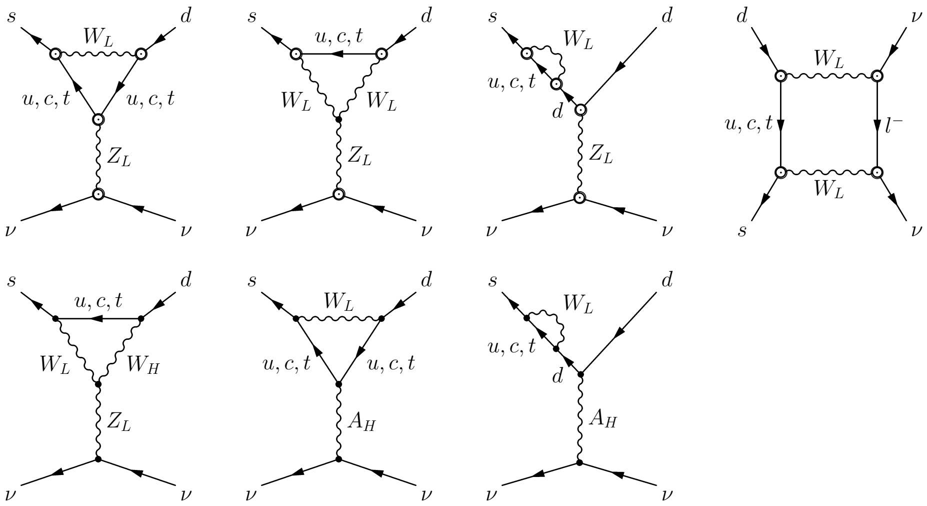
This means for the contribution of the heavy top quark T, $\mathcal{O}(v^4/f^4)$ has to be considered as

$$x_T \propto \frac{f^2}{v^2}\, x_t, \qquad x_i = \frac{m_i^2}{M_{W_L}^2}$$

• Result depends on LH parameters:

- ▶ $\boldsymbol{f/v}$: ratio of the two scales $(5 < f/v < 20)$
- ▶ $\boldsymbol{x_L}$: describes the mixing of t and T $(0.2 < x_L < 0.9)$
- ▶ $\boldsymbol{s}$: sine of the mixing angle of $W_1^\pm$ and $W_2^\pm$ $(0.2 < s < 0.8)$

- No dependence on s'
- Scalar contributions are negligible
- Result differs from the one of Choudhury *et al.* (2005), they find much bigger effects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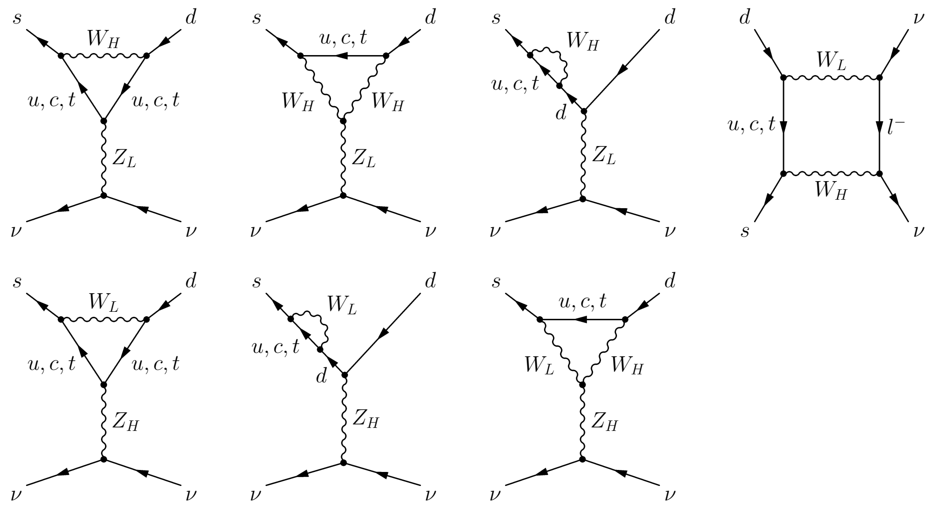
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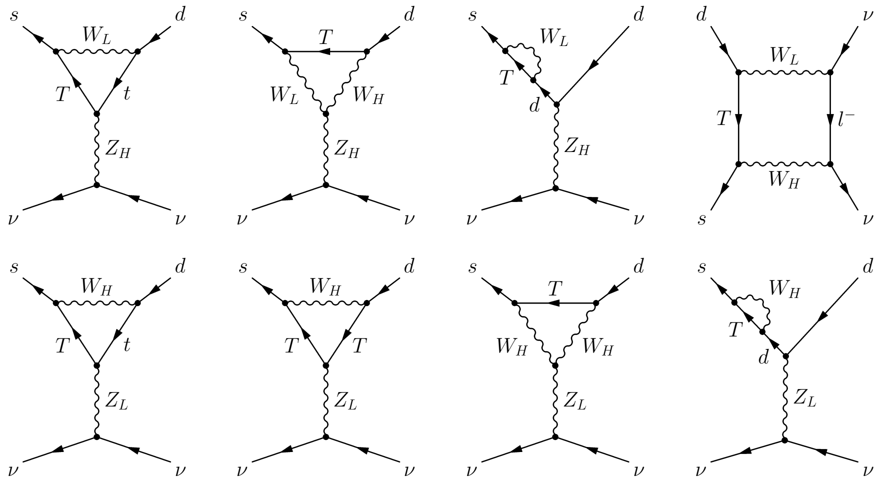
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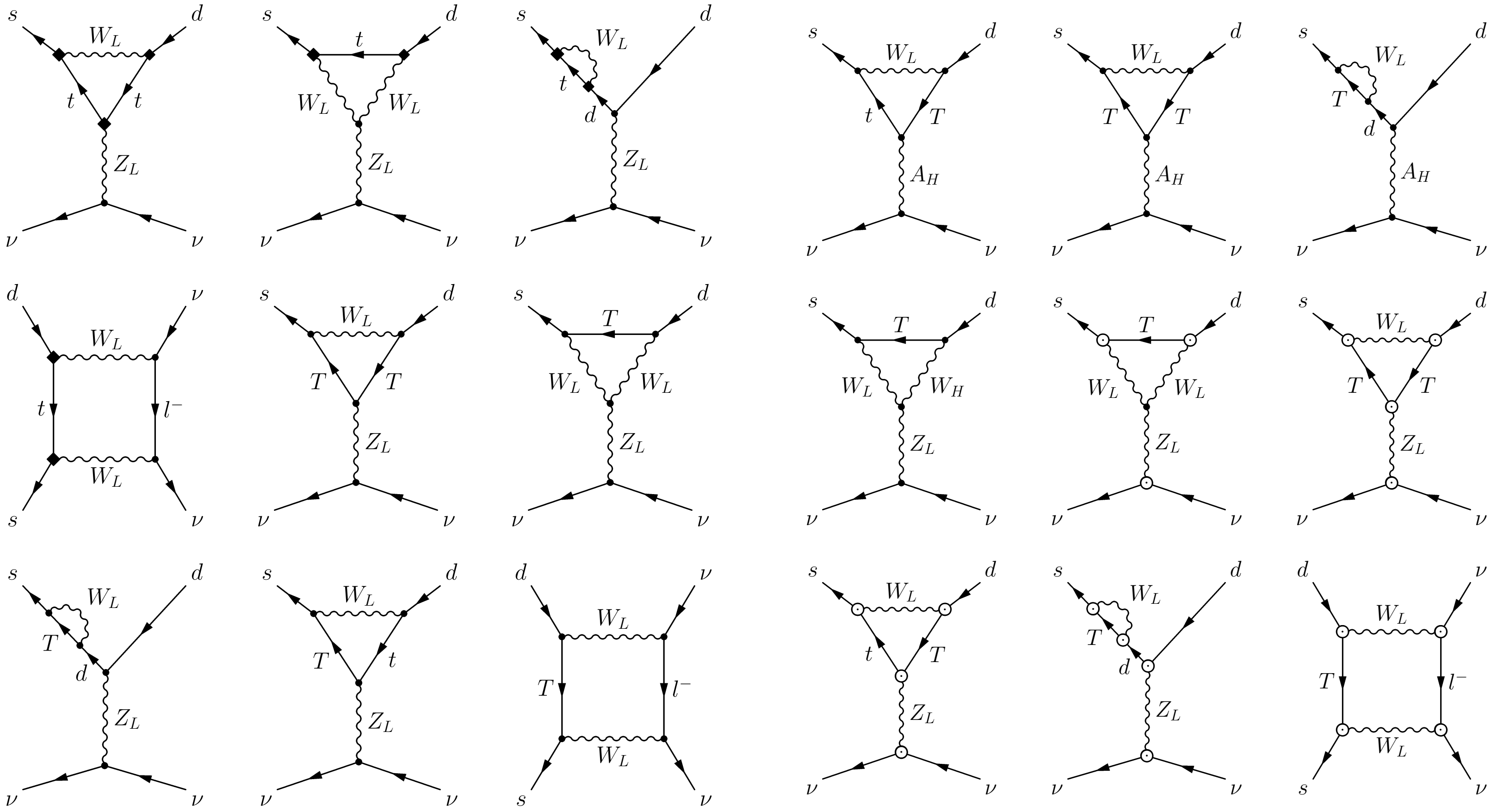
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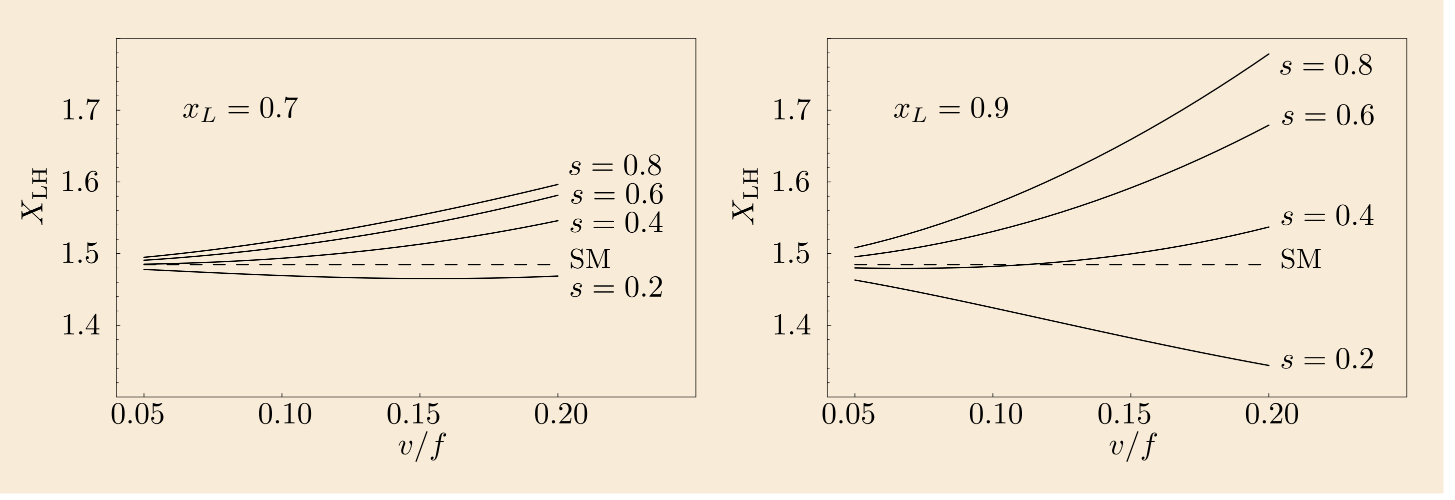
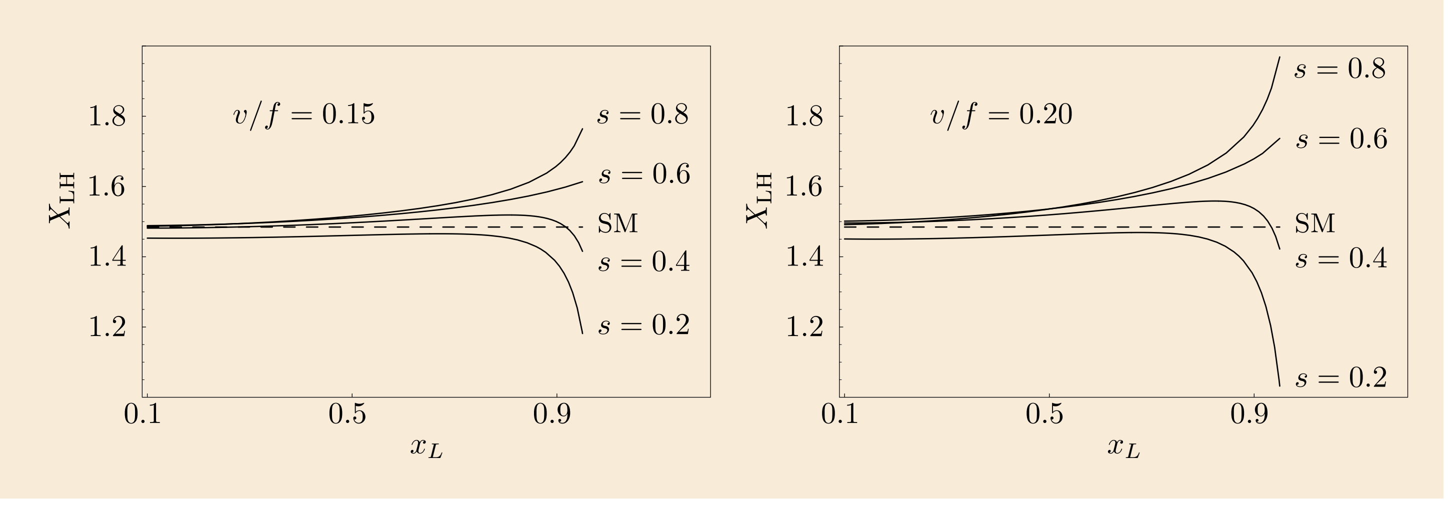
Numerical Results (preliminary)

X Function





- Result for X_{LH} strongly depend on x_L and on v/f
- Significant effects only if x_L and v/f are not too small
- Suppression and enhancement of X possible:
  $v/f = 0.2$, $x_L \approx 1$, $s \approx 1$: largest enhancement
  $v/f = 0.2$, $x_L \approx 1$, $s \approx 0.2$: largest suppression

$$-33\,\% < \frac{\Delta X}{X} < 35\,\%$$

$$0.99 \; < \; X \; < \; 2.00$$

$$X_{\mathrm{SM}} \approx 1.5$$

Size of the Corrections

$$\frac{Br(K_{\rm L} \to \pi^0 \bar{\nu}\nu)_{\rm LH}}{Br(K_{\rm L} \to \pi^0 \bar{\nu}\nu)_{\rm SM}} = \frac{S_{\rm SM}}{S_{\rm LH}} \frac{X^2_{\rm LH}}{X^2_{\rm SM}}$$

- Similar for $K^+ \to \pi^+ \bar{\nu}\nu$ but more complicated due to charm contribution
- The value of $(|V_{td}|)_{\rm LH}$ is not the same as in the SM:

$$\frac{(|V_{td}|)_{\rm LH}}{(|V_{td}|)_{\rm SM}} = \sqrt{\frac{S_{\rm SM}}{S_{\rm LH}}} \ < \ 1$$

S is the Inami-Lim function that governs particle-antiparticle mixing.

Excursion: Particle-Antiparticle Mixing and the S function in the LH Model

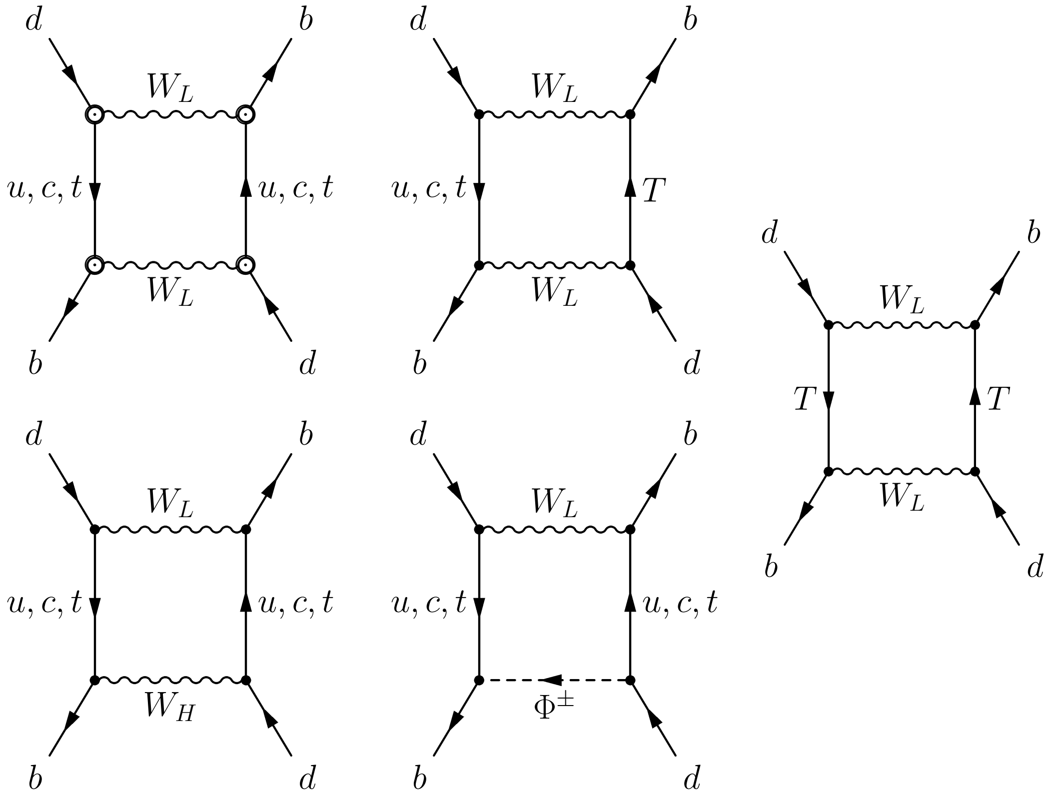
- **Particle-Antiparticle mixing in the SM:**

$$\mathcal{H}_{\text{eff}} = \frac{G_F^2}{16\pi^2} M_W^2 \left[\lambda_c^2 \eta_1 S(x_c) + \lambda_t^2 \eta_2 S(x_t) + 2\lambda_c \lambda_t \eta_3 S(x_c, x_t)\right] (\bar{b}q)_{V-A} (\bar{b}q)_{V-A}$$

  - $\lambda_i = V_{ib}^* V_{iq}$, in the case of $B_q^0 - \bar{B}_q^0$ mixing, $q = d, s$
  - $K^0 - \bar{K}^0$ mixing: $\lambda_i = V_{is}^* V_{id}$, operators: $(\bar{s}d)_{V-A} (\bar{s}d)_{V-A}$
  - η_i correspond to the QCD corrections

- **Particle-Antiparticle Mixing in the LH model**
  - The S function receives a correction due to NP contribution
  - $S_{\text{LH}} = S_{\text{SM}}(x_t) + \Delta S$

- Contributing diagrams



- ΔS is positive in the full range of parameters and the enhancement of S amounts to at most 15% (A.J.Buras, A.Poschenrieder and S.U., Nucl.Phys. B716 (2005), hep-ph/0501230)

► Suppression of $(V_{td})_{\mathrm{LH}}$

The Size of the Corrections (preliminary)

$$0.31 < \frac{Br(K_\mathrm{L} \to \pi^0 \bar{\nu}\nu)_\mathrm{LH}}{Br(K_\mathrm{L} \to \pi^0 \bar{\nu}\nu)_\mathrm{SM}} < 1.14$$

$$0.46 < \frac{Br(K^+ \to \pi^+ \bar{\nu}\nu)_\mathrm{LH}}{Br(K^+ \to \pi^+ \bar{\nu}\nu)_\mathrm{SM}} < 1.10$$

- Size of the corrections $\propto \frac{v^2}{f^2}$. Here: $\frac{v}{f} = 0.2$ ($f \approx 1.2$ TeV). If $\frac{v}{f} = 0.1$ ($f \approx 2.5$ TeV), corrections are about a factor 4 smaller.

Main Messages

- Even for f as low as 1.2 TeV, the effects amount to at most 70% suppression and 15% enhancement for the branching ratio of $K_{\rm L} \to \pi^0 \bar{\nu}\nu$ and to 50% suppression and 10% enhancement for the branching ratio of $K^+ \to \pi^+ \bar{\nu}\nu$.
- The effects due to the Littlest Higgs model are visible and in the full range of parameters consistent with the present data for FCNC.

Mixing Parameter in the Top Sector (x_L)

- Yukawa Lagrangian in the top sector:
  $\mathcal{L} = \lambda_2 f \tilde{t}\tilde{t}' + \lambda_1 t_3 h^0 u_3'^c + \lambda_1 f \tilde{t} u_3'^c + \mathcal{O}(1/f) + h.c., \qquad < h^0 >= v$
- $\chi_i = (b_3, t_3, \tilde{t})$ are the left handed fields replacing the third left handed SM quark doublet
- $u_3'^c$ and $\tilde{t}'^c$ are the corresponding right handed singlets
- $x_L = \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}$
- Field rotation into mass eigenstates

$$t_L = c_L\, t_3 - s_L\, \tilde{t}, \qquad T_L = c_L\, t_3 + s_L\, \tilde{t}$$

$$s_L = x_L v/f + \mathcal{O}(v^3/f^3) \qquad c_L = 1 - v^2/f^2\; 1/2\; x_L^2$$