Hard Spectator Interactions in $B \to \pi\pi$

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Outline

Phenomenological Importance of $B \rightarrow \pi\pi$

QCD-Factorization

Feynman Diagrams for $B \rightarrow \pi\pi$

Hard Spectator Interactions

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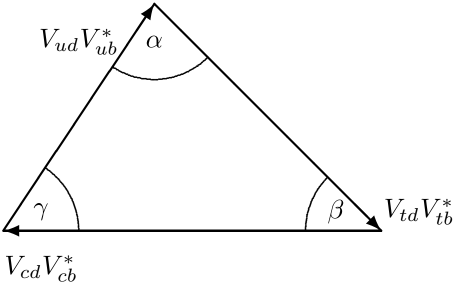

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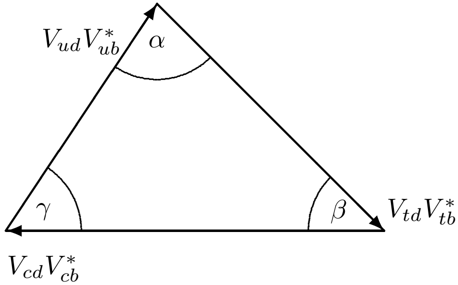

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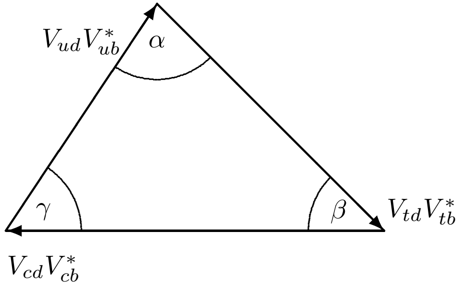

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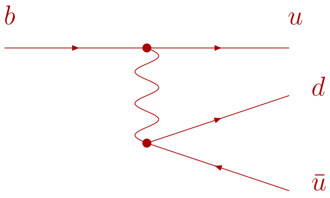
- The angle β is well determined by $B \to J/\psi K_S$
- For the angles α, γ controlling of hadronic quantities in decays like $B \to \pi\pi$ is needed

B-decays and CKM-angles

- Tree diagram



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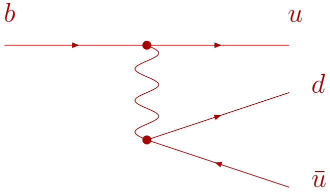

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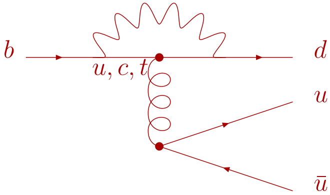
- Penguin diagram



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QCD-Factorization Beneke, Buchalla, Neubert, Sachrajda hep-ph/0006124

- Two energy scales in decays $B \to \pi\pi$:
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$\Rightarrow$ Separation of short- and long-distance effects

When does QCD-Factorization not work?

- Infrared (IR) singularities remain from loop integrals
  - Formally: IR singularities cannot be absorbed into counter terms
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- Endpoint singularites occur from integrals over meson wave functions i.e.

$$\int_0^1 d\xi dv du \, \phi_\pi(u)\phi_\pi(v)\phi_B(\xi)T(\xi, u, v)$$

diverges for $u, v \to 0, 1$ or $\xi \to 0$
$\Rightarrow$ Contribution of soft constituent quarks is dominant

How does QCD-Factorization work?

- Perturbative expansion of the matrix element:

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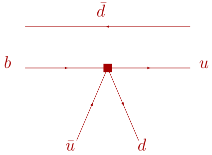
- IR and endpoint singularities of $A^{(1)}$ and $\phi^{(1)}$ cancel each other
  $\Rightarrow$ T stays finite

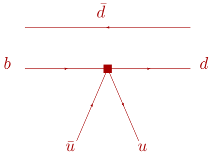
Feynman diagrams for $B \longrightarrow \pi\pi$

- LO:
  - $\bar{B}^0 \longrightarrow \pi^+\pi^-$:



$$\sim C_1 + \frac{C_2}{N_C} \approx 1$$

  - $\bar{B}^0 \longrightarrow \pi^0\pi^0$:



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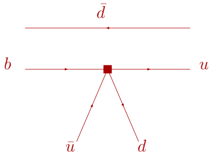

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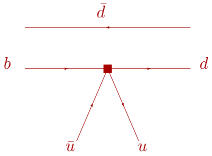
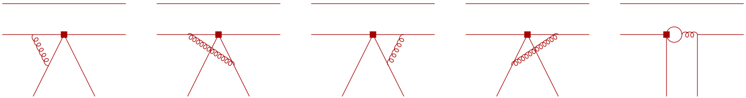


$$\sim C_2 + \frac{C_1}{N_C} \approx 0.1$$

- In the case $\bar{B}^0 \to \pi^0\pi^0$ LO is color suppressed and NLO is expected to be more important

NLO

- Form factor contributions:



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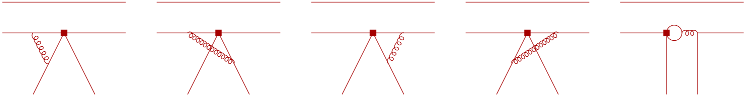

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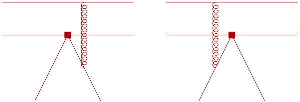

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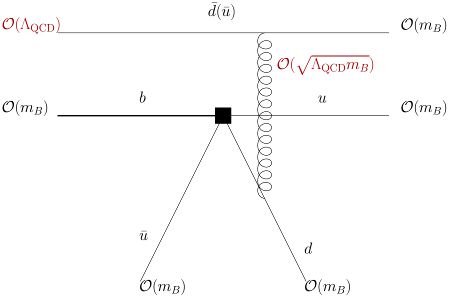

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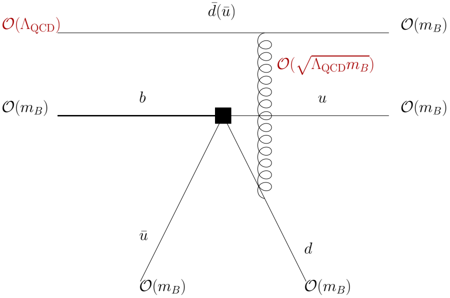

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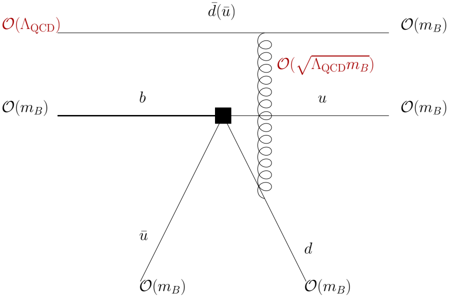

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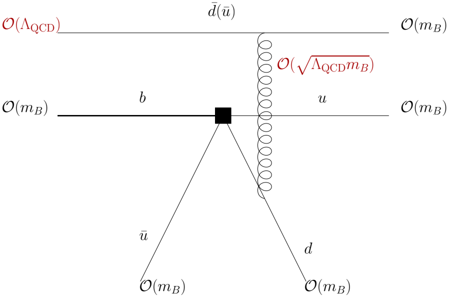

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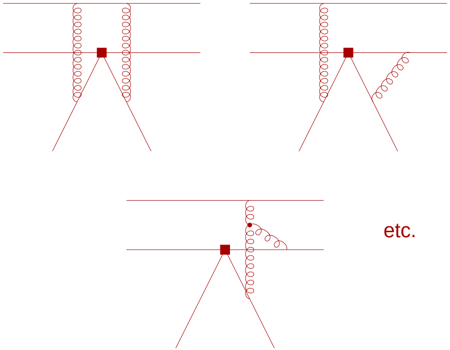
$\Rightarrow$ At NNLO hard spectator interactions are expected to be important

Hard spectator interactions in NNLO



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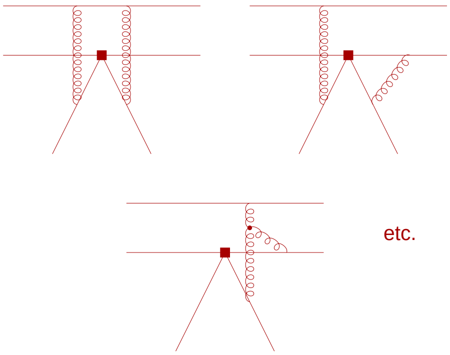

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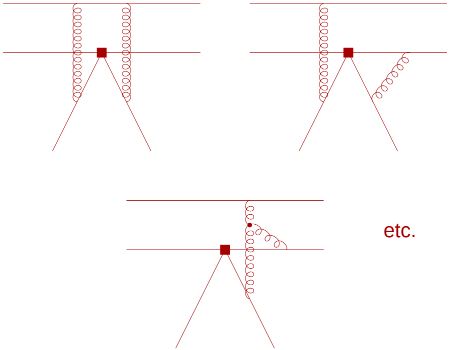

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- Only one loop calculations despite $\mathcal{O}(\alpha_s^2)$
- Feynman integrals contain up to five propagators and three linearly independent momenta
  $\Rightarrow$ Expansion in $\Lambda_{\mathrm{QCD}}/m_B$ on the level of integrands rather than of Feynman integrals

Some numbers Beneke,Jäger hep-ph/0512351 and Beneke, Neubert hep-ph/0308039

- CP averaged branching ratios

Mode	NLO	NNLO	Experiment
$B^- \to \pi^- \pi^0$	5.1	5.5 ± 1.0	5.5 ± 0.6
$\bar{B}^0 \to \pi^+ \pi^-$	5.2	5.0 ± 1.3	5.0 ± 0.4
$\bar{B}^0 \to \pi^0 \pi^0$	0.7	$0.73^{+0.8}_{-0.6}$	1.45 ± 0.3

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- NNLO taken from Beneke and Jäger. My work is still in progress

Summary

- B-decays play an important role for determining the CKM angles
- In leading power QCD-effects in B-decays can be handled in the framework of QCD-factorization
- Hard spectator interactions are important because they introduce large logaritms $\ln \frac{\Lambda_{\text{QCD}}}{m_B}$