String Theory

at the

Max Planck–Institut für Physik

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Outline

1. Introduction

2. Overview over Research in String Theory at Max Planck Institute

Today's well-known theories:

- Standard Model of elementary particle physics:

  Local quantum field theory (gauge theory), pointlike particles

  Known to be correct up to $\mathcal{O}(10^2)$ GeV

  Effective theory (free parameters such as quark masses)

- Einsteins theory of gravity (General relativity)

  Non-renormalizable

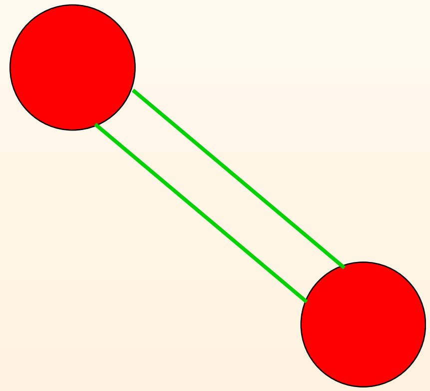
  Quantum effects occur at $M_{Planck} \simeq 10^{19} GeV$

Search for a unified theory of all interactions:

- Quantum theory of Gravity
- Describe all interactions in unified framework
- Provide relations between standard model parameters

Origin of String Theory in the 60's: Regge trajectories of resonances

$J = \alpha' M^2 + \alpha_0$



Strong interactions described by QCD since early 70's

QCD: Low-energy regime not well-understood (confinement)

String theory may provide

non-perturbative description of strong-coupling regime of QCD

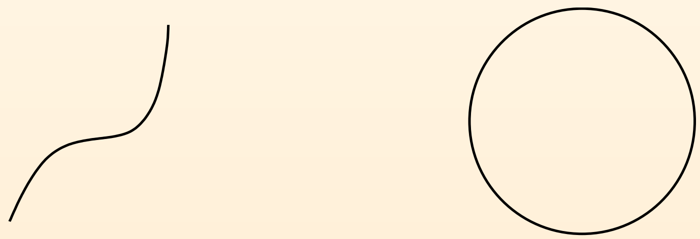
within framework of unified theory

Quantum Theory of Gravity and Unification of Interactions:

Give up locality at very short distances

Natural cutoff: String length

$$l_s \sim \frac{1}{M_{Planck}}, \quad l_s = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35} m$$



Open strings: Gauge interactions   Closed strings: Gravity

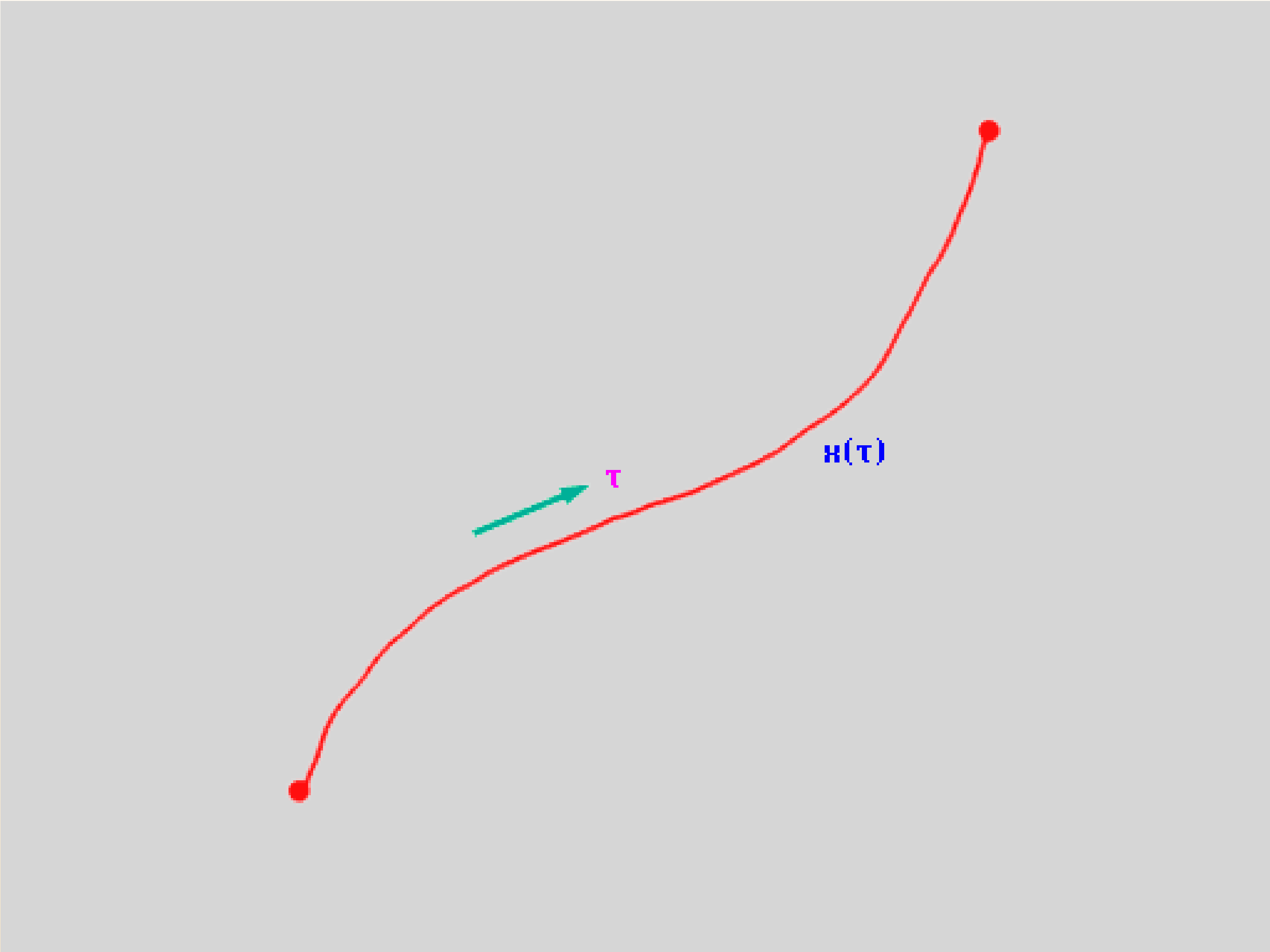
Essentially two possible degrees of freedom – simplification

Higher oscillation modes may be excited

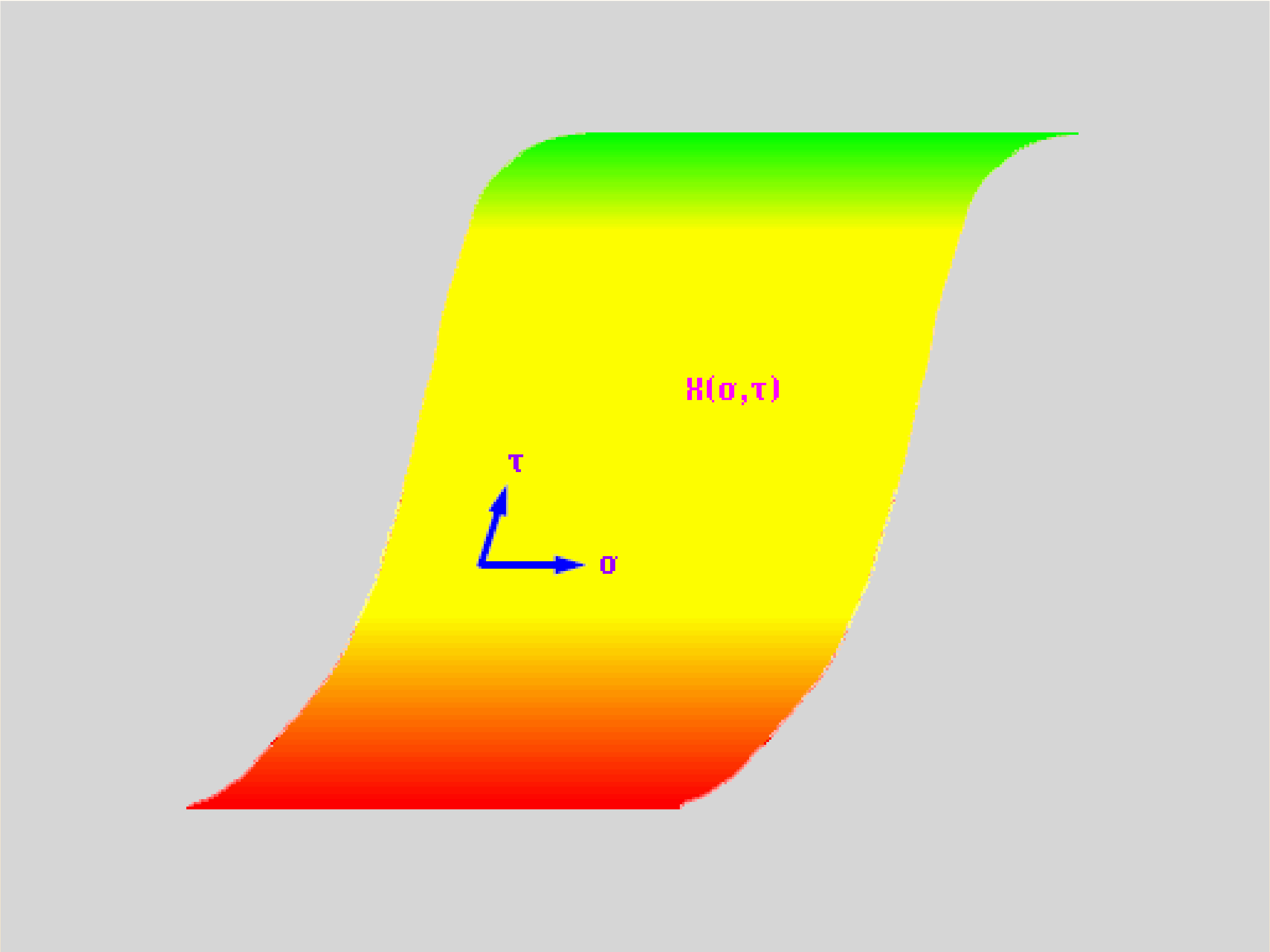
Direct experimental observation difficult or impossible

Indirect proof: Supersymmetry, ...

Relativistic Particle: Action $S = -mc \int d\tau \sqrt{-\dot{X}^{\mu}\dot{X}_{\mu}}$



Relativistic String: Worldsheet



$(\sigma, \tau) \rightarrow X^{\mu}(\sigma, \tau)$

Nambu-Goto action:

$$S_{NG} = -\frac{1}{\alpha'} \cdot Area = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-det\gamma}$$

$$\gamma_{\tau\tau} = \frac{\partial X^{\mu}}{\partial \tau} \frac{\partial X_{\mu}}{\partial \tau}, \ldots$$

Polyakov action:

$$S_p = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma \sqrt{-det\gamma}\gamma^{ab}\partial_a X^{\mu} \partial_b X_{\mu}$$

Quantization possible

Closed strings: Left and right moving components

$$X^\mu(\sigma,\tau) = X_L{}^\mu(\sigma+\tau) + X_R{}^\mu(\sigma-\tau)$$

Expansion in oscillators: $(X^\mu(\sigma+2\pi) = X^\mu(\sigma))$

$$X_L{}^\mu(\tau+\sigma) = \frac{1}{2}x^\mu + \frac{\alpha'}{2}p^\mu(\tau+\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\bar{\alpha}_n{}^\mu e^{-in(\tau+\sigma)},$$

$$X_R{}^\mu(\tau-\sigma) = \frac{1}{2}x^\mu + \frac{\alpha'}{2}p^\mu(\tau-\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n{}^\mu e^{-in(\tau-\sigma)}.$$

Quantization:

$$[x^\mu, p^\nu] = i\eta^{\mu\nu}\,,$$
$$[\alpha^\mu{}_m, \alpha^\nu{}_n] = [\alpha^\mu{}_m, \alpha^\nu{}_n] = m\delta_{m+n,0}\eta^{\mu\nu}$$

α operators: Creators and annihilators of higher string excitations

Graviton: $\alpha^i{}_{-1}\bar{\alpha}^j{}_{-1}|0\rangle$, $m^2 = 0$

Beware of tachyons ! $m^2 < 0$, negative norm states

Bosonic string theory is consistent in 26 dimensions

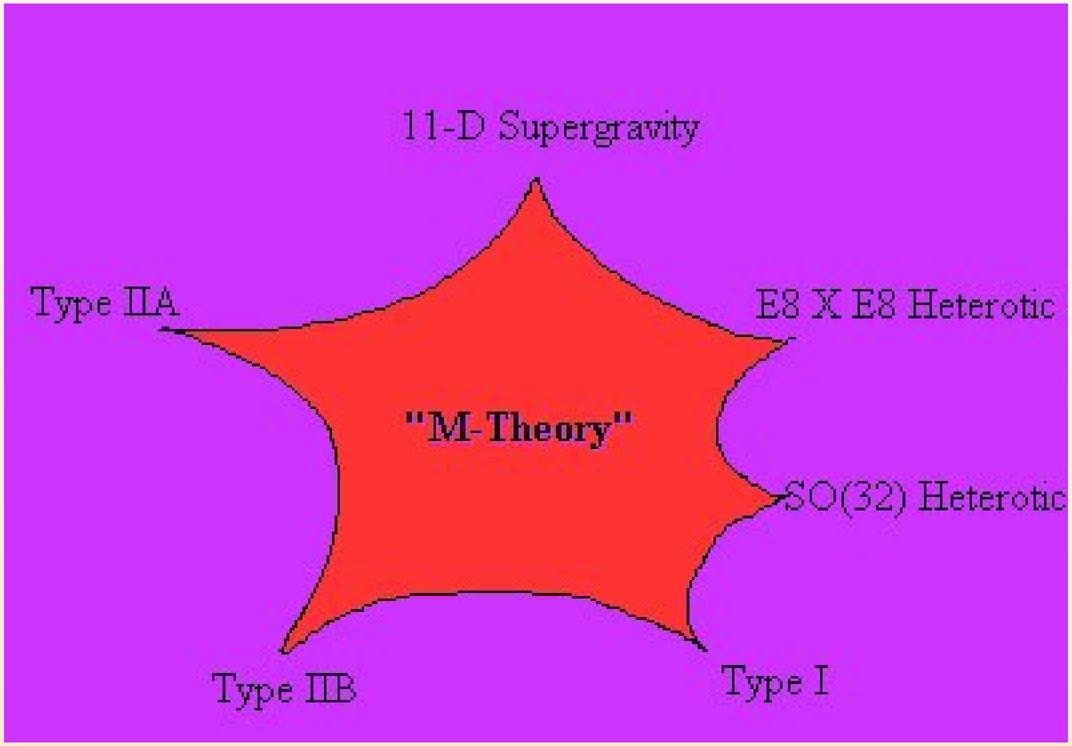
(Spectrum Lorentz invariant)

Superstring theory is consistent in ten dimensions (9 space + 1 time)

Fermions: X^μ have fermionic superpartnes ψ^μ

GSO projection: Ensures well-defined ground state, absence of tachyons

There are five consistent superstring theories in ten dimensions:

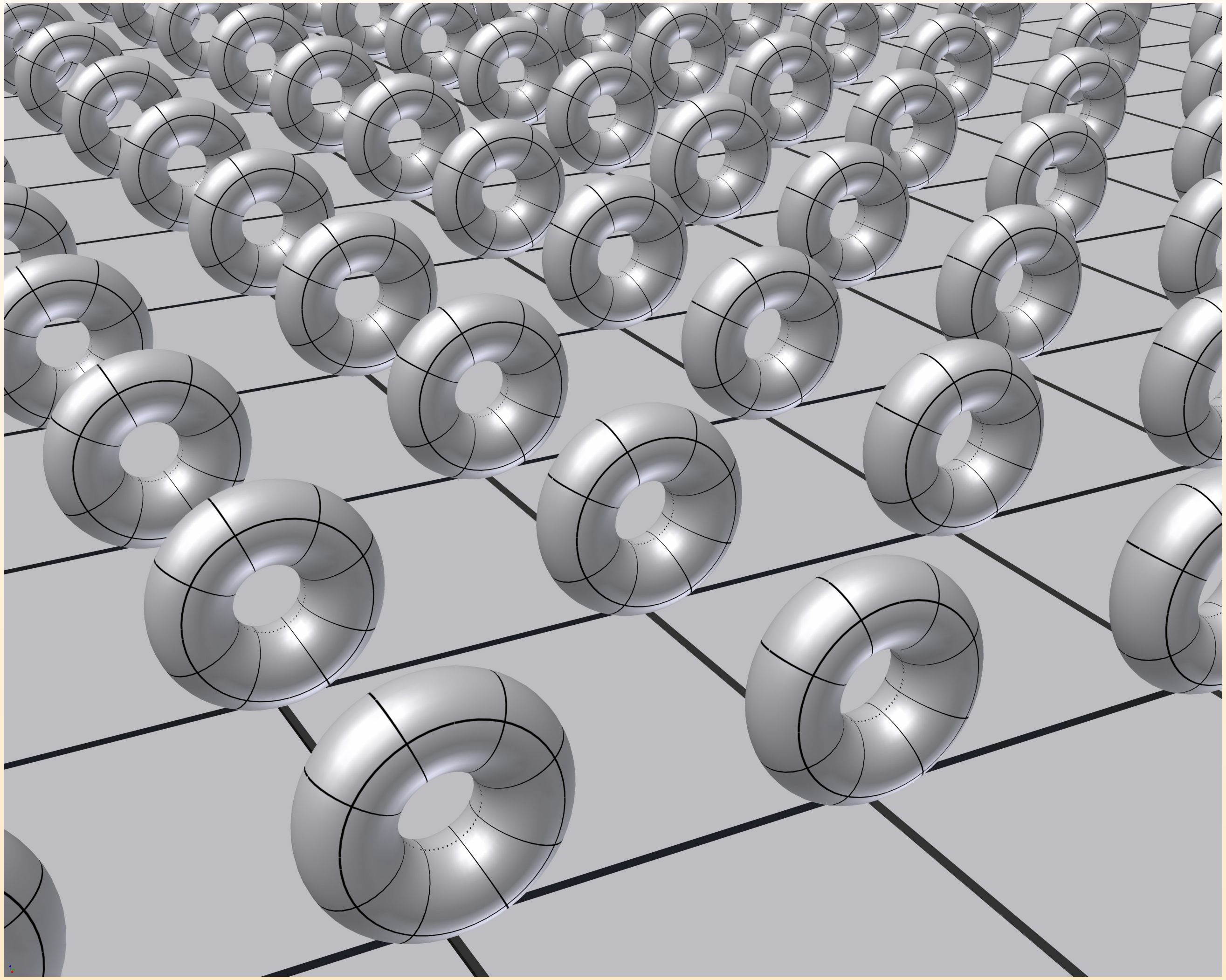


$\Rightarrow$ Consistent quantum theory of gravity in ten dimensions

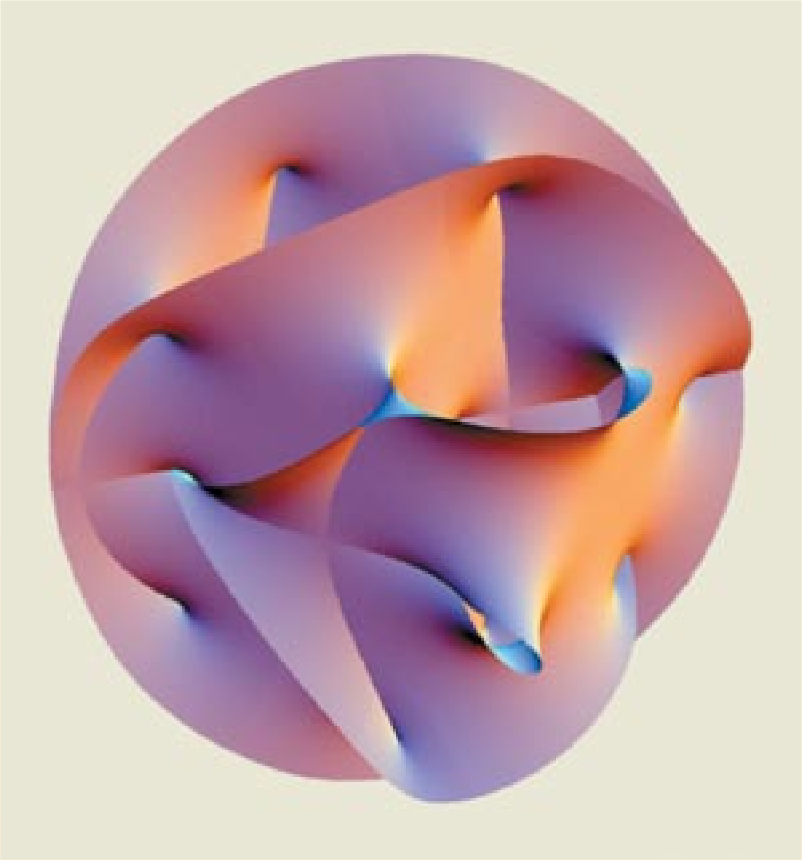
What about the six unobserved dimensions?

1. Idea: They are compactified

Compactification: Torus



Compactification: Calabi-Yau manifold



leads to four-dimensional field theories with $\mathcal{N} = 1$ supersymmetry

Experimental signs of extra dimensions?

Compactification with $R < 10^{-16cm} \Leftrightarrow E > 100GeV$

Periodic boundary conditions $\Rightarrow$ discrete spectrum

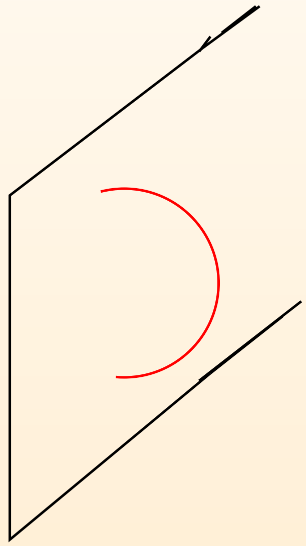
Kaluza-Klein particles

Tower: Different masses, but otherwise same quantum numbers

If seen at collider, direct sign for extra dimensions and strings

2nd idea why extra dimensions have not been observed so far:

D - branes embedded into 10d space (Hypersurfaces)



D3 branes: (3+1)-dimensional hypersurfaces

Open strings can end on D-branes ⇔ Dynamics

Higher order gauge fields couple to D-branes:
For example $A_{\mu\nu\sigma\rho}$ instead of $A_\mu \quad \Rightarrow$ fluxes

Second interpretation of D branes:

Solitonic solutions of supergravity

Heavy objects which curve space around them

Open questions

- M theory
- Low-energy limits: 1. Reproduce Standard Model
- Low-energy limits: 2. Describe non-perturbative regime of strong interactions
- Cosmology

Some research topics at Max Planck Institute

1. Deriving the Standard Model from String Theory

   Blumenhagen, Lüst, Honecker, Gmeiner, Weigand ...

2. Cosmology and supergravity

   Zagermann ...

3. QCD and String Theory

   Erdmenger, Apreda, Große, Höhne, Kaminski ...

Brane worlds and Standard Model

Branes which span all of the four observed dimensions

plus some of the conjectured compact extra dimensions

In the extra dimensions, the branes may intersect

Aim: Deriving the supersymmetric Standard Model (MSSM)

‘intersecting brane worlds’

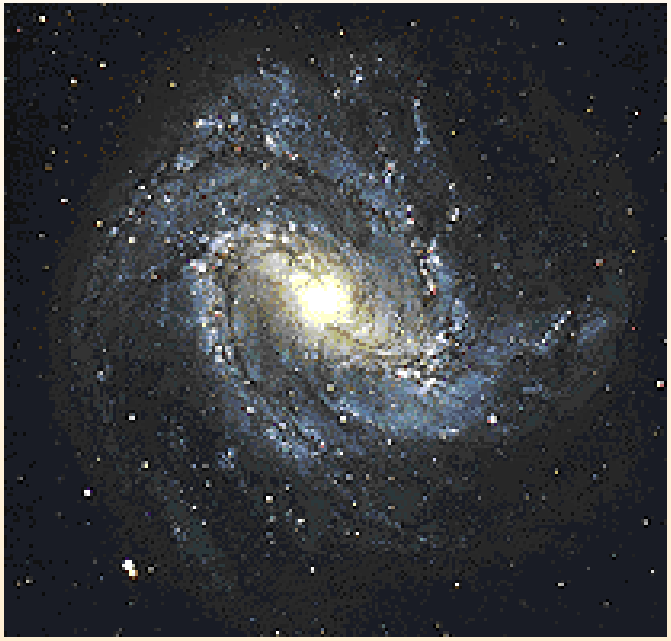
Quarks and Leptons: Open strings attached to intersection points

SUSY breaking calculable (fluxes)

Cosmology and Supergravity

Cosmological implications of string theory and supergravity

(Inflation, time-dependence, vacuum energy, cosmological constant...)



Low energy effective actions and formal aspects of supergravity theories

(Supersymmetric extensions of general relativity)

AdS/CFT Correspondence

(Maldacena 1997, AdS: Anti de Sitter space, CFT: conformal field theory)

- Duality Quantum Field Theory $\Leftrightarrow$ Gravity Theory
- Arises from String Theory in a particular low-energy limit
- Duality: Quantum field theory at weak coupling

  $\Leftrightarrow$ Gravity theory at strong coupling (and vice versa)

Conformal field theory in four dimensions

$\Leftrightarrow$ Supergravity Theory on $AdS_5 \times S^5$

Quantum field theory in which the fields transform covariantly under conformal transformations

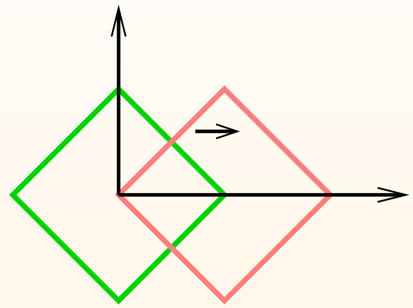
Conformal coordinate transformations: preserve angles locally

$\Rightarrow$ Correlation functions are determined up to a small number of parameters

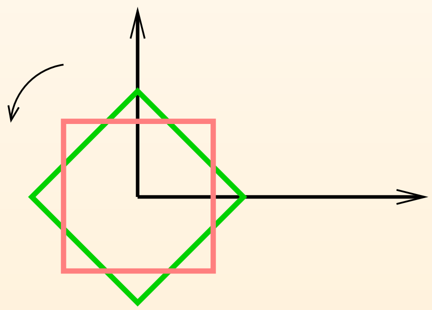
Confinement and conformal symmetry are incompatible!

Conformal coordinate transformations

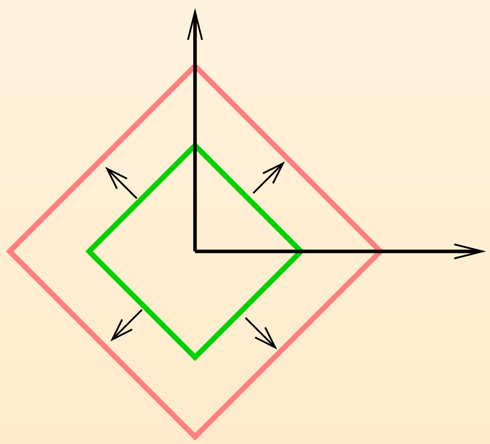
Translation

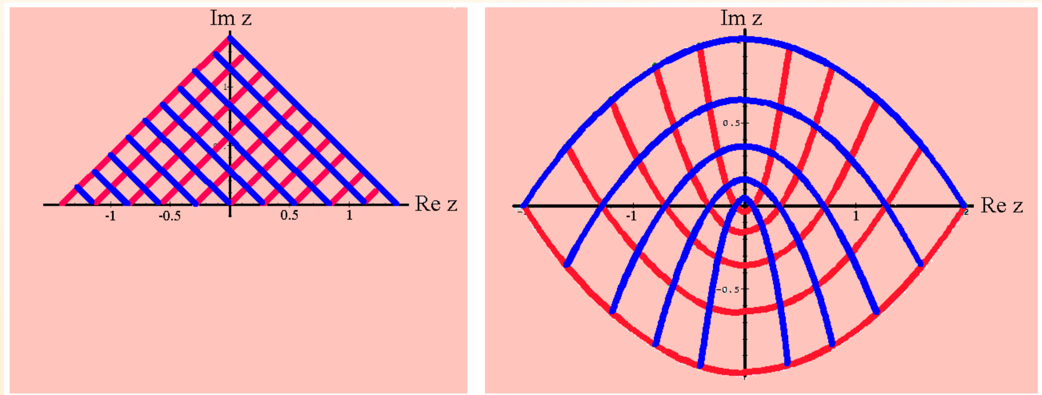


Rotation



Scale transformation

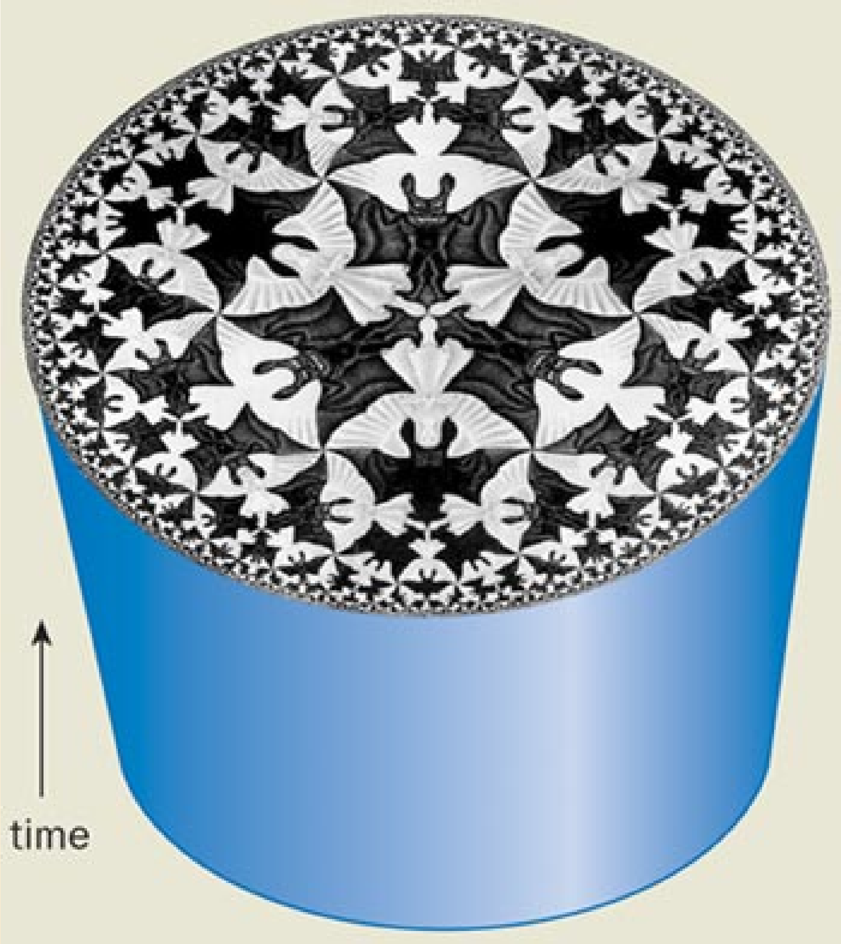




special conformal transformation

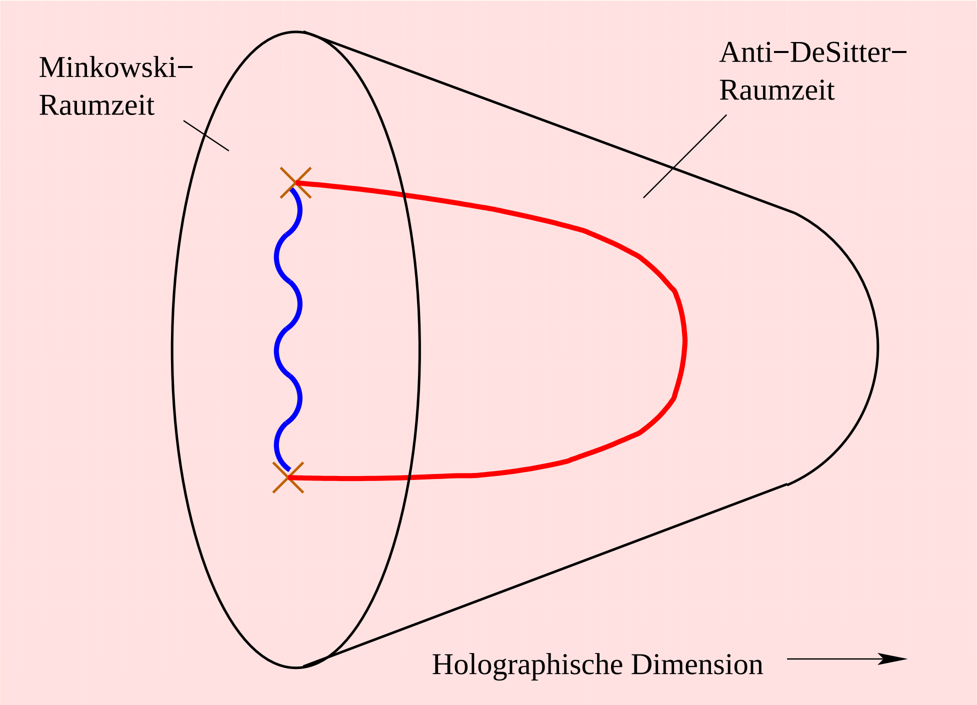
Anti-de Sitter Space

Space of constant negative curvature, has a boundary



Quelle: Institute of Physics, Copyright: C. Escher

AdS/CFT Correspondence



AdS/CFT correspondence:

'Dictionary' Gauge invariant operators in field theory ⇔ Fields in gravity theory

Symmetry properties coincide

Holography

Quantum field theory at the boundary of Anti-de Sitter space:

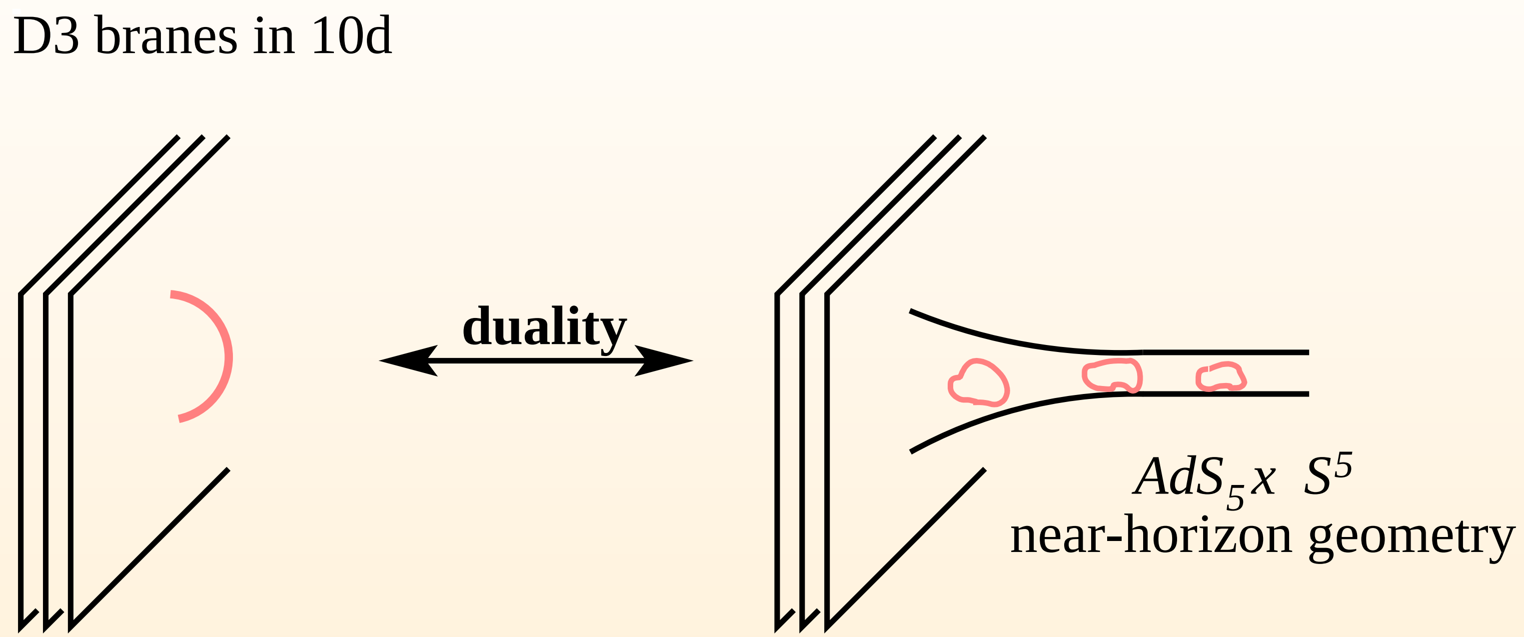
$\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory ($N \to \infty$)

$\beta \equiv 0$, conformal

- 1 vector field A_μ
- 4 complex Weyl fermions $\lambda_{\alpha A}$ ($\bar{4}$ of $SU(4)_R$)
- 6 real scalars ϕ_i (6 of $SU(4)_R$)

all in adjoint representation of gauge group

String theory origin of AdS/CFT correspondence



⇓ Low-energy limit

supersymmetric $SU(N)$ gauge theory in four dimensions ($N \to \infty$)

Supergravity on $AdS_5 \times S^5$

Symmetries

are of vital importance for AdS/CFT correspondence:

Conformal symmetry and supersymmetry of the gauge theory

$\Leftrightarrow$ Isometries of $AdS_5 \times S^5$ space

Generalizations of the AdS/CFT correspondence

$\mathcal{N} = 4\ SU(N)$ SUSY Gauge theory:

- $N \to \infty$
- Supersymmetry
- Conformal symmetry
- All fields in the adjoint representation of the gauge group

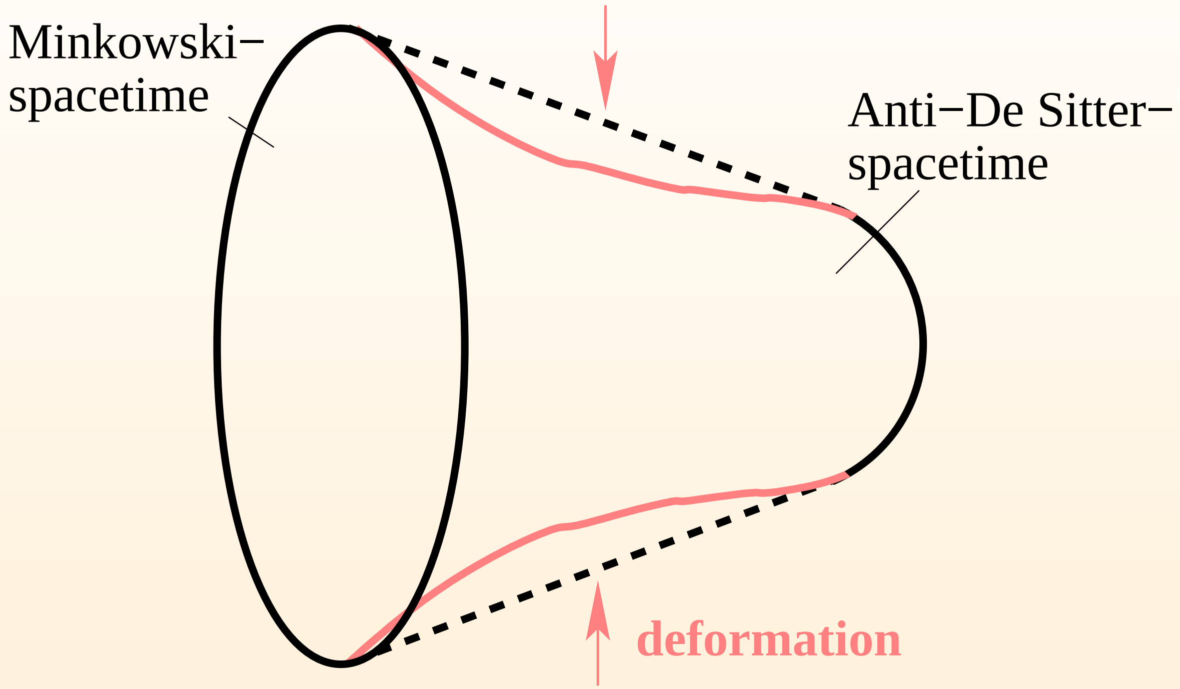
QCD:

- $N = 3$
- No supersymmetry
- Confinement
- Quarks in fundamental representation of the gauge group

Desirable extensions of AdS/CFT:

- Relax $N \to \infty$ limit ($1/N$ corrections)
  $\Leftrightarrow$ String theory instead of supergravity
- Break SUSY and conformal symmetry $\Leftrightarrow$ Deformation of AdS space
- Add quarks in fundamental representation of gauge group

Deformations of AdS space



Fifth Dimension ⇔ Energy scale

Renormalization group flow from supergravity

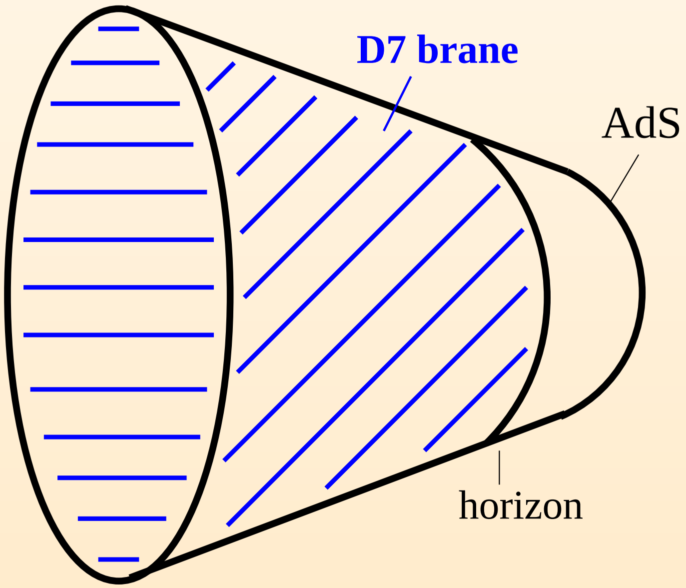
⇒ 'holographic' Renormalization Group flow

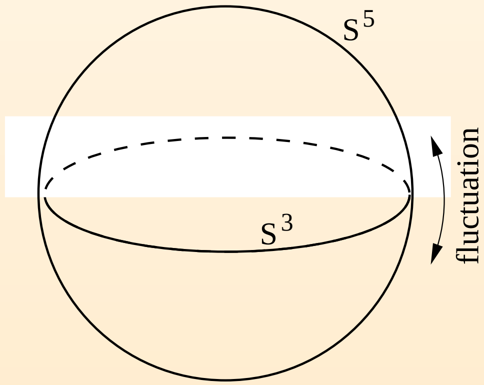
SUSY broken by deformation of S^5

Quarks (fundamental fields) within the AdS/CFT correspondence

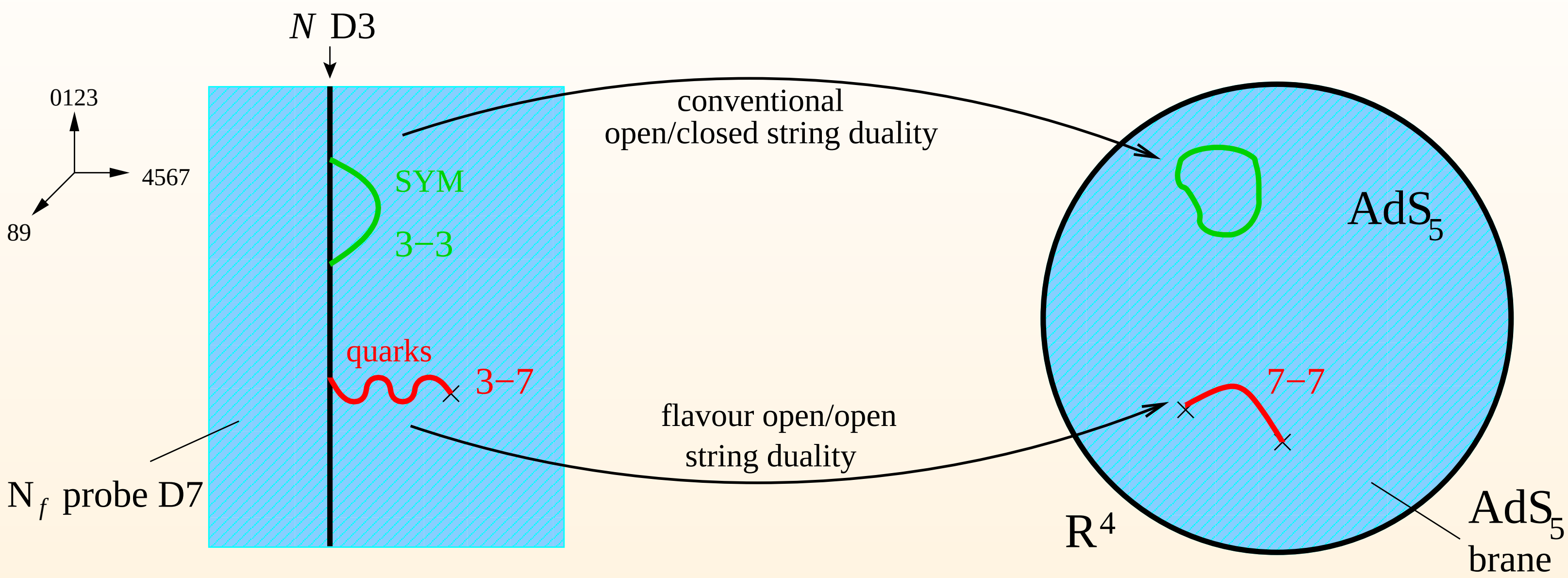
D7 brane probe:

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		





Quarks (fundamental fields) from brane probes



$N \to \infty$, N_f small (probe limit))

duality acts twice:

4d $\mathcal{N} = 4$ SU(N) SUSY gauge theory coupled to fundamental multiplet	$\longleftrightarrow$	Supergravity on $AdS_5 \times S^5$ + Dynamics of the probe brane on $AdS_5 \times S^3$

Chiral symmetry breaking in QCD

Chiral symmetry $SU(3)_L \times SU(3)_R$

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \qquad \psi = \psi_L + \psi_R$$

Explicit breaking of chiral symmetry by mass terms

Spontaneous breaking of chiral symmetry by quark condensate $\langle \bar{\psi}\psi \rangle$

Spontaneous symmetry breaking $\Leftrightarrow$ new massless particles

Goldstone bosons

Mesons $\Leftrightarrow$ Goldstone bosons of chiral symmetry   Example: Pion

Combine the deformation of the supergravity metric

with the addition of brane probes:

Dual gravity description of chiral symmetry breaking and Goldstone bosons

J. Babington, J. E., N. Evans, Z. Guralnik and I. Kirsch,

“Chiral symmetry breaking and pions in non-SUSY gauge/gravity duals”

Phys. Rev. D 69 (2004) 066007

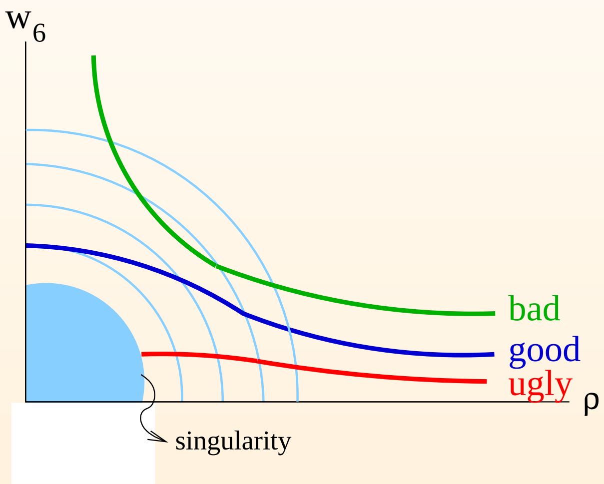
Outline of calculation

- Embedding of brane probe in curved space background:
  - Deriving and solving the equations of motion for the probe brane (from Dirac-Born-Infeld action which describes dynamics of D brane))
- Fluctuations around embedding: Meson spectrum
- Quark mass and condensate:

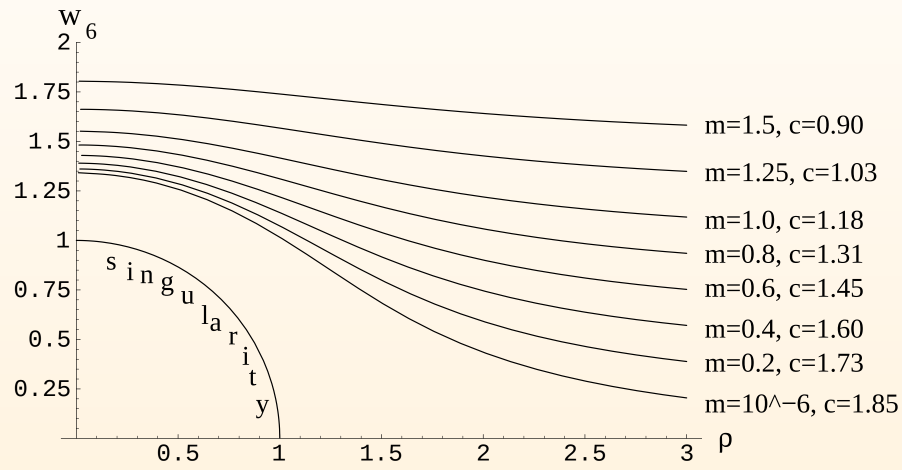
  - are fixed by boundary conditions on the brane embedding at the boundary of the curved space

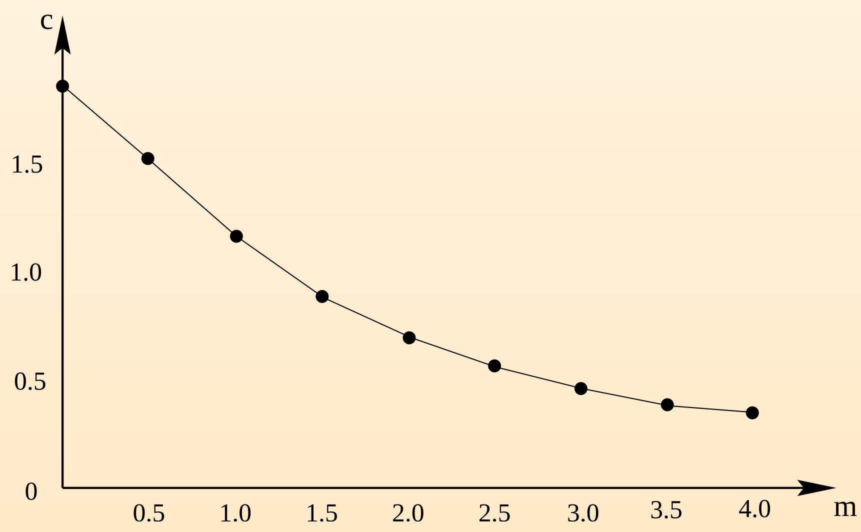
Chiral symmetry breaking

Solution of equation of motion for probe brane



Numerical Result:





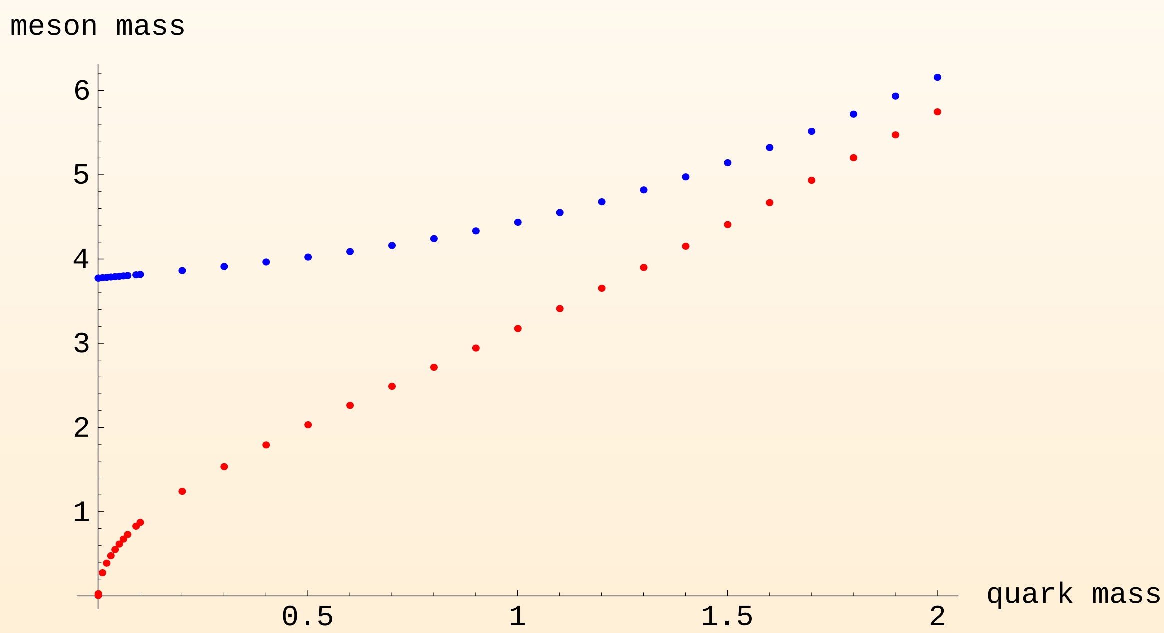
Result:

Screening effect: Regular solutions do not reach the singularity

Spontaneous breaking of $U(1)_A$ chiral symmetry: For $m \to 0$ we have $c \equiv \langle \bar{\psi}\psi \rangle \neq 0$

Meson spectrum

from fluctuations of the probe brane



Goldstone boson (η')

Gell-Mann-Oakes-Renner relation: $M_{Meson} \propto \sqrt{m_{Quark}}$

Comparison to Experiment

$D4/D8/\bar{D}8$ brane model Sakai+Sugimoto 12/2004

Vector and axial vector mesons ρ und a_1

(from gauge field fluctuations on the probe brane)

Meson mass ratio:

Experiment:

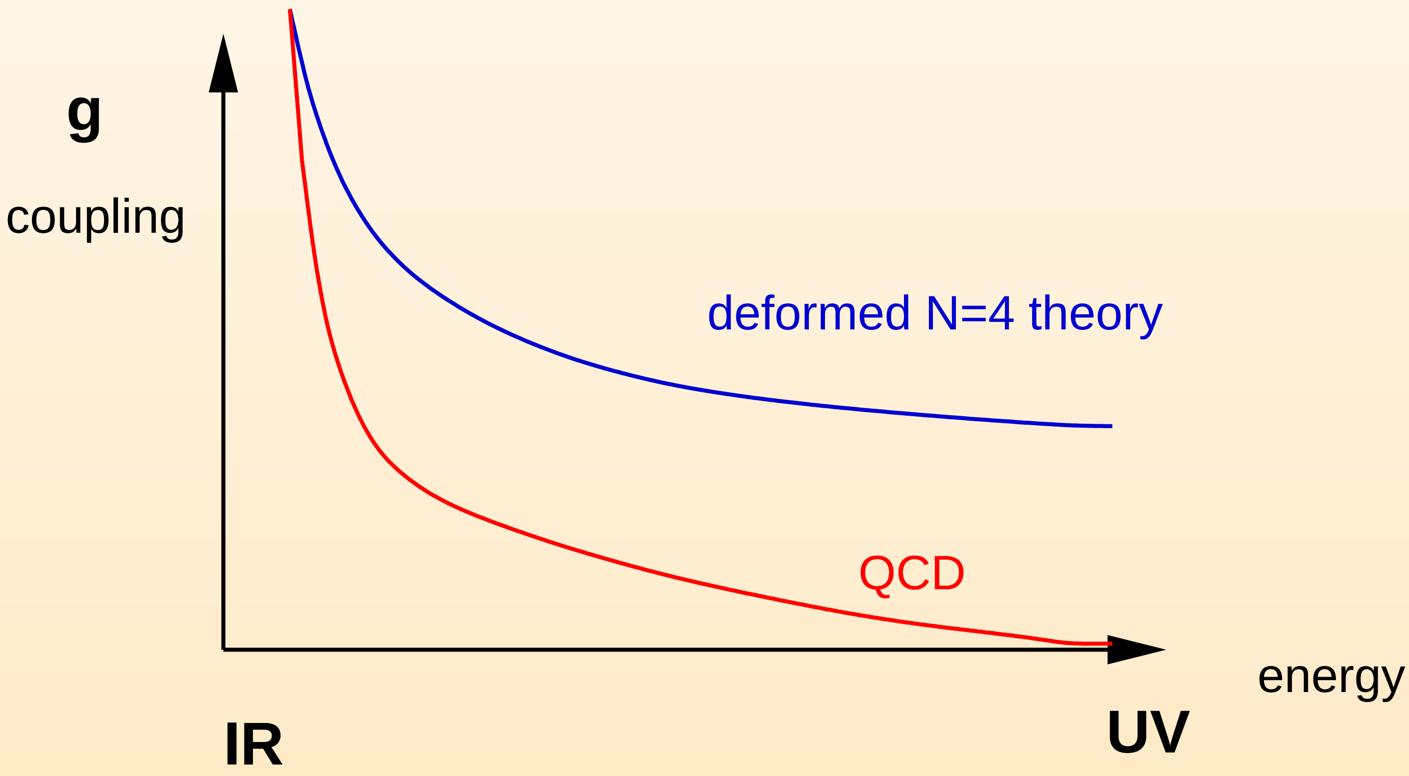
$$\frac{m_{a_1}^2}{m_\rho^2} = \frac{(1230MeV)^2}{(776MeV)^2} = 2.51$$

In string theory model:

$$\frac{m_{a_1}^2}{m_\rho^2} = 2.4$$

Discussion

- New effective description of chiral symmetry breaking and mesons from string theory
- Meson spectrum from fluctuations of a probe brane in curved space background



Outlook

There are many interesting questions to be addressed by string theory...

- LHC physics (SUSY, new developments ?)
- Non-perturbative description of low-energy physics