NEUTRINO PHYSICS — THEORY

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Literature

- Bilenky, Giunti, Grimus: *Phenomenology of Neutrino Oscillations*, `hep-ph/9812360`
- Akhmedov: *Neutrino Physics*, `hep-ph/0001264`
- Grimus: *Neutrino Physics – Theory*, `hep-ph/0307149`

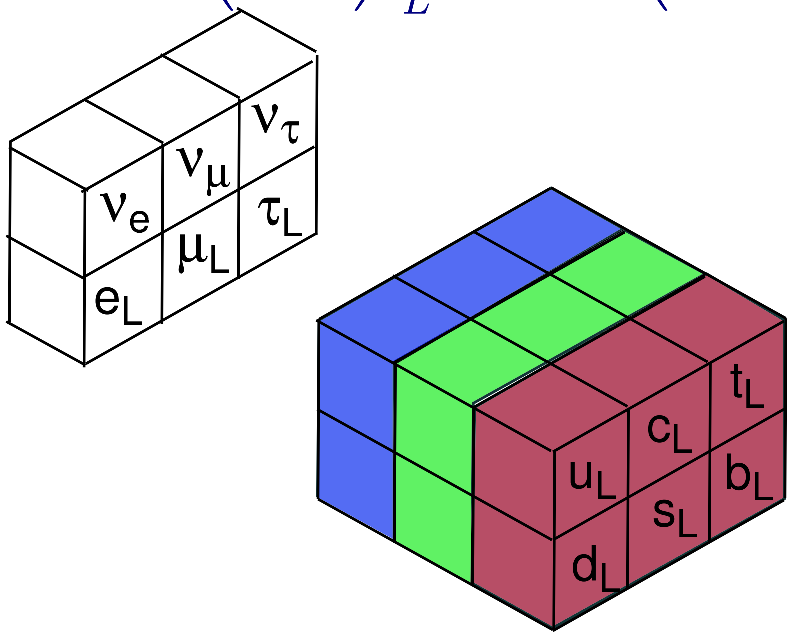
Contents

Introduction

Standard Model of Particle Physics $\leftrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$

$$Q_L^1 = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad Q_L^2 = \begin{pmatrix} c \\ s \end{pmatrix}_L \quad Q_L^3 = \begin{pmatrix} t \\ b \end{pmatrix}_L \qquad \begin{array}{c} u_R,\ c_R,\ t_R \\ d_R,\ s_R,\ b_R \end{array}$$

$$E_L^1 = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad E_L^2 = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad E_L^3 = \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \qquad \begin{array}{c} e_R,\ \mu_R,\ \tau_R \\ \underline{\text{no } (\nu_e)_R,\ (\nu_\mu)_R,\ (\nu_\tau)_R} \end{array}$$



$$m_d = \mathcal{O}(m_u)$$

$$m_s = \mathcal{O}(m_c)$$

$$m_t = \mathcal{O}(m_b)$$

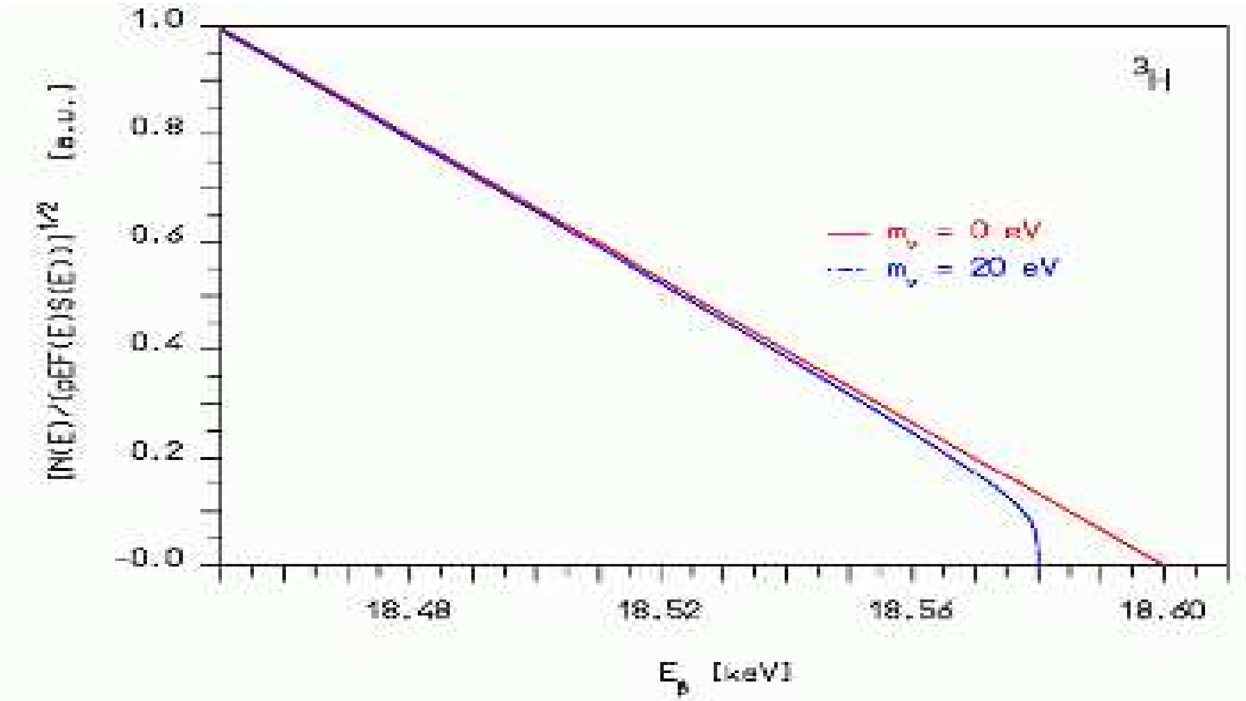
$$m_e \simeq 0.5 \cdot 10^6 \text{ eV} \gg m_{\nu_e} \lesssim \text{eV}$$

$\Rightarrow$ Assumption: neutrinos massless

Limit on Neutrino Masses

Classical Method: Curie–Plot from β–Decay $Z \to (Z+1) + e^- + \overline{\nu_e}$

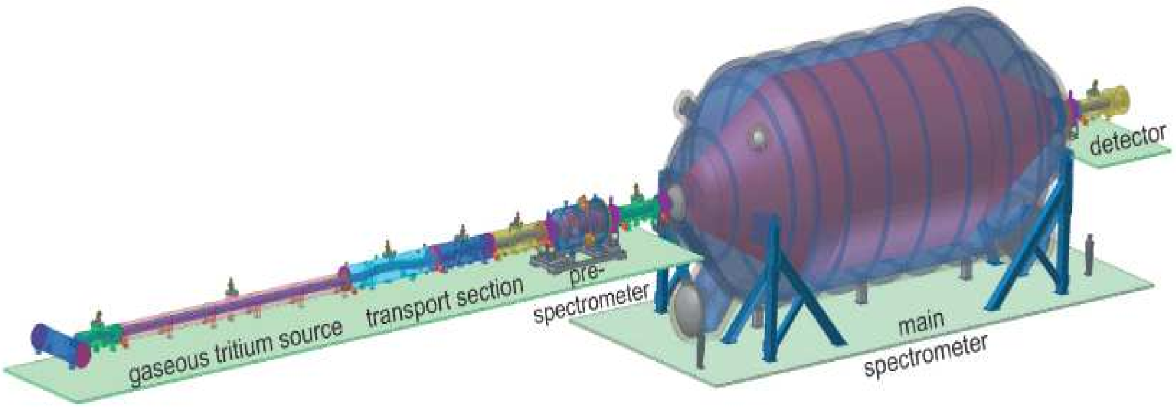
$$K(E_e) = \sqrt{\frac{dN(E_e)/dE_e}{F(Z', E_e)\, E_e \sqrt{E_e^2 - m_e^2}}} \propto \sqrt{(E_0 - E_e)\sqrt{(E_0 - E_e)^4 - m_\nu^2}}$$



Best limit from ^{3}H: $m(\nu_e) \leq 2.3$ eV at 95 % C.L. (Mainz, Troitsk)

Neutrino mass

- Triton decay $^3\text{H} \rightarrow {}^3\text{He} + e^- + \overline{\nu_e} \Rightarrow m(\nu_e) < 2.3$ eV
- future: KATRIN $m(\nu_e) < 0.2$ eV



- cosmology: $\Omega_\nu h^2 = \frac{\sum m_\nu}{92.5 \text{ eV}}$; structure formation and $m_\nu, \ldots$

Bound on $\sum m_\nu$	Data used
0.69 eV	WMAP, 2dF, H_0, Lyα
1.01 eV	WMAP, 2dF, H_0
1.8 eV	WMAP, SDSS

Mass terms

In SM: Higgs Mechanism

$$\mathcal{L} = h_d\,\overline{Q_L}\,\Phi\,d_R + h_u\,\overline{Q_L}\,\Phi^c\,u_R \xrightarrow{SSB} \frac{h_d\,v}{\sqrt{2}}\,\overline{d_L}\,d_R + \frac{h_u\,v}{\sqrt{2}}\,\overline{u_L}\,u_R \equiv m_d\,\overline{d_L}\,d_R + m_u\,\overline{u_L}\,u_R$$

with

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix},\ E_L = \begin{pmatrix} (\nu_e)_L \\ e_L \end{pmatrix} \text{ and } \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{SSB} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

Analogously with leptons:

$$\mathcal{L} = h_\nu\,\overline{E_L}\,\Phi^c\,\nu_R + h_e\,\overline{E_L}\,\Phi\,e_R \xrightarrow{SSB} \frac{h_\nu\,v}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R + \frac{h_e\,v}{\sqrt{2}}\,\overline{e_L}\,e_R \equiv m_\nu\,\overline{\nu_L}\,\nu_R + m_e\,\overline{e_L}\,e_R$$

No mass term for $\nu \Leftrightarrow$ No ν_R

Neutrino Mixing

Suppose Neutrinos have mass:

$$E_L^1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad E_L^2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \quad E_L^3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$e_R^1 = e_R \; , \quad e_R^2 = \mu_R \; , \quad e_R^3 = \tau_R$$

$$\nu_R^1 = (\nu_e)_R \; , \quad \nu_R^2 = (\nu_\mu)_R \; , \quad \nu_R^3 = (\nu_\tau)_R$$

$\Rightarrow$ Mass *Matrices* m_ν and m_ℓ:

$$\mathcal{L} = h_\nu^{ij} \, \overline{E_L^i} \, \Phi^c \, \nu_R^j + h_\ell^{ij} \, \overline{E_L^i} \, \Phi \, e_R^j \xrightarrow{SSB} (m_\nu)_{ij} \, \overline{(\nu^i)_L} \, \nu_R^j + (m_\ell)_{ij} \, \overline{(e^i)_L} \, e_R^j$$

$$\equiv \overline{\nu'_L} \, m_\nu \, \nu'_R + \overline{\ell'_L} \, m_\ell \, \ell'_R$$

with

$$\nu'_{L,R} \equiv \begin{pmatrix} (\nu_e)_{L,R} \\ (\nu_\mu)_{L,R} \\ (\nu_\tau)_{L,R} \end{pmatrix} \text{ and } \ell'_{L,R} \equiv \begin{pmatrix} e_{L,R} \\ \mu_{L,R} \\ \tau_{L,R} \end{pmatrix}$$

Neutrino Mixing

Diagonalization of mass matrices:

$$m_\nu^{\rm diag} = U_L^\dagger\, m_\nu\, U_R \text{ and } m_\ell^{\rm diag} = V_L^\dagger\, m_\ell\, V_R \text{ with } U_{L,R}\, U_{L,R}^\dagger = \mathbb{1} \text{ and } V_{L,R}\, V_{L,R}^\dagger = \mathbb{1}$$

New basis ("flavor basis" → "mass basis")

$$\mathcal{L} = \overline{\nu'_L}\, m_\nu\, \nu'_R + \overline{\ell'_L}\, m_\ell\, \ell'_R + \frac{g}{\sqrt{2}}\, W^\alpha\, \overline{\ell'_L}\, \gamma_\alpha\, \nu'_L$$

$$\overline{\nu'_L}\, U_L\, U_L^\dagger\, m_\nu\, U_R\, U_R^\dagger\, \nu'_R + \overline{\ell'_L}\, V_L\, V_L^\dagger\, m_\ell\, V_R\, V_R^\dagger\, \ell'_R + \frac{g}{\sqrt{2}}\, W^\alpha\, \overline{\ell'_L}\, \gamma_\alpha\, V_L\, V_L^\dagger\, U_L\, U_L^\dagger\, \nu'_L$$

$$\equiv \overline{\nu_L}\, m_\nu^{\rm diag}\, \nu_R + \overline{\ell_L}\, m_\ell^{\rm diag}\, \ell_R + \frac{g}{\sqrt{2}}\, W^\alpha\, \overline{\ell_L}\, \gamma_\alpha\, U\, \nu_L$$

with

$$\nu_L \equiv U_L^\dagger\, \nu'_L\ , \quad \nu_R \equiv U_R^\dagger\, \nu'_R\ , \quad \ell_L \equiv V_L^\dagger\, \ell'_L\ , \quad \ell_R \equiv V_R^\dagger\, \ell'_R\ ,$$

<u>P</u>ontecorvo–<u>M</u>aki–<u>N</u>akagata–<u>S</u>akawa (PMNS) Mixing Matrix

$$U = V_L^\dagger\, U_L$$

Remarks on PMNS

Possible Parametrization:

$$U = V_L^\dagger \, U_L = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & -c_{12}\, s_{23} - s_{11}\, c_{23}\, s_{13}\, e^{-i\delta} & c_{23}\, c_{13} \end{pmatrix}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$

- $\nu_\alpha = U^*_{\alpha i}\, \nu_i$ with $\alpha = e, \mu, \tau$ (flavor states, interacting) and $i = 1, 2, 3$ (mass states, propagating)
- three angles and one phase (CP violation!!)
- analogous to CKM Matrix for Quarks
- a priori $\theta^\nu_{ij} \neq \theta^q_{ij}$ and $\delta^\nu \neq \delta^q$
- If $m_\nu = 0$ then $U = \mathbb{1}$

Consequences of PMNS matrix: Oscillations

At time $t = 0$ flavor state $|\nu_\alpha\rangle$ produced with time evolution

$$|\nu(t)\rangle = U^*_{\alpha i}\, e^{-iE_i\, t}\, |\nu_i\rangle$$

Amplitude for probability of finding state $|\nu_\beta\rangle$ at later time t

$$\langle\nu_\beta|\nu(t)\rangle = U^*_{\alpha i}\, e^{-iE_i\, t}\, \langle\nu_\beta|\nu_i\rangle = U_{\beta j}\, U^*_{\alpha i}\, e^{-iE_i\, t}\, \langle\nu_j|\nu_i\rangle = U_{\beta j}\, U^*_{\alpha j}\, e^{-iE_j\, t}$$

and probability

$$P(\nu_\alpha \to \nu_\beta; t) = \left|U_{\beta j}\, U^*_{\alpha j}\, e^{-iE_j\, t}\right|^2$$

(sum over j!!)

with relativistic neutrinos

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \simeq p + \frac{m_i^2}{2E}$$

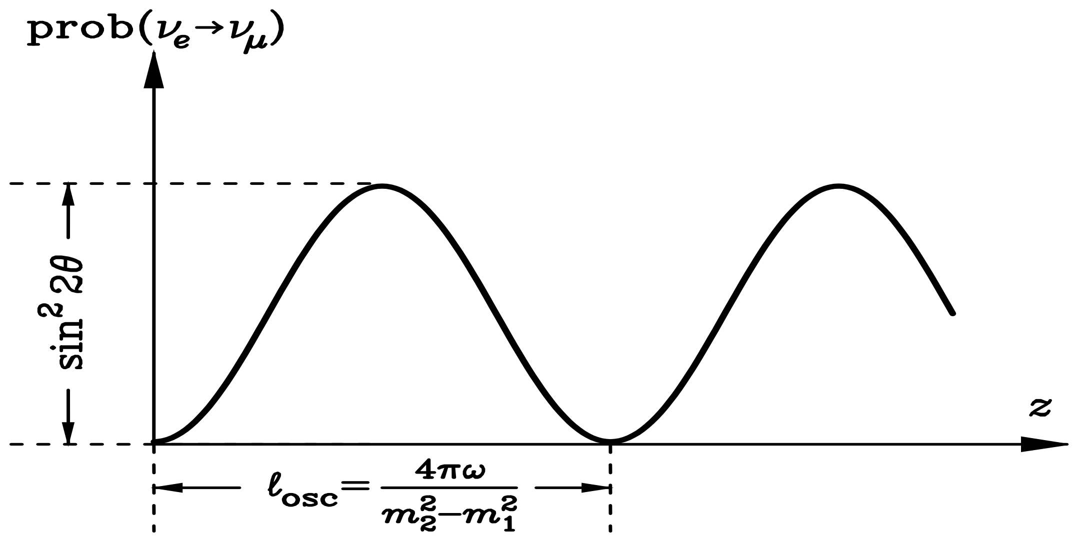
Two flavor case

$$\nu_\alpha = U^*_{\alpha i}\,\nu_i \rightarrow \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

gives

$$P(\nu_e \rightarrow \nu_\mu; t) = \sin^2 2\theta\, \sin^2 \frac{m_2^2 - m_1^2}{4E} t = \sin^2 2\theta\, \sin^2 \frac{\Delta m^2}{4E} t$$

$$= \sin^2 2\theta\, \sin^2 \left(\pi \frac{L}{l_{\rm osc}}\right) = \sin^2 2\theta\, \sin^2 \left(1.27 \frac{\Delta m^2}{\mathrm{eV}^2}\, \frac{L}{\mathrm{km}}\, \frac{\mathrm{GeV}}{E}\right)$$



Experimental Constraints

Nature provides mixing angle θ and mass–squared difference Δm^2

Experiments can "choose" energy E and baseline L

$$(\Delta m^2)_{\min} \sim \frac{E}{L}$$

Source	Flavor	E [GeV]	L [km]	$(\Delta m^2)_{\min}$ [eV^2]
Atmosphere	$\overset{(-)}{\nu}_e$, $\overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 10^2$	$10 \ldots 10^4$	10^{-6}
Sun	ν_e	$10^{-3} \ldots 10^{-2}$	10^8	10^{-11}
Reactor	$\overline{\nu_e}$	$10^{-4} \ldots 10^{-2}$	10^{-1}	10^{-3}
LBL accelerator	$\overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 1$	10^2	$1 \ldots 10$
SBL accelerator	$\overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 1$	10^{-1}	10^{-1}

Oscillations in matter

relativistic limit $E \gg m_i^2$

$$i\,\partial_t\,\Psi = \frac{M^2}{2E}\,\Psi \text{ with } \Psi = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \text{ and } M^2 = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}$$

in matter coherent forward scattering of ν_e
described through effective Hamiltonian for CC interactions
gives potential for ν_e (in flavor basis $U^T\,M^2\,U$!!)

$$V = \sqrt{2}\,G_F\,N_e \quad \text{(neutral, unpolarized matter)}$$

and therefore

$$i\,\partial_t \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\,\cos 2\theta & \frac{\Delta m^2}{4E}\,\sin 2\theta \\ \frac{\Delta m^2}{4E}\,\sin 2\theta & \frac{\Delta m^2}{4E}\,\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} -\frac{\Delta m^2}{4E}\,\cos 2\theta + \sqrt{2}\,G_F\,N_e & \frac{\Delta m^2}{4E}\,\sin 2\theta \\ \frac{\Delta m^2}{4E}\,\sin 2\theta & \frac{\Delta m^2}{4E}\,\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Oscillations in matter

Diagonalizing (constant N_e)

$$H = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}\, G_F\, N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}$$

gives flavor states *in matter:*

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{2E}\sin 2\theta}{\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}\, G_F\, N_e}$$

Maximal mixing ($\theta_m = \pi/4$) if

$2\sqrt{2}\, G_F\, N_e\, E \stackrel{!}{=} \Delta m^2 \cos 2\theta$ *even if θ is small!!*

w.l.o.g: $\Delta m^2 > 0 \Rightarrow$ sensitive to $\theta <$ or $> \pi/4$

Example core of Sun: $0.5 \left(\frac{E}{\mathrm{MeV}}\right) \stackrel{!}{\simeq} \left(\frac{\Delta m^2}{8 \cdot 10^{-5}\ \mathrm{eV}^2}\right)\left(\frac{\cos 2\theta}{0.4}\right)$

MSW Effect

$$\nu_A = \nu_e \cos\theta_m + \nu_\mu \sin\theta_m$$
$$\nu_B = -\nu_e \sin\theta_m + \nu_\mu \cos\theta_m$$

with $\tan 2\theta_m = \dfrac{\dfrac{\Delta m^2}{2E}\sin 2\theta}{\dfrac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}\,G_F\,N_e}$

Sun: ν_e pass through a medium with slowly varying ("adiabatically") density (neutrino is propagation eigenstate all along its trajectory, therefore no $\nu_B \to \nu_A$ transitions)

High density: $\theta_m \simeq \pi/2$ $\nu_B \simeq -\nu_e$

Resonance: $\theta_m \simeq \pi/4$

Low density: $\theta_m \simeq \theta$ $\nu_B \simeq \nu_\mu \cos\theta - \nu_e \sin\theta \Rightarrow P(\nu_e \to \nu_\mu) = \cos^2\theta$

condition for adiabaticity is (density variation small over several oscillation lengths)

$$\gamma = \frac{\Delta m^2 \sin^2 2\theta}{2E\cos 2\theta}\,\frac{1}{\nabla \ln N_e} \gg 1$$

happens indeed for found parameters

Three flavor oscillations

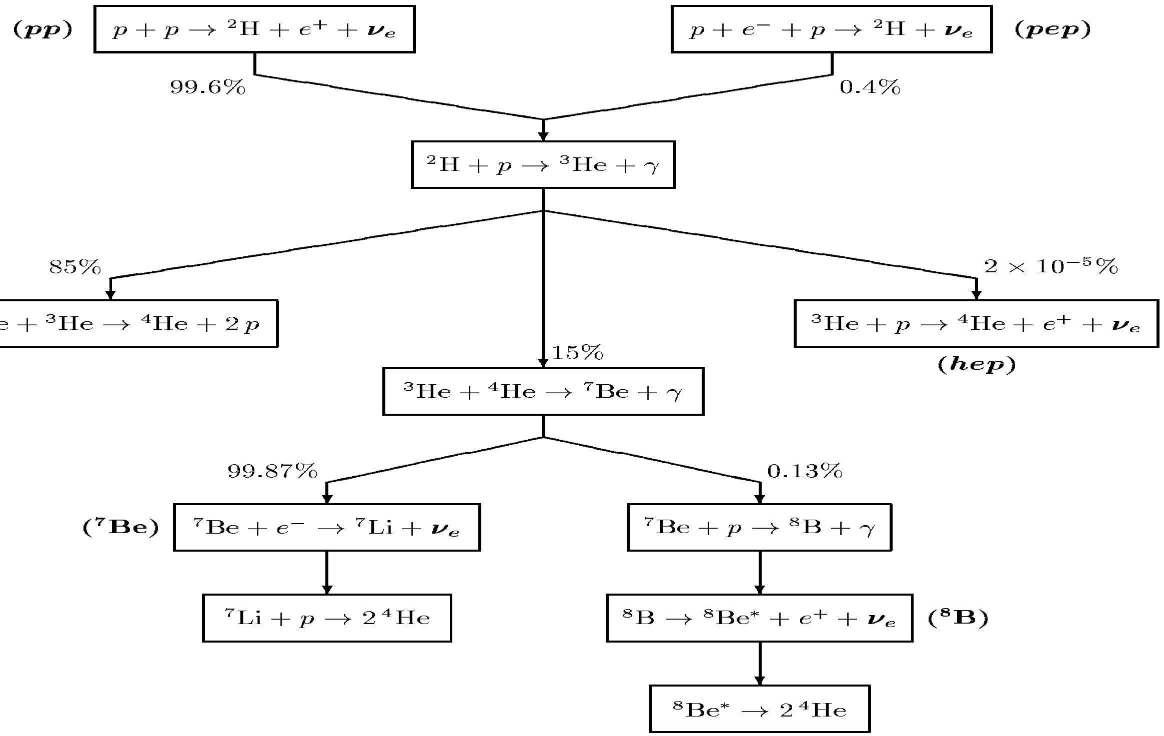
$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 2\,\mathcal{R} \sum_{j>i} U_{\alpha i}\, U^*_{\alpha j}\, U^*_{\beta i}\, U_{\beta j} \left[1 - \exp\left\{ i \frac{\Delta m^2_{ji}}{4E} L \right\}\right]$$

- two independent $\Delta m^2_{ji} = m^2_j - m^2_i$ due to $\Delta m^2_{21} = \Delta m^2_{31} - \Delta m^2_{32}$
- simplifies for $|\Delta m^2_{21}| \ll |\Delta m^2_{32}| \simeq |\Delta m^2_{31}|$ and $|U_{e3}| \ll 1$
- $P(\nu_\alpha \to \nu_\beta) \neq P(\overline{\nu_\alpha} \to \overline{\nu_\beta})$ if there is CP violation

$$U = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & -c_{12}\, s_{23} - s_{11}\, c_{23}\, s_{13}\, e^{-i\delta} & c_{23}\, c_{13} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & s_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Solar Neutrinos

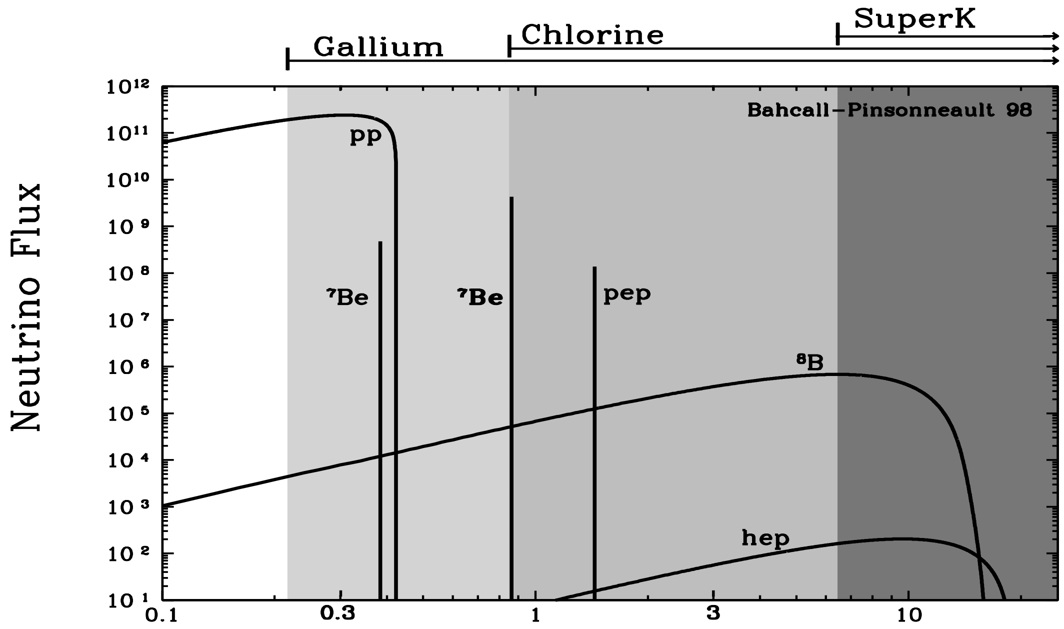


$$4p + 2e^- \to {}^4\mathrm{He} + 2\nu_e + 26.73\ \mathrm{MeV} \quad \Leftrightarrow 10^{10}\ \nu\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$$

$$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{12}\ \sin^2 \frac{\Delta m^2_{21}}{4E}\, L$$

$$(\Delta m^2_{31} \gg \Delta m^2_{21}\ \text{oscillations averaged})$$

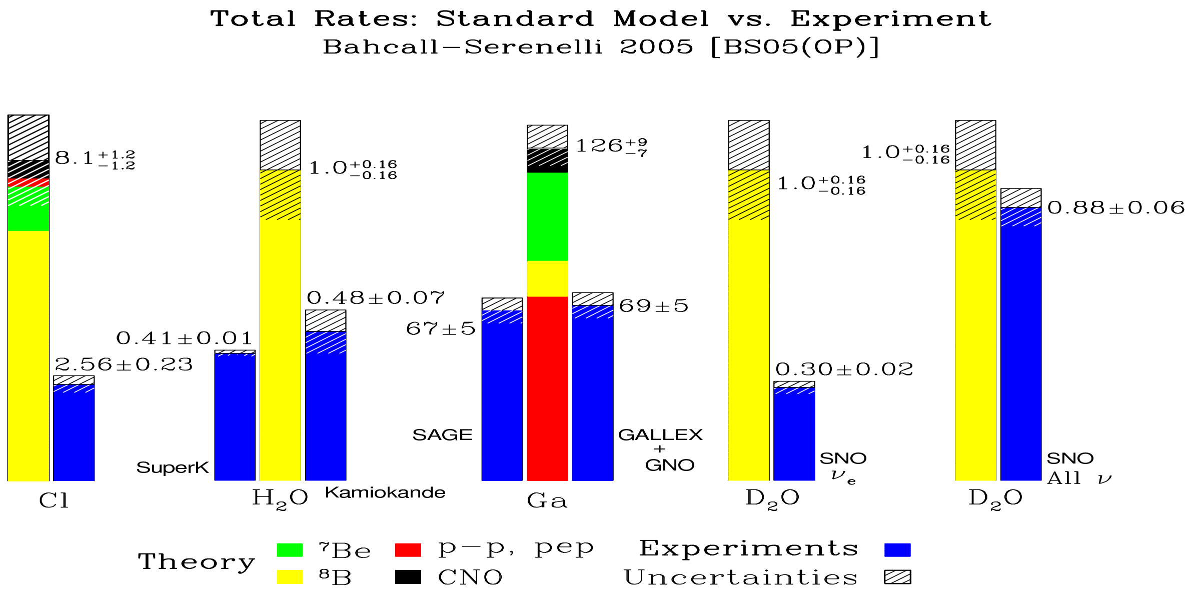
Solar Neutrinos

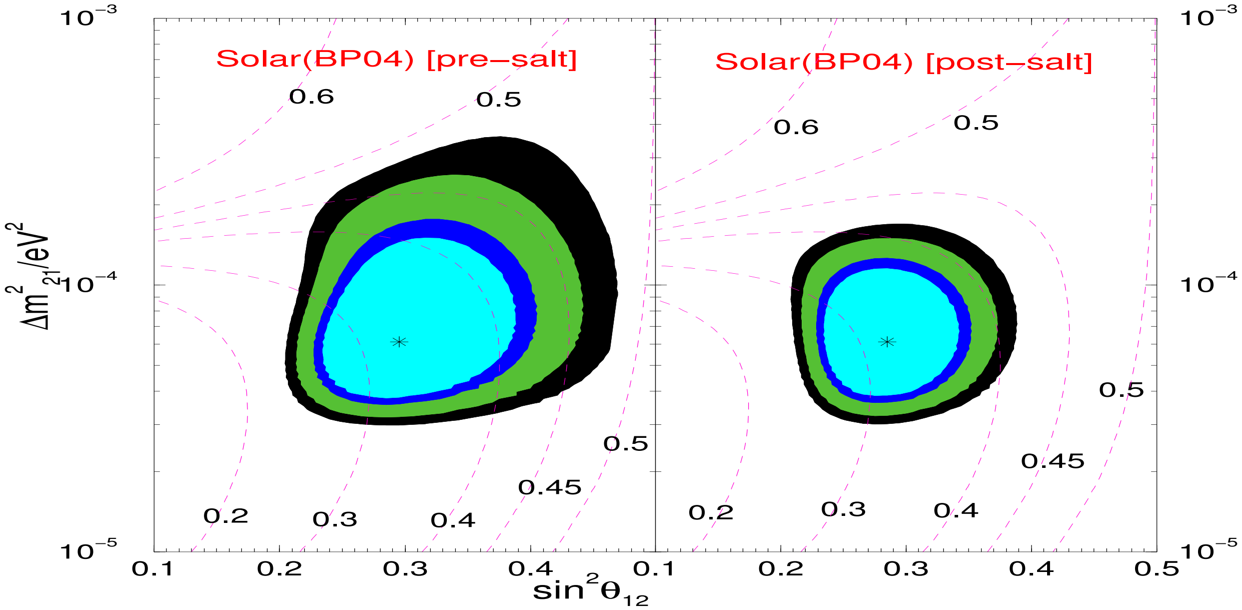


Solar neutrino energy spectrum

Strategies for solar ν detection:

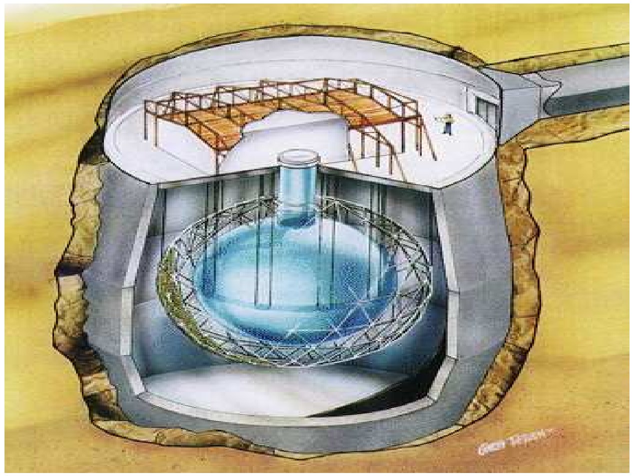
- $\nu_e + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e^-$ (Homestake)
- $\nu_e + {}^{71}\mathrm{Ga} \rightarrow {}^{71}\mathrm{Ge} + e^-$ (SAGE, GALLEX)
- $\nu_e + e^- \rightarrow \nu_e + e^-$ (Kamiokande, SuperKamiokande)

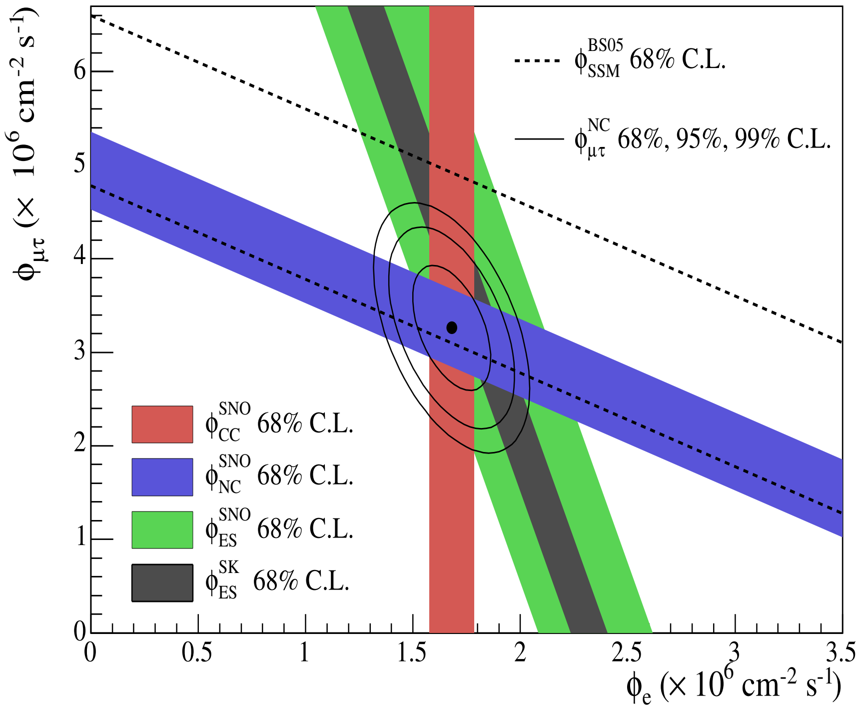




$$\sin^2\theta_{12} \simeq 0.3 \text{ and } \Delta m^2_{21} \equiv \Delta m^2_{\odot} \simeq 8 \cdot 10^{-5}\ \text{eV}^2$$

SNO





- $\nu_e + d \to p + p + e^-$ (CC) $\Rightarrow \Phi_e = P(\nu_e \to \nu_e)\, \Phi^{\rm SSM}$
- $\nu_\alpha + d \to p + n + \nu_\alpha$ (NC) $\Rightarrow \Phi_e + \Phi_{\mu\tau}$
- $\nu_\alpha + e^- \to \nu_\alpha + e^-$ (elastic scattering) $\Rightarrow \Phi_e + 0.16\, \Phi_{\mu\tau}$

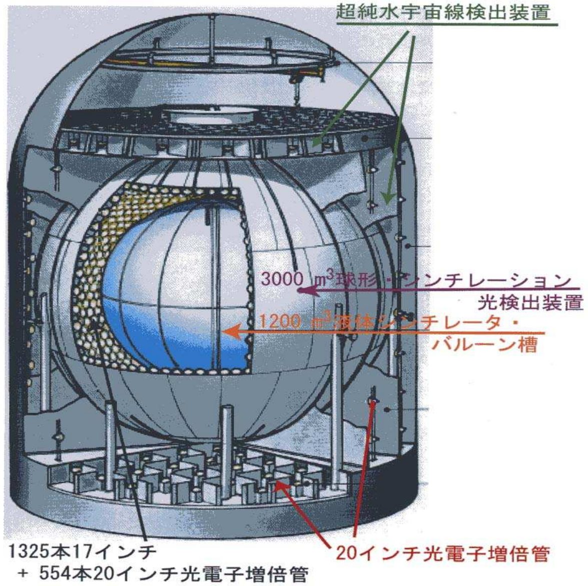
TESTING SOLAR NEUTRINOS WITH REACTORS: KAMLAND

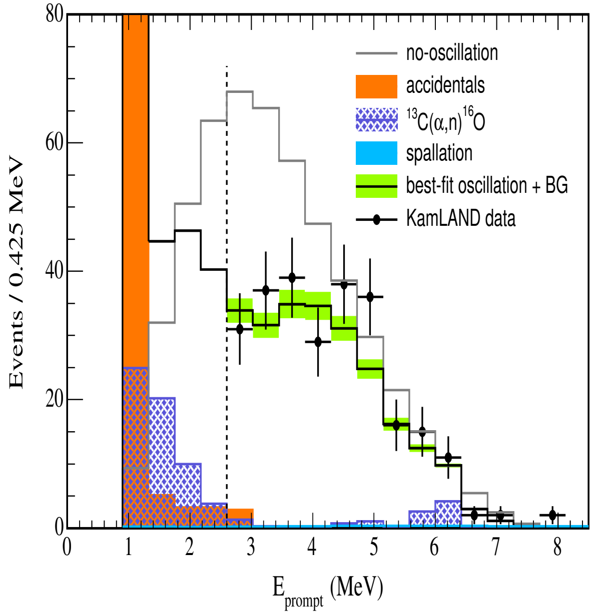
Reactor neutrinos from neutron rich fission products

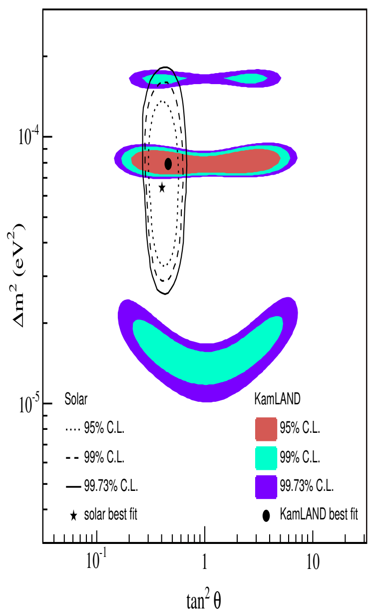
$$n \to p + e^- + \overline{\nu_e} \quad \text{with } E \simeq \text{few MeV}$$

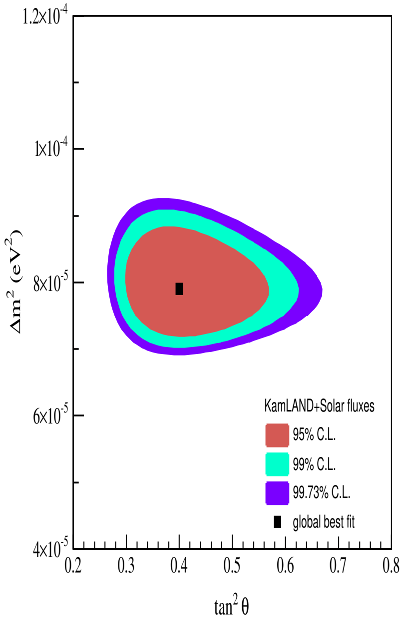
If $L \simeq 100$ km:

$$\frac{\Delta m^2_\odot}{E} L \sim 1 \Rightarrow \text{ solar } \nu \text{ parameters!!}$$





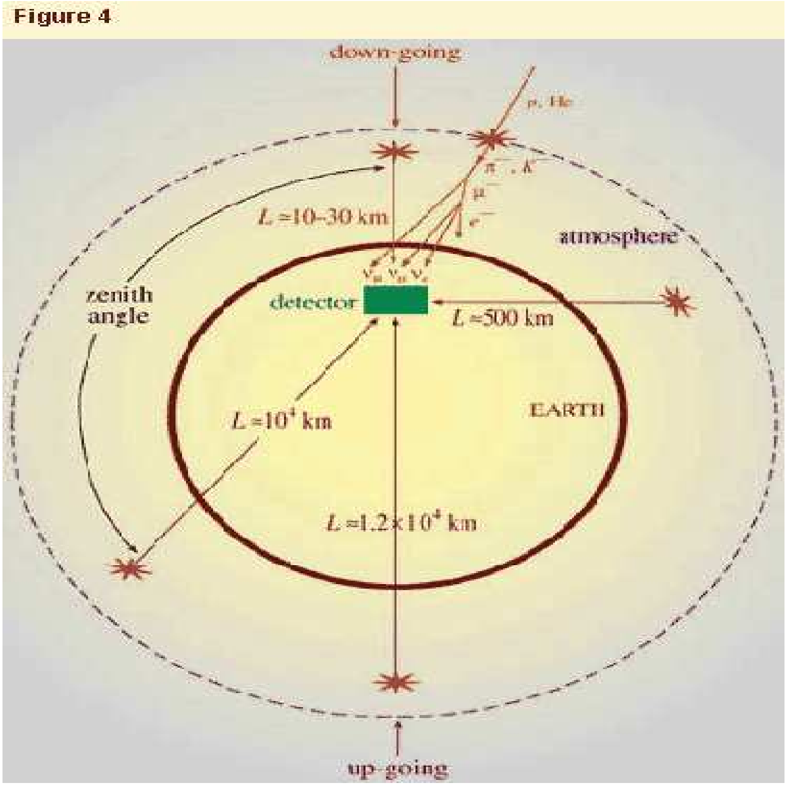




Spectral distortion consistent with oscillations

Parameters consistent with solar neutrinos!!

Atmospheric Neutrinos

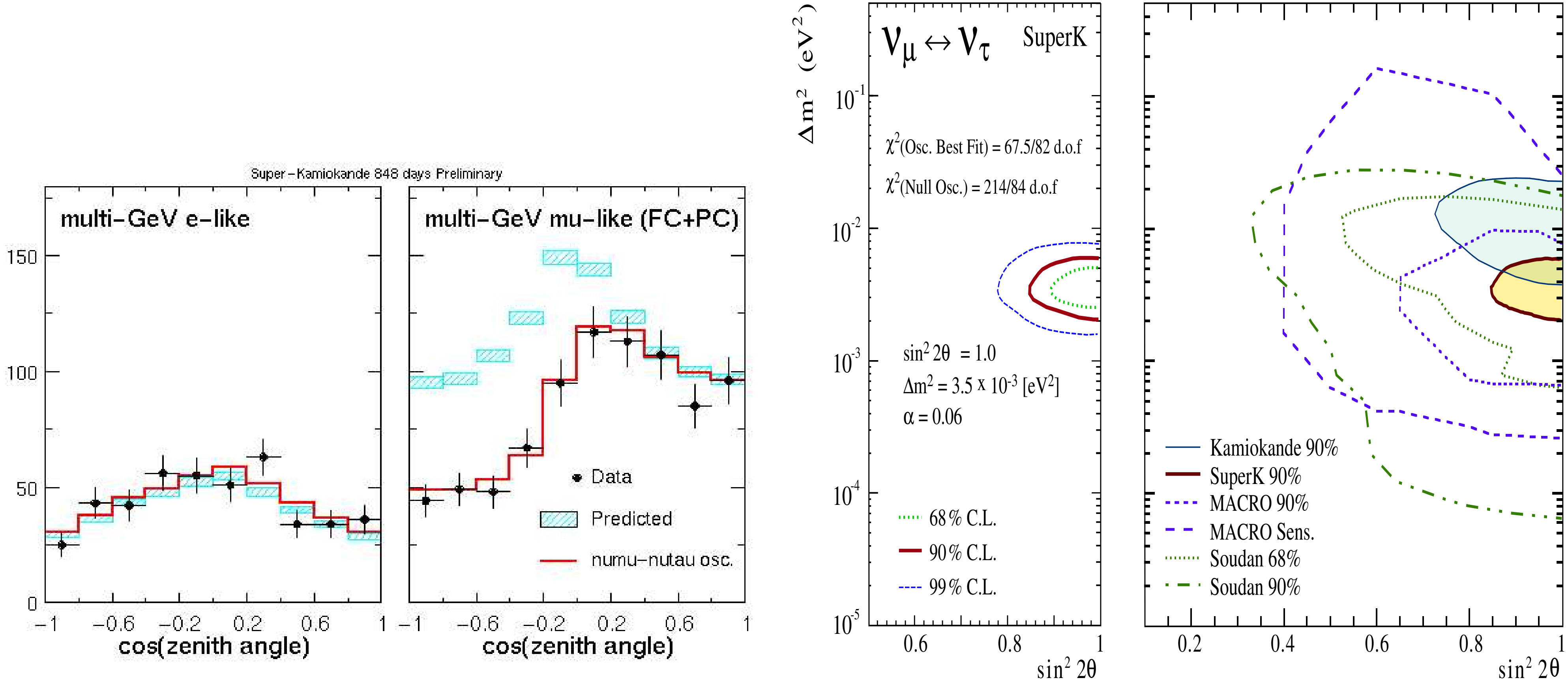


zenith angle $\cos\theta = 1$ $L \simeq 500$ km

zenith angle $\cos\theta = 0$ $L \simeq 10$ km down–going

zenith angle $\cos\theta = -1$ $L \simeq 10^4$ km up–going

Atmospheric Neutrinos

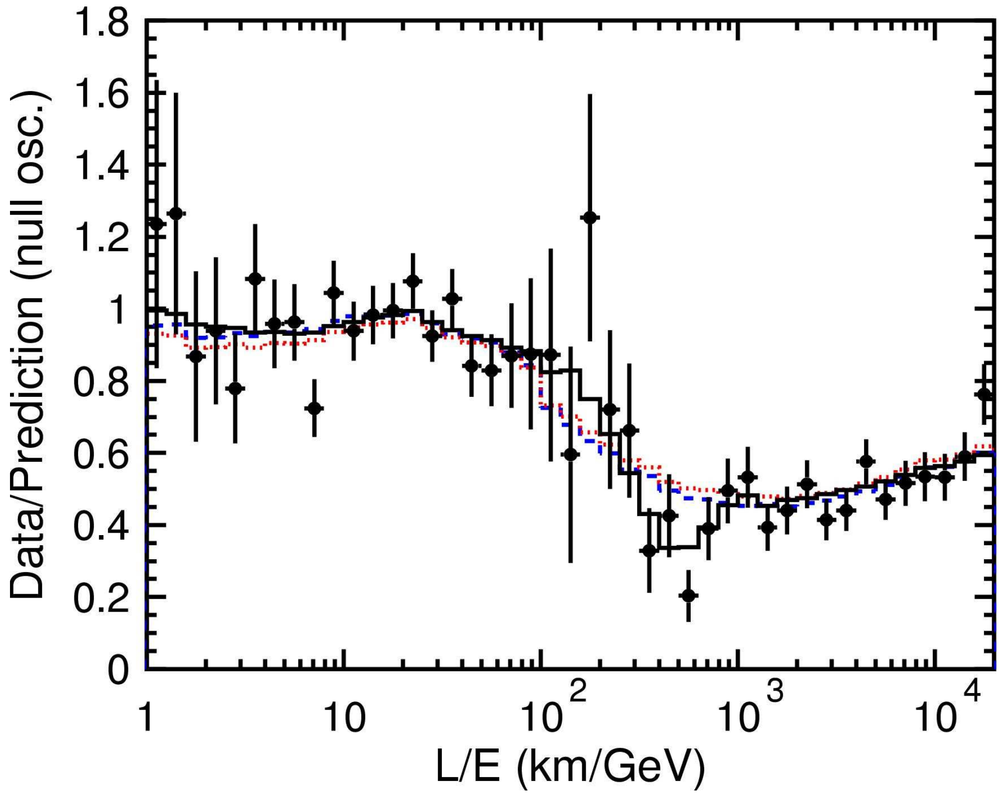


For $L \simeq 10^4$ km: $P(\nu_\mu \to \nu_\tau) \simeq \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{32}}{4E} L$

($\Delta m^2_{21} \ll \Delta m^2_{32}$ oscillations frozen)

with $\theta_{23} \simeq \pi/4$ MAXIMAL MIXING!! and $\Delta m^2_{32} \equiv \Delta m^2_{\rm A} \simeq 2 \cdot 10^{-3}$ eV^2

Atmospheric Neutrinos



Dip at $L/E \simeq 500$ km/GeV $\Rightarrow$ Oscillatory Behavior!!

(No ν_τ observed yet)

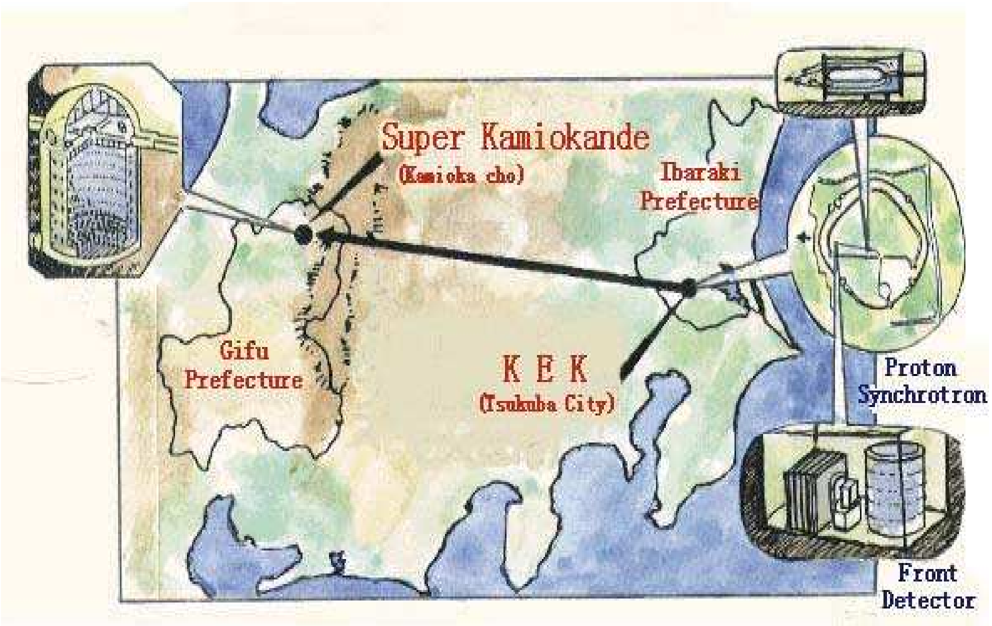
Testing Atmospheric Neutrinos with Accelerators: K2K

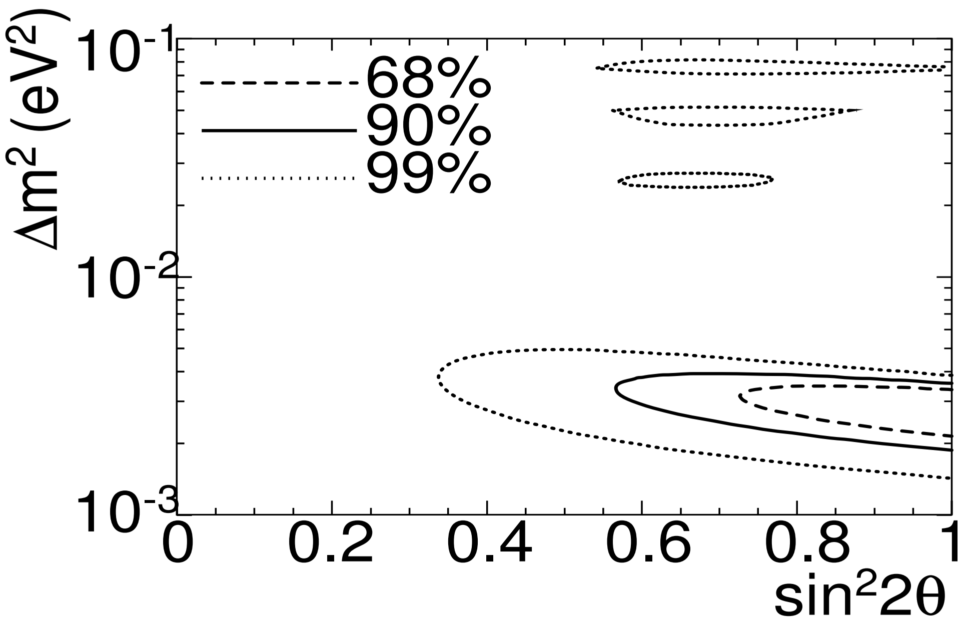
Proton beam

$$p + X \to \pi^{\pm}\,,\ K^{\pm} \to \pi^{\pm} \to \overset{(-)}{\nu}_{\mu} \quad \text{with } E \simeq \text{GeV}$$

If $L \simeq 100$ km:

$$\frac{\Delta m^2_{\rm A}}{E}\, L \sim 1 \Rightarrow \text{ atmospheric } \nu \text{ parameters!!}$$





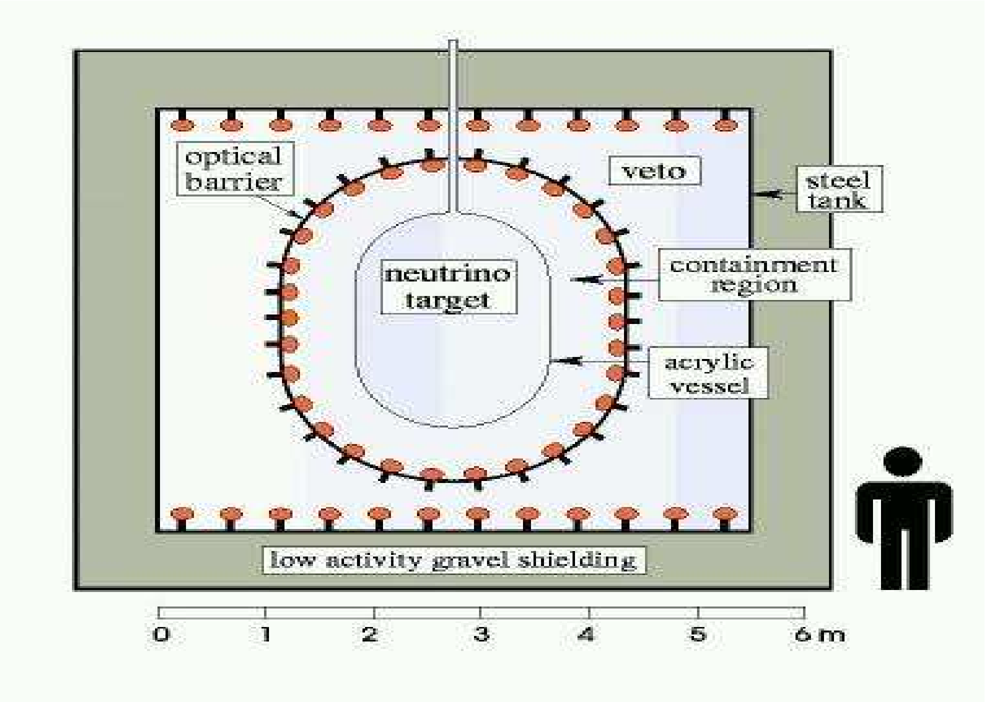
Parameters consistent with atmospheric neutrinos!!

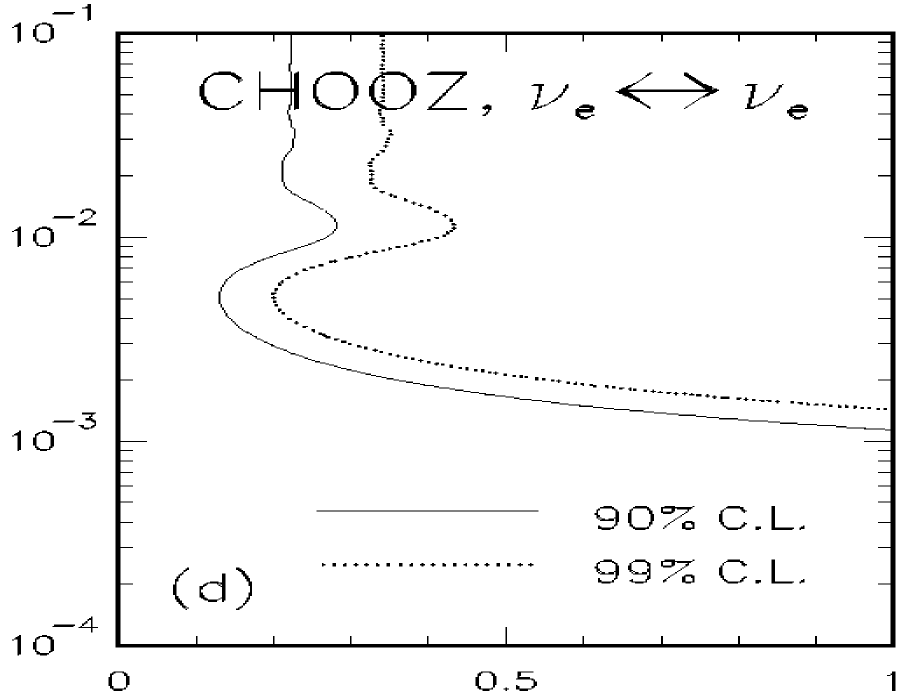
The third mixing: Short–Baseline Reactor Neutrinos

$E_\nu \simeq$ few MeV and $L \simeq 0.1$ km:

$$\frac{\Delta m^2_{\rm A}}{E} L \sim 1 \Rightarrow \text{ atmospheric } \nu \text{ parameters!!}$$

with $P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{32}}{4E} L$





$$\sin^2\theta_{13} = |U_{e3}|^2 \leq 0.05$$

The emerging picture

$$U = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & -c_{12}\, s_{23} - s_{11}\, c_{23}\, s_{13}\, e^{-i\delta} & c_{23}\, c_{13} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & s_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

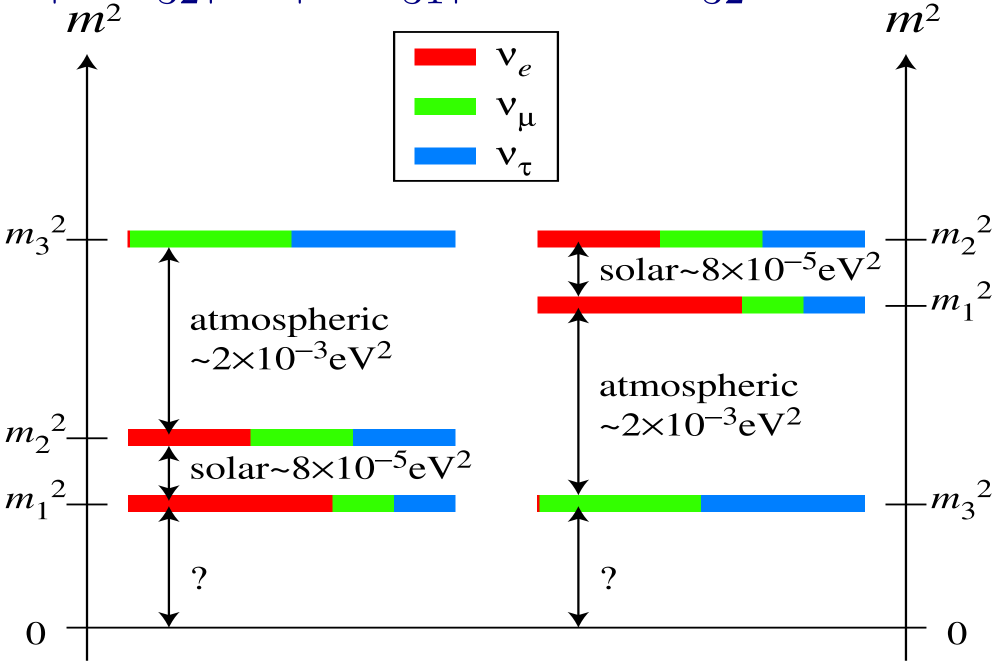
- $\theta_{12} \simeq 33^0 \leftrightarrow$ solar + KamLAND neutrinos
- $\theta_{23} \simeq 45^0 \leftrightarrow$ atmospheric + K2K neutrinos
- $\theta_{13} \lesssim 13^0 \leftrightarrow$ short baseline reactor neutrinos (“CHOOZ angle”, $|U_{e3}|$)
- δ testable in (*three flavor!*) long–baseline oscillations

The emerging picture

$$|U| = \begin{pmatrix} 0.73-0.88 & 0.47-0.67 & 0-0.23 \\ 0.17-0.57 & 0.37-0.73 & 0.56-0.84 \\ 0.20-0.58 & 0.40-0.75 & 0.54-0.82 \end{pmatrix} \overset{BF}{=} \begin{pmatrix} 0.84 & 0.55 & 0 \\ 0.39 & 0.59 & 0.71 \\ 0.39 & 0.59 & 0.71 \end{pmatrix}$$

Hierarchy of mass squared differences and unknown smallest neutrino mass

$$\Delta m^2_{21} \ll |\Delta m^2_{32}| \simeq |\Delta m^2_{31}| \text{ with } \Delta m^2_{32} < 0 \text{ or } \Delta m^2_{32} > 0$$



Two small parameters: $|U_{e3}| \lesssim 0.2$ and $R \equiv \Delta m^2_\odot / \Delta m^2_{\rm A} \simeq 1/25$

Neutrino masses

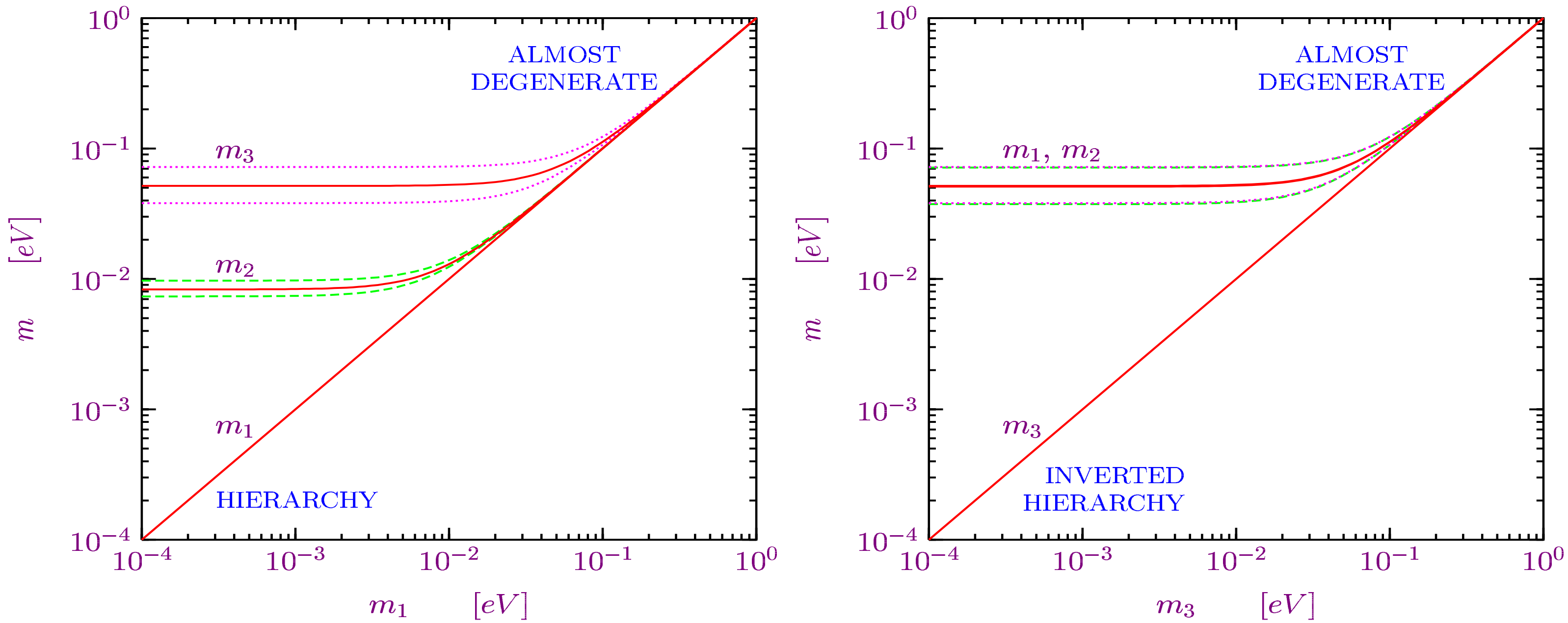
$$|\Delta m^2_{32}| \simeq 2 \cdot 10^{-3}\ \text{eV}^2 \Rightarrow 0.04\ \text{eV} \lesssim m_{\text{heaviest}} \lesssim 2.3\ \text{eV}$$

$$0 \lesssim m_{\text{smallest}} \lesssim 2.3\ \text{eV}$$

normal ordering:
$$\begin{cases} m_{\text{smallest}} = m_1 \\ m_2 = \sqrt{\Delta m^2_{\odot} + m_1^2} \\ m_3 = \sqrt{\Delta m^2_{\text{A}} + \Delta m^2_{\odot} + m_1^2} \end{cases}$$

inverted ordering:
$$\begin{cases} m_{\text{smallest}} = m_3 \\ m_2 = \sqrt{m_3^2 - \Delta m^2_{\text{A}}} \\ m_1 = \sqrt{m_2^2 - \Delta m^2_{\odot}} \end{cases}$$

Neutrino masses



- $m_3 \simeq \sqrt{\Delta m^2_{\rm A}} \gg m_2 \simeq \sqrt{\Delta m^2_{\odot}} \gg m_1$: normal hierarchy (NH)
- $m_2 \simeq \sqrt{\Delta m^2_{\rm A}} \simeq m_1 \gg m_3$: inverted hierarchy (IH)
- $m_3 \simeq m_2 \simeq m_1 \equiv m_0 \gg \sqrt{\Delta m^2_{\rm A}}$: quasi–degeneracy (QD)

The future: open issues for neutrinos oscillations

Look for *three–flavor effects:*

- precision measurements
  - how maximal is θ_{23} ? how small is U_{e3} ?
- sign of Δm^2_{32} ?

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{2E}\,\sin 2\theta}{\frac{\Delta m^2}{2E}\,\cos 2\theta - \sqrt{2}\,G_F\,N_e} = f(\mathrm{sgn}(\Delta m^2))$$

- is there CP violation?

$$\Delta P_{CP} \equiv P(\nu_e \to \nu_\mu) - P(\overline{\nu_e} \to \overline{\nu_\mu})$$

$$= \frac{1}{2}\left(\sin\frac{\Delta m^2_{21}}{2E} + \sin\frac{\Delta m^2_{32}}{2E} - \sin\frac{\Delta m^2_{31}}{2E}\right)\sin 2\theta_{12}\,\sin 2\theta_{13}\,\sin 2\theta_{23}\,\cos\theta_{13}\,\sin\delta$$

- Problems:
  - two *small* parameters: $\Delta m^2_{\odot}/\Delta m^2_{\mathrm{A}} \simeq 1/25$ and $|U_{e3}| \lesssim 0.2$
  - 8–fold degeneracy for fixed L/E and $\nu_{e,\mu} \to \nu_{e,\mu}$ channels

Degeneracies

Expand full *3–flavor* oscillation probabilities in terms of $R = \Delta m^2_\odot / \Delta m^2_{\mathrm{A}}$ and $|U_{e3}|$:

$$P(\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\mu) \simeq \sin^2 2\theta_{13}\ \sin^2\theta_{23}\ \ \frac{\sin^2(1-\hat{A})\Delta}{(1-\hat{A})^2}$$

$$\pm\ \sin\delta \cdot \sin 2\theta_{13}\ R\ \sin 2\theta_{12}\ \cos\theta_{13}\ \sin 2\theta_{23} \sin\Delta \frac{\sin\hat{A}\Delta\ \sin(1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

$$+\cos\delta \cdot \sin 2\theta_{13}\ R\ \sin 2\theta_{12}\ \cos\theta_{13}\ \sin 2\theta_{23}\ \cos\Delta \frac{\sin\hat{A}\Delta\ \sin(1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

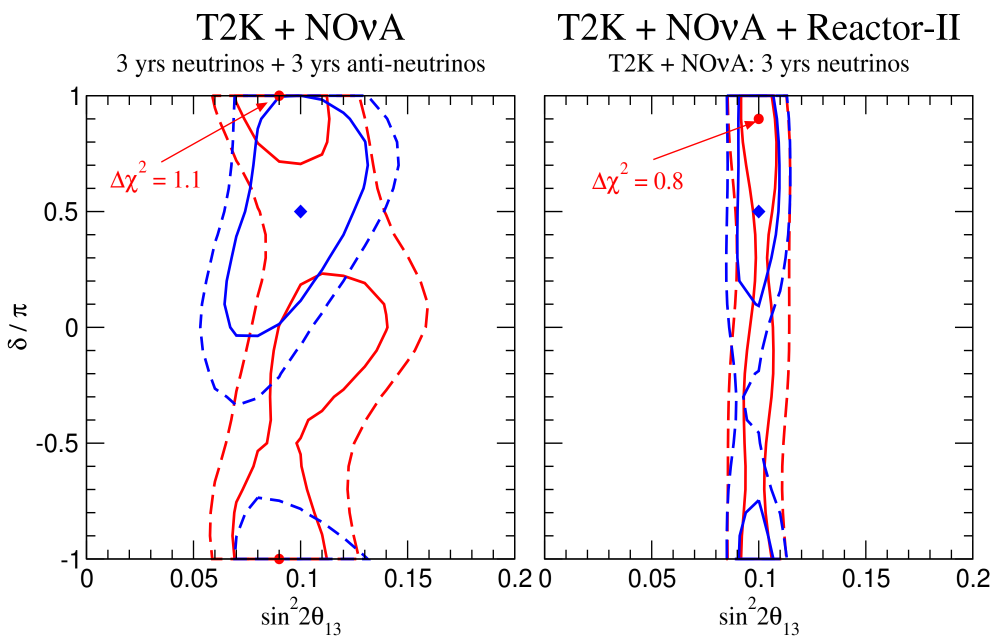
$$+\ R^2\ \sin^2 2\theta_{12}\ \cos^2\theta_{23} \frac{\sin^2\hat{A}\Delta}{\hat{A}^2} \text{ with } \hat{A} = 2VE/\Delta m^2_{\mathrm{A}} \text{ and } \Delta = \Delta m^2_{\mathrm{A}}$$

- $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$ degeneracy
- θ_{13}–δ degeneracy
- δ–$\mathrm{sgn}(\Delta m^2_{\mathrm{A}})$ degeneracy

Solutions: more channels, different L/E, high precision,. . .

Long–baseline Neutrinos

	$\Delta m^2_{\rm A}$	$\sin^2\theta_{23}$
current	88 %	79%
MINOS+CNGS	26%	78%
T2K	12%	46%
Noνa	25%	86%
Combination	9%	42%

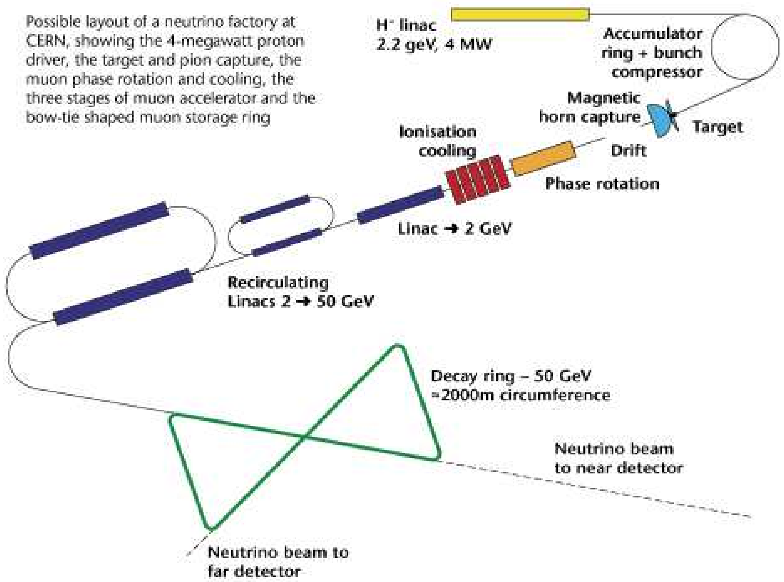


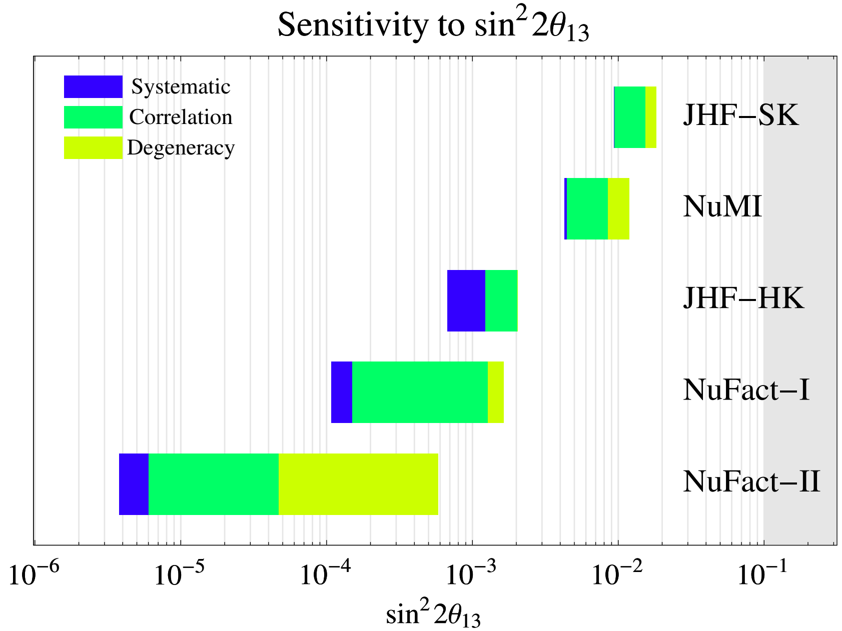
The far far future in a galaxy far far away

β–beams:
$$^{18}\mathrm{Ne} \rightarrow {}^{18}\mathrm{Fe} + e^+ + \nu_e$$
$$^{6}\mathrm{He} \rightarrow {}^{6}\mathrm{Li} + e^- + \overline{\nu_e}$$

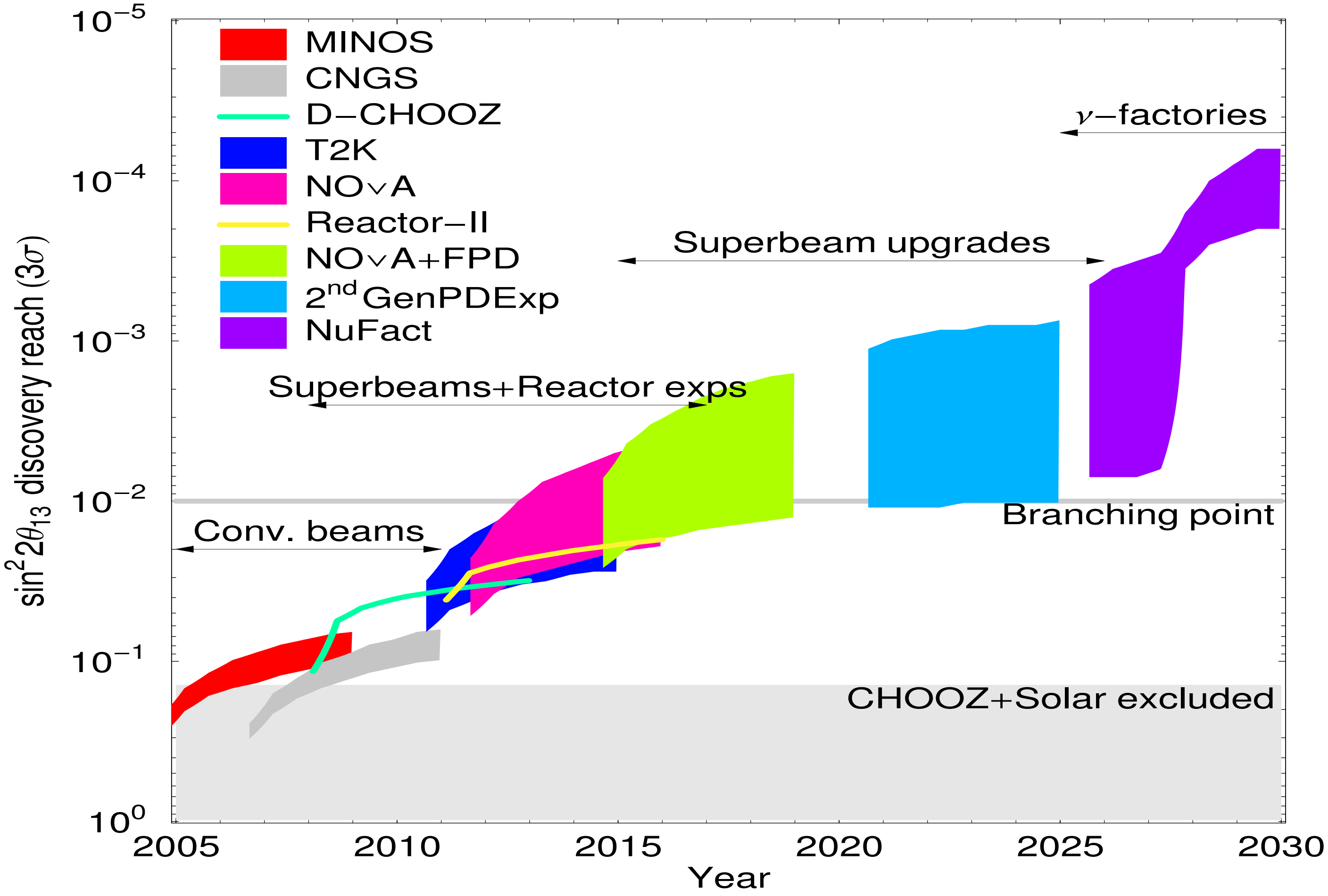
and/or "neutrino factories": $\mu^- \rightarrow e^- + \overline{\nu_e} + \nu_\mu$

flux known exactly; no background



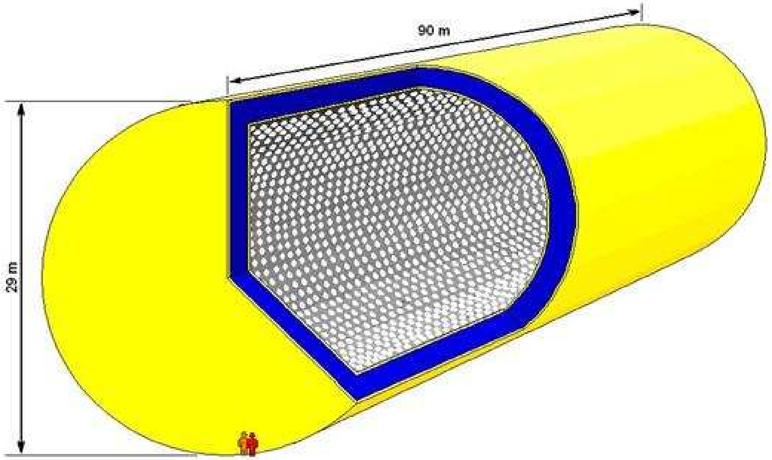


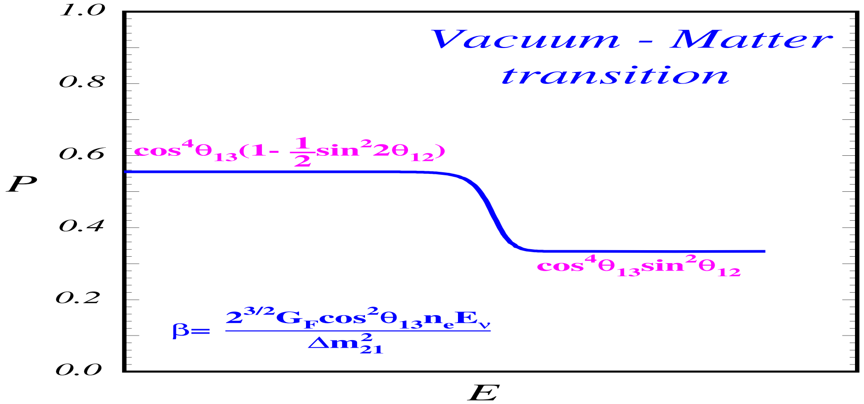
Typical time scale



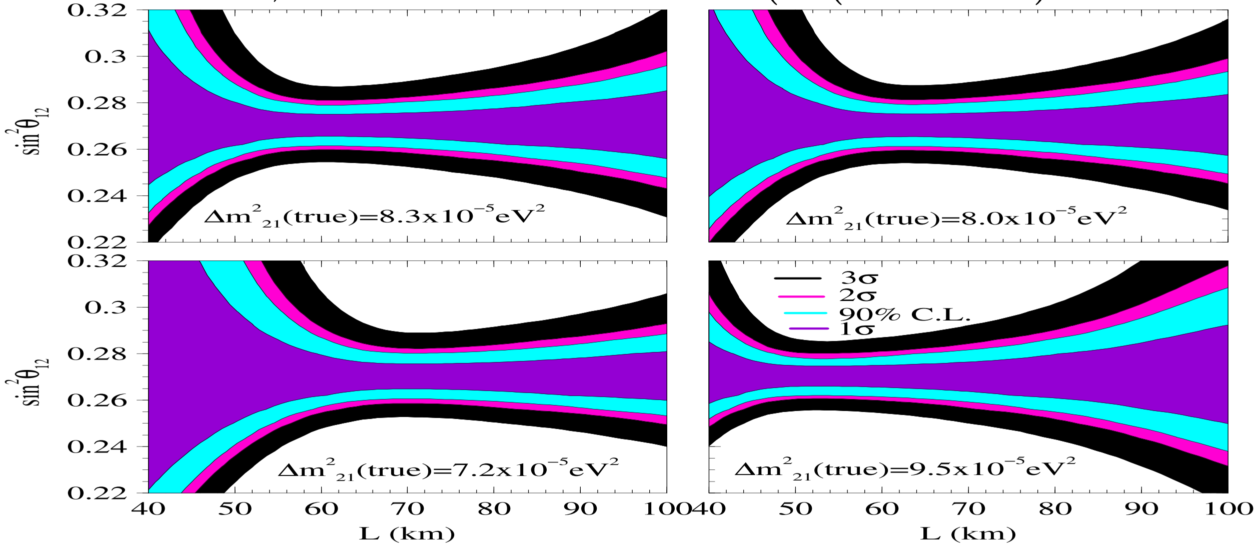
Future of solar neutrino(parameter)s

- low energy neutrinos (^{7}Be, pep, pp) from the Sun (Borexino, LENA, *pp*...)





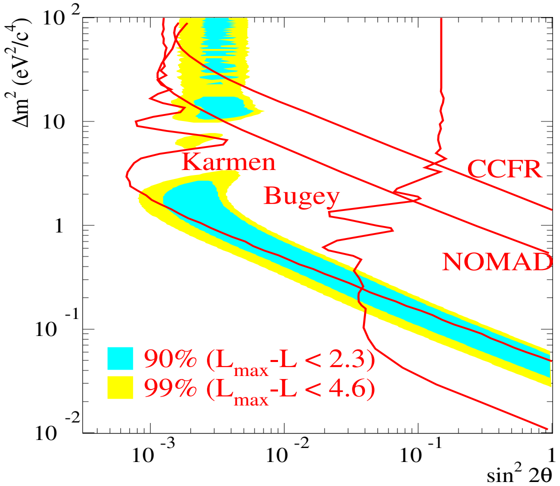
- reactor; located at SPMIN ($P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{12}$)



The black sheep: LS(N)D

Short baseline accelerator neutrinos detected via $\overline{\nu_e} + p \rightarrow n + e^+$

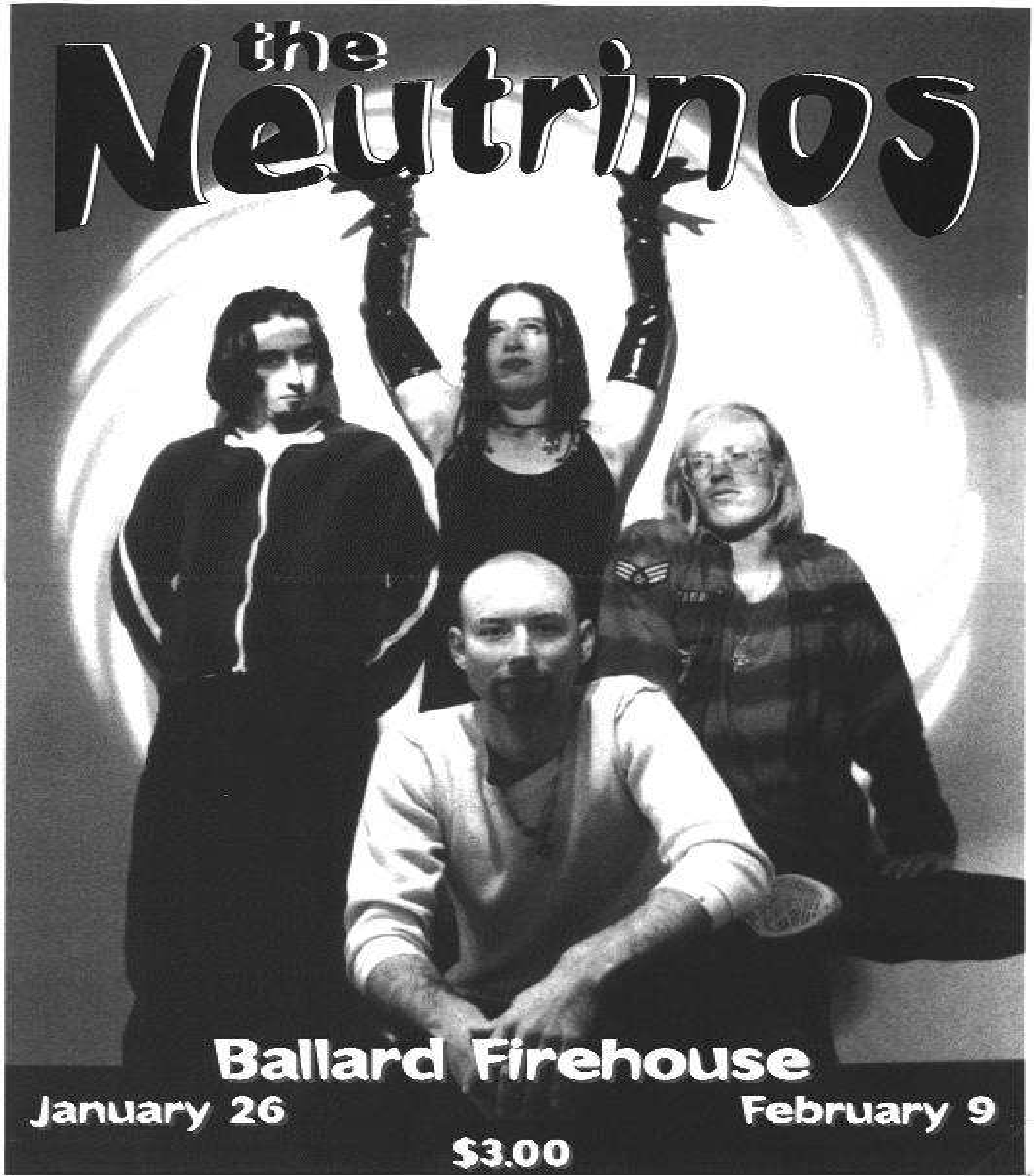
interpreted as $\overline{\nu_\mu} \rightarrow \overline{\nu_e}$ oscillations!!



$\Delta m^2 \simeq \text{eV}^2$!! $\Rightarrow$ since $N_\nu(m_\nu \leq 45 \text{ GeV}) = 3$

$\Rightarrow$ fourth light neutrino: "sterile neutrino" ν_s!!!

- Problems with solar/atmospheric neutrino experiments (2 or more ν_s?)
- Currently tested at MiniBooNE (early 2006?)



Which one is sterile?

A different mass term for Neutrinos

Till now: Dirac mass term for two *independent* neutrino fields ν_L and ν_R (just as for quarks and charged leptons)

$$\mathcal{L}_D = \frac{m_D \sqrt{2}}{v} \overline{\nu_L} \, \Phi^c \, \nu_R \xrightarrow{SSB} m_D \overline{\nu_L} \, \nu_R + h.c.$$

New field ν_R is a SM singlet! $\Rightarrow$

$\mathcal{L}_M = \frac{1}{2} M_R \overline{(\nu_R)^c} \, \nu_R + h.c.$ "Majorana mass term" will appear!

$$\psi \to \psi^c = C \, \overline{\psi}^T \text{ and } \overline{\psi^c} = \psi^T C^T = -\psi^T C$$

Majorana mass M_R has nothing to do with SM or Higgs mechanism

$$\Rightarrow M_R \gg m_D \lesssim m_{\rm top}$$

We even can assume that

$$M_R = M_{\rm GUT} \simeq 10^{16} \text{ GeV}$$

Total mass term is sum of Dirac and Majorana

Dirac + Majorana masses

Properties:

$$\overline{\nu_R^c}\, M_R\, \nu_R = \overline{\nu^c_{R\alpha}}\, (M_R)_{\alpha\beta}\, (\nu_R)_\beta = (\nu_R^T)_\alpha\, C^T\, (M_R)_{\alpha\beta}\, (\nu_R)_\beta$$

$$= -(\nu_R)^T_\beta\, (M_R)_{\alpha\beta}\, C\, (\nu_R)_\alpha = \overline{\nu^c_{R\beta}}\, (M_R)_{\alpha\beta}\, (\nu_R)_\alpha = \overline{\nu^c_{R\alpha}}\, (M_R)_{\beta\alpha}\, (\nu_R)_\beta$$

$$= \overline{\nu_R^c}\, M_R^T\, \nu_R$$

$\Rightarrow$ Majorana mass matrices are symmetric!

Moreover: $\overline{\nu_L}\, m_D\, \nu_R = \overline{\nu_R^c}\, m_D^T\, \nu_L^c$

Put everything together:

$$\mathcal{L} = \mathcal{L}_D + \mathcal{L}_M = m_D\, \overline{\nu_L}\, \nu_R + \frac{1}{2}\, M_R\, \overline{(\nu_R)^c}\, \nu_R$$

$$= \frac{1}{2}\, \overline{n_L^c} \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} n_L + h.c. \text{ with } n_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

$\Rightarrow$ Most general mass term is a Majorana mass term!!

See–saw mechanism

Diagonalize

$$\frac{1}{2}\,\overline{n_L^c}\begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} n_L \text{ with } n_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

with $M_R \gg m_D \Rightarrow$ is almost diagonal

$\Rightarrow$ Ansatz:

$$U^T \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \text{ with } U \simeq \begin{pmatrix} 1 & \rho \\ -\rho^\dagger & 1 \end{pmatrix} + \mathcal{O}(\rho^2)$$

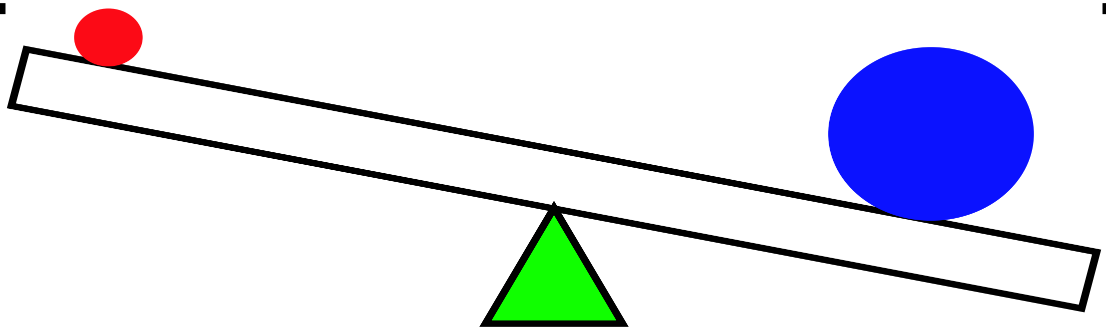
Inserting gives $\rho^* \simeq m_D^T\, M_R^{-1}$ and

$m_1 \simeq -m_D^T\, M_R^{-1}\, m_D + \mathcal{O}(\rho^2)$ three flavor neutrinos $\nu_{e,\mu,\tau}$

$m_2 \simeq M_R + \mathcal{O}(\rho)$ additional heavy neutrinos $N_{1,2,3}$

$m_\nu \simeq m_D^2/M_R \simeq v^2/(10^{15}\text{ GeV}) \simeq 0.01\text{ eV} \simeq \sqrt{\Delta m_{\text{A}}^2} \ll m_D$

explains why neutrinos are so much lighter than quarks and charged leptons!!



$$\mathcal{L} = \overline{(\nu_L)^c}\, m_\nu\, \nu_L = \overline{(\nu^c)_R}\, m_\nu\, \nu_L \sim (\nu_L)^T\, m_\nu\, \nu_L$$

- Mass term couples left–handed to right-handed field
- if independent: Dirac mass term
- if dependent: Majorana mass term
- Then left– and right–handed ν no longer independent:

$$\nu = \nu_L + \nu_R = \nu_L + (\nu_L)^c \Leftrightarrow \nu^c = \nu \text{ “Majorana particle”}$$

- Mass term $\nu^T\,\nu$ not invariant under $\nu \to e^{iL}\,\nu$ (cf. with Dirac term $\overline{\nu}\,\nu$)
  Lepton number violation!!
- Mass term $\nu^T\,\nu \Rightarrow$ two additional phases in PMNS matrix
- (Phenomenological implications of heavy Majoranas → later)

The neutrino mixing matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & s_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

- $\theta_{12} \simeq 33^0 \leftrightarrow$ solar + KamLAND neutrinos
- $\theta_{23} \simeq 45^0 \leftrightarrow$ atmospheric + K2K neutrinos
- $\theta_{13} \lesssim 13^0 \leftrightarrow$ short baseline reactor neutrinos ("CHOOZ angle", $|U_{e3}|$)
- δ testable in (*three flavor!*) long–baseline oscillations
- α, β connected to Majorana nature of neutrinos
  $\Leftrightarrow$ **only observable effects in Lepton Number Violating Processes!!**
- alternative: no Majorana phases but
  $m_1 \to m_1$, $m_2 \to m_2\, e^{2i\alpha}$ and $m_3 \to m_3\, e^{2i\beta}$
  connected to CP parities of the ν_i: CP **conservation if** $\alpha, \beta = 0, \pi/2, \pi$

Two popular cases

$\theta_{23} \simeq 45^0$ and $\theta_{12} \simeq 30^0 \longleftrightarrow$ "Bi–large Mixing"

- $\sin^2\theta_{12} = 1/3$: "Tri–bimaximal Mixing"

$$U = U_{\text{tribimax}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} P$$

- $\sin^2\theta_{12} = 1/2$: "Bimaximal Mixing"

$$U = U_{\text{bimax}} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \\ \frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix} P$$

With $\theta_{13} = 0$ no CP violation in neutrino oscillations...

“Predicting” U_{e3}

Recall charged lepton contribution to PMNS matrix

$$U = U_\ell^\dagger \, U_\nu$$

Assume that $U_\nu = U_{\rm bimax}$ is bimaximal and “quark–lepton symmetry” $U_\ell \simeq V_{\rm CKM}$

$$U_\ell = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & B\,\lambda^3 \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\,\lambda^2 \\ (A-B)\,\lambda^3 & -A\,\lambda^2 & 1 \end{pmatrix} \text{ with } \lambda \stackrel{?}{\simeq} 0.22$$

multiply $U_\ell^\dagger$ from the left to $U_{\rm bimax}$ and obtain the observables:

$$\left.\begin{array}{c} \tan^2\theta_{12} \simeq 1 - 2\sqrt{2}\cos\phi\,\lambda \\ |U_{e3}| \simeq \dfrac{\lambda}{\sqrt{2}} \\ \Delta P_{CP} \propto \sin\phi \end{array}\right\} \Rightarrow \tan^2\theta_{12} \simeq 1 - 4\cos\phi\,|U_{e3}| \stackrel{!}{\simeq} 0.43$$

$\Rightarrow |U_{e3}| \simeq 0.16 \Rightarrow \lambda \simeq 0.22 \simeq \theta_C$ and large CP violation

Structure of the Mixing matrix — Quarks vs. Leptons

$$V_{\text{CKM}} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\,\lambda^3\,(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\,\lambda^2 \\ A\,\lambda^3\,(1-\rho+i\eta) & -A\,\lambda^2 & 1 \end{pmatrix} = \mathbb{1} + \mathcal{O}(\lambda)$$

$$U_{\text{PMNS}} \simeq \begin{pmatrix} \sqrt{\frac{1}{2}}\,(1+\lambda) & \sqrt{\frac{1}{2}}\,(1-\lambda) & A_\nu\,\lambda \\ -\frac{1}{2}\left(1-(1-A_\nu\,e^{i\delta})\,\lambda\right) & \frac{1}{2}\left(1+(1-A_\nu\,e^{i\delta})\,\lambda\right) & \sqrt{\frac{1}{2}}\,(1-B_\nu\,\lambda^2)e^{i\delta} \\ \frac{1}{2}\left(1-(1+A_\nu\,e^{i\delta})\,\lambda\right) & -\frac{1}{2}\left(1+(1+A_\nu\,e^{i\delta})\,\lambda\right) & \sqrt{\frac{1}{2}}\,(1+B_\nu\,\lambda^2)e^{i\delta} \end{pmatrix}$$

$$= U_{\text{bimax}} + \mathcal{O}(\lambda)$$

"Quark–Lepton–Complementarity": $\theta_\odot + \theta_C = \pi/4$

Linked to Quark–Lepton–Symmetry??

CKM in PMNS?

Numerology:

$$\theta_{12} + \theta_C = \sin^{-1}\sqrt{0.3} + \sin^{-1} 0.22 \simeq \pi/4$$

"Quark–Lepton–Complementarity" (QLC)

Possible Realization:

$$\left.\begin{array}{l} U_\nu = U_{\rm bimax} \\ U_\ell = V_{\rm CKM} \end{array}\right\} \Rightarrow U = V_{\rm CKM}^\dagger \, U_\nu \text{ (approximate QLC)}$$

$$m_D = m_{\rm up} \quad \text{from } SO(10)$$

Go to basis in which $m_{\rm up}$ is diagonal, i.e., $U_{\rm up} = \mathbb{1}$

from $U_{\rm up} = \mathbb{1}$ it follows that $U_{\rm down} = U_\ell$

get bimaximal U_ν from special structure of M_R via see–saw

The neutrino mass matrix

Assume $\theta_{23} = \pi/4$ and $\theta_{13} = |U_{e3}| = 0$:

$$U = U(\theta_{23} = \pi/4\,,\ \theta_{13} = 0) = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

and $m_\nu = U\, m_\nu^{\rm diag}\, U^T = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(D+E) & \frac{1}{2}(D-E) \\ \cdot & \cdot & \frac{1}{2}(D+E) \end{pmatrix}$ with

$$A = m_1\ \cos^2\theta_{12} + e^{2i\alpha}\, m_2\ \sin^2\theta_{12}$$

$$B = \frac{\sin\theta_{12}\ \cos\theta_{12}}{\sqrt{2}} \left(e^{2i\alpha}\, m_2 - m_1\right)$$

$$D = \left(m_1\ \sin^2\theta_{12} + e^{2i\alpha}\, m_2 \cos^2\theta_{12}\right)$$

$$E = e^{2i\beta}\, m_3$$

μ–τ Symmetry!!

The neutrino mass matrix if $\theta_{12} = \pi/4$

μ–τ symmetric mass matrix simplifies further for certain mass hierarchies

- NH: $m_3 \simeq \sqrt{\Delta m^2_{\rm A}}$, $m_2 \simeq \sqrt{\Delta m^2_\odot} \simeq \sqrt{\Delta m^2_{\rm A}}\,\sqrt{R}$ and $m_1 \simeq 0$:

$$m_\nu \simeq \frac{\sqrt{\Delta m^2_{\rm A}}}{2} \begin{pmatrix} \sqrt{R} & \sqrt{\frac{R}{2}} & \sqrt{\frac{R}{2}} \\ \cdot & e^{2i(\beta-\alpha)} & -e^{2i(\beta-\alpha)} \\ \cdot & \cdot & e^{2i(\beta-\alpha)} \end{pmatrix} \xrightarrow{R\simeq 0} \frac{\sqrt{\Delta m^2_{\rm A}}}{2} \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix}$$

conserves L_e

- IH: $m_2 \simeq m_1 \simeq \sqrt{\Delta m^2_{\rm A}}$ and $m_3 \simeq 0$:

$$m_\nu \simeq \frac{\sqrt{\Delta m^2_{\rm A}}}{2} \begin{pmatrix} 1+e^{2i\alpha} & \sqrt{\frac{1}{2}}(e^{2i\alpha}-1) & \sqrt{\frac{1}{2}}(e^{2i\alpha}-1) \\ \cdot & e^{i\alpha}\cos\alpha & e^{i\alpha}\cos\alpha \\ \cdot & \cdot & e^{i\alpha}\cos\alpha \end{pmatrix} \xrightarrow{\alpha=\pi/2} \frac{\sqrt{\Delta m^2_{\rm A}}}{2} \begin{pmatrix} 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$$

conserves $L_e - L_\mu - L_\tau$

The neutrino mass matrix if $\theta_{12} = \pi/4$

QD: $m_3 \simeq m_2 \simeq m_1 \equiv m_0$:

$$m_\nu \simeq \frac{m_0}{2} \begin{pmatrix} 1 + e^{2i\alpha} & \sqrt{\frac{1}{2}}(e^{2i\alpha} - 1) & \sqrt{\frac{1}{2}}(e^{2i\alpha} - 1) \\ \cdot & \frac{1}{2}\left(1 + e^{2i\alpha} + 2e^{2i\beta}\right) & \frac{1}{2}\left(1 + e^{2i\alpha} - 2e^{2i\beta}\right) \\ \cdot & \cdot & \frac{1}{2}\left(1 + e^{2i\alpha} + 2e^{2i\beta}\right) \end{pmatrix}$$

$$\xrightarrow{\alpha=\beta=0} m_0 \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 1 & 0 \\ \cdot & \cdot & 1 \end{pmatrix} \quad \text{unit matrix}$$

$$\xrightarrow{\alpha=0\,,\ \beta=\pi/2} m_0 \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix} \quad \text{conserves } L_\mu - L_\tau$$

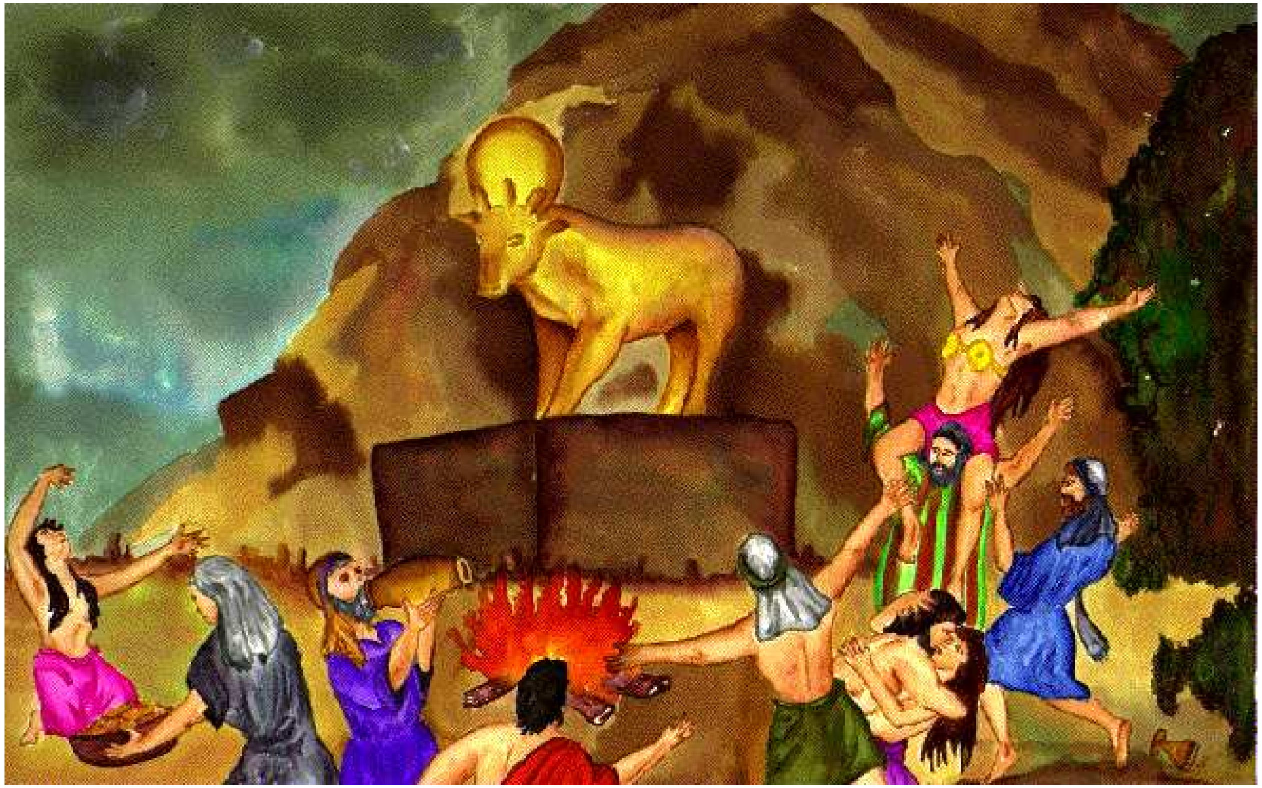
Lepton–Number Violation: Neutrinoless Double Beta Decay

Mass term $\nu^T \nu$ not invariant under $\nu \to e^{iL} \nu \Rightarrow$ Lepton number violation!!

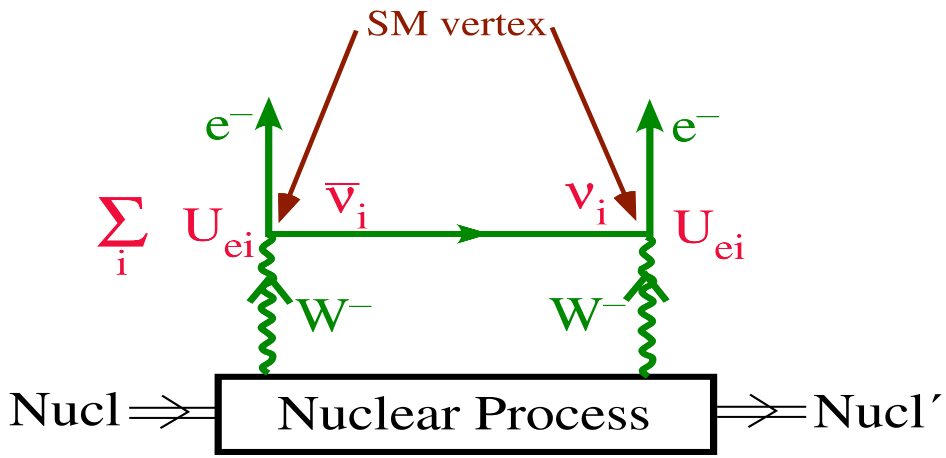
everyone's favorite process:

Neutrinoless Double Beta Decay $(0\nu\beta\beta)$

$$(A, Z) \to (A, Z+2) + 2\, e^- \quad \Delta L = 2$$



Neutrinoless Double Beta Decay



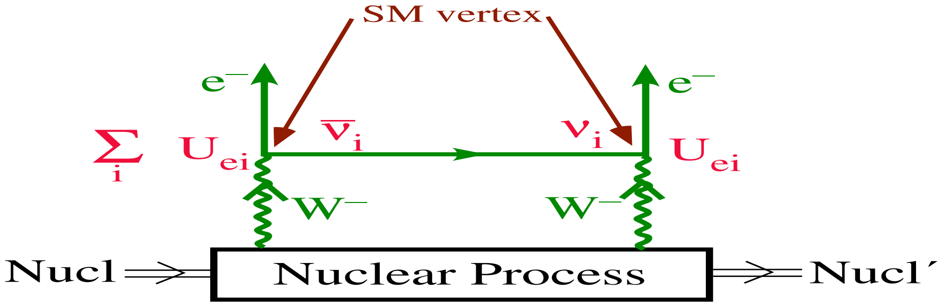
- only works when $\nu = \nu^c$
- only works when $m_\nu \neq 0$
- spin flip $\Rightarrow$ Amplitude $\propto m_\nu / E$

Amplitude proportional to coherent sum:

$$\langle m \rangle \equiv \left|\sum U_{ei}^2\, m_i\right| = \left|c_{12}^2\, c_{13}^2\, m_1 + s_{12}^2\, c_{13}^2\, m_2\, e^{2i\alpha} + s_{13}^2\, m_3\, e^{2i\beta}\right|$$
$$= f\left(\theta_{12}, m_i, |U_{e3}|, \mathrm{sgn}(m_3^2 - m_2^2), \alpha, \beta\right)$$

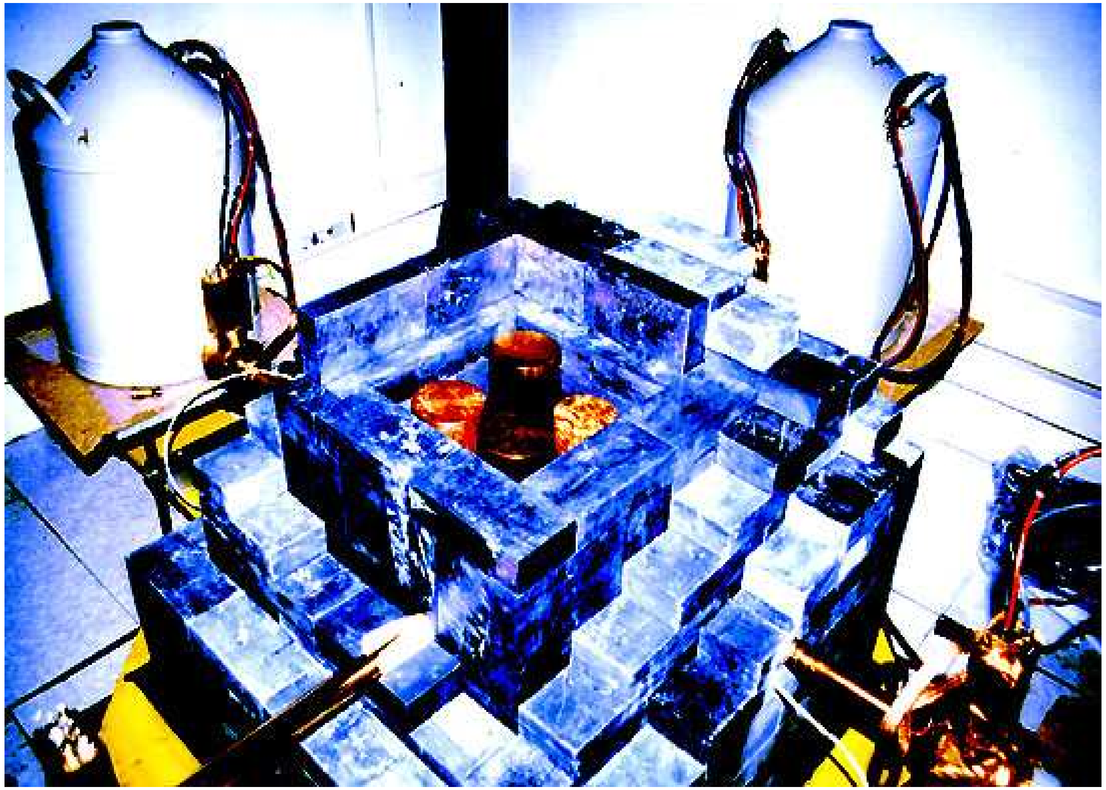
"Effective mass" $\langle m \rangle$

Neutrinoless Double Beta Decay



$$\Gamma(0\nu\beta\beta) = \langle m \rangle^2 \; G(E_0, Z) \; |\mathcal{M}(A,Z)|^2$$

- $\langle m \rangle$: effective mass: neutrino physics
- $G(E_0, Z)$: phase space factor: known
- $\mathcal{M}(A,Z)$: Nuclear Matrix Element: uncertainty of factor $\mathcal{O}(1)$



Best current limit: Heidelberg–Moscow (^{76}Ge)

$$T_{1/2} \geq 1.9 \cdot 10^{25}\ y \Rightarrow \langle m \rangle \lesssim (0.3 \ldots 1.2)\ \text{eV}$$

(part of HM claims evidence corresponding to $\langle m \rangle \simeq (0.1 \ldots 0.9)$ eV)

Neutrinoless Double Beta Decay

Experiment	Source	Detector Description	Sensitivity to $T^{0\nu}_{1/2}$ (y)	Limit on $\langle m \rangle$ (eV)
COBRA	^{130}Te	CdTe semiconductors	1×10^{24}	0.71
DCBA	^{150}Nd	^{enr}Nd layers	2×10^{25}	0.035
NEMO 3	^{100}Mo	several $0\nu\beta\beta$ isotopes	4×10^{24}	0.56
CAMEO	^{116}Cd	$CdWO_4$ crystals	$> 10^{26}$	0.069
CANDLES	^{48}Ca	CaF_2 crystals	1×10^{26}	(0.081)
CUORE	^{130}Te	TeO_2 bolometers	2×10^{26}	0.027
EXO	^{136}Xe	^{enr}Xe TPC	8×10^{26}	0.052
GEM	^{76}Ge	^{enr}Ge diodes	7×10^{27}	0.018
GERDA	^{76}Ge	^{76}Ge in liquid Ar/N	2×10^{26}	0.02
Majorana	^{76}Ge	^{enr}Ge diodes	3×10^{27}	0.025
MOON	^{100}Mo	^{nat}Mo sheets	1×10^{27}	0.036
Xe	^{136}Xe	^{enr}Xe	5×10^{26}	0.066
XMASS	^{136}Xe	liq. Xe	3×10^{26}	0.086

$\Rightarrow$ In $\simeq$ 10 years $\langle m \rangle \simeq \sqrt{\Delta m^2_{\mathrm{A}}}$ probed

$\sqrt{\Delta m^2_{\mathrm{A}}} \leftrightarrow$ 1 t target mass

Mass hierarchies and effective mass

- NH: $m_3 \simeq \sqrt{\Delta m^2_{\rm A}}$, $m_2 \simeq \sqrt{\Delta m^2_\odot} \simeq \sqrt{\Delta m^2_{\rm A}}\,\sqrt{R}$ and $m_1 \simeq 0$:

$$\langle m \rangle^{\rm NH} \simeq \left| \sin^2\theta_{12}\,\sqrt{\Delta m^2_\odot} + \sin^2\theta_{13}\,\sqrt{\Delta m^2_{\rm A}}\,e^{2i(\alpha-\beta)} \right| \lesssim 5 \cdot 10^{-3}\ {\rm eV}$$

$$\text{or } \langle m \rangle^{\rm NH} = \mathcal{O}(\sqrt{\Delta m^2_\odot})$$

- IH: $m_2 \simeq m_1 \simeq \sqrt{\Delta m^2_{\rm A}}$ and $m_3 \simeq 0$:

$$\langle m \rangle^{\rm IH} \simeq \sqrt{\Delta m^2_{\rm A}}\left(1 - \sin^2 2\theta_{12}\,\sin^2\alpha\right) \simeq (0.029 \ldots 0.055)\ {\rm eV}$$

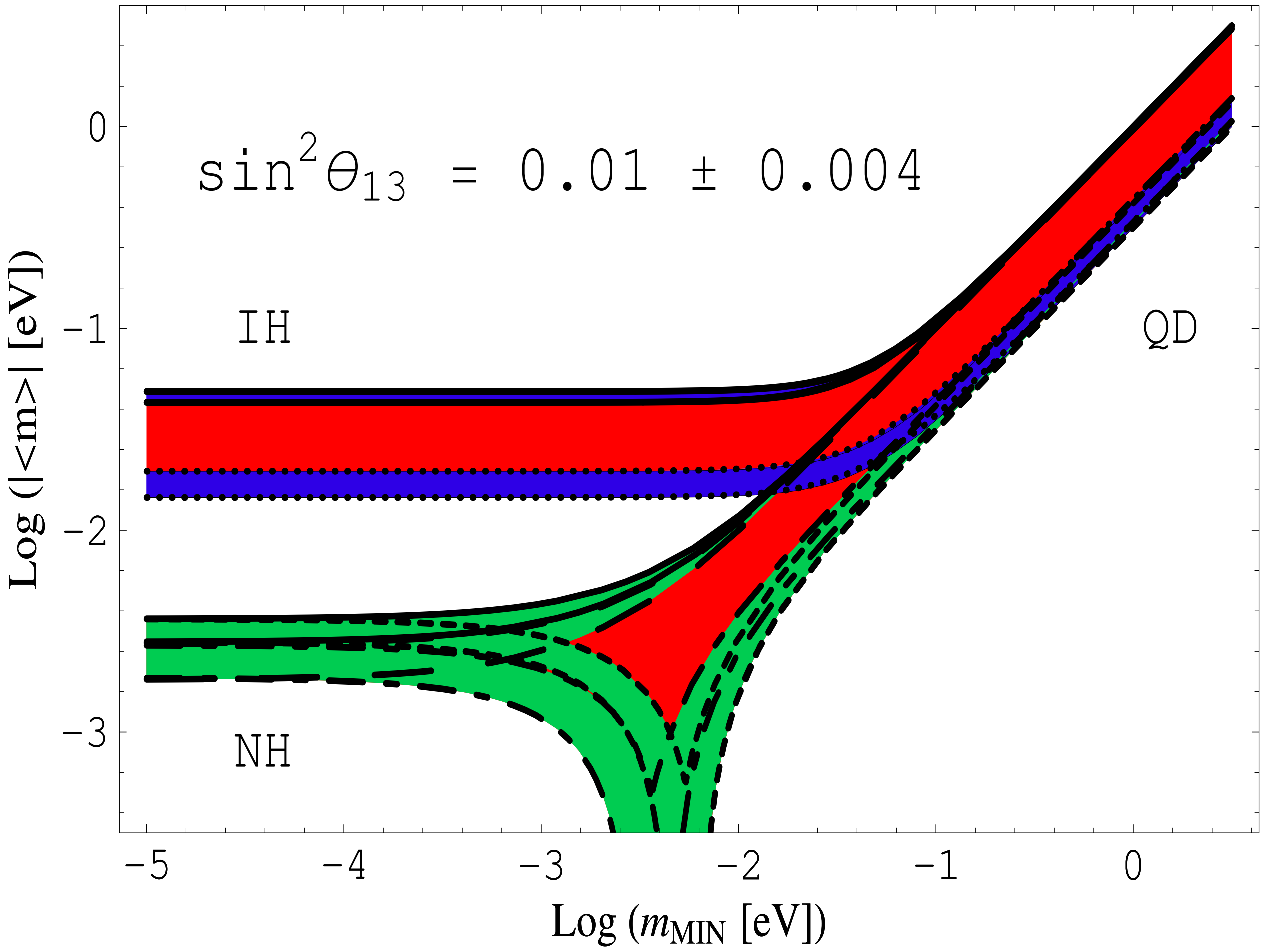
$$\text{or } \sqrt{\Delta m^2_{\rm A}}\,\cos 2\theta_{12} \leq \langle m \rangle^{\rm IH} \leq \sqrt{\Delta m^2_{\rm A}} \text{ or } \langle m \rangle^{\rm IH} = \mathcal{O}(\sqrt{\Delta m^2_{\rm A}})$$

$$\Rightarrow \langle m \rangle^{\rm IH}_{\rm MIN} > \langle m \rangle^{\rm NH}_{\rm MAX} \Rightarrow \text{ Distinguish NH from IH!!!}$$

- QD: $m_3 \simeq m_2 \simeq m_1 \equiv m_0$:

$$\langle m \rangle^{\rm QD} \simeq m_0\left(1 - \sin^2 2\theta_{12}\,\sin^2\alpha\right) \simeq (0.65 \ldots 1)\,m_0$$

$$\text{or } m_0\,\cos 2\theta_{12} \leq \langle m \rangle^{\rm QD} \leq m_0 \text{ or } \langle m \rangle^{\rm QD} = \mathcal{O}(m_0)$$



NH vs. IH works with NME uncertainty $\lesssim 2$ and $m_{\rm smallest} \lesssim 0.01$ eV

What's more to $0\nu\beta\beta$?

- Mass scale: consider QD spectrum

$$m_0 \leq \frac{1+\tan^2\theta_{12}}{1-\tan^2\theta_{12}-2\,|U_{e3}|^2}\,\langle m\rangle^{\rm exp} \lesssim 5 \text{ eV}$$

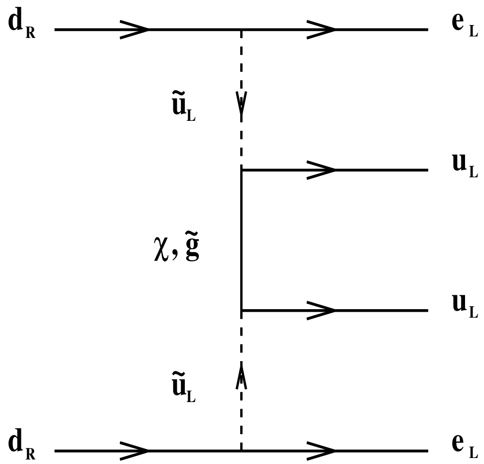
comparable to current ^{3}H limit in the future

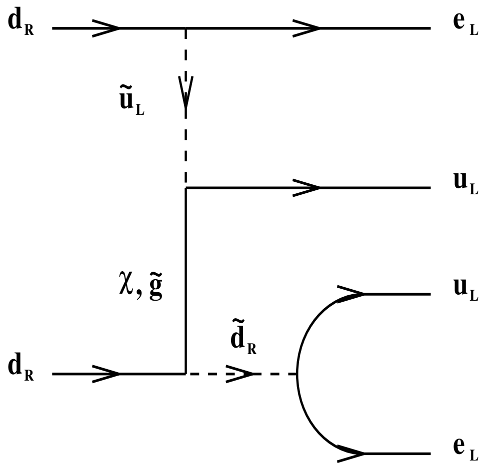
- Majorana phases: consider IH spectrum

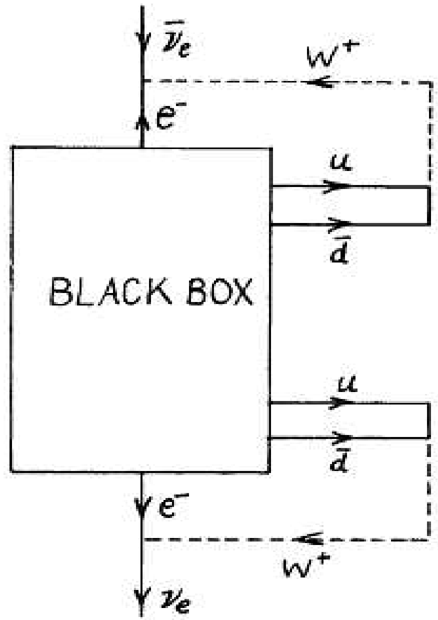
$$\sin^2\alpha = \left(1 - \frac{\langle m\rangle}{\sqrt{\Delta m^2_{\rm A}}\,(1-|U_{e3}|^2)}\right)^2 \frac{1}{\sin^2 2\theta_{12}}$$

extremely challenging unless NME uncertainty $\lesssim 1.5$

OTHER PROCESSES CONTRIBUTING TO $0\nu\beta\beta$





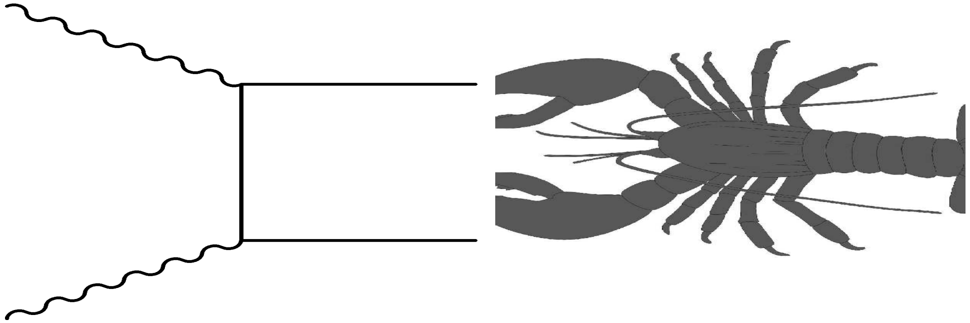


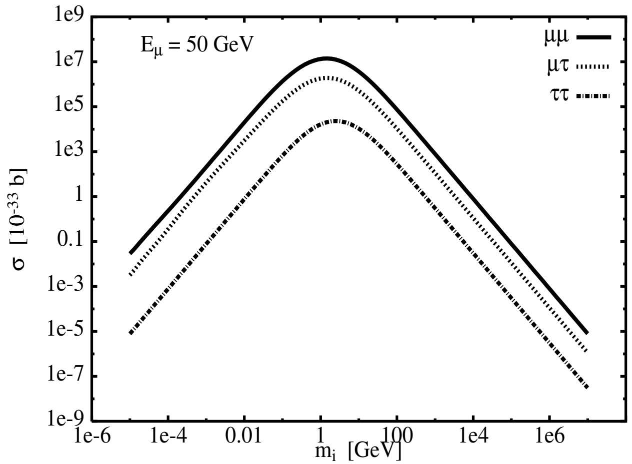
$$2n \to 2p + 2e^- \Rightarrow 2d \to 2u + 2e^- \Rightarrow 0 \to u\bar{d} + u\bar{d} + 2e^-$$

- SUSY
- Higgs triplets
- Right–handed interactions
- Majorons

$\Rightarrow$ limits on masses and couplings

Analogous processes ("The lobster")





- Exotic decays, e.g.,

$$\mathrm{BR}(K^+ \to \pi^- \mu^+ \mu^+) \sim 10^{-30}\ (m_{\mu\mu}/\mathrm{eV})^2 \text{ with } m_{\mu\mu} = \left|\sum U_{\mu i}^2\, m_i\right|$$

- processes at accelerators (νN scattering, ν–fac, HERA "isolated leptons")

$$\mathrm{BR},\ \Gamma,\ \sigma \propto \frac{m^2}{(q^2 - m^2)^2} \simeq \begin{cases} m_i^2 & q^2 \gg m_i^2 \\ m_i^{-2} & q^2 \ll m_i^2 \end{cases}$$

Can we still identify m_ν?

A SIMPLE $U(1)$ FOR m_ν?

L'	matrix	extra
L_e Normal Hierarchy	$\begin{pmatrix} 0 & 0 & 0 \\ \cdot & a & b \\ \cdot & \cdot & d \end{pmatrix}$	$R = \frac{\Delta m^2_\odot}{\Delta m^2_\mathrm{A}} \simeq \lvert U_{e3}\rvert^2$ $\tan^2\theta_\mathrm{atm} \simeq 1 + \lvert U_{e3}\rvert \simeq 1 + \sqrt{R}$ $\langle m\rangle \simeq \sqrt{\Delta m^2_\mathrm{A}}\,\lvert U_{e3}\rvert^2 \simeq \sqrt{\Delta m^2_\odot}$
$L_e - L_\mu - L_\tau$ Inverted Hierarchy	$\begin{pmatrix} 0 & a & b \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$	requires U_ℓ: ideal for QLC $\tan^2\theta_{12} \simeq 1 - 4\,\lvert U_{e3}\rvert \simeq 1 - 2\sqrt{2}\sin\theta_C$ $\langle m\rangle \simeq \sqrt{\Delta m^2_\mathrm{A}}$
$L_\mu - L_\tau$ quasi–degenerate νs	$\begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix}$	in leading order: $U_{e3} = 0$ and $\theta_{23} = \pi/4$ $\langle m\rangle \simeq m_0$

$\Rightarrow$ Let $\langle m\rangle$ decide!

Normal Hierarchy

Matrix m_ν/m_0	comments	correlations
$\begin{pmatrix} a\,\epsilon^2 & b\,\epsilon & d\,\epsilon \\ \cdot & e & f \\ \cdot & \cdot & g \end{pmatrix}$	simple $U(1)$, broken L_e sequential dominance	$\langle m \rangle = c_1\,\sqrt{\Delta m^2_{\rm A}}\,\lvert U_{e3}\rvert^2$ $\lvert U_{e3}\rvert = c_2\,\sqrt{R},\;\; \theta_{23} = \frac{\pi}{4} - c_3\,\sqrt{R}$
$\begin{pmatrix} a\,\epsilon^2 & b\,\epsilon & d\,\epsilon \\ \cdot & 1+\epsilon & 1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$	$\mu\tau$ symmetry broken in e sector	$\langle m \rangle = c_1\,\sqrt{\Delta m^2_{\rm A}}\,\lvert U_{e3}\rvert^2$ $\lvert U_{e3}\rvert = c_2\,\sqrt{R},\;\; \theta_{23} = \frac{\pi}{4} - c_3\,R$
$\begin{pmatrix} a\,\epsilon^2 & b\,\epsilon & b\,\epsilon \\ \cdot & 1+d\epsilon & 1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$	$\mu\tau$ symmetry broken in $\mu\tau$ sector	$\langle m \rangle = c_1\,\sqrt{\Delta m^2_{\rm A}}\,\lvert U_{e3}\rvert$ $\lvert U_{e3}\rvert = c_2\,R,\;\; \theta_{23} = \frac{\pi}{4} - c_3\,\sqrt{R}$
$\begin{pmatrix} 0 & 0 & \epsilon \\ \cdot & a & b \\ \cdot & \cdot & d \end{pmatrix}$	2 zeros also $m_{ee} = m_{e\tau} = 0$	$\langle m \rangle = 0$ $\lvert U_{e3}\rvert = \sqrt{\frac{R}{\cos 2\theta_{12}}\,\frac{\sin 2\theta_{12}}{2\,\tan\theta_{23}}}$ $\theta_{23} = \frac{\pi}{4} - c_1\,\sqrt{R}$
$\begin{pmatrix} a\,\epsilon & b\,\epsilon & d\,\epsilon \\ \cdot & 1+f\,\epsilon & 1+g\,\epsilon \\ \cdot & \cdot & 1+h\,\epsilon \end{pmatrix}$	perturbed m_ν^0	$\langle m \rangle = \frac{\sqrt{\Delta m^2_A}}{2}\,(1 + c_1\;\lvert U_{e3}\rvert)$ $\lvert U_{e3}\rvert = c_2\,\sqrt{R},\;\; \theta_{23} = \frac{\pi}{4} - c_3\,\sqrt{R}$

INVERTED HIERARCHY

Matrix m_ν/m_0	comments	correlations
$\begin{pmatrix} 1+a\,\epsilon & b\,\epsilon & d\,\epsilon \\ \cdot & \frac{1}{2}+f\,\epsilon & \frac{1}{2}+g\,\epsilon \\ \cdot & \cdot & \frac{1}{2}+h\,\epsilon \end{pmatrix}$	perturbed m_ν^0	$\langle m \rangle = \sqrt{\Delta m^2_{\rm A}}\,(1 + c_1\,\lvert U_{e3}\rvert)$ $\lvert U_{e3}\rvert = c_2\,R,\;\; \theta_{23} = \frac{\pi}{4} - c_3\,R$
$\begin{pmatrix} 0 & a & b \\ \cdot & \epsilon^2 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$	broken $L_e - L_\mu - L_\tau$ and $U_\ell \sim V_{\rm CKM}$	$\langle m \rangle = \sqrt{\Delta m^2_{\rm A}}\,\left\lvert \cos 2\theta_{12} + 4i/\sin^2\theta_{23}\,J_{CP}\right\rvert$ $\tan^2\theta_{12} = 1 - 4\,\cos\delta\,\cot\theta_{23}\,\lvert U_{e3}\rvert$
$\begin{pmatrix} a & \sqrt{2}b\,\cos\theta & \sqrt{2}b\,\sin\theta \\ \cdot & d(1+\cos\theta) & d\,\sin\theta \\ \cdot & \cdot & d(1-\cos\theta) \end{pmatrix}$	$2\,N$ see-saw $L_e - L_\mu - L_\tau$ *strongly* broken	$\sqrt{\Delta m^2_{\rm A}}\,\cos 2\theta_{12} \le \langle m \rangle \le \sqrt{\Delta m^2_{\rm A}}$ $U_{e3} = 0,\;\; \theta_{23}$ large

Quasi–degeneracy

Matrix m_ν/m_0	comments	correlations
$\begin{pmatrix} 1 & 0 & 0 \\ \cdot & 1 & 0 \\ \cdot & \cdot & 1 \end{pmatrix}$ + sequential dominance	type II see–saw upgrade	$\langle m \rangle \simeq m_0$ $\lvert U_{e3}\rvert = c_1\sqrt{R},\ \theta_{23} = \frac{\pi}{4} - c_2\sqrt{R}$ phases shrink with m_0
$\begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & -1 \\ \cdot & \cdot & 0 \end{pmatrix}$	$L_\mu - L_\tau$ plus perturbations	$\langle m \rangle = m_0\ (1/\sqrt{2} + c_1\,\lvert U_{e3}\rvert)$ $\lvert U_{e3}\rvert = c_2\,\Delta m^2_{\rm A}/m_0^2 \lesssim 0.1$ $\theta_{23} = \pi/4 - c_3\,\lvert U_{e3}\rvert$
$\begin{pmatrix} a & \epsilon & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & d \end{pmatrix}$	also $m_{e\mu} = m_{\tau\tau} = 0$ and $m_{e\mu} = m_{\mu\mu} = 0$ and $m_{e\tau} = m_{\tau\tau} = 0$	$\langle m \rangle \simeq m_0 \simeq \sqrt{\frac{\Delta m^2_A\ \tan^4\theta_{23}}{1-\tan^4\theta_{23}}}$ $R \simeq \frac{1+\tan^2\theta_{12}}{\tan\theta_{12}}\ \tan 2\theta_{23}\ {\rm Re}\ U_{e3}$ $\Rightarrow \theta_{23} \neq \pi/4$ and Re $U_{e3} \simeq 0$
$r_\nu \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + c_\nu \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 1 & 0 \\ \cdot & \cdot & 1 \end{pmatrix}$	$S(3)_L \times S(3)_R$ democracy	$\langle m \rangle \simeq m_0$, requires $r_\nu \ll 1$ $\lvert U_{e3}\rvert \simeq \sqrt{m_e/m_\mu},\ \theta_{23}$ large depends on $m_{e,\mu,\tau}$ and breaking
$\begin{pmatrix} a & b & d \\ \cdot & e & f \\ \cdot & \cdot & g \end{pmatrix}$	anarchy	$\lvert U_{e3}\rvert$ close to upper bound, θ_{23} close to bound extreme hierarchy unlikely

Baryogenesis

Baryon Asymmetry of the Universe (BAU)

$$Y_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq (6.5\,^{+0.4}_{-0.3}) \cdot 10^{-10} \text{ (WMAP)}$$

Three necessary (Sakharov–)conditions to generate it

1. Baryon number violation (Y_B)
2. C and CP violation ($\Gamma(B \nearrow) \neq \Gamma(B \searrow)$)
3. Departure from thermal equilibrium ($\langle B \rangle_T \neq 0$)

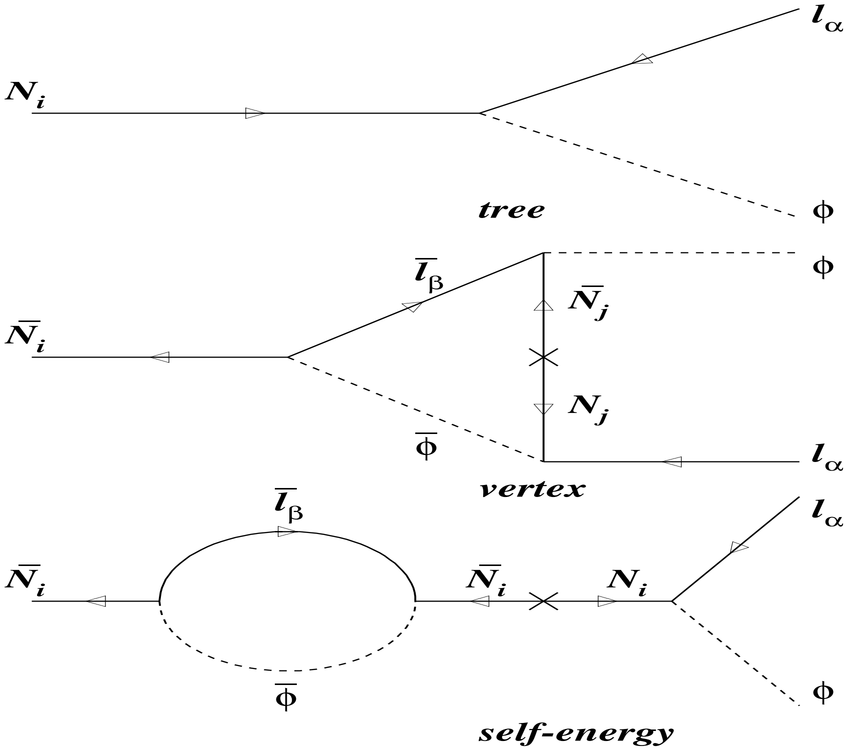
- 3. Requires 1st order phase transition:
  $\leftrightarrow m_H \lesssim 50$ GeV...
- CP Violation in SM too small
- SUSY parameter space very restricted

$\Rightarrow$ New physics!

Leptogenesis

One–loop corrections to decay of heavy Majorana neutrinos:

$$\mathcal{L} = \mathcal{L}_{\rm EW} + \frac{1}{2} M_{ij} \overline{N_i^c} N_j + \frac{(m_D)_{ij}}{v} \overline{L^i} \phi^c N_j + \text{h.c.}$$



Leptogenesis

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \phi\, l^c) - \Gamma(N_1 \to \phi^\dagger\, l)}{\Gamma(N_1 \to \phi\, l^c) + \Gamma(N_1 \to \phi^\dagger\, l)} \propto \sum_{j \neq i} \mathrm{Im}(m_D\, m_D^\dagger)^2_{1j}\, f(M_j^2/M_1^2)$$

- Out–of–equilibrium and CP violation easy to fulfill
- Decay asymmetry $\to$ Baryon asymmetry through SM processes ("Sphalerons")
- $Y_B \sim 10^{-4}\, \varepsilon_1 \Rightarrow \varepsilon_1 \sim 10^{-6}$
- $\varepsilon_1 \propto M_1/M_j$ for $M_{3,2} \gg M_1$
- ε_1 depends on $m_D\, m_D^\dagger$

Can we measure/proof Leptogenesis through neutrino properties??

No we can't

Experimentally accessible

$$m_\nu = U^* \, m_\nu^{\mathrm{diag}} \, U^\dagger = -m_D^T \, M_R^{-1} \, m_D$$

Parametrize:

$$m_D = i \sqrt{M_R} \, R \sqrt{m_\nu^{\mathrm{diag}}} \, U^\dagger \text{ with } R \, R^T = \mathbb{1}$$

Then leptogenesis depends on:

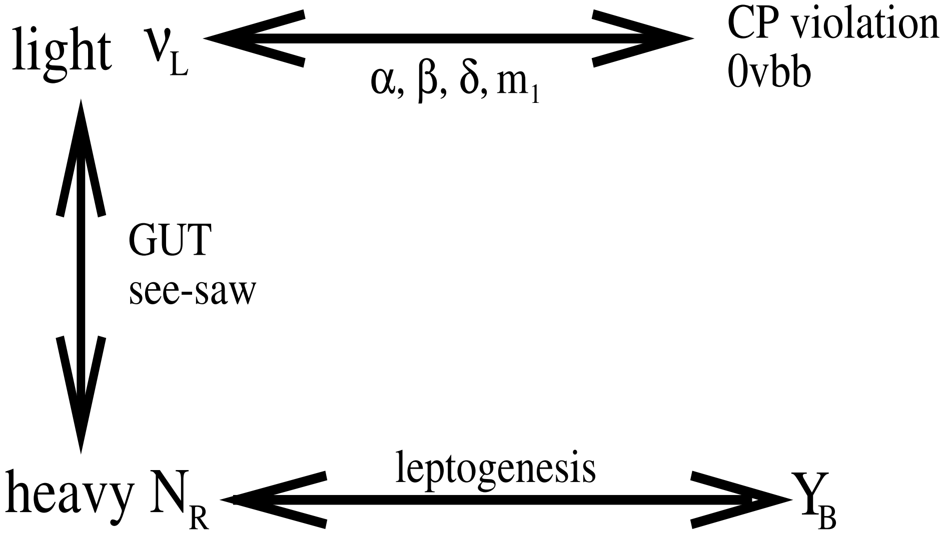
$$m_D \, m_D^\dagger = \sqrt{M_R} \, R \, m_\nu^{\mathrm{diag}} \, R^\dagger \sqrt{M_R} \; (\Rightarrow m_\nu \lesssim 0.1 \text{ eV})$$

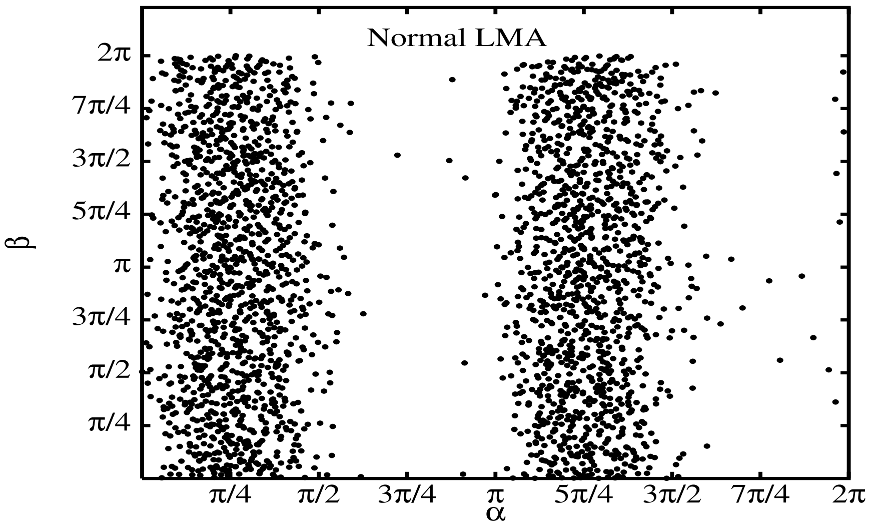
independent on U and the low energy phases!!

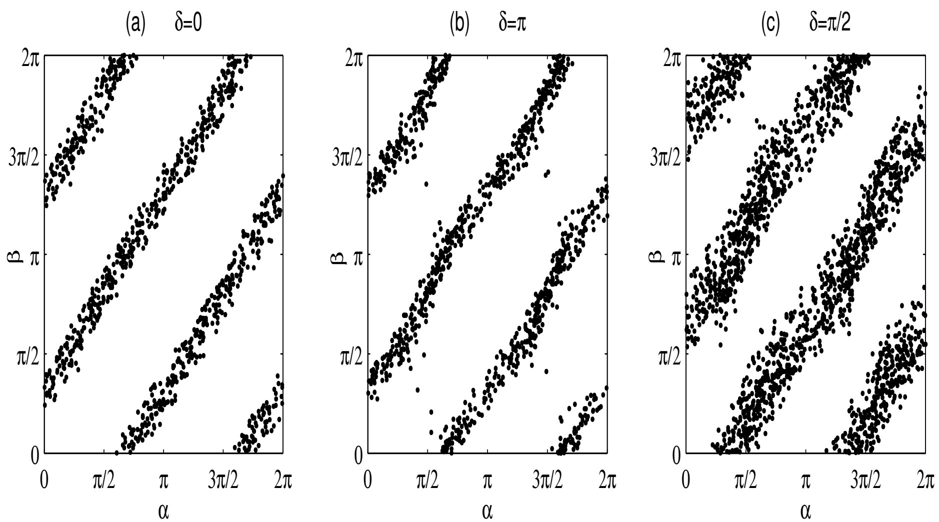
⇒ There is no direct connection between low and high energy CP violation!!!

- If phases in U all zero and phases in R non–zero…
  "Leptogenesis with no low energy CP violation"
- Parameter counting: M_R and m_D contain 12 + 6 parameters, m_ν only 6 + 3

Connection to low energy observables





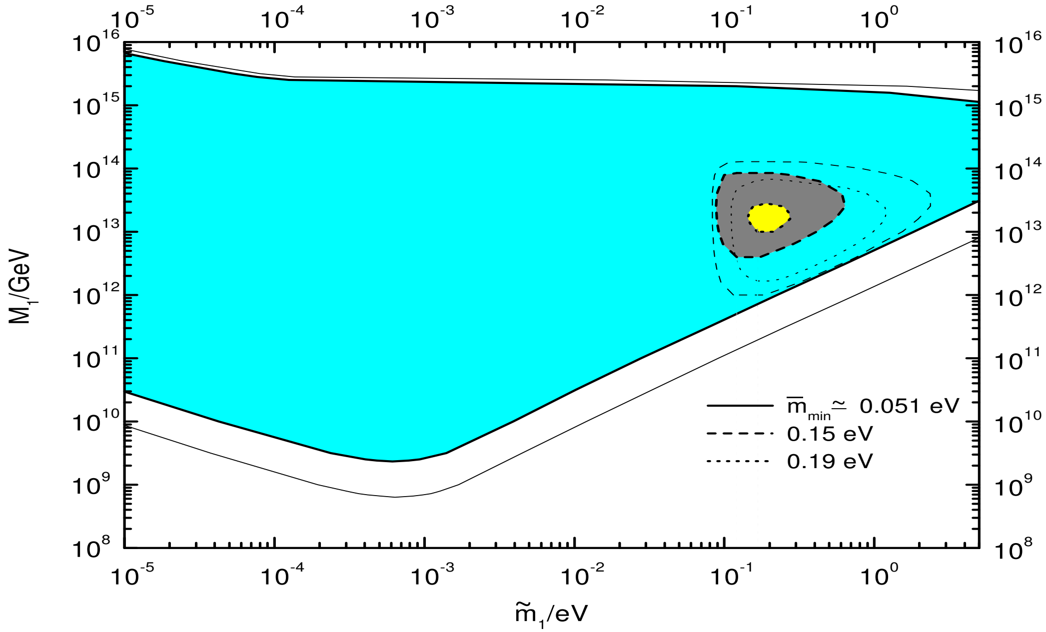


A bound on light neutrino masses from leptogenesis

with analytical limit on ε_1

$$|\varepsilon_1| \lesssim \frac{3\,M_1}{8\,\pi\,v^2}(m_3 - m_1) \simeq \frac{3\,M_1}{8\,\pi\,v^2}\sqrt{\Delta m^2_{\rm A}}$$

obtain $Y_B^{\rm max}(M_1, \tilde{m}_1, \varepsilon_1, \overline{m})$ where $\overline{m}^2 = \sum m_i^2$

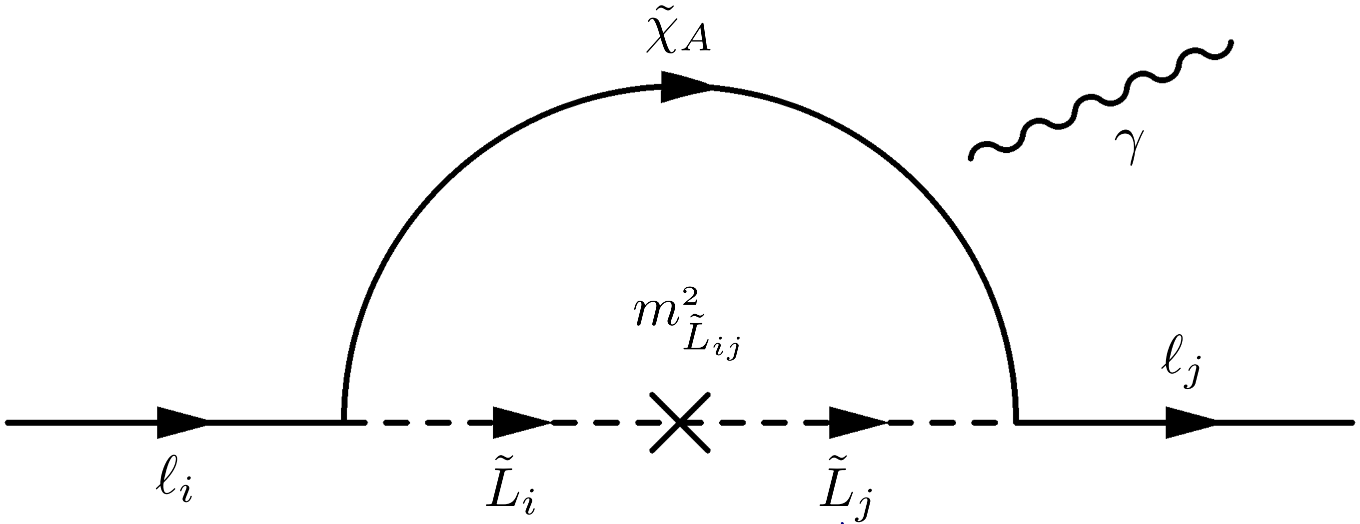


$\overline{m} < 0.2$ eV $\Rightarrow m_i \leq 0.12\ (0.11)$ eV $\Rightarrow$ Quasi–degenerate light neutrinos disfavored!

Limit on heavy Majorana mass: $M_1 \gtrsim 2 \cdot 10^9$ GeV (gravitino problem) ...

Reconstruction of see-saw parameter space

SUSY see–saw has more observables, in particular LFV via off–diagonal entries in slepton mass matrix



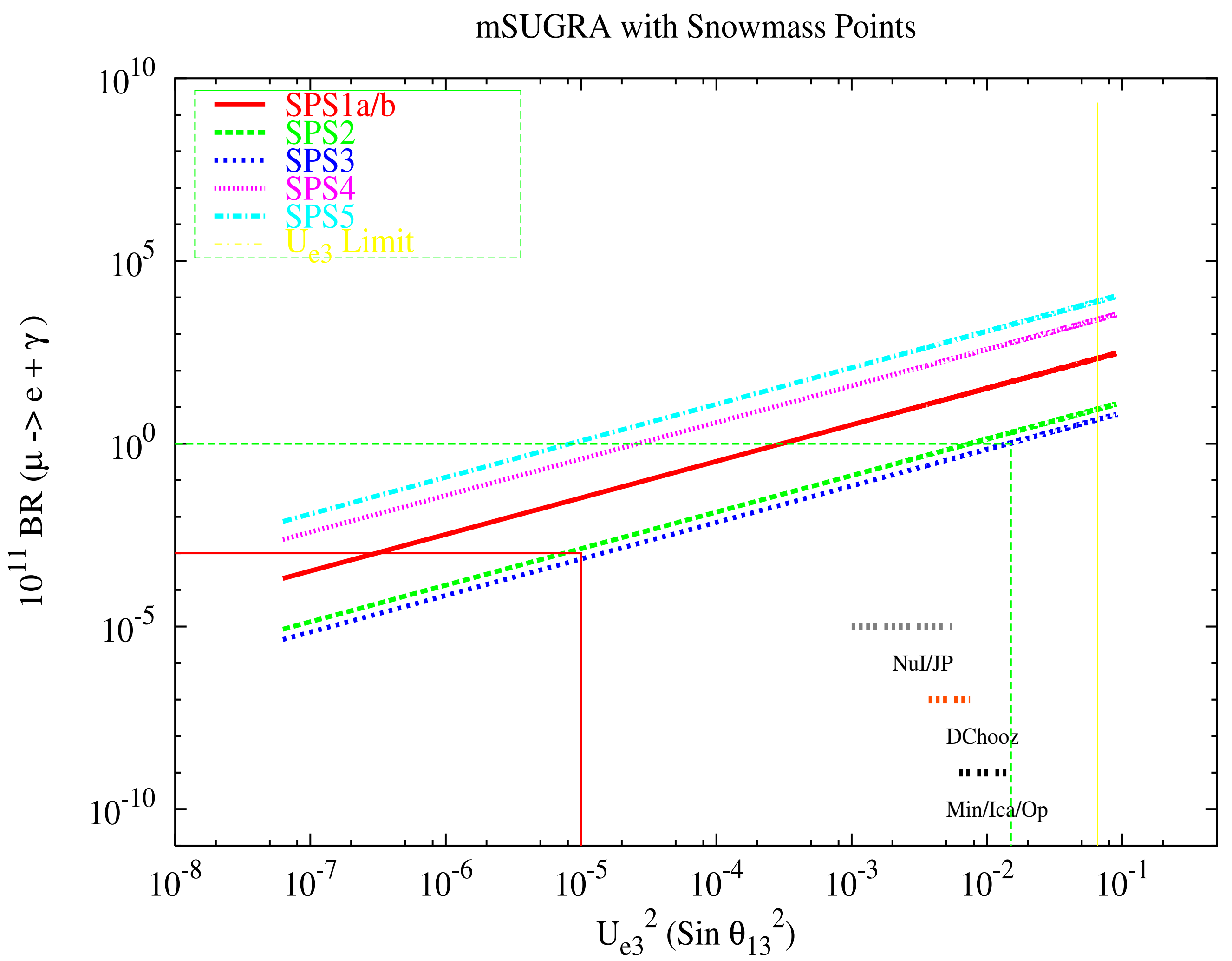
$$BR(\mu \to e + \gamma) \propto |(m_D^\dagger\, m_D)_{12}|^2$$

Useful parametrization:

$$m_D = U_R\, m_D^{\rm diag}\, U_L^\dagger \quad \Rightarrow \quad \begin{array}{ll} Y_B & m_D\, m_D^\dagger = U_R\,(m_D^{\rm diag})^2\, U_R^\dagger \\ \text{LFV} & m_D^\dagger\, m_D = U_L\,(m_D^{\rm diag})^2\, U_L^\dagger \end{array}$$

Experiments: PSI, B–factories, EDMs, "slepton–oscillations", LHC(!!),...

Lepton Flavor Violation and Neutrinos



Current limit: $1.2 \cdot 10^{-11}$

Future limit: $10^{-13} \ldots 10^{-14}$

Topics not covered

- Cross sections (νN)
- Renormalization of neutrino mass and mixing
- Supernovae
- Geo–neutrinos
- Cosmic rays and neutrinos
- Cosmic neutrino background
- ...

Summary

- Neutrinos massless in SM
  - Oscillations discovered ⇒ New physics!!
  - Consistent picture with solar + KamLAND, atmospheric + K2K and short–baseline reactor neutrinos: "Bi–large" mixing scenario
  - $U_{\rm PMNS} \neq V_{\rm CKM}$
  - Still relations between $U_{\rm PMNS}$ and $V_{\rm CKM}$ implied $(\theta_{12} + \theta_C = \pi/4)$
  - Dozens of new experiments upcoming...
- Small neutrino mass explained by see–saw mechanism
  - Neutrinos are Majorana particles
  - Lepton Number Violation ⇒ $0\nu\beta\beta$
- Model–dependent aspects of see–saw
  - Leptogenesis!!
  - Lepton Flavor Violation beyond Neutrinos, $\mu \to e\gamma$

Exciting future ahead!!